C P 91

MINISTRY OF SUPPLY

AERONAUTICAL RESEARCH COUNCIL

CURRENT PAPERS

An Assessment of the Probable Causes of Variation of the Speed Correction Coefficient of Aircraft Thermometers

By

D. D. Clark,

of the Meteorological Office.

LONDON · HER MAJESTY'S STATIONERY OFFICE

1952

FOUR SHILLINGS NET

C.P. No. 91.

M.R.P. 677

13th Sept., 1951

An assessment of the probable causes of variation of the speed correction coefficient of aircraft thermometers.

by

G. D. Clark. (Meteorological Office)

Summary

The variations in the speed correction coefficient α of aircraft thermometers reported in M.R.P. 527 may arise from any of the following causes in greater or less degree as described in the following table.

Possible sources of variations in α	Form of Variation + increase - decrease o no change with altitude	with speed	Importance
Variations in c_p, the specific heat of air (constant pressure)	+	o	Nil
Variations in Prandtl Number.	-	o	Nil
Variations in angle of attack (Thermometer on lower surface)	+	-	Important in bad sites
Ambient temperature error in indicator	+ or -	- or +	Important where it exists.
Variation in heat exchange in the thermometer (pitot-head only)	-	+	Most important
Transition from laminar to turbulent boundary layer	-	+	Most important

Notation.

u_o	air speed in cm/sec clear of aircraft.
V	air speed in knots.
u_1	air speed near the aerofoil where the thermometer is mounted, cm/sec,
u_2	air speed close to the thermometer itself, cm/sec.
u_m	mean speed inside the pitot-head thermometer $= 1/5\ U_o$.
T_o	undisturbed air temperature.
T_T	total or stagnation temperature of arrested air in °C.
T	temperature at the surface of an aerofoil.
T_w	temperature of the outside wall of the pitot-head thermometer.
T_c	temperature at any point of the platinum wire temperature. element in the pitot-head thermometer.
T_a	temperature of the air at any distance x inside the pitot-head tube.
$\overline{T}_c$	mean temperature of the platinum wire element.
$\overline{T}_a$	mean air temperature inside the pitot-head tube.
T_m	mean temperature of the air in a channel.

ΔT temperature rise of thermometer in °C.
ρ air density in gm/cm^3.
G mass of air entering the pitot-head thermometer per second.
b fore and aft width of a flat plate.
D diameter of inside of pitot-head tube in cm.
d diameter of circle of equivalent area to cross-section of temperature element and former, in cm.
a half width of a channel.
l effective length of inside of pitot-head tube = 3.7 cm.
α speed correction coefficient for °F. and knots, $\Delta T = \alpha(V/100)^2$ °F.
β speed correction coefficient for °C and cm/sec., $\beta = 0.2097 \times 10^{-7}\alpha$
λ defined as $\lambda = 2\ c_p\ \beta$
c_p specific heat of air at constant pressure in units stated.
k coefficient of thermal conductivity of air in units stated.
μ coefficient of viscosity of air in c.g.s. units.
ν kinematic viscosity = μ/ρ
σ Prandtl number = $\mu c_p/k$
R Reynolds number = xu/ν
Nu Nusselt number.
A_w rate of exchange of heat between air and wall inside pitot-head thermometer per unit length of tube.
A_c rate of exchange of heat between air and platinum wire inside pitot-head thermometer per unit length of tube.
k_c rate of loss of heat, per unit length of tube, from platinum wire by conduction through the frame.
C_f skin friction coefficient.
Q_o amount of heat transferred from unit area per second, ergs/sec.
k_H heat transfer coefficient defined in the text.

1. Introduction.

In R.R.P. 527 (1) Shellard has summarised the evidence indicating that a variation takes place in the speed correction coefficient of aircraft thermometers both with altitude and, in one particular case, with air speed. He also mentions some possible reasons for the variations.

In what follows an attempt has been made to list all factors which could possibly affect the speed correction coefficient and to examine each thoroughly in turn.

Air which is arrested in the boundary layer of an aerofoil experiences a rise in temperature due to the conversion of kinetic energy into heat. Some of the acquired heat will be lost by conduction through the boundary layer to the main flow but, at the same time, more heat is being created in the boundary layer by friction which offsets the losses by conduction. The heat balance between the loss by conduction and the gain by friction depends on the non-dimensional parameter, the Prandtl number, σ, defined as

$$\sigma = \frac{\mu\ c_p}{k}$$

If the Prandtl number is unity the two effects cancel out and the innermost layer, which is stationary relative to the aerofoil, acquires the stagnation adiabatic temperature rise. When air is the medium, however, $\sigma < 1$, and there is a net loss of heat by conduction; so that, in this case, the temperature rise at the surface is somewhat less.

For laminar flow, Pohlhausen (2), assuming constant density, has shown that the temperature rise at a flat plate, where there is no heat exchange at the surface, is expressible as

$$\Delta T = f(\sigma)\frac{u_o^2}{2\ c_p} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

where $f(\sigma)$ is a function of σ only, being equal to a close approximation to $\sigma^{1/2}$.

Applying equation (1) to the flow past an aerofoil, where T_1 is the temperature, and u_1 the air speed, near the surface, and T is the temperature of the surface

$$T - T_1 = f(\sigma) \frac{u_1^2}{2 c_p}$$

whence, since

$$T_1 + \frac{u_1^2}{2 c_p} = T_o + \frac{u_o^2}{2 c_p},$$

$$T - T_o = \Delta T = \frac{u_o^2}{2 c_p} \left[1 - \left(\frac{u_1}{u_o}\right)^2 \left\{1 - f(\sigma)\right\}\right]$$

Substituting $f(\sigma) = \sigma^{1/2}$ this formula becomes

$$\Delta T = \frac{u_o^2}{2 c_p} \left[1 - \left(\frac{u_1}{u_o}\right)^2 \left\{1 - \sigma^{1/2}\right\}\right] \quad \ldots\ldots\ldots\ldots(2)$$

Crocco (3) has shown that this formula is valid also for laminar flow in a compressible medium and therefore can be applied at all air speeds.

Dryden (1936) (4) has shown that, in the boundary layer, on an aerofoil, three states of flow can exist, depending, for any given fluid, on the pressure gradient, the state of the surface, the Reynolds number and the degree of turbulence in the air stream. At low Reynolds numbers the flow is laminar, as the Reynolds number increases however, an eddy flow appears which finally develops into turbulent flow. The region of eddy flow, usually referred to as the transition zone, is of complex structure and varies erratically with time.

A more recent article by Emmons (11) describes in more detail the mechanism of transition as revealed by recent experiments. The turbulence according to Emmons starts in spots where the induced oscillations in the laminar layer become unstable; these spots then move separately downstream growing and increasing in number at the same time until all the boundary layer is turbulent.

In problems where both laminar and turbulent flow are considered it is customary to neglect the length of the transition zone and postulate an instantaneous transition. Dryden (4) found that the point of transition was very sensitive to small pressure gradients.

The equivalent expression for the temperature rise for a turbulent boundary layer has been given by Squire (5) in the form

$$\Delta T = \frac{u_o^2}{2 c_p} \left[1 - \left(\frac{u_1}{u_o}\right)^2 \left\{1 - \sigma^{1/3}\right\}\right] \quad \ldots\ldots\ldots\ldots(3)$$

There is every reason to believe that this formula also holds at all speeds provided no marked pressure gradients are present.

Formulae (2) and (3) can be applied to describe the flow over a flat plate thermometer. If u_1 refers to the flow past the thermometer and u_2 to the flow in the immediate vicinity of the thermometer surface then the speed correction factor β, defined as $\Delta T = \beta u_o^2$ is obtained from (2) by replacing u_1 by u_2, and is

$$\beta = \frac{1}{2c_p} \left[1 - \left(\frac{u_2}{u_o}\right)^2 \left\{1 - \sigma^{1/2}\right\}\right] \quad \ldots\ldots\ldots\ldots(4)$$

$$= \frac{1}{2c_p} \left[1 - \left(\frac{u_2}{u_1}\right)^2 \left(\frac{u_1}{u_o}\right)^2 \left\{1 - \sigma^{1/2}\right\}\right] \quad \ldots\ldots(5)$$

for a laminar boundary layer

and from (3)

$$\beta = \frac{1}{2c_p} \left[1 - \left(\frac{u_2}{u_o}\right)^2 \left\{1 - \sigma^{1/3}\right\}\right] \quad \ldots\ldots\ldots(6)$$

$$= \frac{1}{2c_p} \left[1 - \left(\frac{u_2}{u_1}\right)^2 \left(\frac{u_1}{u_o}\right)^2 \left\{1 - \sigma^{1/3}\right\}\right] \quad \ldots\ldots(7)$$

for a turbulent boundary layer.

Neither (2) nor (3) however are adequate to describe the conditions inside a pitot-head thermometer where the temperature changes due to heat losses are significant.

The effect on β of variations in all the quantities on the R.H.S. of (5) and (7) will now be considered, including the effect of the change in the exponent of σ with transition in the boundary layer.

2. Variations in c_p.

Various theoretical and empirical formulae for c_p have been given from time to time, all of which differ from each other. The following two selections quoted by Partington (6) show the pressure dependence of c_p

$$c_p = 0.23702 + 0.0015504\,(p - 1) \quad \text{(Lussana)}$$

$$c_p = 0.2414 + 0.000286\,p \quad \text{(Holborn and Jakob)}$$

where c_p is measured in calories per gram and p in atmospheres.

The changes in c_p with pressure indicated by these formulae are too small to affect our results. It will therefore be assumed that c_p for air does not vary with pressure changes in the atmosphere.

Partington (6) also gives a selection of formulae showing the temperature dependence of the molecular heat per mol at constant pressure for various gases, including oxygen and nitrogen. From these formulae an expression in terms of temperature for the specific heat of air at constant pressure has been calculated, assuming an oxygen to nitrogen ratio by weight of 1 : 3, and ignoring the presence of the rarer atmospheric gases.

The formulae for C_p, the molecular heat at constant pressure, for oxygen and nitrogen, are given in the following form

$$C_p = a + bT + cT^2$$

C_p being measured in calories per mol, and T in °K.

The following set of values for a, b and c, quoted by Partington, are due to Spencer and Justine (1934).

	a	b x 10^3	c x 10^7
O_2	6.0954	3.2533	- 10.171
N_2	6.4492	1.4125	- 0.807

and the following to Bryant and Taylor (1934)

	a	b x 10^3	c x 10^7
O_2	6.25	2.746	- 7.70
N_2	6.30	1.819	- 3.45

The corresponding values of a, b and c for the specific heat of air at constant pressure in calories per gram per °C. have been worked out from these formulae and are given below

Air	a	b x 10^3	c x 10^7
S and J	0.22037	0.06325	- 0.10108
B and T	0.21765	0.07018	- 0.15257
mean	0.2190	0.0667	- 0.1268

The values of c_p for air using the mean values deduced above for a, b and c have been calculated for four separate temperatures, three of which correspond in the I.C.A.N. scale to the pressure levels 900, 500 and 300 mb. and one in the tentative N.A.C.A. scale (7) to the isothermal stratosphere up to 32 km. These values of c_p are given in column 6 of Table I both in c.g.s. units and in calories. They show that the variation in c_p alone is too small too affect β appreciably and that in any case its variation is opposed to that required to account for the observed α variation.

3. Variations in the Prandtl Number.

The Prandtl number is defined as

$$\sigma = \frac{\mu c_p}{k}$$

For air, $\sigma < 1$, and variations in σ could cause variations in the speed correction factor. We have just seen that small variations with temperature occur in c_p, it remains to be seen therefore what happens to the ratio μ/k at low temperatures.

On the assumption of equi-partition of energy in molecular collisions, μ/k can be expressed, to a second approximation, as a linear function of γ , - the ratio of the two specific heats - considered constant over the temperature range in question. In order to test the validity of the assumption of equi-partition of energy in molecular collisions for the lowest temperatures reached, the ratio μ/k has been calculated from independently determined experimental values of both μ and k. The values used for μ are those obtained by Johnston and McCloskey (1940) (8) in America, and for k, those by Taylor and Johnston (1946) (9), also in America. Both sets of values are claimed to be accurate to $\frac{1}{2}$°/$_o$. Only the variations with temperature are considered as the change with pressure in both μ and k is many times too small to have any effect.

The experimental values of μ and k, interpolated from the tables from the sources quoted, are given in columns 3 and 4 of Table I, and the calculated ratio μ/k is given in column 5. The values of σ, $\sigma^{1/2}$ and $\sigma^{1/3}$ formed from these values of μ/k together with the previously calculated values of c_p, appear in columns 7, 8 and 9. The values of $\sigma^{1/2}$ and $\sigma^{1/3}$ at the different altitudes scarcely change, but the differences between $\sigma^{1/2}$ and $\sigma^{1/3}$ at any one level are significant. This point will be discussed later.

4. Variations in (u_1/u_o) and the significance of u_2/u_1.

The ratio u_1/u_o at any part of an aerofoil does not change with the Reynolds number but does change with the inclination of the aerofoil to the airstream. At the distance from the surface at which the thermometer is situated, however, only a small part of the total variation in u_1 is felt. For an under-wing or under-fuselage mounting, u_1/u_o decreases as the angle of incidence of the aerofoil increases, which implies that any variation in β from this cause would consist of an increase with altitude and a decrease with speed, which is the opposite of that reported.

If a variation from this cause were present, then mounting the thermometer on an extension clear of the aircraft ought to remove it.

This may explain why the α measured for the under-nose mounting on aircraft ST 796 (Table III of M.R.P. 527, reproduced as Table II) remains constant at different altitudes whilst, on the same aircraft, a thermometer on an extension platform under the nose gives a decrease in α with altitude. It is difficult to see, however, why the same effect should not be present also on the other Halifax ST 817 for a similar under-nose mounting.

Since variations in u_1/u_o affect the flat-plate thermometer, but not the pitot-head thermometer, it is worth noting that the pitot-head thermometer on Halifax ST 796, mounted under the nose, indicates a decrease in α with altitude, whereas, as reported, the flat-place thermometer, under the nose, has no α variation.

The ratio u_2/u_1 for any given thermometer should remain constant, on the average, and will depend on the thermometer thickness and the shape of the leading edge. Although the ratio u_2/u_1 is not, in itself, a cause of a variation in α, a high value of u_2/u_1 accentuates changes due to transition and augments any variation already present from this cause, as is shown in the increase in the disparity between the entries against pressure for laminar and turbulent boundary layer conditions, of the value of β for $(u_2/u_1)^2 = 2$ as against $(u_2/u_1)^2 = 1$, (assuming $u_1 = u_0$) given in Table I.

5. Changes in boundary layer flow.

We can regard the flat plate thermometer as a thin section aerofoil with parallel sides. If u_2 is the air speed at any point near the surface of the thermometer and u_1 the air speed past the thermometer then u_2/u_1 is constant except at the edges. Since u_2 is small at the edges the rate of exchange of heat there, with the air, is also small, which means that the edges will contribute only slightly to the mean temperature of the thermometer surface.

Since the Reynolds number decreases with height, transition from laminar to turbulent flow, if present, should occur during descent. It could also occur at any level, with an increase of speed, provided the critical Reynolds number is passed.

Transition tends to occur behind regions of minimum pressure. On the flat-plate thermometer such regions would occur behind the leading edge and also behind any corrugation on the sides, since the sides could not be expected to be completely flat and smooth. Transition therefore will not be a gradual process but can be expected to proceed in stages as one favourable region after another becomes operative, until the whole surface behind the leading edge has a turbulent boundary layer.

Applying these theories to the data from the Mosquito flights plotted in curve 1 of Figure 1 (reproduced from M.R.P. 527) we obtain the following results.

Value of α at 300 kts = 1.82
corresponding value of λ at 300 mb. = 0.75
Value of α at 200 kts = 1.55
corresponding value of λ at 300 mb. = 0.64

If we assume that at the lowest speeds the boundary layer is wholly laminar, then, from equation 5, since

$$\lambda = 2 c_p \beta$$

$$\lambda = 1 - \left(\frac{u_2}{u_1}\right)^2 (1 - \sigma^{1/2})$$

Putting $\lambda = 0.64$, $c_p = 0.9772 \times 10^7$ c.g.s. units, $\sigma^{1/2} = 0.8443$. (the two latter from Table I)

$$\therefore \left(\frac{u_2}{u_1}\right)^2 = 2.312 \; ; \qquad \frac{u_2}{u_1} = 1.521$$

If now the proportion of the total surface area at any time under a laminar boundary layer be 'a_1' and under a turbulent boundary layer be 'a_2' ($a_1 + a_2 = 1$) then for the whole thermometer

$$\lambda = a_1\lambda_1 + a_2\lambda_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

where λ_1 and λ_2 refer to laminar and turbulent boundary layers respectively*.

From (5) and (7) remembering that $\lambda = \beta \times 2c_p$ and that $a_1 + a_2 = 1$

*Equation (8) is only strictly true if the surface is a heat insulator but for the small temperature differences between surface and boundary layer which do arise from thermal conduction along the surface, the effects of the differential rates of heat exchange, as between laminar and turbulent portions, may be neglected.

$$\lambda = 1 + \left(\frac{u_2}{u_1}\right)^2 \left[a_1\sigma^{1/2} + (1 - a_1)\sigma^{1/5} - 1\right]$$

whence

$$a_1 = \frac{1 - \lambda - \left(\frac{u_2}{u_1}\right)^2 (1 - \sigma^{1/2})}{\left(\frac{u_2}{u_1}\right)^2 (\sigma^{1/5} - \sigma^{1/2})} \quad \ldots\ldots\ldots\ldots\ldots\ldots(9)$$

Putting $\lambda = 0.75$, and from Table I, $\sigma^{1/2} = 0.8486$ and $\sigma^{1/5} = 0.8962$ and using the value of $(u_2/u_1)^2$ just found, we obtain $a_1 = 0.091$.

In this case, therefore, the reported variation in α with air speed could be explained if almost complete transition to turbulence took place between the least and greatest air speeds. The corresponding Reynolds numbers u_2b/ν where b = 2 cm. are

$$9.4 \times 10^4 \quad \text{and } 1.4 \times 10^5.$$

It is interesting to note that the lowest critical Reynolds number at which Dryden (4) found transition to take place on a flat plate was 9×10^4. Curve 2 of Fig.1 shows no variation with speed, although the corresponding values in Table II show a variation with altitude. Let us assume therefore, that, for this particular thermometer, the flow has remained laminar throughout the speed range at 500 mb.

The mean value of α taken from curve 2 is 1.64, corresponding to $\lambda = 0.68$ at 500 mb. At 900 mb, from Table II, $\alpha = 1.78$, i.e. $\lambda = 0.74$.

Assuming laminar flow at 500 mb, from equation (5) we obtain

$$\left(\frac{u_2}{u_1}\right)^2 = 2.084; \qquad \frac{u_2}{u_1} = 1.444$$

and from equation (9)

$$a_1 = 0.43$$

which means that, at 900 mb., transition would have to occur over a little more than half the thermometer surface to explain the observed change in α with height. The highest Reynolds number u_2b/ν at 500 mb., corresponding to speed 236 knots, is 1.5×10^5, and the lowest Reynolds number at 900 mb. is 1.8×10^5. The conclusion is therefore that, either transition occurs later on this thermometer than on the one on the Mosquito, or else, that a counter-acting tendency is present, caused, say, by variations in u_1/u_0, as has been discussed above.

A similar treatment can be applied to the other observations in Table II which show variations with altitude of the meaned values with respect to airspeed of α at the different pressure levels.

In each case if we assume a laminar boundary layer at the lowest Reynolds numbers and transition or partial transition to a turbulent boundary layer at the highest Reynolds numbers, then all the observations in the tables can be accounted for except those for the under-nose mounting on Halifax No. ST 817 (Table II). In this case, proceeding on the same lines as before, but this time assuming turbulent flow at 900 mb, we get $\alpha = 2.05$, i.e. $\lambda = 0.85$, at 900 mb.

$$\therefore \left(\frac{u_2}{u_1}\right)^2 = 1.442$$

whence the corresponding value of α, assuming a purely laminar boundary layer at 500 mb, should be 1.93, whereas, in fact, it is 1.70. In this case therefore none of the reasons so far discussed can account for all of the variation. The possibility that instrumental inaccuracies may affect the answer will now be discussed.

6. Variations arising within the thermometer itself.

Apparent variations in the speed correction factor could have the following instrumental origins:-

(i) Changes in the thermometer temperature-indicator reading with changes in ambient temperature not allowed for in the calibration.

(ii) Changes of heat exchange efficiency inside the thermometer with changes in Reynolds number. Applicable only to the pitot-head thermometer.

Consider the first possibility, namely the existence of errors arising from ambient temperature effects. In the type of indicator used with the thermometers on which the α variations were detected, the specification allows a maximum variation of ±0.4°F. in reading for an ambient temperature range from +120°F to 0°F. Although it is unlikely that the ambient temperature varied by as much as 120°F during these flights, nevertheless, as a test case, we could assume that at the lowest temperature which the indicators experienced, the readings did suffer a + or - 0.4°F error, not allowed for in the calibration done at ground temperature. An examination of the laboratory test reports of this type of indicator showed that, whereas the majority acquire a positive error at the lowest ambient temperature, there are some which show a negative error: to explain a decrease in α with height a negative error would be necessary.

In fact if

ΔT_1 is the true temperature rise

ΔT_2 is the indicated temperature rise

then the corresponding speed correction factors α_1 and α_2 are given by

$$\Delta T_1 = \alpha_1 \left(\frac{V}{100}\right)^2$$

$$\text{and} \qquad \Delta T_2 = \alpha_2 \left(\frac{V}{100}\right)^2$$

$$\text{But if} \qquad \Delta T_1 = \Delta T_2 + 0.4$$

$$\text{then} \qquad \alpha_1 - \alpha_2 = 0.4 \left(\frac{100}{V}\right)^2$$

Since α_1 is the same at all levels $\alpha_1 - \alpha_2$ represents the variation in α. If V = 200 kts, $\alpha_1 - \alpha_2 = 0.1$.

Although it is unlikely that the permitted maximum error is ever present, except perhaps in a few cases, it is seen that, at speeds of 200 kts, its presence as a negative error would be sufficient to account for a large part of any reported decrease of α with altitude, but that, with increasing speeds, its influence would diminish.

The presence of an ambient temperature error can also cause an apparent variation of α with speed.

Let α_1, ΔT_1 and α_2, ΔT_2 be the values of α and ΔT at the two speeds V_1 and V_2 respectively, and assume again that the error is negative, then we have -

$$\left.\begin{aligned} \Delta T_1 &= \alpha_1 \left(\frac{V_1}{100}\right)^2 \\ \Delta T_2 &= \alpha_2 \left(\frac{V_2}{100}\right)^2 \end{aligned}\right\}$$

Let accents denote true values of ΔT_1 and ΔT_2

$$\therefore \quad \left.\begin{aligned} \Delta T_1' - 0.4 &= \alpha_1 \left(\frac{V_1}{100}\right)^2 \\ \Delta T_2' - 0.4 &= \alpha_2 \left(\frac{V_2}{100}\right)^2 \end{aligned}\right\}$$

i.e.

$$\left.\begin{aligned} a\left(\frac{V_1}{100}\right)^2 - 0.4 &= a_1\left(\frac{V_1}{100}\right)^2 \\ a\left(\frac{V_2}{100}\right)^2 - 0.4 &= a_2\left(\frac{V_2}{100}\right)^2 \end{aligned}\right\}$$

$$\therefore \quad a_2 - a_1 = 0.4\left[\left(\frac{V_2}{100}\right)^2 - \left(\frac{V_1}{100}\right)^2\right]\left(\frac{100}{V_1}\right)^2\left(\frac{100}{V_2}\right)^2$$

$$= 0.4\left[\left(\frac{100}{V_1}\right)^2 - \left(\frac{100}{V_2}\right)^2\right]$$

e.g. if V_1 = 200 kts, V_2 = 300 kts

$$a_2 - a_1 = 0.4\ (5)\ \frac{1}{9 \times 4}$$

$$= 0.06$$

The smaller V_1 the greater the variation in a will be. However, over the speed range of the Halifax flights the effect would be very small, but a significant variation would arise over the speed range of the Mosquito. The variation can only occur when the ambient temperature is least and can have any sign depending on the sign of the indicator error.

The second possibility, namely the variations in efficiency of thermal exchange, only applies to the pitot-head thermometer, because the air flow past the flat plate is sufficiently great to dominate the temperature indicated, whereas, in the interior of a pitot-head, the arrested air is surrounded by potential thermal exchangers, and the temperature which the sensitive element acquires depends on the balance of thermal exchange finally reached.

7. The Pitot-head thermometer.

The principle of the pitot-head thermometer is to measure the temperature of air which has been arrested in the interior of a pitot-tube or similar device. Because the arrested air is at a higher temperature than its surroundings, great precautions have to be taken to prevent heat losses from the air before its temperature can be measured, and at the same time, an efficient thermal exchange with the sensitive temperature element must exist. Lack of attention to these details will result in inaccurate measurements.

The pitot-tube thermometer used in the test described in M.R.P. 527 was the resistance bulb, impact type IT 3-1 as shown in Fig. 2. In the prototype of this thermometer, the body of the tube was made of a thermally insulating material, but, in the production models such as were used, the material was brass. The temperature element was platinum wire (SWG 47) wound on a star-sectioned former of synthetic resin fabric, which was supported on a metal rod projecting from the base of the tube.

In a design of this character the following avenues of heat exchange will have to be considered:-

(i) Losses from the arrested air to the metal walls of the tube

(ii) Losses from the platinum wire by conduction through the synthetic bonded-resin former to the metal supporting rod.

Radiation losses are very small and have been neglected.

The equations of thermal balance are therefore

$$Gc_p(T_T - T_a) = \int_0^x A_e(T_a - T_e)dx + \int_0^x A_w(T_a - T_w)dx \quad ....(10)$$

$$A_e(T_a - T_e) = k_e(T_e - T_w) \quad ....(11)$$

The boundary conditions are $T_a = T_T$ at $x = 0$ and the solution to equations (10) and (11) is

$$T_T - T_a = (T_T - T_W)(1 - e^{-\eta x})$$

where

$$\eta = \frac{k_c A_c + k_c A_W + A_c A_W}{G c_p (k_c + A_c)}$$

If $\overline{T}_a$ represents the mean value of T_a over the whole length (l) of the inside of the tube then

$$T_T - \overline{T}_a = (T_T - T_W)\left[1 - \frac{1 - e^{-\eta l}}{\eta l}\right] \quad \text{..........(12)}$$

whence

$$T_T - \overline{T}_c = (T_T - T_W)\left[1 - \frac{A_c}{k_c + A_c}\left(\frac{1 - e^{-\eta l}}{\eta l}\right)\right] \quad \text{...(13)}$$

where $\overline{T}_c$ is the average temperature of the platinum wire.
Now

$$T_T - \overline{T}_c = \frac{(1 - \lambda_c)}{2c_p} u^2 \quad \text{and} \quad T_T - T_W = \frac{(1 - \lambda_W)}{2c_p} u^2$$

$$\therefore \quad 1 - \lambda_c = (1 - \lambda_W)\left[1 - \frac{A_c}{k_c + A_c}\left(\frac{1 - e^{-\eta l}}{\eta l}\right)\right] \quad \text{......(14)}$$

where λ_c refers to the thermometer as a whole and
λ_W refers only to the outside wall.

To apply this formula to the thermometer in question, values for A_W, A_c and k_c were calculated.
To find A_W we proceed as follows

Define $\quad k_H = \dfrac{Q_c}{\rho c_p u_m (T_m - T_W)}$

whence $\quad A_W = k_H \rho c_p u_m \pi D$

k_H is the heat transfer coefficient for flow in a channel. T_m is the mean temperature in the channel which in this case is $\overline{T}_a$.
In order to express k_H in terms of known quantities, Kármán's generalisation of the formulae based on Reynolds' analogy for heat transfer and skin friction was used, as given by Goldstein (10) p. 657, namely -

$$\frac{1}{k_H} = \frac{2}{c_f} + 5\sqrt{\frac{2}{c_f}}\left[\overline{\sigma - 1} + \log_e\left[1 + 0.83(\sigma - 1)\right]\right] \quad \text{...(15)}$$

Putting $\sqrt{2/c_f} = 5.1\, R^{1/8}$, where $R = 2au_m/\nu$, which is the value which Prandtl found for flow in channels, (15) becomes

$$\frac{1}{k_H} = 5.1\, R^{1/8}\left[5.1\, R^{1/8} + 5\left[\overline{\sigma - 1} + \log_e(1 + 0.83\, \overline{\sigma - 1})\right]\right] \quad \text{..}$$

Now D = 1.2 and 'a' = 0.1

$\therefore$ at 900 mb. $\quad R^{1/8} = 2.663, \quad \dfrac{1}{k_H} = 183.25$

$$\underline{A_W = 4.54 \times 10^5}$$

$\therefore$ at 500 mb. $\quad R^{1/8} = 2.536, \quad \frac{1}{k_H} = 166.1$

$$\underline{A_w = 3.09 \times 10^5}$$

To evaluate A_e we use the relationship

$$A_e = Nu\, k\, \pi\, n\, r$$

where n is the number of turns of wire per centimetre length of tube and r is the length of one turn in cm. Given the fundamental interval of the resistance element is 40 ohm and the S.W.G. = 47

$$\therefore n = 19.2$$

Also $\quad r = 2\sqrt{2}$

Using Hilpert's values of Nusselt number against Reynolds number for a circular cylinder in transverse flow given on p.637 of ref. 10 we obtain,

at 900 mb. $\quad R = 64.3, \quad Nu = 0.50$

$$\therefore \underline{A_e = 2.066 \times 10^5}$$

at 500 mb. $\quad R = 43.5, \quad Nu = 0.18$

$$\therefore \underline{A_e = 0.682 \times 10^5}$$

For k_c only tentative values can be found based on plausible assumptions; for example making the following assumptions.

The wire is in contact with the support for 0.3 cm. of its length at each of the four points and that it is in contact for half its circumference. The synthetic resin former was assumed to conduct like a rectangular parallelepiped whose base is the projection of the area of contact with the wire and whose depth to the metal rod is 0.3 cm. The coefficient of conductivity of the material of the former was assumed to be 10^{-3} cal. per °C. across a cm^3. On the basis of these assumptions, and, remembering that there are four points of contact and 19.2 turns per cm.

$$k_c = 1.66 \times 10^4$$

Substituting these values for A_w, A_e and k_c and $l = 3.7$ in (14) we get

At 900 mb. $\quad 1 - \lambda_e = 0.1315(1 - \lambda_w)$

$$\text{i.e.} \quad \lambda_e = 0.8685 + 0.1315\, \lambda_w \quad \ldots\ldots\ldots\ldots(17)$$

and at 500 mb. $\quad 1 - \lambda_e = 0.2501(1 - \lambda_w)$

$$\text{i.e.} \quad \lambda_e = 0.7499 + 0.2501\, \lambda_w \quad \ldots\ldots\ldots\ldots(18)$$

Unfortunately the values of α found for the pitot-head thermometer as given in M.R.P. 527 are suspect because the greatest value, that of 2.52 for aircraft ST 796 at 900 mb., is greater than the maximum theoretical value, assuming no heat losses, which at 900 mb. has the value 2.407, corresponding with $\lambda = 1$. The same is also true of the value formed from the entries of $\alpha_{ph} - \alpha_{fp}$ and of α_{fp} for the same aircraft, ST 796.

e.g. at 900 mb. $\quad \alpha_{ph} - \alpha_{fp} = 0.71$

$$\alpha_{fp} = 1.78$$

$$\therefore \quad \alpha_{ph} = 2.49$$

No values at all are given for ST 817 at 900 mb.

If it is assumed that no change takes place in λ_w then from (17) and (18)

$$\left(\frac{\lambda_c}{2c_p}\right)_{900} - \left(\frac{\lambda_c}{2c_p}\right)_{500} = 0.0571\ 10^{-7} - 0.0708\ 10^{-7}\ \lambda_w$$

putting $\lambda_w = \sigma^{1/2} = 0.85$

$$\therefore \quad \left(\frac{\lambda_c}{2c_p}\right)_{900} - \left(\frac{\lambda_c}{2c_p}\right)_{500} = 0.0054\ 10^{-7}$$

$$\text{i.e.} \quad \alpha_{c_{900}} - \alpha_{c_{500}} = 0.03$$

If on the other hand λ_w also suffers a decrease with altitude similar to the flat plate the variation of α would be correspondingly increased.

In fact if we assume that the maximum change corresponding to complete transition takes place, then, if $\lambda_{w_{900}}$ and $\lambda_{w_{500}}$ represent the values of λ_w at 900 and 500 mb. respectively,

$$\lambda_{w_{900}} = 1 - \left(\frac{u_2}{u_1}\right)^2 (1 - \sigma^{1/2})$$

$$= 1 - \left(\frac{u_2}{u_1}\right)^2 (0.1038)$$

$$\lambda_{w_{500}} = 1 - \left(\frac{u_2}{u_1}\right)^2 (1 - \sigma^{1/2})$$

$$= 1 - \left(\frac{u_2}{u_1}\right)^2 (0.1535)$$

$$\therefore \quad \lambda_{w_{900}} = 0.3238 + 0.6762\ \lambda_{w_{500}}$$

whence from (17) and (18) after converting the λs to αs

$$\alpha_{c_{900}} - \alpha_{c_{500}} = 1.551 - 0.646\ \alpha_{c_{500}}$$

If we put $\alpha_{c_{500}} = 2.22$ (the average value from Table II)

$$\alpha_{c_{900}} - \alpha_{c_{500}} = 0.12$$

which would make $\alpha_{c_{900}} = 2.34$.

The only value given for $\alpha_{c_{900}}$ is 2.52 which, as already stated, must be suspect.

To test the effect of other values of the conductivity loss k_c, $\alpha_{c_{900}} - \alpha_{c_{500}}$ has been calculated also for $k_c = 10^3$, 5×10^4, 10^5 for $\alpha_{c_{500}} = 2.22$ and the results given below.

When $k_c = 10^3$, $\quad \alpha_{c_{900}} - \alpha_{c_{500}} = 0.08$

" $k_c = 5 \times 10^4$ $\quad \alpha_{c_{900}} - \alpha_{c_{500}} = 0.11$

" $k_c = 10^5$ $\quad \alpha_{c_{900}} - \alpha_{c_{500}} = 0.12$

From these calculated values it is seen that only a small change in the variation is achieved by reducing the leak coefficient k_c from 10^5 to 10^3. Other points to note are that the loss to the frame, of heat, from the air, although contributing to the temperature deficiency of the thermometer, does not introduce any variation over and above that due to transition on the outside of the tube. The two factors affecting the α variation are the balance between the rate of gain of heat by the platinum wire element and the rate of loss from the platinum wire to the frame, and the variations in frame temperature due to transition in flow pattern on the outside. If the insulation of the element to frame were improved and if the tube were made of a non-thermal conductor, as in the prototype, then it ought to be possible to reduce the α variation for this type of thermometer to negligible quantities.

8. Conclusions and recommendations.

A pitot-head thermometer in which care has been taken to eliminate heat losses from the sensitive element to the frame ought not to suffer variations in α even if it does not record the full stagnation temperature rise owing to heat losses from the arrested air.

A flat-plate type of thermometer ought to be mounted on a platform clear of the aircraft to ensure an airstream whose speed relative to the thermometer is unaffected by the proximity of the aircraft.

Care should be taken in the design to maintain a laminar boundary layer over the sensitive surface area throughout the range of Reynolds numbers likely to be encountered. A completely laminar boundary layer is preferable to a completely turbulent one because, apart from questions of drag, it is not always possible to ensure complete turbulence right from the leading edge and, moreover, it is believed that there may be degrees of turbulence.

With these points in view it is considered that a conical or wedge shaped thermometer bulb should give good results.

9. References.

(1) H. C. Shellard, Variation with altitude of the speed correction coefficients of aircraft thermometers, M.R.P. 527, Dec. 1949.

(2) E. Pohlhausen, Z. f. Angew. Math. Mech. I (1921) 120, 121.

(3) L. Crocco, Monografie Scientifiche de Aeronautica, Dec. 1946.

(4) H. L. Dryden, N.A.C.A. report No. 562, (1936)

(5) H. B. Squire, R.and M.No.1986 (1942)

(6) J. R. Partington, An Advanced Treatise on Physical Chemistry Vol. I, Longmans, Green and Co.

(7) N. A. C. A. Tech. Note No. 1200.

(8) H. L. Johnston and K. E. McCloskey, J. Phys. Chem. 44, Dec. 1940, pp. 1038-1057.

(9) W. J. Taylor and H. L. Johnston, J. Chem. Phys. 14, April 1946, pp. 219-233.

(10) S. Goldstein (Ed.), Modern Developments in Fluid Dynamics, Clarendon Press, Oxford.

(11) H. W. Emmons, J. Aeron. Sci. 18, July 1951.

Table I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
									← $\beta \times 10^7$					→		
									$(u_2/u_1)^2 = 0$		$(u_2/u_1)^2 = 1$		$(u_2/u_1)^2 = 2$			
P	T	$\mu \times 10^7$	k	$\mu/k \times 10^7$	$c_p \times 10^{-7}$	$\mu c_p/k = \sigma$	$\sigma^{1/2}$ (1)	$\sigma^{1/3}$ (2)	(1)	(2)	(1)	(2)	(1)	(2)	$\frac{\rho}{\rho_o}$	$\rho/\mu = 1/\nu$
900	281 (I.C.A.N.)	1760.8	2423 (5789 x 10^{-8} cal)	0.7268	0.9906 (0.2367 cal)	0.7199	0.8486	0.8952	0.505	0.505	0.428	0.452	0.352	0.400	0.909	5.330
500	252 (I.C.A.N.)	1617.8	2221 (5306 x 10^{-8} cal)	0.7285	0.9835 (0.2350 cal)	0.7164	0.8465	0.8948	0.508	0.508	0.430	0.455	0.352	0.401	0.565	4.282
300	228.5 (I.C.A.N.)	1493.9	(2047) (4893 x 10^{-8} cal)	0.7298	0.9772 (0.2335 cal)	0.7131	0.8443	0.8933	0.512	0.512	0.432	0.457	0.352	0.402	0.375	3.077
-	218 (N.A.C.A.)	1436.6	1966 (4699.4 x 10^{-8} cal)	0.7307	0.9747 (0.2329 cal)	0.7122	0.8439	0.8922	0.513	0.513	0.433	0.458	0.353	0.402		

Table II

Mean values of α_{fp}, α_{ph}, $\alpha_{ph}-\alpha_{fp}$ from Halifax runs.

(from Table III of M.R.P. 527.)

1. Mean values of α_{fp}

Mounting and Aircraft.	Low Level 900 mb. approx.	Medium Level 700 mb. approx.	High Level 500 mb. approx.
Under nose ST 817	2.05 (1)	-	1.70 (1)
.. .. ST 796	1.77 (1)	-	1.77 (1)
On extension under nose ST 796	1.92 (1)	-	1.79 (3)
Under wing ST 796	1.78 (2)	-	1.64 (7)
.. .. ST 817	-	1.71 (1)	1.61 (1)
All mountings	1.86 (5)	1.71 (1)	1.69 (13)
All under wing mountings	1.78 (2)	1.71 (1)	1.64 (8)

2. Mean values of α_{ph}

Mounting and Aircraft.	Low Level 900 mb. approx.	Medium Level 700 mb. approx.	High Level 500 mb. approx.
Under nose ST 796	2.52 (1)	-	2.27 (2)
.. .. ST 817	-	2.32 (1)	2.11 (1)
All mountings	2.52 (1)	2.32 (1)	2.22 (3)

3. Mean values of $\alpha_{ph} - \alpha_{fp}$

Mounting and Aircraft.	Low Level 900 mb. approx.	Medium Level 700 mb. approx.	High Level 500 mb. approx.
ST 796 (fp under wing) (ph under nose)	0.71 (3)	0.64 (1)	0.59 (2)
ST 817 (.. .. ..)	0.66 (2)	0.63 (6)	0.59 (6)
All mountings	0.69 (5)	0.63 (7)	0.59 (8)

N.B. Bracketed figures indicate number of measurements.

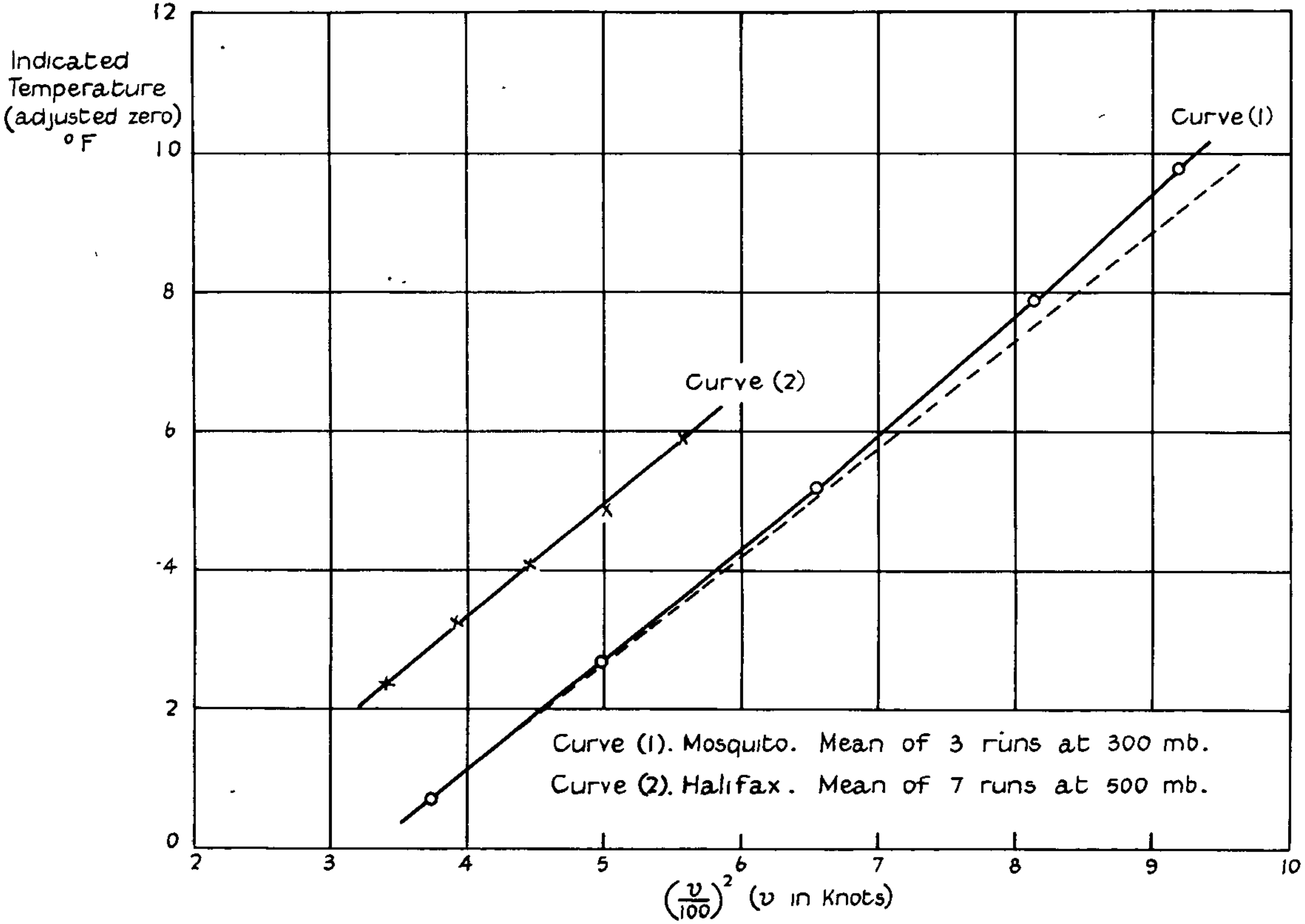
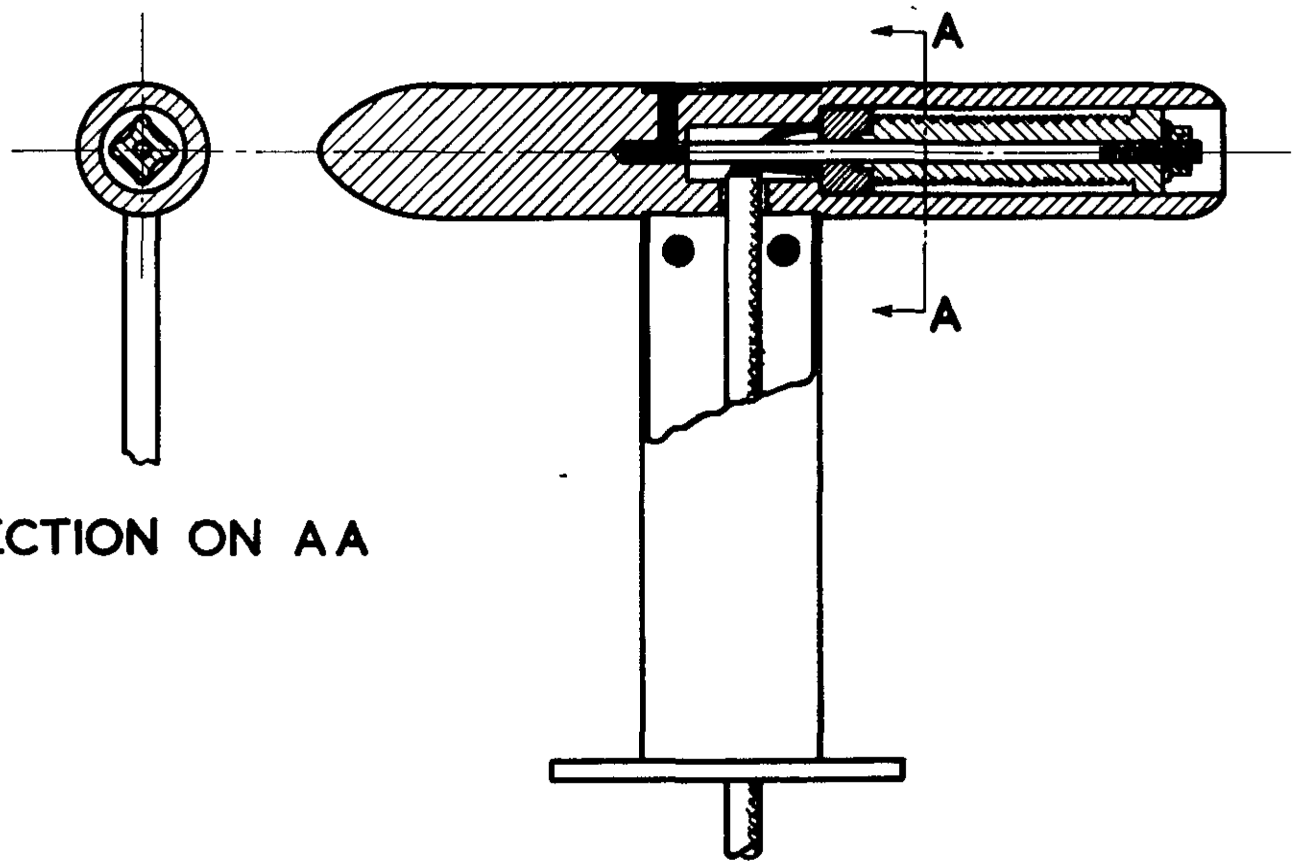

Indicated Temperature (adjusted zero) °F
12
10
8
6
4
2
0
2
3
4
5
6
7
8
9
10
$\left(\frac{v}{100}\right)^2$ (v in Knots)
Curve (1)
Curve (2)
Curve (1). Mosquito. Mean of 3 runs at 300 mb.
Curve (2). Halifax. Mean of 7 runs at 500 mb.

FIG 1

FIG. 2

RESISTANCE BULB FOR AIR THERMOMETER – EXPERIMENTAL



SECTION ON AA

PRINTED AND PUBLISHED BY HER MAJESTY'S STATIONERY OFFICE

To be purchased from

York House, Kingsway, LONDON, W.C.2 423 Oxford Street, LONDON, W.1

P.O. Box 569, LONDON, S.E.1

13a Castle Street, EDINBURGH, 2 1 St. Andrew's Crescent, CARDIFF

39 King Street, MANCHESTER, 2 Tower Lane, BRISTOL, 1

2 Edmund Street, BIRMINGHAM, 3 80 Chichester Street, BELFAST

or from any Bookseller

1952

Price 4s 0d net

PRINTED IN GREAT BRITAIN