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The Correlation and Prediction of Pressure Distributions on Compression Surfaces of Sharp-Edged Conical Bodies at High Supersonic Speeds

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SUMMARY

A method for calculating the pressure distributions around conical bodies at high supersonic speeds, based on a local cone-flow theory, has been developed from the successful correlation of pressures at a symmetrical ridge line.

The method is shown to provide a means for the accurate estimation of pressures on compression surfaces of conical bodies with sharp leading edges, providing the shock wave is detached from the edge.

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1. Introduction

At high supersonic Mach numbers there is a need for a simple method of calculating pressures on conical bodies with arbitrary cross-sections. Existing methods (tangent-cone, Newtonian, etc.) are inaccurate[1], particularly on bodies having sharp edges or those consisting of flat facets. These methods are seriously in error in the vicinity of sharp leading edges and ridge lines and are unable to take account of variations of the included angle of the ridge or leading edge.

The present note is concerned with pressure measurements near ridge lines and leading edges when the shock wave surrounding the body is detached from the edges. The technique for predicting these pressures is then extended to treat complete compression surfaces.

2. Correlation of Measured Pressures at a Ridge Line

One important aspect of the flow over conical bodies that has defied attempts at analysis is the flow in the region of the lower ridge line on the undersurface of bodies with diamond and triangular cross-sections. It is well known[2] that the included angle of the ridge is a dominant parameter, the pressure at the ridge line increasing as the included angle of the ridge is increased. This effect is illustrated for a free-stream Mach number of 4.0 in Fig.1, in which the pressure coefficient at the lower ridge line of various bodies is plotted against the inclination of the ridge to the free-stream direction (δ_r). The data for a ridge angle (2ζ) of 60° include measurements from both triangular and diamond cross-sections indicating that, for a given aspect ratio, the pressure variation at the lower ridge line is independent of the shape of the upper surface.

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The data in Fig.1 are for bodies of planform aspect ratio less than 4/3 because the effects of increasing aspect ratio are significant for the larger ridge angles. These effects are shown in more dètail in Fig.2 demonstrating that a larger aspect ratio increases the pressure at the ridge line when the ridge angle exceeds 90°. The data are coincident, for a fixed value of δ_r, when the ridge angle is less than 90° and the aspect ratio is less than 2. For small ridge angles shock attachment will occur at the ridge and the flow properties can be calculated using the infinite yawed-wing concept and the oblique shock relations. As expected, the dependence on aspect ratio diminishes as shock attachment is approached. Further experimental data[2] suggest that even when the ridge angle approaches 180° (i.e., a flat delta surface) the aspect ratio is only significant if it is greater than unity. There are insufficient data available at Mach numbers other than 4.0 to determine whether the criteria for the aspect ratio are independent of Mach number or whether they are a function of the Mach angle, as would appear likely.

The separate experimental results for each ridge angle shown in Fig.1 can be correlated with the parameter $\delta_r(\sin \zeta)^{\frac{1}{2}}$. The acceptable correlation achieved with the parameter is demonstrated in Fig.3, in which the data points are transferred from Fig.1.

3. Prediction of Pressures at a Ridge Line

A prediction of the pressure coefficient at the ridge line can be made by deriving an equivalent conical body with a circular cross-section and semi-vertex angle σ, such that

$$\sigma = \delta_r(\sin \zeta)^{\frac{1}{2}} \qquad \ldots (1)$$

The pressure coefficient at the ridge line can then be calculated from the flow properties existing on the circular cone at zero incidence in a flow with the same free-stream Mach number as the original body. A theoretical curve based on this concept is also shown in Fig.3 which demonstrates the effectiveness of the method. Two conditions must be satisfied before the method can be utilised:-

(i) the shock wave at the ridge line must be detached, and the flow must be conical;

(ii) a limit must be placed on the maximum aspect ratio, - for free-stream Mach numbers near 4 the limiting aspect ratio can be approximated by

$$AR \leq \frac{1}{\sin^2 \zeta} .$$

The conical flow analysis predicts little variation of the pressure coefficient at the ridge line with Mach number when this is greater than 4. The prediction is verified by experimental data[3] for models tested at Mach numbers of 6.85 and 8.6, shown in Fig.4. Only minor variations in the data are evident for the Mach number range 4 - 8.6 which is contrary to that expected from the parameter $\left(\sqrt{M_\infty^2 - 1} . \tan \delta\right)$ used by Küchemann[4] for correlating ridge line pressures at a Mach number of 4.

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4. Prediction of Pressure Distributions over Compression Surfaces

The success of the present correlation and prediction method for ridge lines of conical models indicates a procedure for calculating the pressure distribution over the complete compression surfaces of a body, providing the surfaces are free from separations at the edges. It is suggested that a generator on the surface of a conical body can be represented by an equivalent circular cone at zero incidence, with the semi-angle of the equivalent cone (σ) given by

$$\sigma = \delta(\sin \lambda)^{\frac{1}{2}} , \qquad \ldots (2)$$

where δ is the angle between a generator in the surface and the free-stream direction, and λ is the local inclination of the surface with respect to the free-stream plane through the generator. Approximations to the angles δ and λ are shown in Fig.5; for the sake of clarity λ is projected onto the base of the body and labelled $\bar{\lambda}$.

To illustrate the application of the technique the surface pressures predicted from equation (2), with the aid of cone-flow tables[5], are compared in Fig.6 with the experimental pressure distributions over the lower surfaces of the model sketched in Fig.5, at a Mach number of 4.0. The agreement is good and represents a considerable improvement over Newtonian theory or tangent-cone theory. Unlike methods that rely on perturbations about a known flow field, e.g., linearised characteristics[6], the technique maintains a relatively high accuracy when the body is pitched through large angles of attack.

Despite the fact that the present prediction is basically a local theory it is found to be satisfactory on bodies having geometrically similar surfaces on one side of an edge but different surface geometries on the other side of the edge. This is demonstrated in Fig.7 in which the pressure at the leading edge of a triangular section at yaw is compared with the pressure at the ridge line of a diamond section (basically two triangular sections). The two sets of pressures are in exceptionally good agreement if the small reduction in pressure outboard of 80% semi-span on the triangular section is neglected (c.f., Fig.6, $\theta = 12°$, $y/b = -1$). This reduction in pressure near the leading edge is due to an expansion around the edge and subsequent separation on the adjacent surface, but it is surprising that these edge effects have a very localised field of influence on compression surfaces.

5. Conclusions

A correlation of experimental pressures at ridge lines of conical bodies has been developed to take account of variations of the ridge angle and a method has been devised for predicting these pressures, applicable to ridge lines and leading edges of conical bodies with detached shock waves.

The technique has been extended to calculate pressure distributions over complete compression surfaces of bodies and comparisons with experimental data for a sharp-edged conical body show a significant improvement over existing theories. The present technique is simple in application and provides an accurate estimation of pressure distributions on conical bodies over a wide range of attitudes.

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The preceding methods are applicable to the lower (compression) surfaces of bodies but the upper surfaces contribute to the normal force. There are no satisfactory methods available for predicting upper surface pressure distributions. Techniques for estimating these pressure distributions are therefore being sought so that the magnitude of the total normal force can be calculated reliably.

References

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2	L. C. Squire	Pressure distributions at M = 4.0 on some 'flat' delta wings. RAE Tech. Note Aero.2865. A.R.C. R. & M.3373, 1963.
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6	L. C. Squire	Applications of linearised characteristics to sharp-edged conical bodies. A.R.C.25 725. March, 1964.

EM

FIG. 1

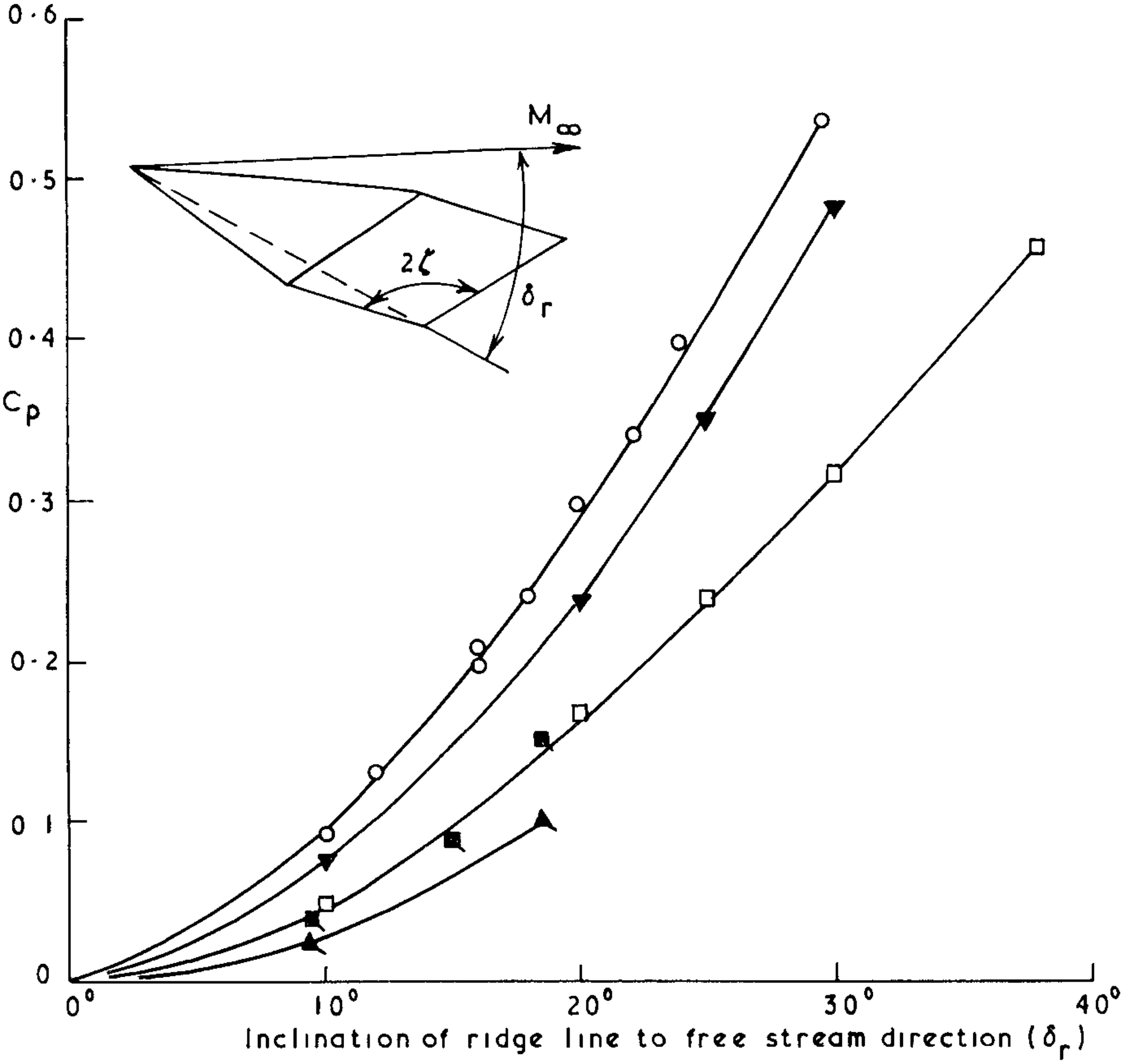
Symbol — Ridge angle (2ζ)

- ○ 180°
- ▽ 120°
- □ 60°
- △ 30°

180°, 60°: NPL data for triangular section

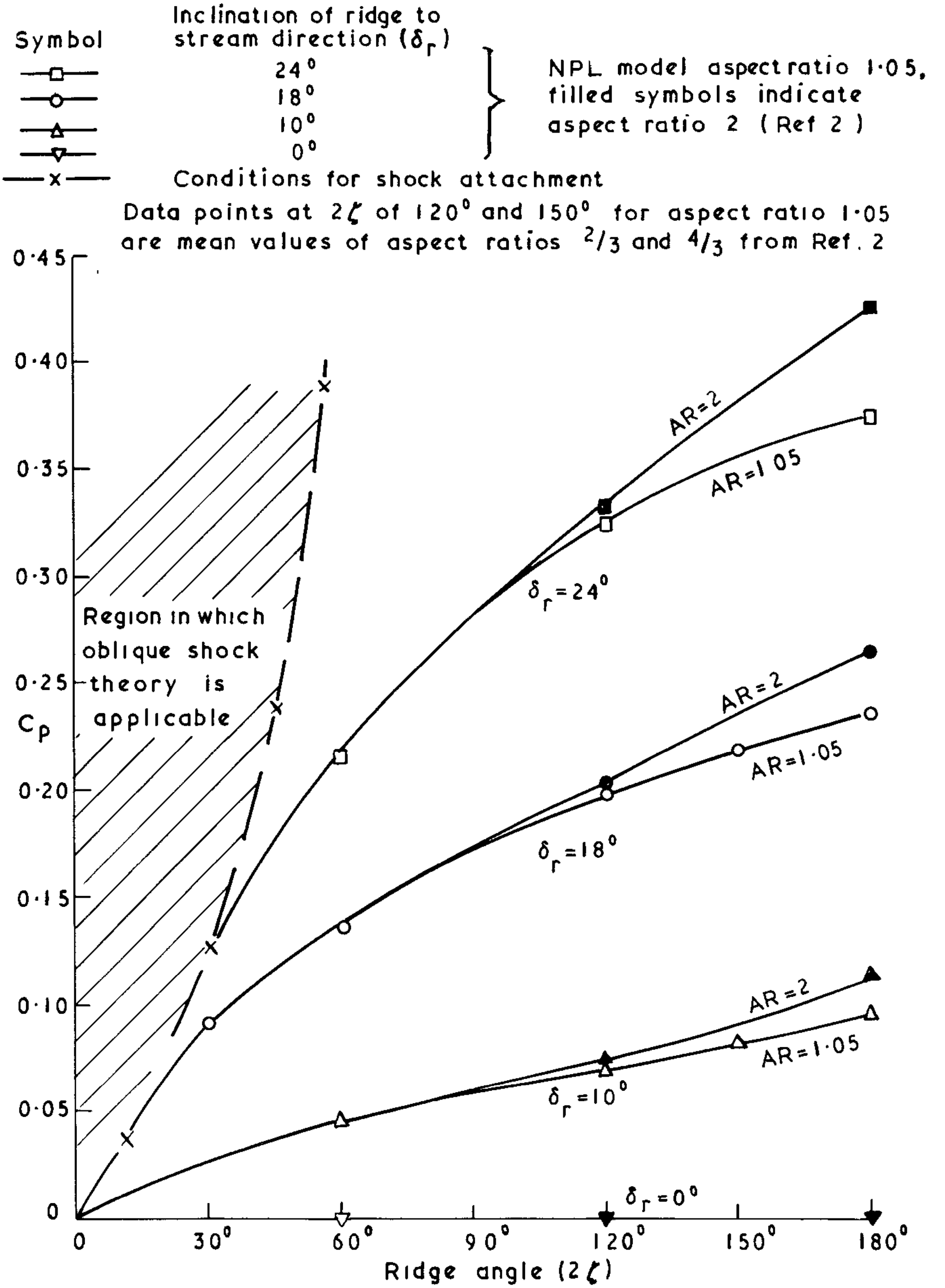
Filled symbols indicate measurements by Squire[2] for triangular and diamond section bodies of aspect ratio < 4/3

Flagged symbols indicate measurements at leading edges of diamond sections[2]



Pressure coefficient at ridge line on low aspect-ratio conical bodies at $M_\infty = 4$

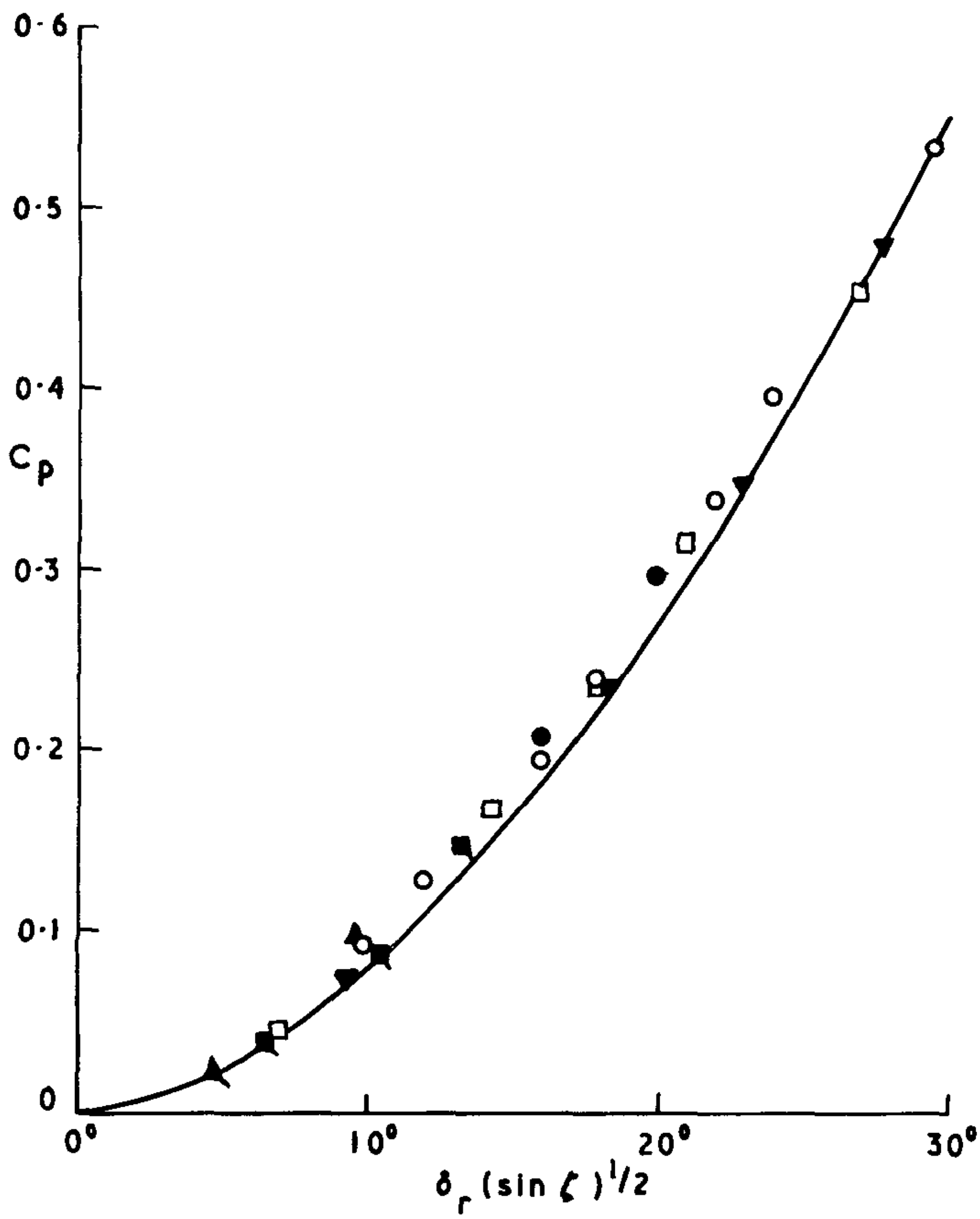
FIG. 2



Effects of aspect-ratio on the variation of pressure coefficient at ridge line with ridge angle at $M_\infty = 4$

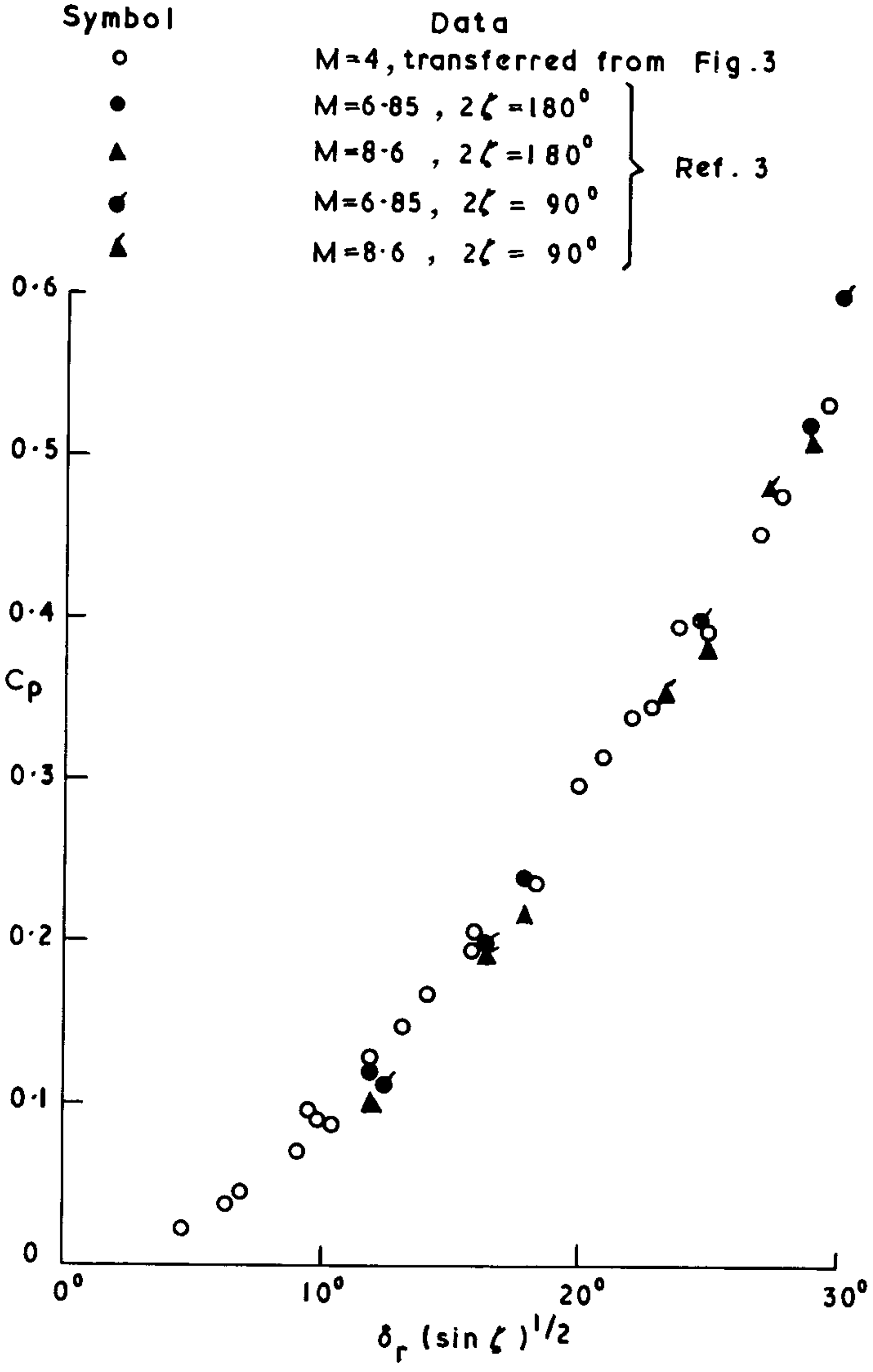
FIG. 3

Symbol	Ridge angle (2ζ)
○	180°
▽	120°
□	60°
△	30°
———	Pressure on cone of semi-apex angle σ, $[\sigma = \delta_r (\sin \zeta)^{1/2}]$



Correlation of pressure coefficient at ridge line of conical bodies at $M_\infty = 4$

FIG. 4



Effects of free stream Mach number on pressure coefficient at ridge line

N.B. The angle $\bar{\lambda}$ is shown in the base of the body whereas the angle λ, required for Eqn. (2) is measured in a plane perpendicular to the intersection of the free-stream plane ONP, and the surface OAPB

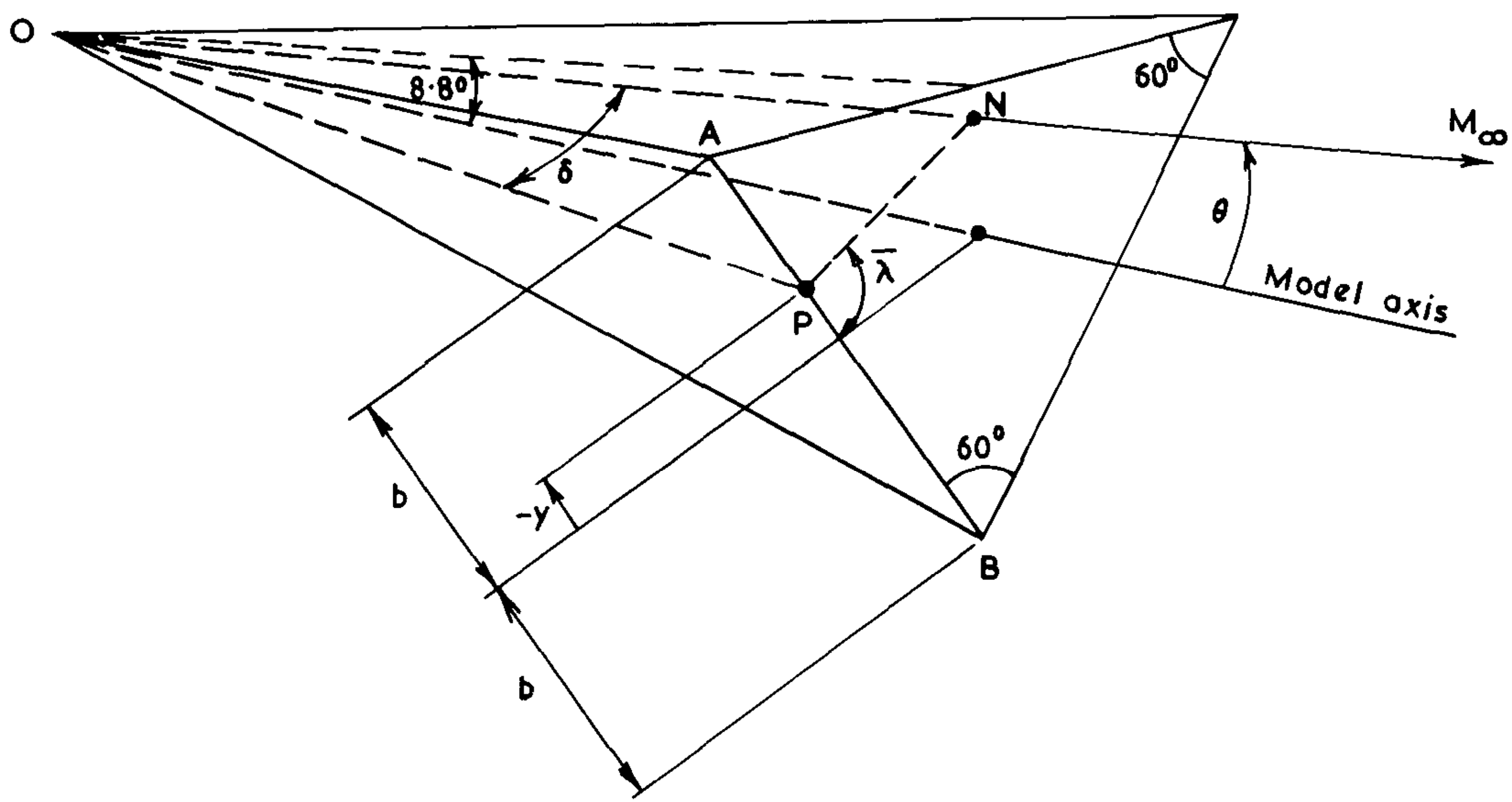
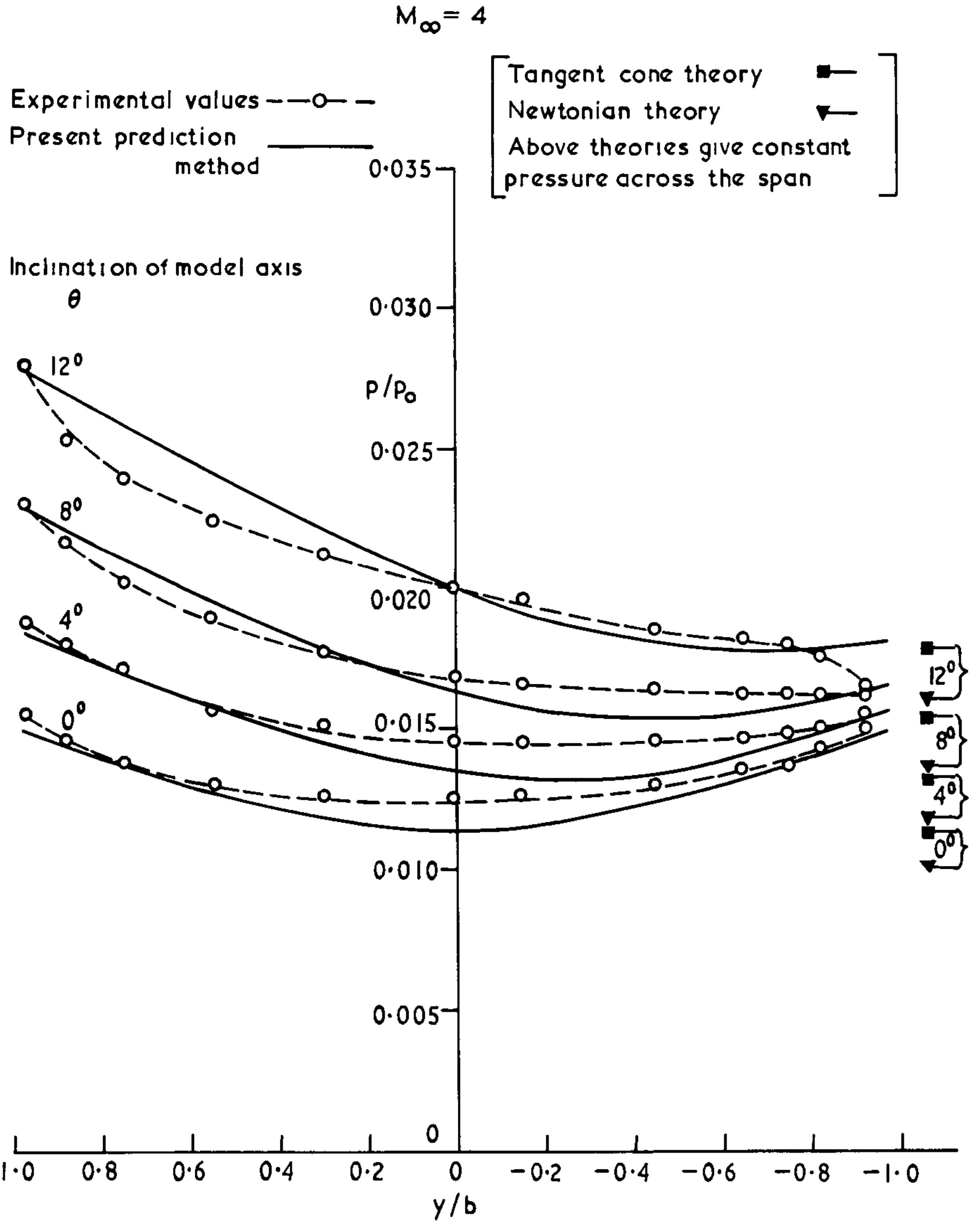


FIG. 5

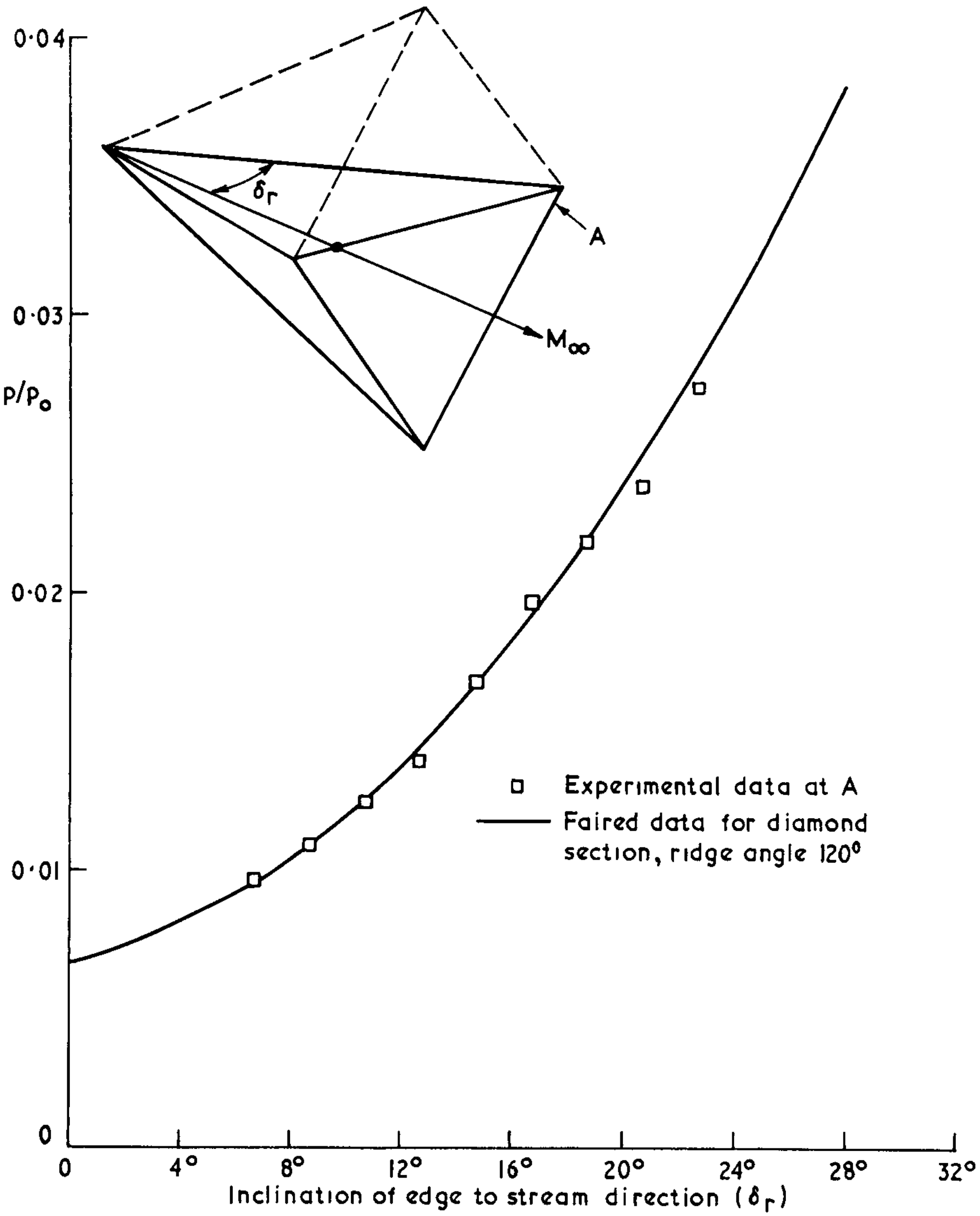
Definition of angles λ and δ and details of NPL model

FIG. 6



Comparison of experimental pressure distributions with the predicted values (Eqn.2) for the lower surface of the NPL model

FIG. 7



Comparison of pressures at leading edge of triangular section and at ridge line of diamond section ($M_\infty = 4$)

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