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A Critical Examination of the use of a Two-Dimensional Turbulent Profile Family to Represent Three-Dimensional Boundary Layers

by

N. A Cumpsty,

Cambridge University Engineering Dept

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A critical examination of the use of a two-dimensional turbulent profile family to represent three-dimensional boundary layers

by N.A. Cumpsty†

Cambridge University Engineering Department

Summary

The streamwise velocity profile is established as the most suitable basic profile for the calculation of three-dimensional turbulent boundary layers. Measured streamwise profiles are compared with Thompson's two-dimensional profile family and it is shown that the discrepancies produced by the variation of flow direction within the boundary layer, the pressure gradient normal to the external flow and the convergence or divergence of the flow are generally small. The result of the streamwise pressure gradient (which is as much a two-dimensional as a three-dimensional effect) can, however, be very appreciable. The four effects listed above are expressed as non-dimensional parameters and limits are suggested within which the streamwise profile is likely to be moderately well represented by Thompson (or similar) two-dimensional profiles. Some consideration is given to the associated problem of estimating the coefficient of skin friction in three-dimensional boundary layers and some alternative methods are compared.

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*Replaces A.R.C.30 781.

†Now at Rolls-Royce, Ltd., Hucknall, Nottingham.

1. Introduction

There are serious shortcomings in our knowledge of three-dimensional turbulent flows and calculation methods for three-dimensional turbulent boundary layers have generally used, wherever possible, descriptions of the flow developed for two-dimensional boundary layers. This had made it necessary to adopt a streamline coordinate system*. With this system the velocity within the boundary layer can be divided into a cross-flow component (in a direction normal to the external flow) having zero velocity at the boundary layer edge as well as at the surface, and a basic profile (which is very often the streamwise profile) having free stream velocity at the boundary layer edge and zero velocity at the surface. This is illustrated in Figure 1.

When the three-dimensional effects, including crossflow, are small, the use of two-dimensional descriptions for the flow is natural and needs no justification. In general, however, the effects are not small and the approximation has been justified by the observation that the basic profiles generally resemble the form of two-dimensional profiles and have been quite well described by two-dimensional profile families. The paper investigates the use of a reliable profile family to describe the three-dimensional

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* This is a system of orthogonal curvilinear coordinates in which one coordinate, in our notation s, is measured along the projection on to the surface of the flow just outside the boundary layer. n is measured normal to s in the plane of the surface and ζ is measured normal to the surface, as indicated in Figure 1.

basic profile in a more systematic way than has been tried before, and an attempt is made to provide a guide to the limits within which the approximation may be used with some confidence. The associated use of two-dimensional skin friction relations to determine the coefficient of skin friction in three-dimensional flows is also considered.

It is not easy, however, to make a quantitative assessment of the degree of similarity between the two and three-dimensional profiles. This would require the comparison of measured three-dimensional profiles with measured two-dimensional profiles of identical form parameter and Reynolds number (assuming that these two parameters are sufficient to describe the profiles fully) and with the experimental data available this is quite impracticable. The inner region of two-dimensional turbulent boundary layers, in the absence of strong pressure gradients, has a particular mean velocity distribution usually known as the law of the wall. It is therefore possible to compare the inner region of three-dimensional basic profiles with the more or less universal two-dimensional velocity distribution. This is particularly useful, since two-dimensional profile families rely upon the validity of the law of the wall over an appreciable part of the boundary layer thickness; good agreement between measured profiles and the profile family is likely only when the measured profile satisfies the law of the wall, at least approximately, for a considerable part of the boundary layer thickness. Comparison of measured basic profiles with the law of the wall shows, in fact, that the streamwise profile is the basic profile most likely to be well described by two-dimensional relations. Such comparisons are also used to explain discrepancies between measured profiles and the profile families where these occur.

2. The two-dimensional profile family

Coles[1] and more recently Thompson[2] have proposed models for the mean velocity profiles of two-dimensional turbulent boundary layers. In the absence of strong pressure gradients both are capable of representing two-dimensional profiles very accurately. To preserve continuity with earlier work by the present author, Thompson profiles have been used here, but the conclusions should be equally applicable to Coles' model.

Thompson's profile family uses the concept of intermittency (or pseudo-intermittency). It is assumed that, while turbulent, the mean velocity is given by the law of the wall, $u_f = u_\tau . f\left(\frac{u_\tau . \zeta}{\nu}\right)$, and while non-turbulent by the free stream velocity, U_s. The average mean velocity is then given by

$$u = u_f + (1 - \gamma).U_s \quad ,$$

where γ, a function of distance from the surface, is the intermittency distribution and u_τ is the friction velocity. The intermittency as deduced from measured velocity profiles using the above equation, is found to take a more or less universal form, although this differs significantly from the intermittency distributions measured with hot wires.

Thompson constructed his family so that the profile $u/U_s = u/U_s(\zeta/\theta)$ is presented in terms of the form parameter $H(= \delta^*/\theta)$, and the Reynolds number based on momentum thickness, $R_\theta(= \theta U_s/\nu)$. Using the law of the wall, Thompson produced skin friction relationships based on the same assumptions as the profile family, the coefficient of skin friction, c_f, also being given in terms of H and R_θ. The results of Thompson's skin friction law are in good agreement with predictions by the well-known Ludwieg-Tillmann[3] relation.

3. Three-dimensional considerations

a) The choice of basic profile

The basic profile must have zero velocity at the wall and free-stream velocity at the boundary layer edge and in a skewed boundary layer there are several possibilities which satisfy these requirements. Since we are looking for close agreement between the two-dimensional representation and the basic profile it is profitable to find which of the possible profiles most closely resembles the corresponding two-dimensional profile.

In the absence of strong pressure gradients, the inner region of two-dimensional turbulent boundary layers shows a unique relation for the mean velocity known as the law of the wall. It is therefore expected that to be satisfactorily represented by a two-dimensional profile family the three-dimensional basic profile must show reasonable agreement with this law. The measure of agreement can be readily tested by plotting the profile in the manner suggested by Clauser[4] and is shown by the accuracy with which the experimental points define a line having slope and position compatible with the contours drawn according to the law of the wall.

The simplest of the basic profiles (and the one most convenient for the purpose of boundary layer calculations) is the streamwise profile, u/U_s. Also very straightforward is the resultant profile, $(u^2 + v^2)^{\frac{1}{2}}/U_s$, where for this purpose the profile is considered in one plane. Comparatively recently Perry and Joubert[5] suggested that the developed profile

$\int_0^u \frac{(1 + (\frac{dv}{du})^2)^{\frac{1}{2}}}{U_s} \cdot du$ should be used. This form was derived from a simple mixing length analysis.

In Figure 2 the streamwise, resultant and developed profiles are plotted on Clauser charts for three measured boundary layers. It can be seen that the streamwise and developed profiles are equally compatible with the contours of the chart, only the resultant profile is clearly inferior. Because of its much greater simplicity the streamwise profile has been used in the remainder of the paper and is recommended as the basis of three-dimensional calculations.

b) Factors which can affect three-dimensional boundary layers

It is well known that in strong pressure gradients the two-dimensional boundary layer ceases to be satisfactorily described by Thompson's (or Coles') profile family, and the region in which the law of the wall appears valid becomes very small. It is clearly of great interest to know under what conditions the two-dimensional description of a three-dimensional profile (say the streamwise profile) becomes inaccurate and the two-dimensional law of the wall ceases to be even approximately correct in the inner region. It should be pointed out that, although quite severely skewed boundary layers show a clear region in which the two-dimensional law of the wall appears valid, no explanation for this observation has been proposed. The agreement with the law of the wall may be entirely fortuitous or it may be that our understanding of three-dimensional turbulent flows is too limited to allow us to predict it in skewed flows; evidence of the law of the wall is simply presented as an indication of the closeness of the agreement between the two-dimensional and three-dimensional velocity profiles and is not intended to imply a common flow mechanism.

It is to be expected that the form of the three-dimensional boundary layer will be affected by the pressure gradient in the streamwise direction, $\frac{\partial p}{\partial s}$, and by the pressure gradient in the crosswise direction, $\frac{\partial p}{\partial n}$. It will also be affected by the variation of flow direction within the boundary layer (the skewing) and by the convergence or divergence of the flow.

The effect of the pressure gradients is most obvious close to the surface and it is therefore appropriate to non-dimensionalise them with respect to inner-region variables. The streamwise and crosswise pressure gradient parameters, Δ_s and Δ_c, are given by $\frac{\nu}{\rho u_\tau^3} \frac{\partial p}{\partial s}$ and $\frac{\nu}{\rho u_\tau^3} \frac{\partial p}{\partial n}$, where $u_\tau = \sqrt{\tau_w/\rho}$ is the streamwise friction velocity.* The streamwise friction velocity is clearly appropriate for consideration of the streamwise velocity profile. The convergence or divergence of the flow, $\frac{1}{h_2} \frac{\partial h_2}{\partial s}$, (where h_2 may be thought of as a characteristic length separating adjacent external streamlines in the plane of the surface) has the dimensions $(\text{length})^{-1}$. It has been non-dimensionalised with respect to the streamwise momentum thickness, $\theta_{11} = \int_0^\infty \frac{u}{U_s}(1 - \frac{u}{U_s})d\zeta$, to give $\frac{\theta_{11}}{h_2} \frac{\partial h_2}{\partial s}$. Finally the skewing of the boundary layer is measured by the angle between the flow at the boundary layer edge and the limiting direction as the surface is approached. It is denoted by β.

There is little reason to suppose that the effects of four parameters Δ_s, Δ_c, $\frac{\theta_{11}}{h_2} \frac{\partial h_2}{\partial s}$ and β are linearly related

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* Perry and Joubert[5] used a single parameter containing the vectorial resultant skin friction and the vectorial pressure gradient. Here, where the intention is to consider the pressure gradient in the streamwise and crosswise directions separately, two independent parameters are more convenient.

to their magnitudes, or that their combined effects may be simply added. An ideal investigation would isolate each particular effect or hold three constant while varying the fourth. It would also repeat this for a range of values of H and $R_{\theta 11}$ because the effect of the other four parameters may well depend on these. In practice, data suitable for such an investigation do not exist and in this paper the effect of each parameter is assumed independent of the other three and independent of the form and Reynolds number of the streamwise profile. Although this has no rigorous justification it does allow the importance of the effects to be assessed.

Because of the dependence of the three-dimensional boundary layer on $\triangle_s$, $\triangle_c$, $\frac{\theta_{11}}{h_2} \frac{\partial h_2}{\partial s}$, and β it is convenient to group them all as three-dimensional effects. In fact $\triangle_s$ is not really a three-dimensional effect but is the familiar two-dimensional pressure gradient parameter extended to three-dimensional flows. This distinction is important because it will be shown that most of the large discrepancies between the measured streamwise profiles and the profile family is attributable to the streamwise pressure gradient.

4. The data used

A large number of three-dimensional velocity profiles have been measured and it would be quite impracticable in this report to compare all of them with Thompson profiles having identical H and R_θ values. Profiles have therefore been selected either to show the effect of a particular parameter or combination of parameters, or to show the typical results of a particular geometry. It must be emphasised, however, that no selection has been made in order to obtain good agreement between the streamwise and Thompson profiles. The data are summarised below.

Kehl[6] included in his measurements in diffusers boundary layer developments in convergent and divergent channels where the aspect ratio was adjusted to give either zero or very small pressure gradients. The channels were straight so that there was no crosswise pressure gradient or corresponding crossflow. Only two profiles are considered here, one measured in diverging flow, the other in converging flow. In each case the values of $\left|\frac{\theta_{11}}{h_2}\frac{\partial h_2}{\partial s}\right|$ were almost equal and the Reynolds numbers were comparable.

Cumpsty and Head[7] measured the velocity profiles on the attachment line of a long swept wing. For this singular flow there is no streamwise pressure gradient or growth and no crossflow, but strong flow divergence. Typical values of $\frac{\theta_{11}}{h_2}\frac{\partial h_2}{\partial s}$ for the fully turbulent boundary layer were around 10 times the values observed by Kehl.

Francis and Pierce[8] measured the boundary layer developments in two curved ducts each of different, but constant, radius. Some additional measurements were made in a straight duct downstream of the one of smaller radius. There was virtually no streamwise pressure gradient along the duct so that the skin friction and the value of Δ_c remained approximately constant for each duct radius. The crossflow grew initially very rapidly and profiles with different values of β could be compared at the same value of Δ_c. After a fairly short distance along the duct, however, the crossflow began to show the effects of the side walls and the boundary layers can no longer be taken as typical of those found in external flows.

The well known data of Hornung and Joubert[9] were measured in the highly disturbed region in front of a circular cylinder standing on a flat plate. Figure 3 shows the isobars and external streamlines on which are superimposed a map of the locations at which profiles were measured. The numbers refer to the number of the run measured at the adjacent position, shown by a cross. The external flow streamline pattern is approximate and was deduced from measured flow directions at the boundary layer edge. It is clear from Figure 3 that the boundary layer was exposed to a variety of streamwise and cross-wise pressure gradients. Furthermore, the magnitudes of the corresponding values of Δ_s and Δ_c were in some cases very large. Thus, with Δ_s and Δ_c changing along each streamline, and with large crossflows and divergence, these measurements are particularly unsuited to analysis where the effects of the individual parameters are desired. It is used here principally to assess the general effects of large values of these parameters, but precise conclusions are impossible.

The recent measurements on a swept wing by Cumpsty and Head[10] have also been used. These were made on a long wing of large thickness-chord ratio, swept at 61°. Because of the large thitkness-chord ratio they tend to represent conditions on a practical wing at an angle of attack rather than in the cruise attitude, and therefore give an idea of the upper values of Δ_c likely to be encountered on swept wings. Five profiles are considered, extending from the position of minimum pressure to within a short distance of the separation line.

The data is summarised in Table 1 together with values of the parameters from other geometries for comparison. In some cases the values of Δ_s and Δ_c could be found only very approximately and the values obtained should be used with some caution. The values of Δ_c were obtained for Hornung and

Joubert's data by measuring the external streamline radius on a figure such as Figure 3, (the value of Δ_c on Altman and Hayter's[11] swept wing was obtained in the same way), and similarly the values of Δ_s for Hornung and Joubert's data were found by measuring distances along streamlines between isobars. For Cumpsty and Head's measurements on the rear of the swept wing, the values of Δ_s, Δ_c and $\frac{\Theta_{11}}{h_s}\frac{\partial h_2}{\partial s}$ were deduced from the measured pressure distribution, but as these authors point out, there is some uncertainty associated with this procedure. From Table 1 it can be seen, however, that the values of Δ_c and Δ_s are generally considerably greater for the data of Hornung and Joubert. Furthermore, the values of Δ_c for Francis and Pierce's results are seen to depend almost entirely on the radius of the duct.

5. Discussion of measured streamwise velocity profiles

In Figure 4 two velocity profiles measured by Kehl are compared with the Thompson profiles having the same values of H and R_Θ as the measured profiles. The measurements at station 14 of channel AK1c were made in a converging flow with $\frac{\Theta_{11}}{h_2}\frac{\partial h_2}{\partial s} = -3.5\ 10^{-4}$, and the measurements at station 8 in channel K2 were made in a diverging flow with $\frac{\Theta_{11}}{h_2}\frac{\partial h_2}{\partial s} = 3.9\ 10^{-4}$. There was no pressure gradient at either station. It can be seen that the agreement between the Thompson and measured profiles is good in each case. Examining the inner region on the Clauser plot shown in Figure 5, we find a rather short logarithmic region, for which probable explanation is that Kehl used a round pitot tube of diameter 0.6 mm but made no correction for the displacement of the effective centre. When the familiar correction of 0.18 times the outside diameter is applied to some of the measurements for profile 14 of channel AK1c the linear region on the Clauser plot is extended, but this correction seems to be rather too large. In spite of this uncertainty it can be concluded that convergence with values of $\frac{\Theta_{11}}{h_2}\frac{\partial h_2}{\partial s}$ in the range $\pm 3\ 10^{-4}$ has, at most, a very marginal effect on the logarithmic part of the inner region and on the form of the streamwise velocity profile. Good agreement with the law of the wall is also shown by Cumpsty and Head's attachment line results at Reynolds numbers which are sufficiently high to be outside the transition region. This suggests that $\frac{\Theta_{11}}{h_2}\frac{\partial h_2}{\partial s}$ has a negligible effect on the form of the velocity profile up to values of at least $3\ 10^{-3}$.

Three boundary layer profiles measured by Francis and Pierce are compared with the corresponding Thompson profiles in Figure 6. The profiles were selected so that one pair have approximately equal values of Δ_c($\Delta_c \approx 0.01$) but with one crossflow much larger than the other, while another pair have comparable crossflow (β = 15.9° and 17.3°), but with one value of Δ_c only about 40% of the other. The agreement is very good in each case with no suggestion that the profile with largest Δ_c and β shows poorest agreement. The same profiles are shown in a Clauser plot in Figure 7, together with a profile measured in the straight portion of duct downstream from the duct of smaller centreline radius. (This particular profile shows such an unusual form that it seems likely that the restraint imposed by the duct walls means that it has little relevance to external flow boundary layers). In each case there is a very considerable linear region on the Clauser plots and even for the profile with the largest value of Δ_c and β this is quite as long as that found for normal two-dimensional flat-plate boundary layers.* The slopes of the linear regions on the Clauser plots are perfectly compatible with the constant c_f contours. It therefore appears that in the absence of streamwise pressure gradients and very large flow convergence or divergence, the form of the mean streamwise velocity profile is indistinguishable from a two-dimensional profile in the inner region up to $\beta = 27°$ and $\Delta_c = 0.01$.

For reasons outlined earlier, the data of Hornung and Joubert are not amenable to such straightforward interpretation. The measurements are, however, useful here because the values of Δ_c and β are very large and because they introduce strong streamwise pressure gradients. Figure 8 shows a number of streamwise profiles compared with the corresponding Thompson profiles, and Figure 9 shows a selection of the profiles on a Clauser plot. Runs 9,7,8,5 and 6 were made along a line normal to the plane of symmetry, with Run 9 measured on the centreline as indicated in Figure 3. Considerations of flow symmetry suggest for Run 9 that $\Delta_c = 0$ and $\beta = 0$, and the measured crossflow was indeed very small. The agreement between the Thompson and measured profile at this position is nevertheless the poorest shown. The Clauser plot of the profile shows an extremely short linear region, indicative of a very severe adverse pressure gradient. In fact, an estimate for Δ_s is 0.24, even larger than the values corresponding to Stratford's[12] zero-shear-stress layer in two dimensions. The disagreement between the measured profile for Run 9 and the corresponding Thompson profile is indeed

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* It should be noted that Francis and Pierce, using the resultant profile, concluded from their data that the logarithmic law of the wall is not a satisfactory assumption for three-dimensional flows. They found no law of the wall region comparable to that which they found in the two-dimensional profiles they measured.

quite similar to those shown by Thompson[2] for Stratford's measured profiles. Poor agreement of the Thompson profiles with the measured profiles is, of course, to be expected when the law of the wall region is very short, since the Thompson profiles rely on its validity over an appreciable part of the boundary layer thickness. In Figure 10, u/U_s has been plotted against (distance from surface)$^{\frac{1}{2}}$ and, as Stratford and Townsend[13] have predicted for two-dimensional boundary layers in very severe adverse pressure gradients, there is an extensive linear region.

Referring back to Figure 8 it can be seen that for Run 8, when $\beta \approx 35°$, the agreement is better than for Run 9 but still not good. The discrepancy is of a similar type to that for Run 9 and it seems most probable that it is still the strong adverse pressure gradient in the streamwise direction that is responsible for the discrepancy. By Run 6, however, the streamwise pressure gradient is greatly reduced, (see Figure 3 and Table 1) and the agreement between measured and Thompson profiles is comparatively good, despite the fact that β and Δ_c are still large. There is also a comparatively long linear region on the Clauser plot for Run 6, although at a slightly different slope to the contours. Even for Run 22, when $\beta = 45°$ and $\Delta_c = 0.043$, the Thompson profile provides a tolerable representation of the profile and there is some linear region on the Clauser plot although at a different slope to the contours. This is particularly remarkable since, as well as the large values of Δ_c and β at the measuring position of Run 22, the flow has passed through a region of strong positive streamwise pressure gradient before entering the strong negative streamwise pressure field existing at the measuring position.

The Clauser plots of Hornung and Joubert's data do suggest that it is no longer reasonable to assume that the two-dimensional inner law is valid. This is very noticeable for those profiles in strong streamwise pressure gradients, such as Runs 9 and 8, and also for the profile shown in a strong crosswise pressure gradient, Run 22, where the best line through the data in the inner region is rotated relative to the contours of the Clauser plot. This had been clearly shown by Perry and Joubert[5]. These authors, in restricting their attention to Hornung and Joubert's data, failed to note that with many other important flows the conditions are very much less severe; the law of the wall, and the profile families based on it, are therefore good approximations for many cases of practical interest. Furthermore, it is noteworthy that even when the inner region of the streamwise profile differs noticeably from the law of the wall in two-dimensional flows (as for example in Run 22), the profile family can provide a fair representation of the overall profile. It is profiles measured in streamwise pressure gradients large enough to have produced poor agreement in two-dimensional flows that compare worst with the Thompson profile family.

All the streamwise profiles measured by Cumpsty and Head on the rear of a swept wing show excellent agreement with the Thompson profiles as indicated in Figure 11. The Clauser plots of Figure 12 show that for most of the profiles there are moderate regions in which the logarithmic law of the wall is valid. (Close to the surface there is a trend for u/U_s to be too large, which is almost certainly due to inaccuracy of measurement). The values of Δ_c must only be regarded as approximate since they were obtained from the measured pressure distribution with some unverified assumptions. Moreover, for the profiles near to the separation line the flow curvatures deduced were uncertain. However, the profiles show convincingly that even on this thick and highly swept wing the combinations of streamwise and crosswise pressure gradients, convergence and crossflow are not sufficient either to cause appreciable discrepancies between the streamwise profile and the Thompson profile, or to alter radically the form of the streamwise inner region.

6. The coefficient of skin friction

The direct measurement of skin friction in boundary layers is difficult and subject to error. Accurate measurements of skin friction in two-dimensional turbulent boundary layers have generally been made by calibrating a device in fully developed pipe flow, where the skin friction is known with considerable accuracy. On the assumption that a universal law of the wall is valid for pipe and boundary layer flow, the device (Stanton tube, Preston tube, boundary layer fence or razor blade) can be used to obtain c_f in the boundary layer. In the presence of a pressure gradient the universal inner law breaks down and estimates of c_f will be in error by an amount depending on the depth of the device d, the strength of the pressure gradient and the skin friction, i.e. on the non-dimensional parameter $\frac{d}{\tau_w} \frac{dp}{dx}$. For this reason, the razor blade technique is particularly suitable for boundary layers in pressure gradients because the overall depth can be made very small.

The razor blade technique is also very suitable for three-dimensional boundary layer measurements and East[14] has described a method for obtaining the magnitude and direction of the wall shear stress. Because of the small depth of the razor blade the effect of the change in direction away from the surface can be made relatively small. East[15] has found that the results using the razor blade technique in a severely disturbed flow of similar geometry to that used by Hornung and Joubert[9] agree closely with the estimates from Clauser plots.

East's results support the assumption made here that a linear region on a Clauser plot provides a reasonable estimate for the skin friction, emphasis being given to the position of the line rather than its slope. (This is particularly important if the resultant profile is considered, for, as Figure 2 shows, the slope of this profile does not generally satisfactorily match that of the contours.)

Because the flow is laminar in the immediate vicinity of the surface, the resultant velocity and shear stress both tend to the same direction as the surface is approached. The streamwise component of skin friction coefficient is then simply given by $c_{f1} = c_f \cos\beta$, where c_f is the resultant coefficient and β is defined as the angle between the limiting surface streamline and the flow direction at the boundary layer edge.

It seems, a priori, that the streamwise velocity profile will provide an estimate for the streamwise component of the skin friction coefficient, c_{f1}, whereas the resultant or developed profiles will provide an estimate for c_f. To test this, three profiles have been used to obtain the resultant coefficient of skin friction from Clauser plots and the results are shown in Table 2. It can be seen that the agreement is generally very good. For Hornung and Joubert's Run 22, when the agreement is less good, it will be recalled that positioning the linear region is somewhat arbitrary, as Figure 2 shows. Cumpsty and Head[16] have likewise shown that the Thompson skin friction law gives nearly identical estimates for c_f when applied to the streamwise and resultant profiles.

In Table 3 the estimates for the streamwise coefficient of skin friction obtained from Clauser plots and the Thompson skin friction law are compared for the boundary layers discussed previously. As expected, the estimates agree closely for those profiles which show a considerable linear region on the Clauser plots and good agreement between the measured velocity profiles and Thompson profiles. The profiles showing largest discrepancy are those measured by Hornung and Joubert in which the streamwise pressure gradient was very severe, in particular Run 9.

7. Conclusions

1. The streamwise velocity profile provides the most suitable basic velocity profile for integral calculation techniques because of its simplicity and because, for a wide range of three-dimensional effects, the measured streamwise profiles have been found to have inner region velocity distributions very similar to those of two-dimensional boundary layers.

2. The parameters Δ_s, Δ_c, $\frac{\theta_{11}}{h_2}\frac{\partial h_2}{\partial s}$ and β provide a measure of the severity of the effects to which the three-dimensional boundary layer is subjected. Without such parameters it is possible to arrive at misleading conclusions regarding the applicability of particular approximations.

3. The streamwise velocity profile is closely approximated by the Thompson profile family (and, by inference, by other reliable profile families, such as Coles') for a wide range of conditions.

a) No discrepancy is discernible for $\Delta_c = 0.01$ and $\beta = 27°$ and, even when $\Delta_c = 0.043$ and $\beta = 45°$, the agreement is sufficiently satisfactory to provide a fair approximation to the real profile form.

b) No discrepancy is evident when $\frac{\theta_{11}}{h_2} \frac{\partial h_2}{\partial s} = 3\ 10^{-4}$, and it is inferred that this would be true up to at least $3\ 10^{-3}$.

c) The streamwise pressure gradient is observed to have a quite marked effect on the agreement between the profile family and measured streamwise profile, comparable to that found for two-dimensional flows. In fact the instances of greatest disagreement are almost entirely attributable to the effect of large streamwise pressure gradients. The streamwise pressure gradient, it should be noted, is not a specifically three-dimensional effect and many of the large discrepancies observed would have been found in two-dimensional flows with pressure gradients of comparable severity. For values of Δ_s greater than, say, 0.05 a noticeable discrepancy between the profile family and measured profile is to be expected, accompanied by a short law of the wall region.

4. The results of East[15] suggest strongly that the Clauser plot gives a reliable indication of the coefficient of skin friction even for severely distorted three-dimensional flows.

5. Provided the three-dimensional effects are not too large (particularly Δ_s) the estimates for the resultant coefficient of skin friction (accepting that the streamwise profile gives the streamwise component of skin friction, which must be corrected to give the resultant skin friction) all agree within a few per cent using:

a) a reliable skin friction law (e.g. that due to Ludwieg and Tillmann or Thompson) applied to the streamwise or resultant profile,

and b) Clauser plots of the streamwise, resultant or developed velocity profiles.

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Table 1

Summary of Data

		Radius of external streamline (inches)	Δ_c	Δ_s	$\frac{\theta_{11}}{h_2} \frac{\partial h_2}{\partial s}$	β Degrees
Francis and Pierce	Run 212DOC	25	.009	0	0	15.9
	224DOC	25	.010	0	0	27.1
	506DOC	55	.0036	0	0	7.3
	518DOC	55	.0042	0	0	17.3
	206IOC	∞	0	0	0	15.8
Hornung and Joubert	Run 5	50	0.030	+0.04	-	30
	6	50	0.017	+0.03	-	24
	8	75	0.035	+0.07	-	35
	9	∞	0	+0.24	-	0
	22	40	0.043	-0.06	-	45
Cumpsty and Head (Rear of swept wing)	x = 0	-	0.0007	+0.008	$- 0.7\ 10^{-4}$	- 3.0
	0.217 ft	-	0.0044	+0.049	$- 6.3\ 10^{-4}$	+ 7.5
	0.466	-	0.0041	+0.030	$- 8.3\ 10^{-4}$	16.0
	0.650	-	0.0036	+0.027	$-10.1\ 10^{-4}$	21.5
	0.813	-	0.0052	+0.032	$-11.6\ 10^{-4}$	31.5
Kehl	AK1c, 14	0	0	0	$- 3.5\ 10^{-4}$	0
	K2 8	0	0	0	$+ 3.9\ 10^{-4}$	0
Cumpsty and Head (Swept wing attachment line)	$C^* \doteqdot 2.10^5$	0	0	0	$+ 3\ 10^{-3}$	0
Altman and Hayter (Wing swept at 45°, C_L = 1.0, x/c = 0.5)		120	0.001	0.0021	-	20° approx.
Gruschwitz III 10		26.6	0.019	0	0	16.5°

Table 2

Resultant skin friction coefficient based on different assumptions for the appropriate velocity

	Coefficient of Resultant Skin Friction from Clauser chart based on		
	Streamwise profile u/U_s	Resultant profile $(u^2+v^2)^{\frac{1}{2}}/U_s$	Developed profile $\frac{1}{U_s}\int_0^u \left(1+\left(\frac{dv}{du}\right)^2\right)^{\frac{1}{2}} du$
Hornung and Joubert, Run 5	0.0018	0.0018	0.0018
Hornung and Joubert, Run 22	0.0021	0.0024	0.0024
Francis and Pierce, Run 224DOC	0.0031	0.0032	0.0032

Table 3

Comparison of coefficient of streamwise component of skin friction from the Thompson skin friction law and the Clauser plot

	Coefficient of streamwise component of skin friction	
	From Thompson c_f law	From Clauser plot
Hornung and Joubert, Run 5	0.0018	0.0016
22	0.0016	0.0015
12	0.0020	0.0017
6	0.0022	0.0020
8	0.0015	0.0010
9	0.0010	0.0005
Francis and Pierce, Run 212DOC	0.0030	0.0030
224DOC	0.0027	0.0027
506DOC	0.0033	0.0032
518DOC	0.0028	0.0028
Kehl, Channel AK1c Station 14	0.0033	0.0032
Channel K2 Station 8	0.0034	0.0032
Cumpsty and Head x = 0.0	0.0042	0.0041
0.217 ft	0.0029	0.0029
0.466 ft	0.0023	0.0023
0.650 ft	0.0019	0.0019
0.813 ft	0.0014	0.0012

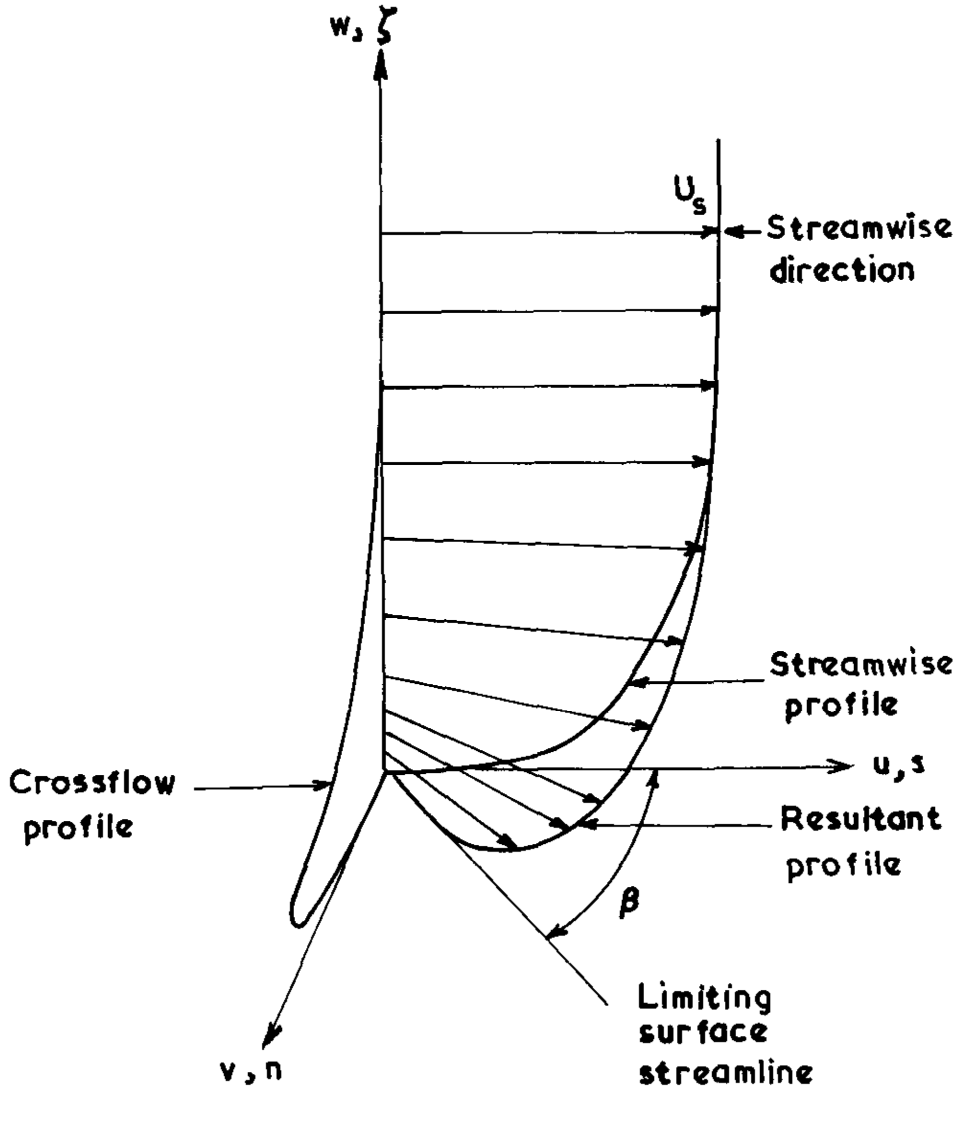


FIG .1 A diagrammatic skewed profile showing the basic notation

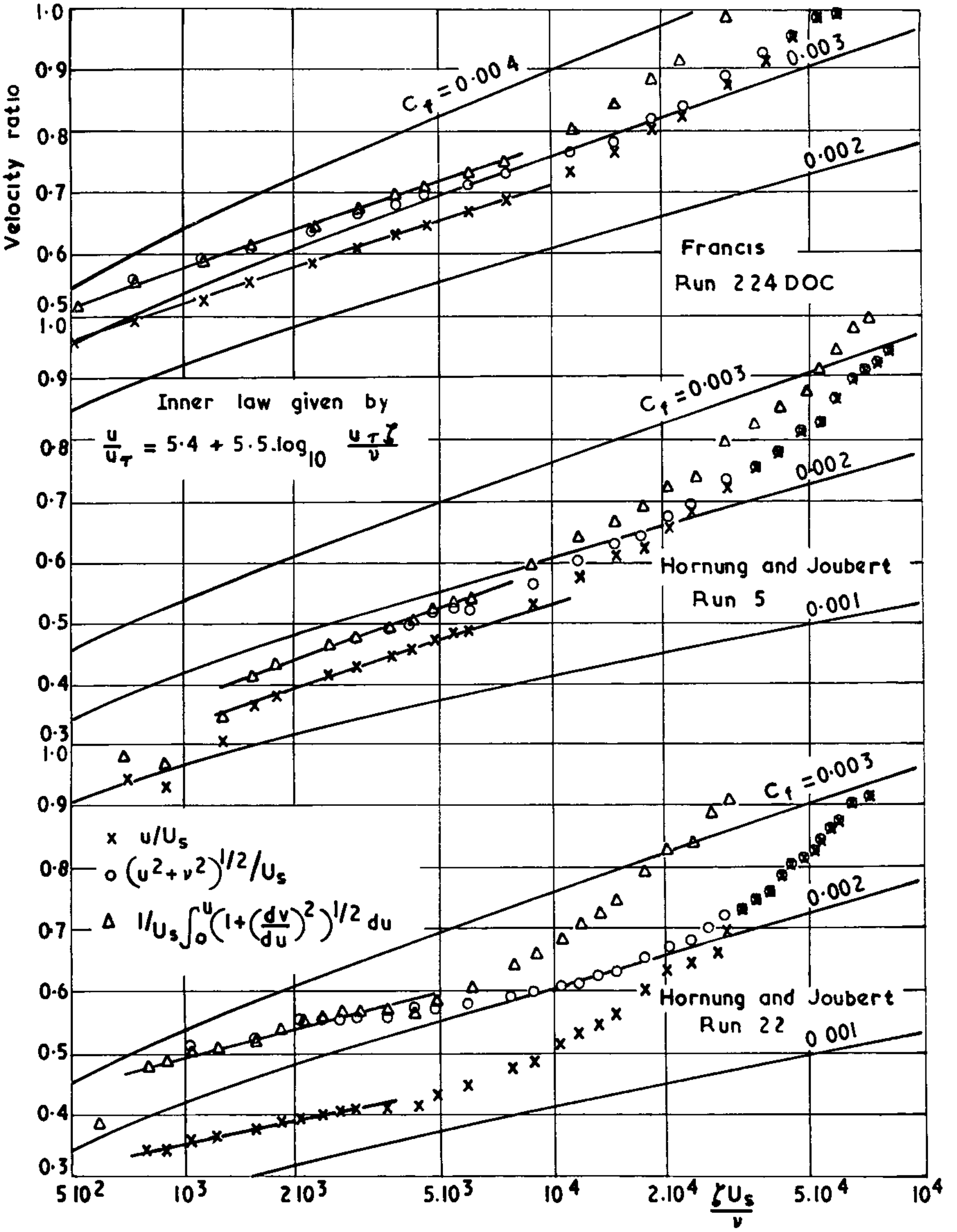
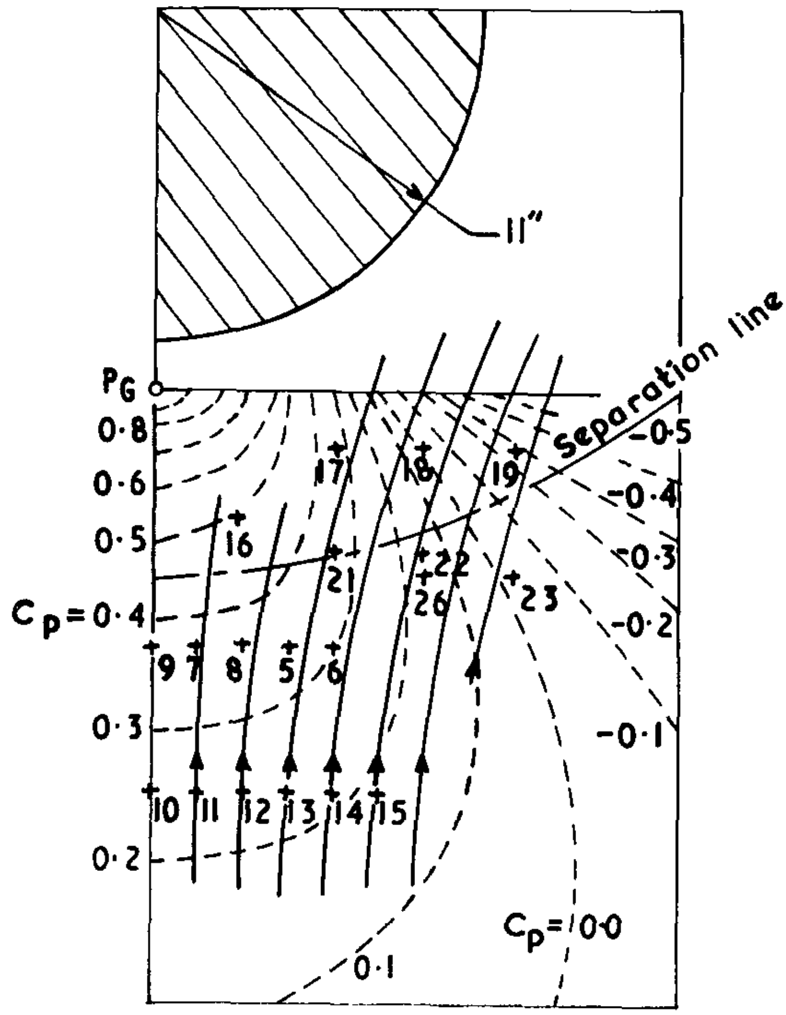


FIG. 2 Clauser plots of three profiles comparing different velocity ratios



Solid lines show streamlines
deduced from flow directions
Broken lines show isobars
Numbers refer to runs made
at the adjacent crosses

$$C_p = \frac{p - p_{ref}}{p_G - p_{ref}}$$

FIG 3. Approximate streamline patterns and isobars measured by Hornung and Joubert

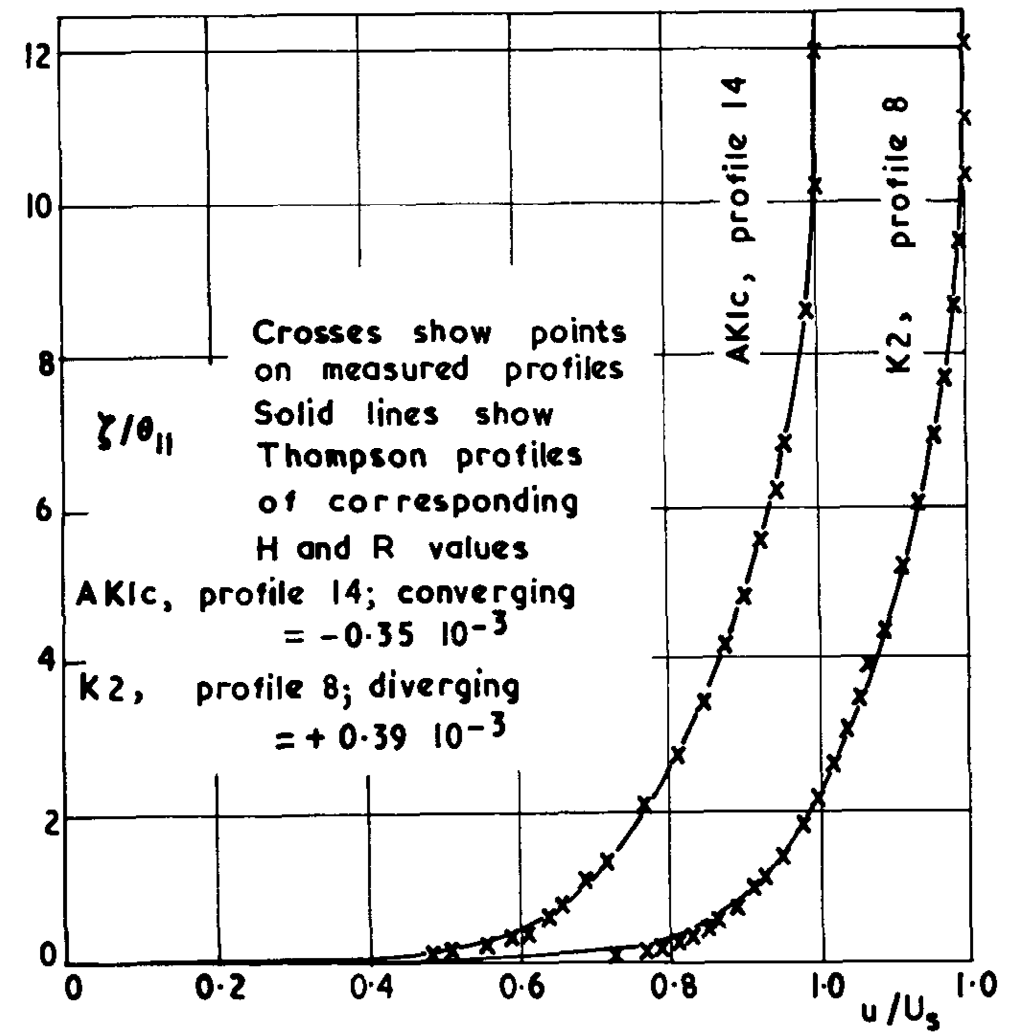


FIG. 4 Two velocity profiles measured by Kehl in zero pressure gradient

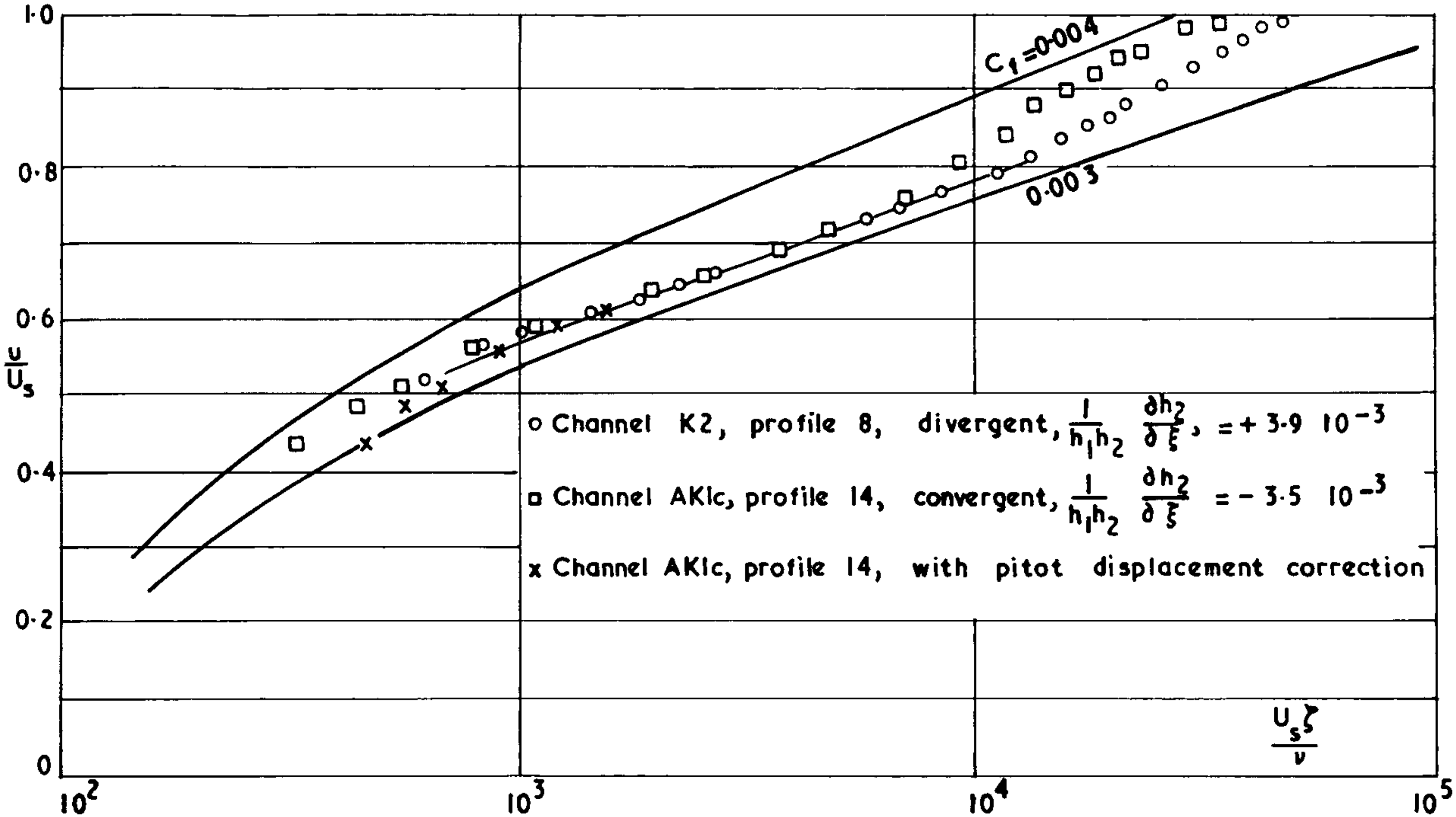


FIG.5 Clauser plots of profiles measured by Kehl

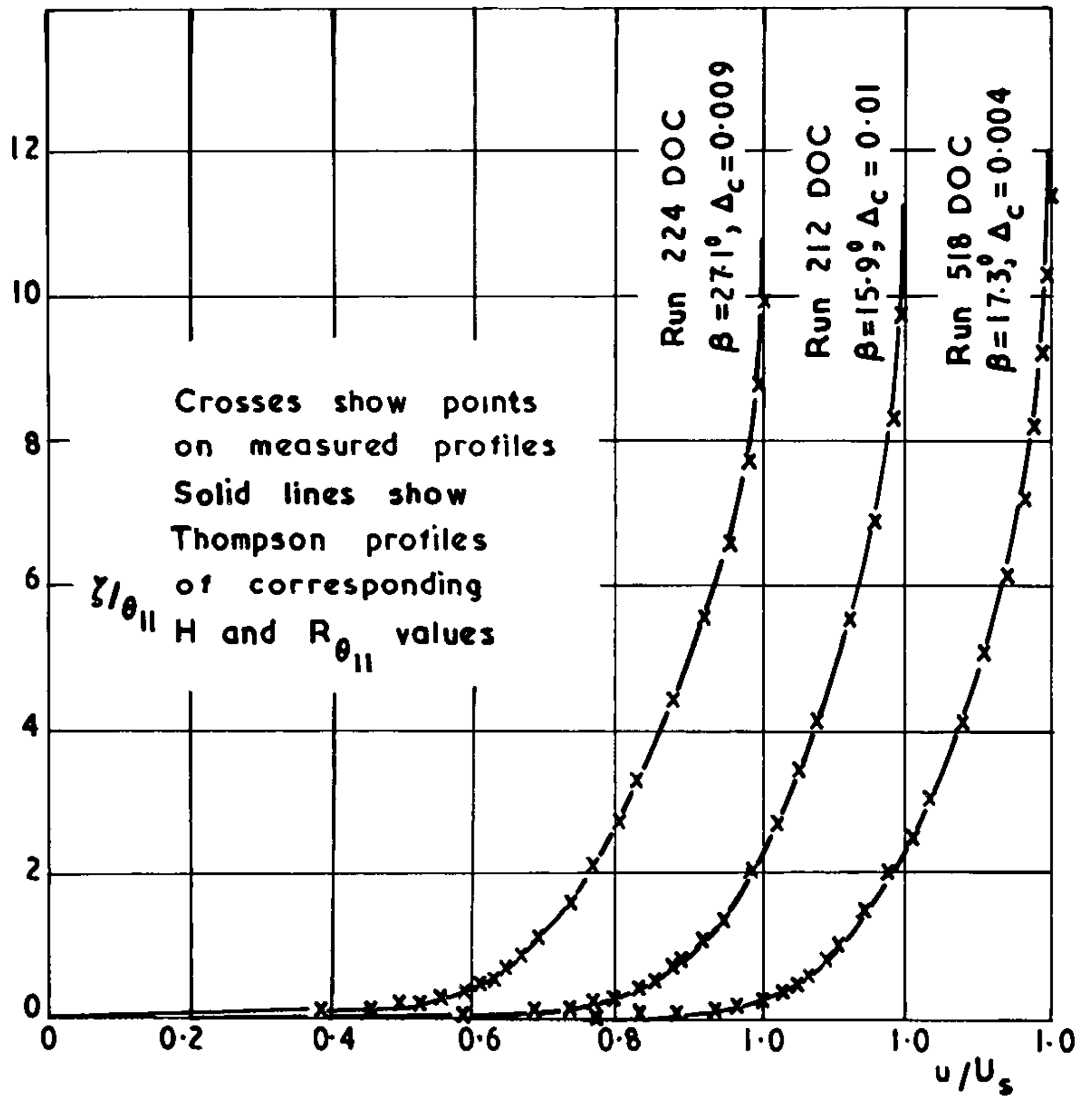


FIG. 6 Three streamwise velocity profiles measured by Francis and Pierce

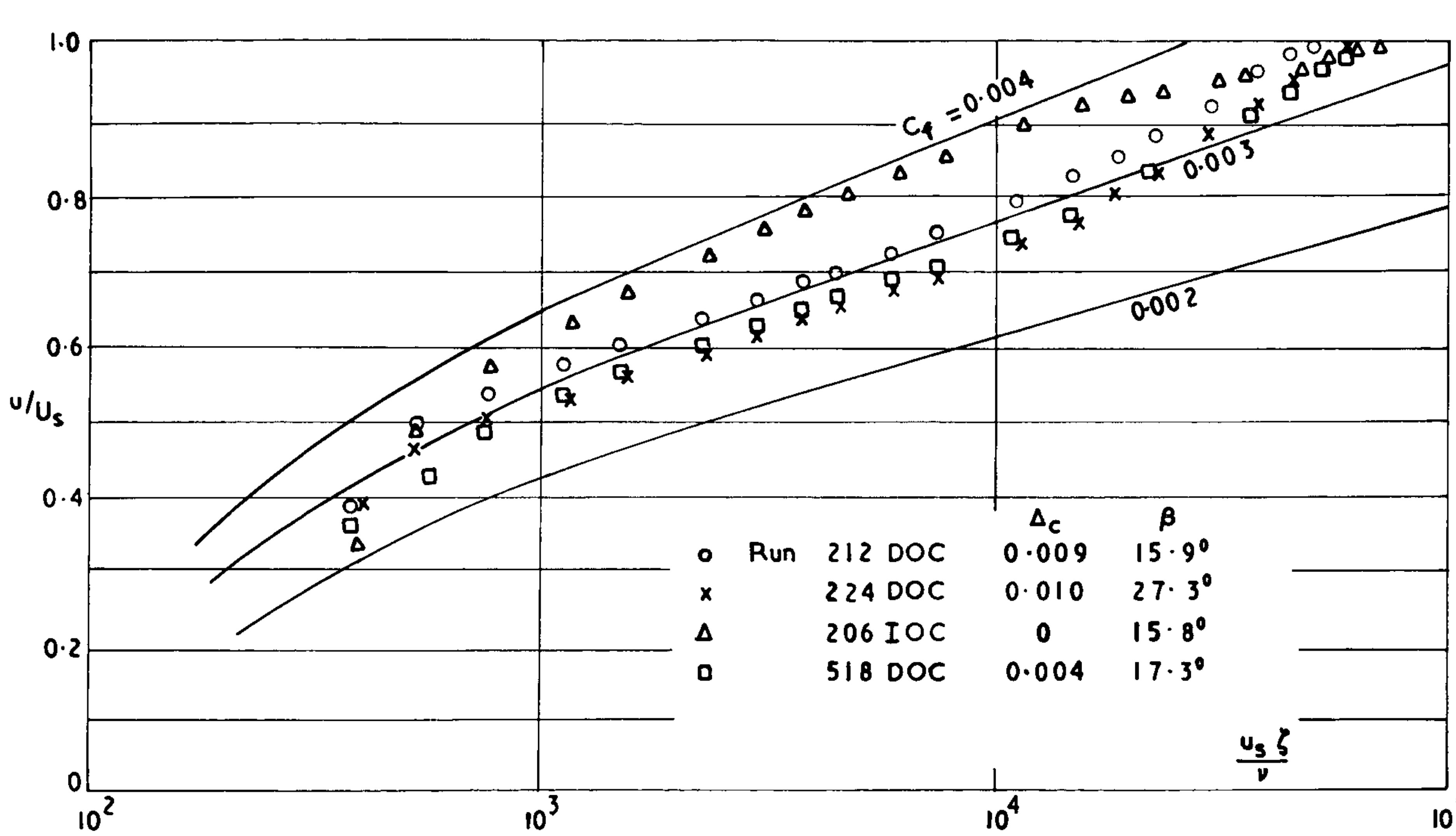


FIG. 7 Clauser plots of streamwise profiles measured by Francis

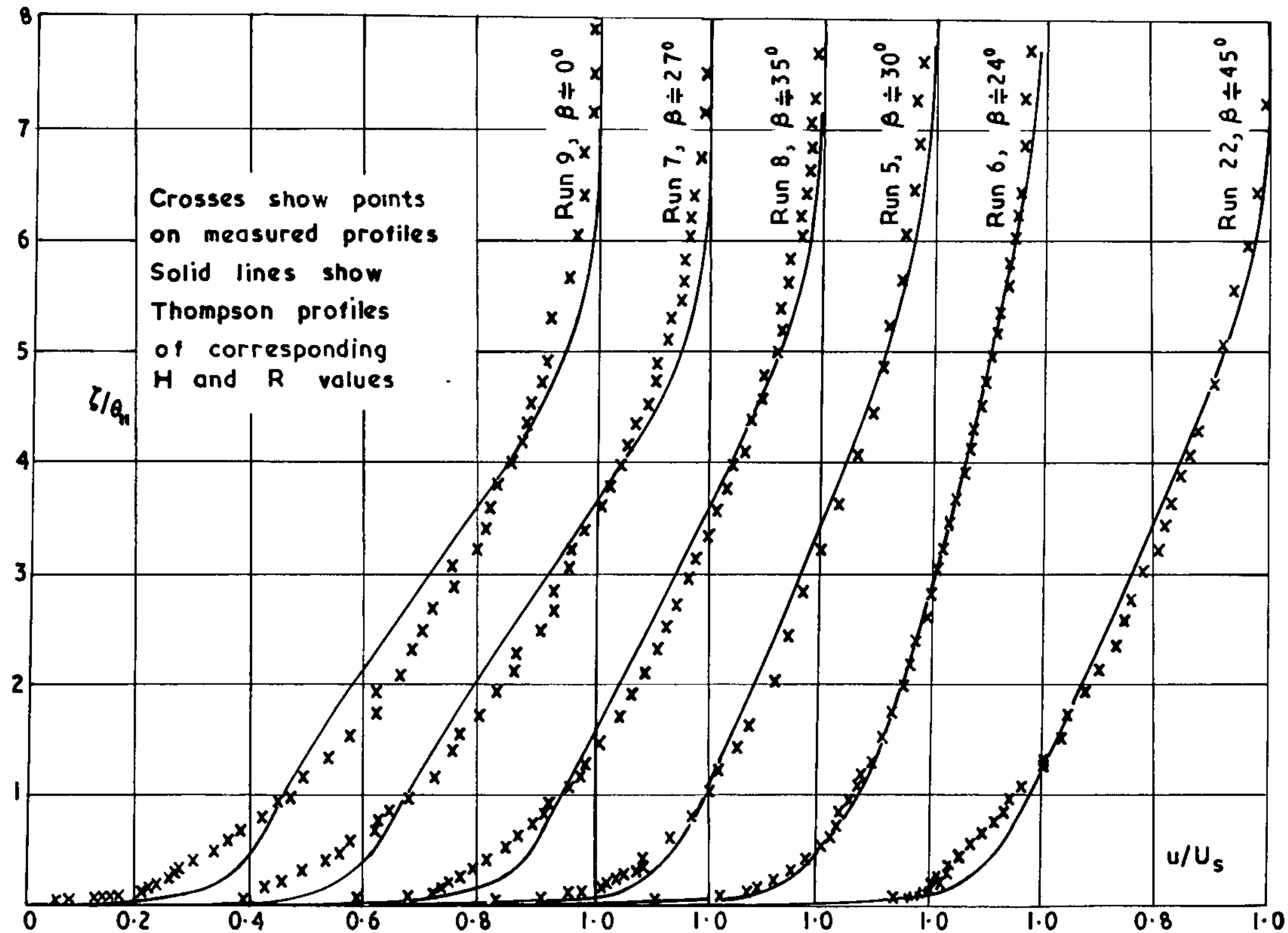


FIG.8 Streamwise velocity profiles measured by Hornung and Joubert

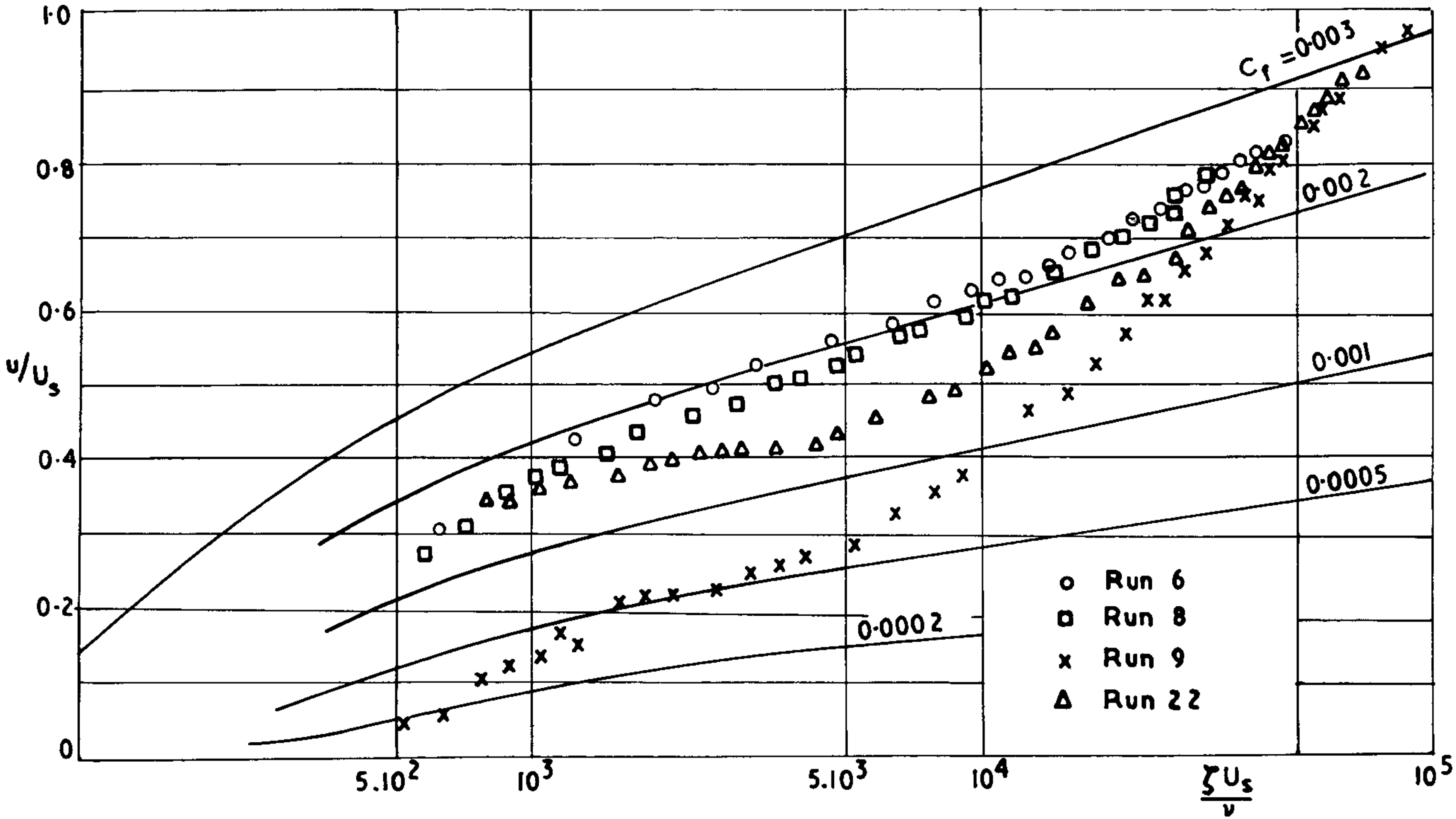


FIG. 9 Clauser plots of streamwise profiles measured by Hornung and Joubert

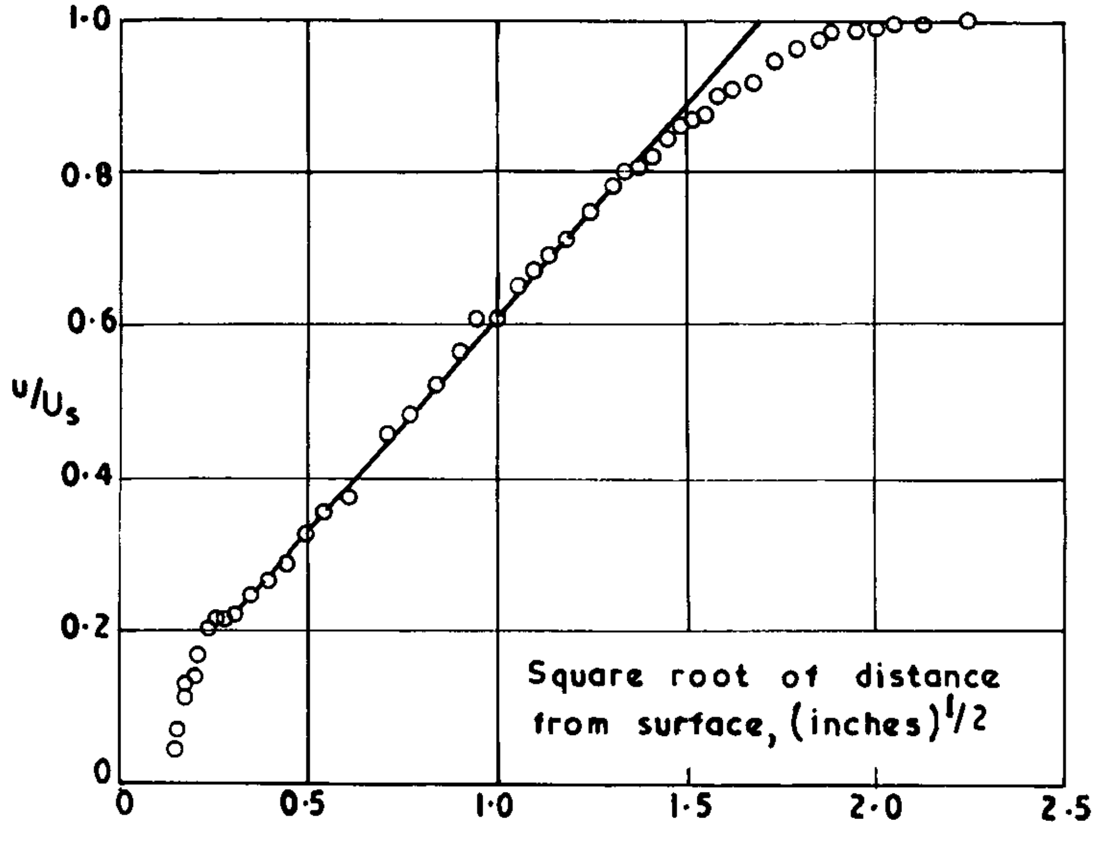


FIG.10 Velocity profile of Hornung and Joubert, Run 9, plotted against square root of distance

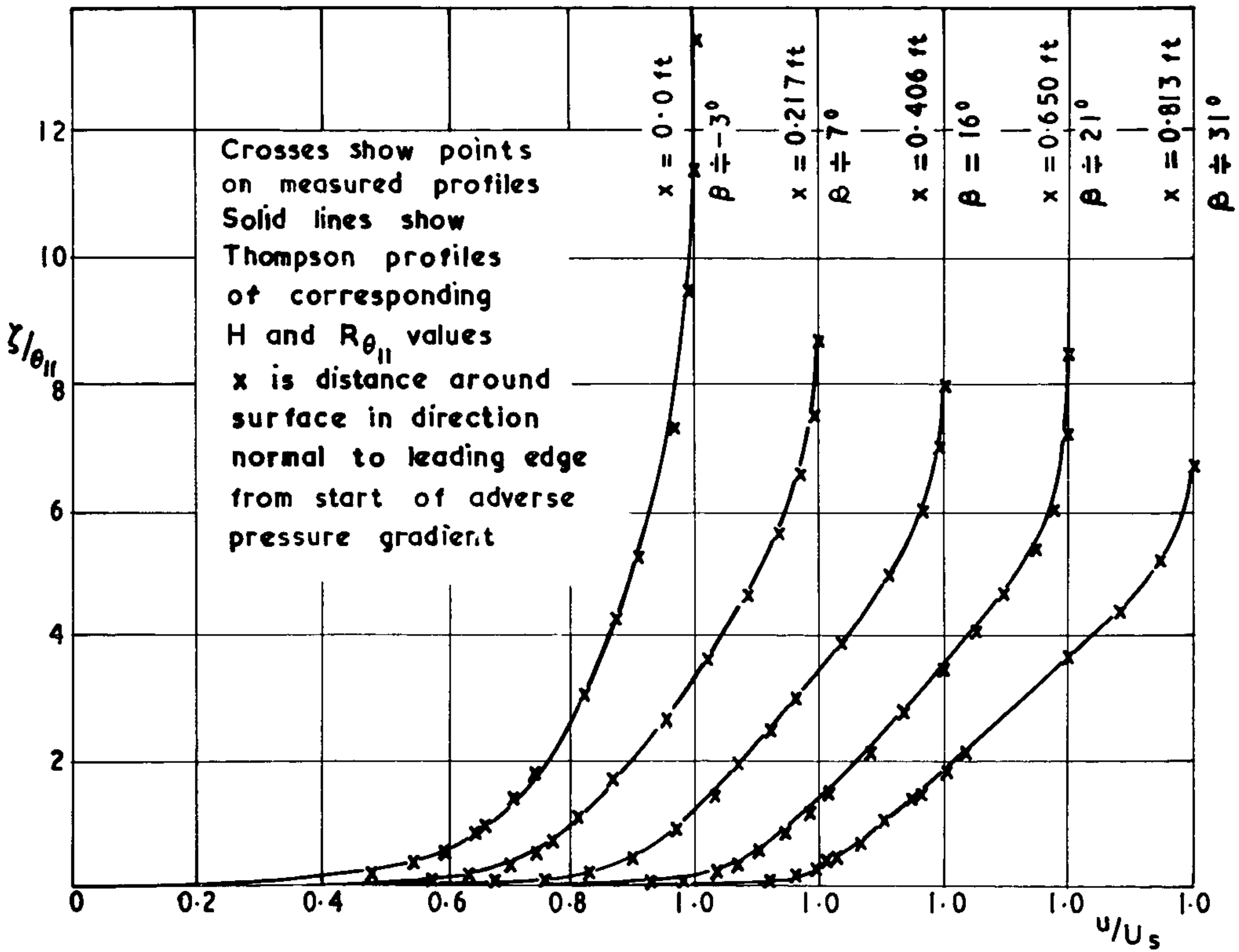


FIG. 11 Streamwise velocity profiles measured by Cumpsty and Head

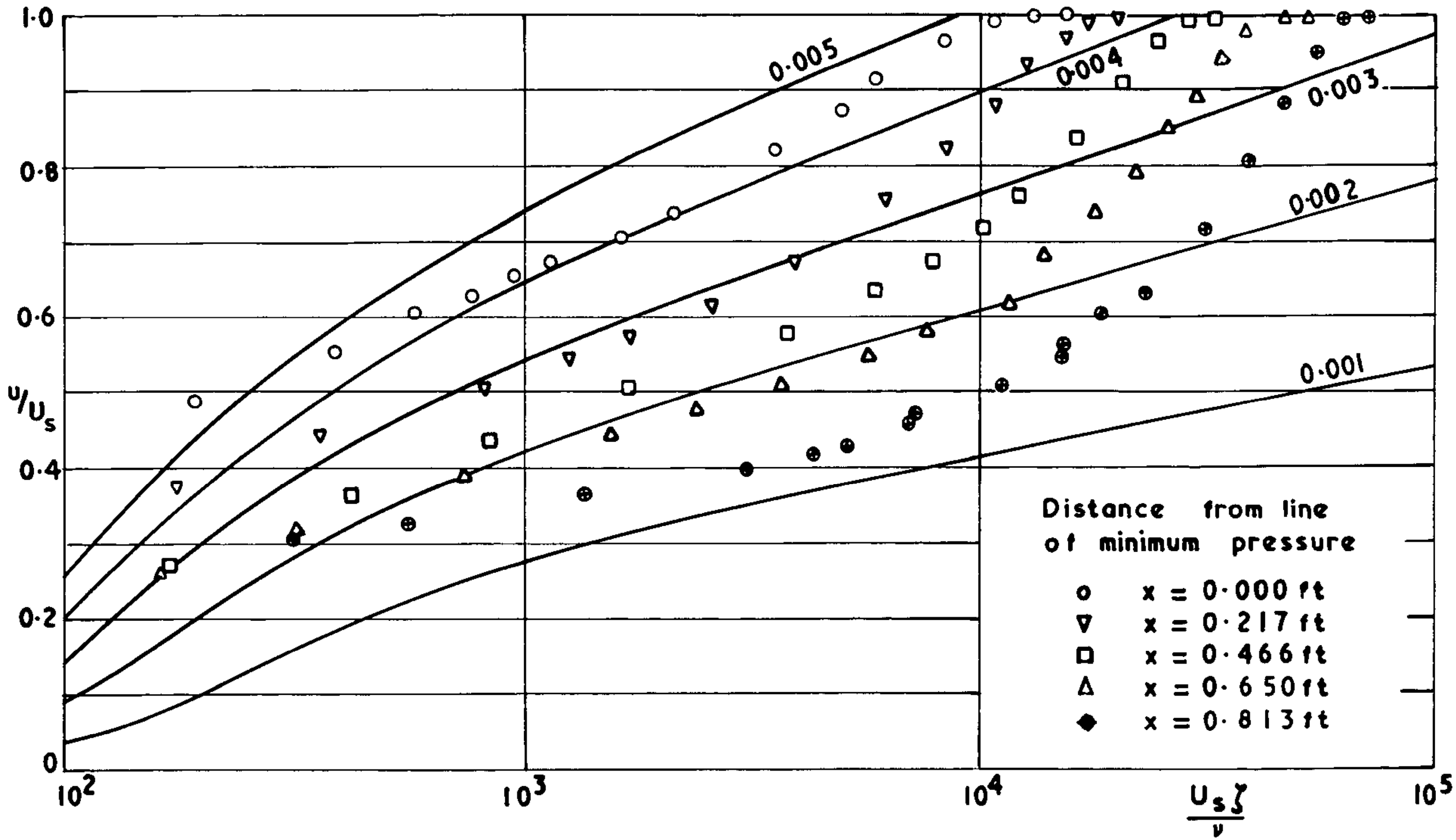


FIG.12 Clauser plots of streamwise velocity profiles measured by Cumpsty and Head

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The streamwise velocity profile is established as the most suitable basic profile for the calculation of three-dimensional turbulent boundary layers. Measured streamwise profiles are compared with Thompson's two-dimensional profile family and it is shown that the discrepancies produced by the variation of flow direction within the boundary layer, the pressure gradient normal to the external flow and the convergence or divergence of the flow are generally

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