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MODIFICATION OF A NORMAL SHOCK BY ELECTROSTATIC FORCES

by

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SUMMARY

The suggestion that the sonic bang may be alleviated by electrostatic means is examined and several objections to the idea are put forward. At the same time, some theoretical aspects of supersonic electrogasdynamics are investigated and the results presented.

* Replaces R.A.E. Technical Report 69040 - A.R.C. 31245

CONTENTS

1 INTRODUCTION

Early in 1968, a paper[1] was presented at the A.I.A.A. Sixth Aerospace Sciences meeting, suggesting the possibility of altering supersonic flows by the use of electrostatic forces. In particular great interest was aroused by the suggestion that the sonic bang could be alleviated in this way, and several articles appeared in the non-technical press at the time. The paper in question, which is the work of M.S. Cahn and G.M. Andrew of the Northrop Corporation, consists mainly of a qualitative description of a series of experiments they have carried out. The manner in which they propose to reduce the sonic bang is by spraying electric charge ahead of the aircraft and charging the aircraft nose similarly, so that the oncoming charged stream anticipates its presence. In this way, they argue, the discontinuity of the shock could be smoothed out*.

The idea of influencing supersonic flows by electrostatic forces has a certain immediate appeal, but a short calculation of the electrostatic energy density that can be achieved in practice shows that the prospects are in fact, poor. This calculation is described in section 2 and in section 3 a more detailed one-dimensional analysis is given, which confirms the fact that a significant reduction of shock intensity cannot be produced by electrostatic forces of a practical magnitude.

Some further points may be noted here. Firstly, even if a significant reduction in the strength of the discrete shock could be achieved so that part of the near field pressure rise were accommodated by infinitesimal Mach waves, these would eventually coalesce with the shock to strengthen it again in the far field. Secondly, the spreading out of the shock could also be brought about aerodynamically by designing Prandtl-Meyer compression surfaces (although again there would be a far-field coalescence) and hence the electrostatic device would have to be shown to be superior in some way to an aerodynamic fairing. Finally, the weight and volume of the electrostatic generators and the interference with radio communications from any corona discharge would need to be kept down to an acceptable level.

2 THE ELECTRICAL ENERGY DENSITY

There is a natural limitation on the use of electrostatic forces in a gas in that the electric field, E, reaches a value, E_{BR}, at which any further increase merely causes electrical breakdown. At a height of 60000 ft,

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*A very similar suggestion to this was made by O'Neil[2], in a private communication.

for instance, $E_{BR} \simeq 3 \times 10^5$ volt/m for air and if this value were exceeded electric currents would flow to reduce the field. It can therefore be seen that the maximum achievable energy density of the electrostatic forces is given by $\frac{1}{2} \varepsilon E_{BR}^2$, where ε is the permittivity of free space, and with the value of E_{BR} given above, this is approximately 0.4 joule/m^3, compared with the energy density of a supersonic flow which is of the order 10^4 joule/m^3. From this it follows that if the electrostatic forces act over a length of 1 metre, their effect would be equivalent to a pressure change of order $10^{-4} p_s$, where p_s is the stagnation pressure. Since a typical pressure change through the nose shock of a supersonic aircraft at a Mach number of 2 is about $0.05 p_s$, the maximum possible electrostatic forces seem to be too small to have a significant effect on the shock strength.

3 A ONE-DIMENSIONAL MODEL

As far as is known, little theoretical work has previously been undertaken in the field of supersonic electrogasdynamics, and this would seem to be a good opportunity to explore, in a limited way, some of the ideas involved. We consider only a very simple, one-dimensional model with a normal shock of negligible thickness at the origin of the flow coordinate, $x = 0$. A normal shock is of course, a particularly severe example to choose, but it still serves as a guide to the effectiveness of electrostatic forces in such flows.

Electric charge is inserted into the flow ahead of the shock and withdrawn again behind it. (The exact physical nature of such a process is not dwelt upon, but a system of charge sprays and collectors could, no doubt, be arranged in a suitable way.) The fluid medium is taken to be a perfect gas, which always remains electrically non-conducting, and the flow far upstream and downstream is uniform. No ionisation occurs at the shock and all dissipative effects are neglected. The length scale, L, associated with the electrostatic phenomena is large compared with the shock thickness (see section 3.3) so that the Rankine-Hugoniot relations may be applied across the plane $x = 0$. It is also assumed that the electrostatic charge is associated with particles of negligible mass, so that a change in charge density does not imply any change in the mass flow rate.

3.1 Flow equations

The equations of motion of one-dimensional gas flow with a body force, comprising conservation of mass, momentum and energy are[3]:

$$\rho u = \text{constant} \quad , \tag{1}$$

$$\rho u \frac{du}{dx} = -\frac{dp}{dx} + F \quad , \tag{2}$$

$$\frac{F}{\rho} = \frac{d}{dx}\left(h + \tfrac{1}{2}u^2\right) \quad , \tag{3}$$

where u, p, ρ are the velocity, pressure and density of the gas respectively, x is the coordinate in the direction of motion, F is the body force per unit mass acting in the x-direction and h is the specific enthalpy.

In the case of an electrostatic body force, we have

$$F = \rho_e E \quad , \tag{4}$$

and

$$\rho_e = \varepsilon \operatorname{div} \underline{E} = \varepsilon \frac{dE}{dx} \quad , \tag{5}$$

where ρ_e is the charge density, $\underline{E}$ is the electric field and ε is the permittivity of the gas (taken to be that of free space).

Using (4) and (5), the momentum and energy expressions (2) and (3) become,

$$\rho u \frac{du}{dx} = -\frac{dp}{dx} + \varepsilon E \frac{dE}{dx} \quad , \tag{6}$$

$$\frac{\varepsilon E}{\rho}\frac{dE}{dx} = \frac{d}{dx}\left(h + \tfrac{1}{2}u^2\right) \quad . \tag{7}$$

The governing equations (1), (6) and (7) may be reduced to non-dimensional form by writing:

$$\bar{\rho} = \frac{\rho}{\rho_1}, \quad \bar{u} = \frac{u}{u_1}, \quad \bar{p} = \frac{p}{p_1}, \quad \bar{E} = \frac{E}{E_1}, \quad \xi = \frac{x}{L} \quad ,$$

where the subscript 1 denotes upstream $(x \to -\infty)$ values, and L is a typical length associated with the system.

Taking into account that $h = \frac{\gamma}{\gamma - 1}\frac{p}{\rho}$ for a perfect gas, (where γ is the ratio of the principal specific heats), we write equations (1), (6) and (7) as

$$\bar{\rho}\,\bar{u} = 1 \qquad , \tag{8}$$

$$\frac{d\bar{u}}{d\xi} = -\frac{1}{\gamma M_1^2}\frac{d\bar{p}}{d\xi} + \frac{2\lambda_1}{\gamma M_1^2}\bar{E}\frac{d\bar{E}}{d\xi} \qquad , \tag{9}$$

$$\frac{2\lambda_1}{\gamma M_1^2}\frac{\bar{E}}{\bar{\rho}}\frac{d\bar{E}}{d\xi} = \frac{d}{d\xi}\left(\frac{1}{(\gamma - 1)\,M_1^2}\frac{\bar{p}}{\bar{\rho}} + \tfrac{1}{2}\bar{u}^2\right) \qquad , \tag{10}$$

where $M_1 = \left(\frac{\rho_1\, u_1^2}{\gamma p_1}\right)^{\frac{1}{2}}$ is the upstream Mach number and $\lambda_1 = \frac{\epsilon\, E_1^2}{2p_1}$ is the ratio of the upstream electrical energy density and the upstream pressure.

Using the upstream conditions, equation (9) may be integrated immediately to give

$$\bar{p} = 1 + \gamma M_1^2\,(1 - \bar{u}) - \lambda_1\,(1 - \bar{E}^2) \qquad . \tag{11}$$

Now, by making use of (8), (9) and (11), $\bar{\rho}$ and $\bar{p}$ may be eliminated from (10) to yield after some manipulation,

$$\frac{d\bar{u}}{d\xi} = \frac{2\lambda_1\,\bar{u}\,\bar{E}\,\frac{d\bar{E}}{d\xi}}{\gamma\,\{M_1^2\,(\gamma + 1)\,\bar{u} + \lambda_1\,(1 - \bar{E}^2) - M_1^2\,\gamma - 1\}} \qquad . \tag{12}$$

In general, the denominator is non-zero, so that once a suitable electric field distribution has been selected, equation (12) may be integrated to determine the flow velocity at any station ξ. It is not usually possible to find analytical solutions of (12) and therefore, in most cases, a numerical integration procedure has to be employed.

Across the shock, at $\xi = 0$, the Rankine-Hugoniot conditions apply, so that

$$\bar{u}_B = \bar{u}_A\,\frac{(\gamma - 1)\,M_A^2 + 2}{(\gamma + 1)\,M_A^2} \qquad , \tag{13}$$

where A, B refer to conditions ahead of and behind the shock, i.e. at $\xi = 0-$ and $0+$ respectively.

3.2 Selection of appropriate electric fields

Since we are attempting to "smooth out" the discontinuity at the shock, we would like the Mach number, M, to decrease smoothly from the upstream value (> 1) to a value approaching unity at the shock; and then to decrease smoothly to a suitable downstream value (see section 3.4).

For a gas flow acted upon by a body force, F, it can be shown[4] that the change in Mach number, $\Delta M = k(M)\, F/(1 - M^2)$. As the factor of proportionality, k(M), always remains greater than zero, we have the following conditions:

$$\left.\begin{array}{l} M > 1\ ,\quad F \text{ negative} : M \text{ decreases} \\ M > 1\ ,\quad F \text{ positive} : M \text{ increases} \\ M < 1\ ,\quad F \text{ negative} : M \text{ increases} \\ M < 1\ ,\quad F \text{ positive} : M \text{ decreases} \end{array}\right\} \qquad (14)$$

It can therefore be seen that to obtain the desired effect, we need a negative body force upstream of the shock and a positive body force behind the shock. Since $F = \rho_e E$, this implies that E must change sign at the origin. (A sign change in the charge density, ρ_e, has obvious difficulties associated with it.) It is also desirable that ρ_e should be zero for upstream and downstream, reaching a maximum at the shock. These requirements can be met, in general, if charge is inserted into the flow ahead of the shock and withdrawn again behind it. In order to satisfy the condition that E changes sign at the origin, however, an external electric field over and above that due to the charge distributions usually has to be introduced as well.

With these considerations in mind, two electrostatic charge distributions have been selected for investigation. The first was chosen mainly for mathematical convenience, while the second was felt to be more satisfactory in a practical sense.

Case (a)

Consider a charge density distribution given by

$$\rho_e = \begin{cases} \frac{k_A}{L} \exp(-\xi^2) , & \xi < 0 , \\ \frac{k_B}{L} \exp(-\xi^2) , & \xi > 0 , \end{cases}$$

where k_A, k_B are to be chosen to satisfy continuity of charge across the shock. In nondimensional form

$$\bar{\rho}_e = \frac{\rho_e L}{\varepsilon E_1} = \frac{d\bar{E}}{d\xi} = \begin{cases} \frac{k_A}{\varepsilon E_1} \exp(-\xi^2) , & \xi < 0 , \\ \frac{k_B}{\varepsilon E_1} \exp(-\xi^2) , & \xi > 0 . \end{cases} \tag{15}$$

The electric field configurations associated with (15) are, for zero field at the shock,

$$\bar{E} = \frac{k_A \sqrt{\pi}}{2\varepsilon E_1} \operatorname{erf} \xi , \quad \xi < 0 , \tag{16}$$

$$\bar{E} = \frac{k_B \sqrt{\pi}}{2\varepsilon E_1} \operatorname{erf} \xi , \quad \xi > 0 . \tag{17}$$

In order to satisfy this zero field condition, an external electric field, $\bar{E}_E$, has to be introduced. Since the field at $\xi = 0$ due to the charge distribution (15) is given by

$$\int_0^\infty \frac{\bar{\rho}_e}{2} d\xi - \int_{-\infty}^0 \frac{\bar{\rho}_e}{2} d\xi = \frac{\sqrt{\pi}}{4\varepsilon E_1} (k_B - k_A) ,$$

the external field is

$$\bar{E}_E = \frac{\sqrt{\pi}}{4\varepsilon E_1} (k_B - k_A) . \tag{18}$$

The external field can be made zero by taking $k_B = k_A$, but this implies, in general, that charge is annihilated at the shock. In the limiting case of vanishing shock strength, however, $\bar{E}_E \to 0$.

For $\xi \to -\infty$, application of (16) gives

$$k_A = -\frac{2\varepsilon E_1}{\sqrt{\pi}} . \tag{19}$$

Across the shock the law of conservation of electric charge applies so that

$$\bar{\rho}_{eA}\,\bar{u}_A = \bar{\rho}_{eB}\,\bar{u}_B \quad .$$

At $\xi = 0$, $\bar{\rho}_{eA} = k_A/\varepsilon\, E_1$, $\bar{\rho}_{eB} = k_B/\varepsilon\, E_1$, whence

$$k_B = \frac{k_A\,\bar{u}_A}{\bar{u}_B} \quad . \tag{20}$$

The use of such a Gaussian charge distribution and associated error function field implies a continuous rate of change of charge flux, Z, where

$$Z = \operatorname{div}\,(\rho_e\,\underline{u}) = \frac{d}{dx}\,(\rho_e\,u) \quad .$$

In nondimensional form,

$$\bar{Z} = \frac{ZL}{\varepsilon\,E_1\,u_1} = \frac{d}{d\xi}\,(\bar{\rho}_e\,\bar{u}) \quad . \tag{21}$$

Since ρ_e is specified, Z has to be calculated from the solution for the velocity, which is derived from the non-linear differential equation (12). It would seem to be more reasonable from a physical point of view, although less convenient mathematically, to specify the rate of change of charge flux instead of the charge density itself. For this reason we analyse a second case:

Case (b)

In which charge is inserted into the flow at a constant rate Z_A between stations ξ_1 and ξ_2 upstream of the shock and withdrawn at a constant rate Z_B between stations ξ_3 and ξ_4 downstream of the shock. Hence,

$$\frac{d\bar{E}}{d\xi} = \bar{\rho}_e = \begin{cases} \bar{Z}_A\,(\xi - \xi_1)/\bar{u} & , \quad \xi_1 \le \xi \le \xi_2 \\ \bar{Z}_A\,(\xi_2 - \xi_1)/\bar{u} & , \quad \xi_2 < \xi < \xi_3 \\ \bar{Z}_B\,(\xi - \xi_3)/\bar{u} & , \quad \xi_3 \le \xi \le \xi_4 \end{cases} \quad . \tag{22}$$

In order that there shall be no residual charge in the flow for $\xi > \xi_4$, we must impose the condition,

$$\bar{Z}_B\,(\xi_4 - \xi_3) + \bar{Z}_A\,(\xi_2 - \xi_1) = 0 \quad . \tag{23}$$

It can be seen that, in general, $\bar{E}$ can no longer be given as a simple function of ξ and we now have to solve the non-linear simultaneous differential equations (12) and (22) to determine $\bar{u}$ and $\bar{E}$.

The external field needed to ensure that the field at the origin is zero, is given by

$$\bar{E}_E = -\tfrac{1}{2}\left\{ \int_{\xi_3}^{\xi_4} \frac{\bar{Z}_B\,(\xi - \xi_3)}{\bar{u}}\, d\xi + \int_0^{\xi_3} \frac{\bar{Z}_A\,(\xi_2 - \xi_1)}{\bar{u}}\, d\xi - \int_{\xi_2}^{0} \frac{\bar{Z}_A\,(\xi_2 - \xi_1)}{\bar{u}}\, d\xi - \int_{\xi_1}^{\xi_2} \frac{\bar{Z}_A\,(\xi - \xi_1)}{\bar{u}}\, d\xi \right\} \quad . \quad (24)$$

This can be calculated from the solution for $\bar{u}$ for different sets of conditions; it no longer vanishes, however, as the shock strength tends to zero.

3.3 Method of solution

Case (a)

The integration of equation (12), with the error function field distributions given by (16)-(19) was carried out on an ICL 1907 computer, using the Runge-Kutta method. The error function was assumed to reach its asymptotic values at $\xi = \pm 3$, $(\text{erf}\,(\pm 3) = \pm 0.99996)$.

Starting from the upstream value of $\bar{u} = 1$ at $\xi = -3$, the expression (16) for $\bar{E}$ was used and equation (12) integrated step-by-step until the shock at $\xi = 0$ was reached. At this point condition (13) was applied across the shock. The integration then proceeded from the value $\bar{u}_B$ given by (13) at $\xi = 0$ to $\xi = 3$, using the expression (17) for the electric field distribution.

The value of k_A (and hence, by (19), the upstream electric field, E_1) was chosen to be just large enough to make the discontinuity at $\xi = 0$ vanish when starting from an upstream Mach number of 2. This resulted in $k_A = 4.44 \times 10^{-4}$ coulomb/m^2 and $E_1 = -4.45 \times 10^7$ volt/m ($|E_1|$ should be compared with the breakdown value for air of 3×10^5 volt/m). Also, as initial values for (12), M_1 and λ_1 must be specified. It can easily be seen that, once E_1 has been selected, λ_1 is completely defined by the upstream pressure, p_1. This was taken to be 7.23×10^3 N/m^2, which is the value for air at 18300 m (60000 ft), a typical cruising height for a Mach 2 transport aircraft.

It follows that $\lambda_1 = \varepsilon E_1^2/2p_1 = 1.21$. If ρ_1 is also taken for air at the same altitude, we have $\rho_1 = 0.116$ kg/m^3 and u_1 may then be determined from the upstream Mach number, M_1.

Case (b)

Here the simultaneous differential equations (12) and (22) have to be integrated. For this the Runge-Kutta method is again suitable and the integration proceeded as in (a).

The stations $(\xi_1, \xi_2, \xi_3, \xi_4)$ were taken to be (-3, 0, 0, 3)* and the rate of entry of charge, $\bar{Z}_A$, was chosen for each different M_1, so that the electric field vanished at the shock. From this, equation (23) gives the corresponding rate of withdrawal of charge $\bar{Z}_B$ as $-\bar{Z}_A$. The values of λ_1 and ρ_1 were taken to be the same as in case (a).

The essential data for both cases is summarised in Table 1 below.

Table 1

Essential data

		$M_1 = 2$	$M_1 = 3$
Case (a)	$\bar{E}_E$	0	- 1.0
	k_A	4.44×10^{-4} coulomb/m^2	4.44×10^{-4} coulomb/m^2
	k_B	4.44×10^{-4} coulomb/m^2	15.3×10^{-4} coulomb/m^2
	u_1	591 m/sec	885 m/sec
Case (b)	$\bar{Z}_A$	- 0.172	- 0.206
	$\bar{Z}_B$	0.172	0.206
	$\bar{E}_E$	- 0.50	- 4.63
	u_1	591 m/sec	885 m/sec
Both cases	λ_1	= 1.21	
	E_1	= -4.45×10^7 volt/m	
	p_1	= 7.23×10^3 N/m^2	
	ρ_1	= 1.16×10^{-1} kg/m^3	
	γ	= 1.4	
	ε	= 8.85×10^{-12} farad/m	

*The case (-3, -1, 1, 3) was also considered, but the results were very similar to those for (-3, 0, 0, 3) and are not presented here.

At this stage, it is convenient to consider the length scale, L, associated with the system. This can be seen from equation (21) to depend on the rate at which charge can be inserted or withdrawn from the flow. For example, in case (b) for $M_1 = 2$, the actual value of $ZL = \varepsilon E_1 u_1 \bar{Z}$ used in the computation was 4×10^{-2} coulomb m^{-2} sec^{-1}. Now for an electrostatic generator of the Van de Graaf type[5], a typical value of Z is 4×10^{-5} coulomb m^{-3} sec^{-1}, which imposes a length scale, L, of order 10^3 m. A similar calculation for the other computed cases gives results of the same order. It requires little emphasis that the need for such an immense length scale only adds a further point to the difficulties associated with the aeronautical applications of electrogasdynamics.

3.4 Discussion of results

Both cases were computed for a series of upstream Mach numbers with $M_1 \geq 2$. The results for $M_1 = 2$ and 3 are shown in Figs.1 to 7, in which $\bar{\rho}_e$, $M_1\bar{E}$, $\bar{F} = \bar{\rho}_e \bar{E}$, $\bar{Z}$, M, $M_1\bar{u}$, $M_1\bar{p}$ are plotted against ξ. For reference, in Figs.5 to 7, the normal shock values without any electrostatic forces are also included. The profiles in Figs.1 to 7 are generally as expected. It can be seen that the Mach number, velocity and pressure profiles are smoothed out, completely for $M_1 = 2$ and partially for $M_1 = 3$, with the electric fields selected. The large downstream values of the electric field, body force and pressure can be accounted for by the sudden build up of the charge density owing to charge continuity through the shock. It can also be seen from Fig.4a that the mathematically convenient charge density of case (a) does not produce a very convenient profile for Z, the rate of change of charge flux.

It should be noted that the principle of total enthalpy conservation no longer operates when body forces are acting[3]. Therefore, even though no energy is, in fact, added to the flow in those cases with zero external electric field, the downstream conditions differ from those for a normal shock of the same strength without a body force. When an external electric field is introduced, work is done on the system and one expects the downstream conditions to be different.

A short investigation was also carried out into the effects of varying λ_1. For case (a) with the error function field distribution, it was decided to examine the relationship between the upstream Mach number, M_1, and the value of λ_1 (denoted by λ_1') which caused the shock to disappear. This was achieved by the integration of (12) starting from M_1 and varying λ_1

systematically until the Mach number at the shock was unity. Several values of M_1 between 1.25 and 10 were used. The surprising result of these computations is that there appears to be a simple relationship between λ_1' and M_1 of the form

$$\lambda_1' = \mu (M_1 - 1)^2$$

where $\mu = 1.26$. That such a relationship should hold is not immediately apparent from the form of equation (12); however, in view of the nature of the whole project, a more detailed examination was not thought to be justified.

4 CONCLUSIONS

An investigation has been made of how a normal shock might in principle be modified by electrostatic forces induced by spraying charge into a supersonic airstream and acting on it with an electric field. With a sufficiently high value of the electric charge density the discontinuity at the shock can be completely removed and the pressure and velocity charged smoothly over a chosen length scale. In practice, however, the electric field involved is likely to lead to voltages far greater than the breakdown voltage, so that the charge distribution would be unstable, and the length scale would be of the order of kilometres for known means of charge production. It therefore seems unlikely that electrostatic forces can be used to effect major flow changes, although their possible application in certain boundary layer situations, for example, is not necessarily excluded.*

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* Since the completion of this report, the author's attention has been drawn to a similar analysis by Cheng and Goldburg[6], in which the same conclusions as to the effectiveness of the concept are reached.

SYMBOLS

E	electric field
E_{BR}	breakdown electric field
E_E	external electric field
F	body force per unit mass
L	length scale
M	Mach number
Z	rate of change of electric charge density flux
h	specific enthalpy
k	constant associated with Gaussian charge distribution
p	pressure
u	velocity
x	flow coordinate
γ	ratio of specific heats
ε	permittivity of free space
λ_1	$\frac{\varepsilon E_1^2}{2p_1}$
μ	constant
ξ	nondimensional flow coordinate
ρ	density
ρ_e	charge density

Indices

$(\bar{\ })$	nondimensional value
$(\ )_1$	upstream value
$(\ )_A$	value ahead of the shock
$(\ )_B$	value behind the shock

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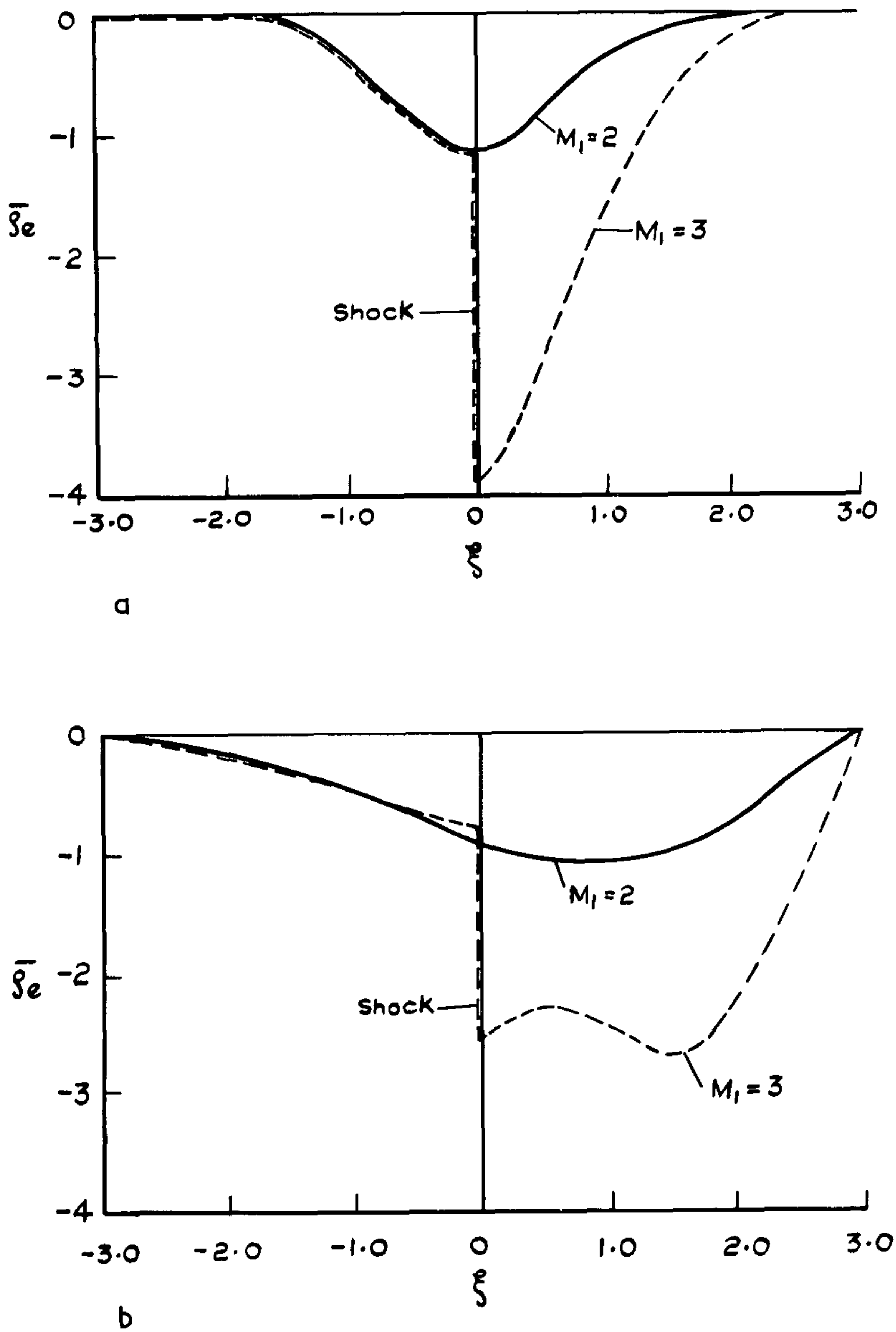


Fig. 1 a & b Variation of non-dimensional charge density, $\bar{\varrho}_e$ with ξ for cases (a) and (b)

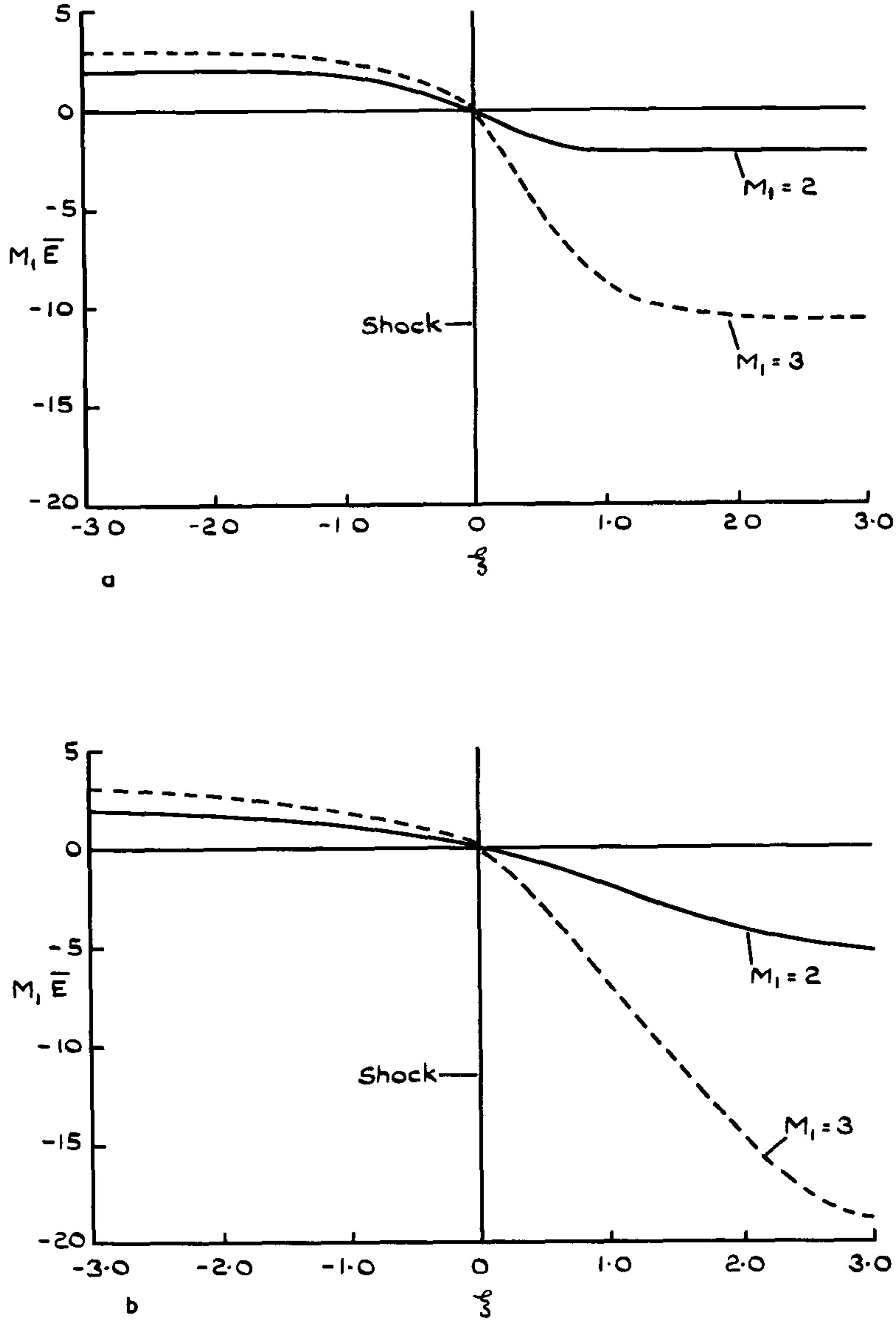
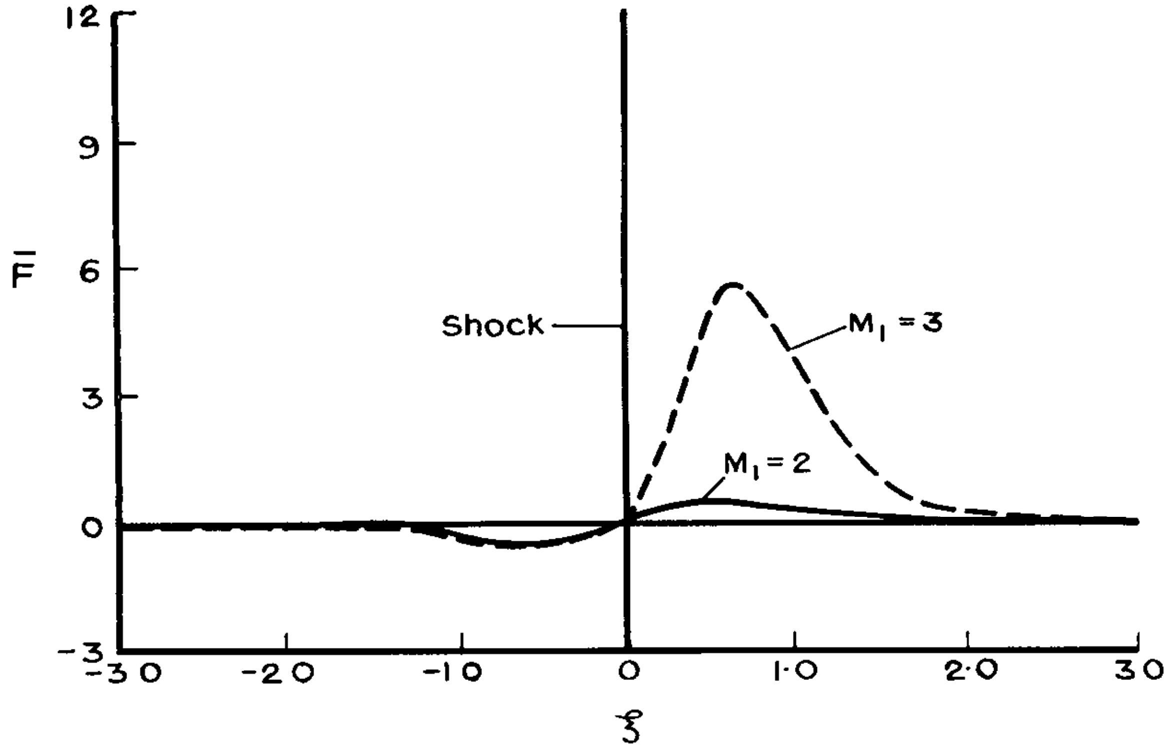


Fig. 2 a & b Variation of non-dimensional electric field, $M_1\bar{E}$, with ξ for cases (a) and (b)



a

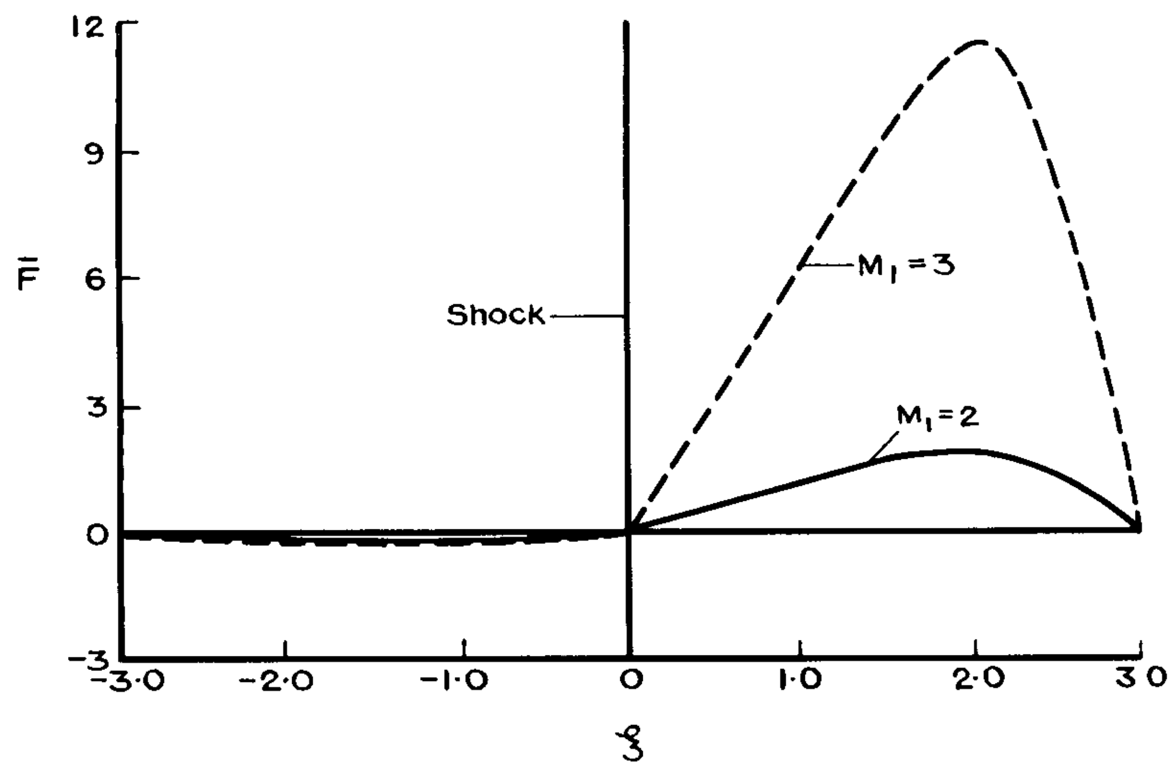


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Fig. 3a & b Variation of non-dimensional electrostatic force, $\bar{F} = \overline{\rho_e}\,\bar{E}$, with ξ for cases (a) and (b)

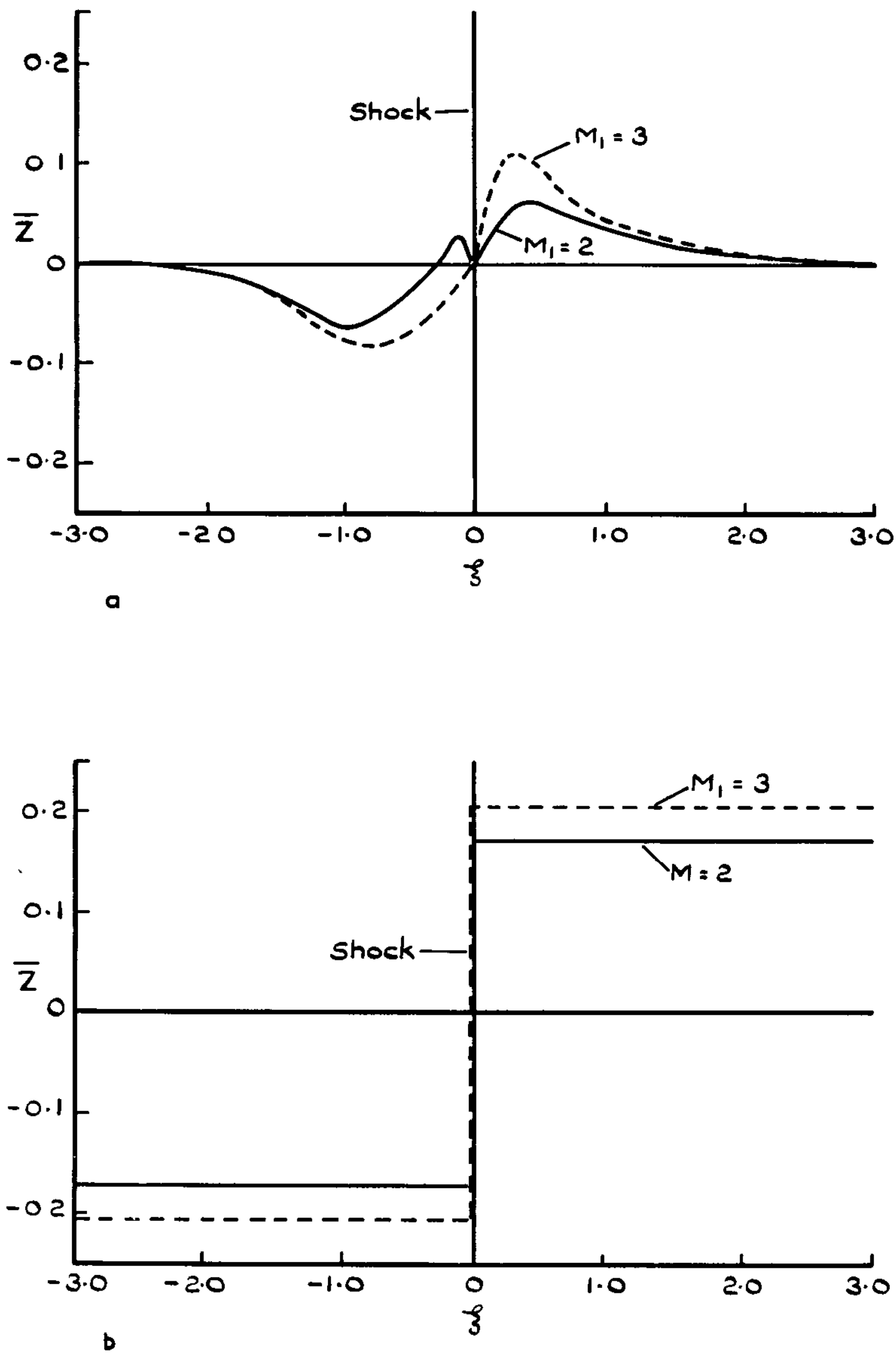
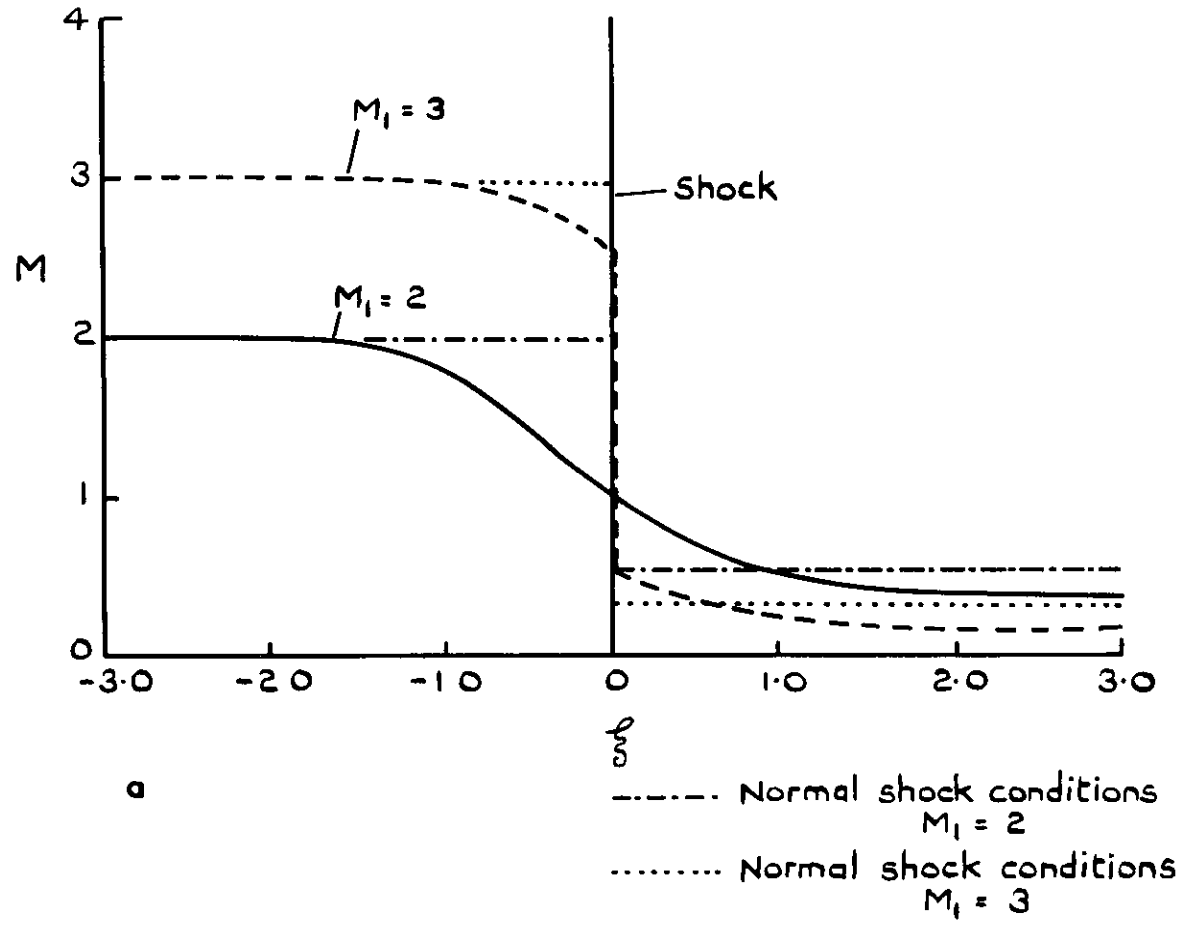


Fig 4 a & b Variation of $\overline{Z} = \frac{d}{d\xi}(\bar{\rho}_e \overline{U})$ with ξ for cases (a) and (b)



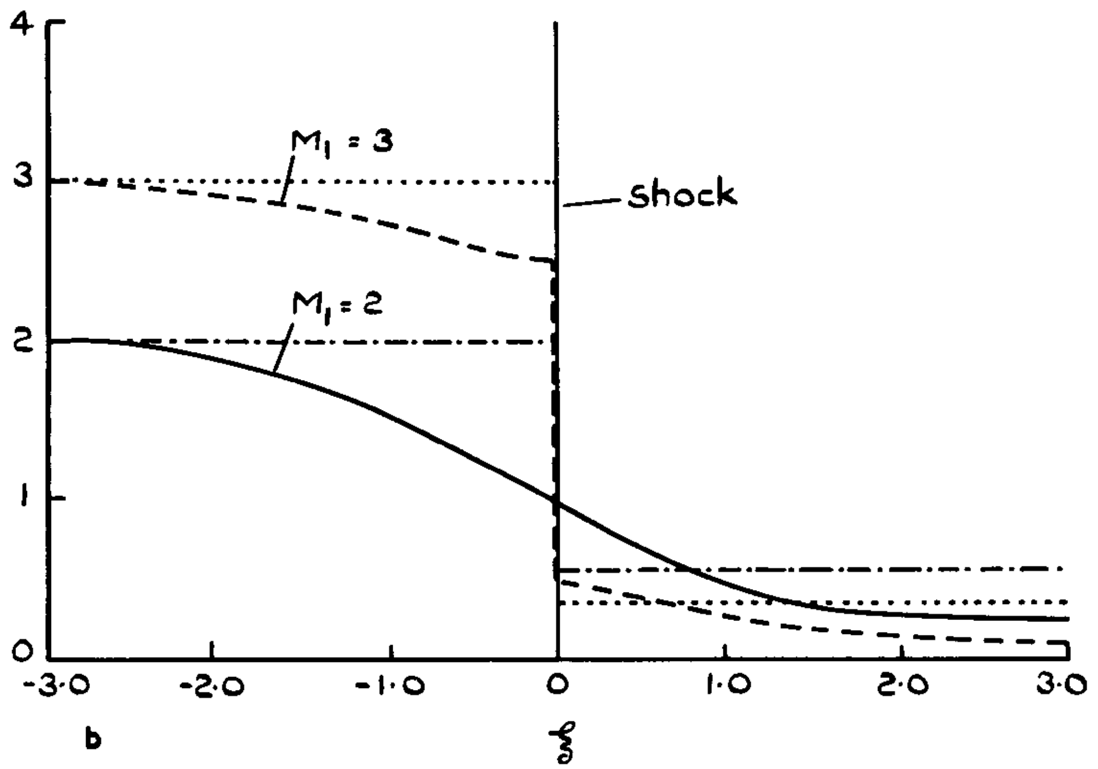


Fig. 5 a & b Variation of Mach number, M, with ξ for cases (a) and (b)

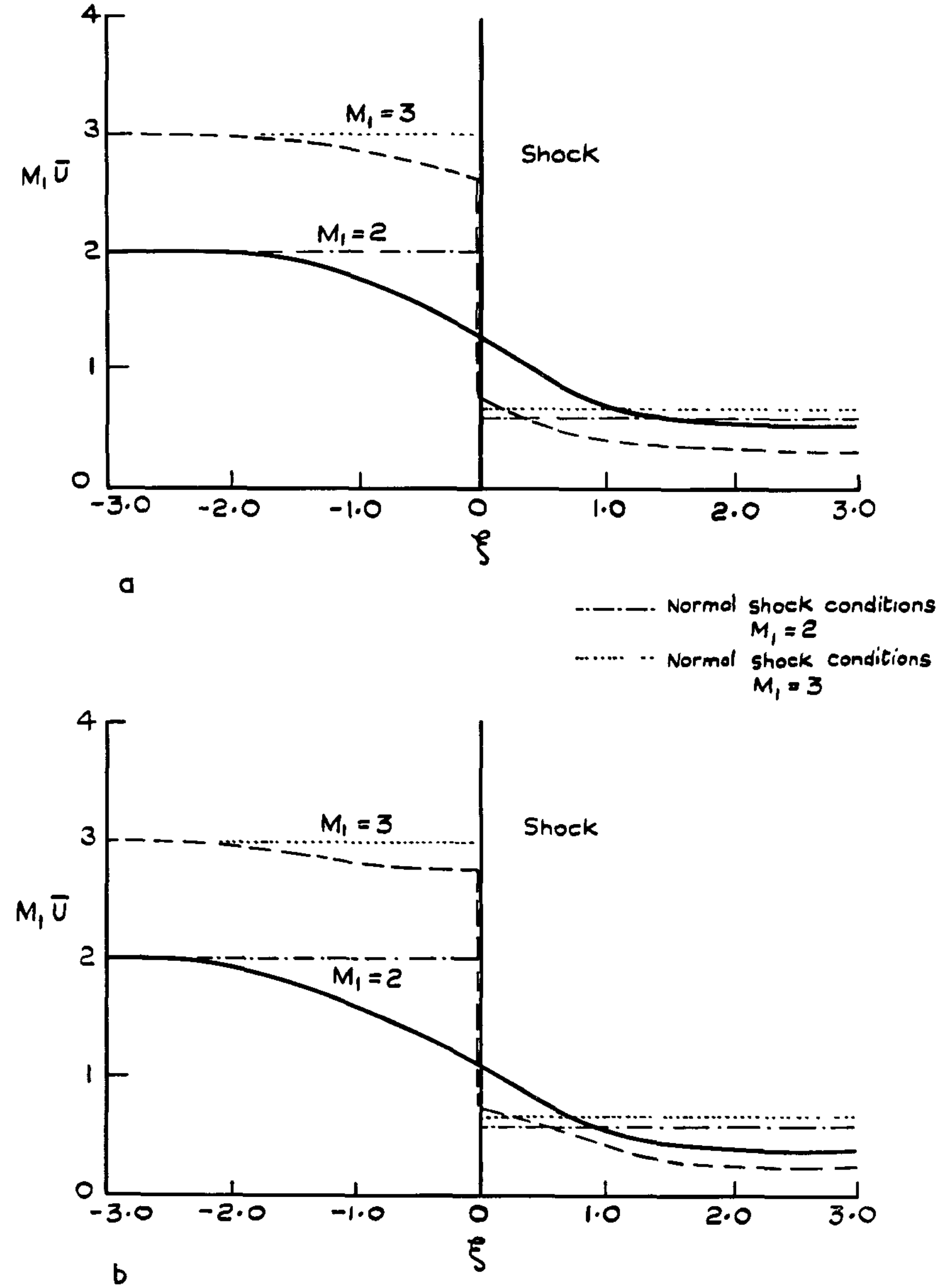
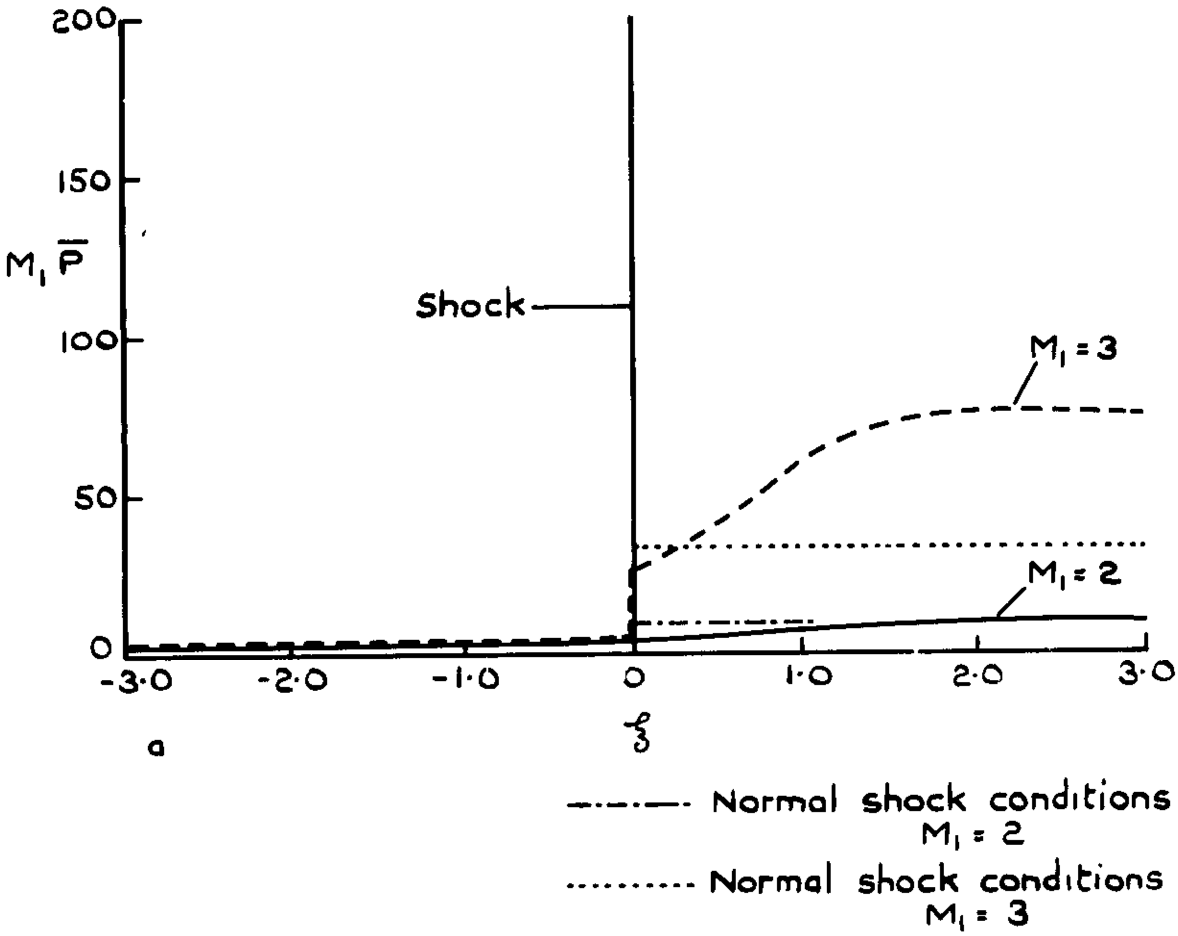


Fig. 6 a & b Variation of non-dimensional flow velocity, $M_1\bar{U}$, with ξ for cases (a) and (b)



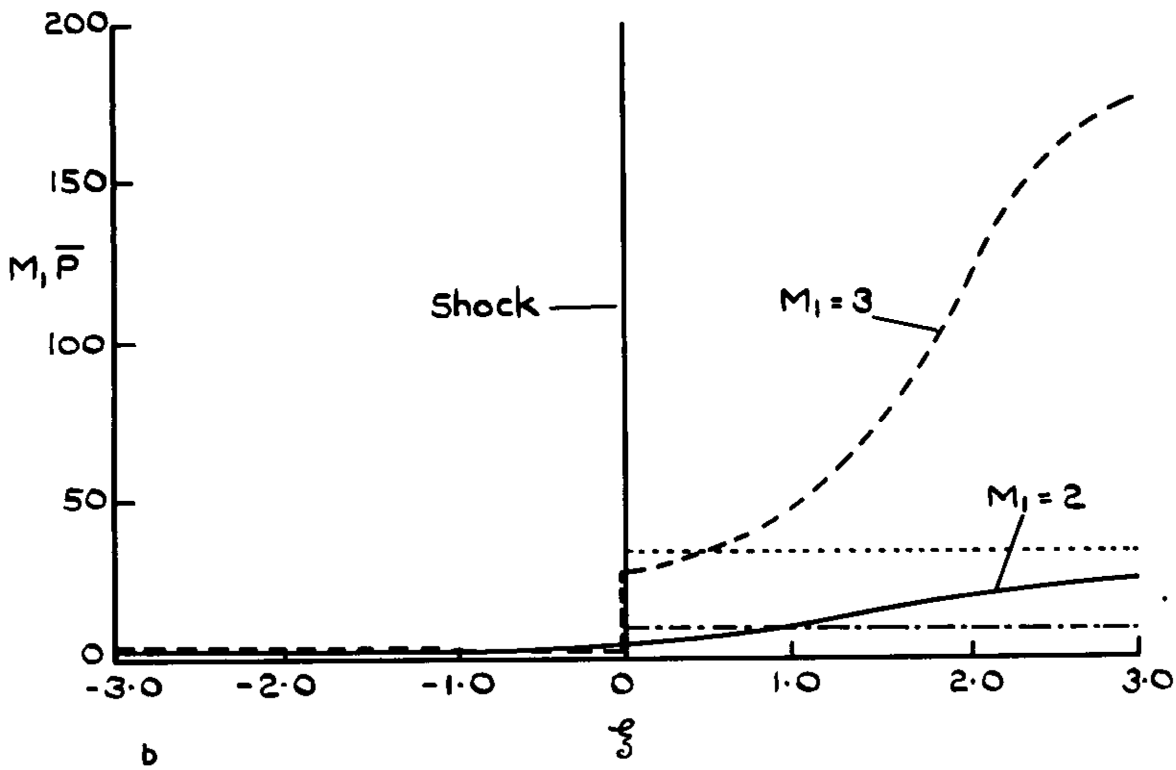


Fig 7 a & b Variation of non-dimensional pressure, $M_1\bar{P}$, with ξ for cases (a) and (b)

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