C.P. No. 998

C.P. No. 998

MINISTRY OF TECHNOLOGY

AERONAUTICAL RESEARCH COUNCIL

CURRENT PAPERS

Gradient Properties of a Model of Stationary Random Turbulence

by

J. G. Jones

LONDON: HER MAJESTY'S STATIONERY OFFICE

1968

SIX SHILLINGS NET

U.D.C. 551.551 : 519.217 : 519.242

C.P. No.998*

June 1967

GRADIENT PROPERTIES OF A MODEL OF STATIONARY RANDOM TURBULENCE

by

J. G. Jones

SUMMARY

Histograms for changes in turbulence velocity over given distances are presented, based on a standard power spectral model. It is proposed that these be used as the basis of a comparison between the gradient properties of the model and the measured gradient properties of samples of atmospheric turbulence. An application of the histograms to the gust response of an aircraft with an autothrottle is described.

* Replaces R.A.E. Technical Report 67134 - A.R.C. 29531

CONTENTS

1 INTRODUCTION

One of the ways of representing atmospheric turbulence is as a quasi-stationary Gaussian random process defined by its power spectrum. It is possible in this way to construct an analytical model of turbulence, with parameters determined by matching the model against measured samples, which can be used to estimate the effects of turbulence upon aircraft loads, difficulty of control, etc. Atmospheric turbulence is of course neither truly stationary nor Gaussian and the Gaussian model needs to be used with some care. For example, it appears that it has little value as a means of predicting the frequency of encounter with the very large gusts in and around storms that may induce aircraft loads above the proof stress level. For such purposes a more promising approach, mentioned by Zbrozek[1], is the description of atmospheric turbulence in terms of a two-dimensional probability distribution of discrete gusts of variable gradient (shear) and length. However, particularly at low altitudes, turbulence often appears in a fairly continuous form which might be adequately described by the power spectral model for many purposes.

One of the most important properties of turbulence about which the aircraft engineer requires statistical information is the magnitude of velocity gradient, or change in velocity over a prescribed distance. This is because such quantities as aircraft loads, or airspeed changes when the aircraft is controlled by pilot or autopilot, depend not only upon the magnitudes of the gust velocities but also to a large extent upon their rate of change. This is a consequence of aircraft response decreasing the effects of the low frequency components of turbulence: what usually matters is the probable change in turbulence velocity in an interval of the order of the aircraft response time.

In the present Report we describe certain gradient properties of the Gaussian random model of atmospheric turbulence. These gradient properties are expressed in terms of conditional probability distributions which determine the change in turbulence velocity to be expected over some prescribed distance. Two possible applications of these results are stressed. One is as a means of converting existing experimental data on the power spectral model to a form which, by expressing the probability of gradients explicitly, allows the order of magnitude of some aircraft responses to be estimated without resort to a full computation of the power spectrum of the response variable in question. A simple example, which is discussed in section 5, is the assessment of the effectiveness, in controlling airspeed in turbulence, of an autothrottle system with known response time. The other, and more fundamental, application of a knowledge of the gradient properties of the spectral model is as a means of comparing the model with actual turbulence samples. It is proposed to find the best fit of the statistical model to the measured power spectrum of a sample of low altitude turbulence and then to check how well the estimated gradients taken from the statistical model compare with the measured gradients taken directly from the sample. Good agreement would not necessarily follow even if the first order amplitude probability distribution of the sample appeared to be consistent with a Gaussian distribution. For instance, it is possible that the structure of turbulence is such that velocity gradients over short distances are determined by shear layers which are only adequately described in terms of higher order probability distributions (see section 5.2). Good agreement between measured gradients and those predicted by the model should considerably support confidence in the power spectrum method as a means of estimating the response of aircraft in turbulence.

2 THE SPECTRAL MODEL

The gradient analysis described in the present Report is based on the assumption that, in the relevant frequency range, atmospheric turbulence can be modelled by a Gaussian process with power spectrum described either by

$$\Phi(\Omega) = \sigma^2 \frac{L}{\pi} \frac{1}{1 + (L\Omega)^2} \qquad (1)$$

or by the asymptotic form of equation (1):

$$\Phi(\Omega) = \frac{1}{\pi} \frac{\sigma^2}{L} \frac{1}{\Omega^2} \ . \qquad (2)$$

Here, σ^2 is the mean square turbulence velocity, L is known as the 'scale length', and Ω is spatial frequency in rad/ft. Φ has the dimensions of $(\text{velocity})^2$ per rad/ft. For the purpose of discussing velocity gradients over relatively short distances the approximation, equation (2), is often adequate.

For surveys of various possible forms for the spectrum of atmospheric turbulence see, for example, Refs.1 to 7. We review the more relevant aspects of possible models in the present section.

Equation (1) is usually quoted as one of a pair of 'Dryden' spectra, the other form being

$$\Phi = \sigma^2 \frac{L}{2\pi} \frac{1 + 3\,(L\Omega)^2}{\{1 + (L\Omega)^2\}^2} \quad . \tag{3}$$

Equation (1) is usually quoted for the streamwise component and equation (3) for the transverse components. These spectra are commonly employed in aeronautical engineering and values of σ^2 and L are chosen to give a good fit of the model spectra to measured spectra over the frequency range of aeronautical interest. For large Ω both forms become asymptotically proportional to Ω^{-2}. More fundamental studies of the structure of turbulence suggest that over this range the slope should be given by $\Omega^{-5/3}$ (the Kolmogoroff similarity theory). However, since measured spectra show a scatter in asymptotic slope which makes it impossible to choose between the two forms, Ω^{-2} and $\Omega^{-5/3}$, the former is often employed by aircraft engineers because of the associated simplification introduced into aircraft response calculations, despite the more scientifically respectable background of the latter.

For many practical purposes, including the present gradient study, it is possible to go even further in simplification and to use equation (1) for all three components of turbulence. The usual practice is to use the more complicated form of equation (3) for the transverse components of atmospheric turbulence, largely because the correlation functions $f(r)$ and $g(r)$, equivalent to the spectra given by equations (1) and (3) respectively, together satisfy a basic equation for isotropic turbulence[8]

$$g(r) = f(r) + \frac{1}{2}\, r\, \frac{\partial f(r)}{\partial r} \quad . \tag{4}$$

Such refinement is, however, rather inconsistent with the experimental basis for the two forms in the case of atmospheric turbulence: equation (1) can be made to give just as good a fit to measured transverse spectra as equation (3). Etkin[3] has directly compared the two forms (Fig.1 of Ref.3) and in fact, when suitably scaled, they only differ by a small amount which lies in the low frequency region where experimental results are not particularly reliable (Fig.1).

There is some justification, then, for basing the present work on the spectral form, equation (1). Any experimental data presented in terms of equation (3) can be converted to an equivalent expression in the form of equation (1) by use of Etkin's[3] conversion formula which we have quoted in

Appendix A for convenience. Further, except in the case of the vertical component of turbulence at low altitude (where the boundary condition at the ground introduces a bound on the scale length) it will usually be sufficient to employ asymptotic results of the form of equation (2), since these cover the frequency range within which aircraft response usually lies. In this case the only significant parameter is σ^2/L, which is defined by the power per unit band at any frequency within the asymptotic region. Values of L and σ for low altitude turbulence have recently been presented by Pritchard[7], who covers a range of terrain types and degrees of atmospheric stability.

In order to use the spectral model to obtain results on gradient probabilities we need also to assume that the turbulence can be approximated by a Gaussian random process. This is a big assumption to make, but it is forced upon us if we wish to use the power spectrum to obtain any information about crossings of levels, etc. The only relevant experimental information that has been discussed in the past concerns the (first order) amplitude probability distribution. The evidence is (to quote Pritchard[7]) 'that for short data runs of the order of 4 minutes, the probability density of the fluctuations of a gust velocity component is closely Normal except for the large amplitudes where the measured density is usually much larger than predicted by the Normal curve! No data exists on higher order probability distributions. As has been pointed out in the Introduction, one of the applications of the present Report is to check if the gradient properties based on a measured spectrum and Gaussian random process assumptions are significantly different from the observed gradient properties.

We have been discussing turbulence spectra in terms of spatial frequency in radians per foot. In order to relate these to the time dependent random process experienced by an aircraft travelling with velocity V we employ Taylor's hypothesis through the relation

$$\Omega = \omega/V \qquad (5)$$

where ω is frequency in rad/sec. This assumption is valid for the speeds at which conventional aircraft fly (but is not valid for low velocities, typical of VTOL aircraft, very near the ground).

3 GRADIENT PROPERTIES OF THE STATISTICAL MODEL

3.1 Background

As discussed in the previous section we consider a stationary random Gaussian process u(s) (where s is distance, in feet) which is completely defined by its power spectrum

$$\Phi(\Omega) = \sigma^2 \frac{L}{\pi} \frac{1}{1 + (L\Omega)^2} \quad . \tag{1}$$

This particular random process is also a scalar Markov process, i.e. the probability law of the 'future' development of the process (in terms of increasing values of s), once it has taken some fixed value at a given point, depends only on that value and not upon the previous 'history' of the process. Mathematically, the conditional probability of $u(s_2)$ given u(s) for all $s < s_1$, where $s_1 < s_2$, is equal to the conditional probability of $u(s_2)$ given $u(s_1)$.

As a result, the gradient properties we require can be expressed simply in terms of the conditional probability of u(s) at $s = s_2$ when some constraint is put on the value of u(s) at $s = s_1$. We will consider two basic types of constraint at $s = s_1$. In the first case (section 3.2) we assume that $u(s_1) = 0$ and determine the probability distributions of $\Delta u = u(s_2)$ for a sequence of values of $s_2 > s_1$. That is, we determine probability distributions which define how quickly u(s) 'gets away' from zero value following a zero crossing (Fig.2). In the case of the second type of constraint (section 3.3) we assume that s_1 is chosen in a completely arbitrary way (so that $u(s_1)$ is a random variable) and determine the probability distributions of $\Delta u = u(s_2) - u(s_1)$ for a sequence of values of $s_2 > s_1$ (Fig.3).

In section 3.4 we consider analogous results in the limiting case where the power spectrum, equation (1), is replaced by its asymptotic form, equation (2). These asymptotic results will be valid whenever the distance d over which gradients are to be considered is small relative to the scale length L of the process.

3.2 The case of zero initial value

In this section we are concerned with the Gaussian random process defined by equation (1) and we are given that $u(s_1) = 0$ (Fig.2). Under this condition we wish to find the probability distribution of the change in turbulence velocity $\Delta u = u(s_2)$ at a position $s_2 > s_1$. Since the process $u(s)$ is Gaussian it follows that this conditional probability distribution is Gaussian (Normal) and hence is completely defined by its mean and variance. By symmetry the mean is zero and so all we are required to find is the variance σ_d^2 where $d = s_2 - s_1$. It is shown in Appendix B that

$$\frac{\sigma_d^2}{\sigma^2} = 1 - e^{-\frac{2d}{L}} \tag{6}$$

where σ^2 and L are factors in equation (1). The presentation of results derived from equation (6) is discussed in section 4.

3.3 The case of random initial value

Here we are concerned (Fig.3) with the case of arbitrary initial position s_1 and hence $u(s_1)$ is a random variable (with Gaussian amplitude distribution of zero mean and variance σ^2). Then the condition probability distribution of $\Delta u = u(s_2) - u(s_1)$, $(s_2 > s_1)$, is Gaussian with zero mean (by symmetry) and variance $\bar{\sigma}_d^2$ (say). It is shown in Appendix C that

$$\frac{\bar{\sigma}_d^2}{\sigma^2} = 2(1 - e^{-\frac{d}{L}}) \quad . \tag{7}$$

Results derived from equation (7) are discussed in section 4.

3.4 Asymptotic results for small d/L

For distances $d = s_2 - s_1$ small compared with the scale length L of the process the gradient properties can be written in a form dependent only on the distance d and the coefficient σ^2/L which appears in the asymptotic spectrum (equation (2)). These results will, moreover, be valid if the asymptotic spectrum form, equation (2), has been fitted directly to experimental data and hence a scale length L need not be assumed to exist. This may be useful particularly in the case of spectra of high altitude turbulence (or even horizontal and lateral components of turbulence at low altitude) where the measured spectra often show no signs of departing from the Ω^{-n} form with constant (logarithmic) slope even at the lowest frequencies measured. Here we assume that, with a suitable choice of σ^2/L, equation (2) can be fitted with adequate accuracy to the measured spectrum over the frequency range of interest, even if the measured exponent n is not exactly equal to 2.

To first order in d/L equations (6) and (7) give the same result for the variance of Δu. The more useful form is that referring to an arbitrary initial point s_1, in which case equation (7) gives, to first order:

$$\bar{\sigma}_d = \left(\frac{2\sigma^2}{L}\right)^{\frac{1}{2}} d^{\frac{1}{2}} \quad . \tag{8}$$

This result is discussed further in the following sections.

4 SUMMARY OF RESULTS

4.1 Zero initial turbulence velocity

The turbulence velocity $u(s)$ is taken to be a stationary Gaussian process with zero mean and power spectrum given by equation (1). Then the change in turbulence velocity Δu over a distance $d = s_2 - s_1$, in the case where the initial turbulence velocity $u(s_1)$ is zero (Fig.2), is Gaussian with zero mean and variance given by equation (6). Histograms, based on this Gaussian distribution, for the random variable $\frac{\Delta u}{\sigma}$, for a sequence of values of d/L, are presented in Figs.4a to 4f. The corresponding non-dimensional gust gradients $\left(\frac{\Delta u}{\sigma}\right) \Big/ \left(\frac{d}{L}\right)$ are also illustrated in Fig.4, in conjunction with each histogram. Equation (6) has also been used to construct a graph (Fig.5) illustrating the expected shear $\Delta u/d$ in a given gradient distance d in the case of zero initial turbulence velocity. The results (Fig.5) are expressed in terms of non-dimensional velocity change $\Delta u' = \Delta u/\sigma$ and non-dimensional gradient distance $d' = d/L$.

4.2 Random initial turbulence velocity

A different result is obtained if we consider the change in turbulence velocity $u(s)$ over a given distance in the case where the initial value is random (Fig.3). The initial point s_1 is chosen in a random manner (so that $u(s_1)$ is a random variable with Gaussian distribution of zero mean and variance σ^2) and results for $\Delta u = u(s_2) - u(s_1)$ are obtained by averaging over all such initial values $u(s_1)$. Then the corresponding change in turbulence velocity Δu over a distance $d = s_2 - s_1$ (see Fig. 3) is Gaussian with zero mean (by symmetry) and variance given by equation (7). Histograms, based on this Gaussian distribution, for the random variable $\Delta u/\sigma$, are presented in Figs. 6a to 6c. Comparison of equations (6) and (7), and of Figs. 4 and 6, illustrates the larger average velocity change Δu in a given distance d subsequent to a random initial value. This is to be expected since when the initial value is random, cases where the initial value is of large magnitude are included (weighted in the appropriate way) and in such cases one would expect relatively large velocity changes Δu towards the mean value $u = 0$.

For small values of d/L (Figs. 4a and 6a) the histograms for the two cases, zero initial turbulence velocity and random initial velocity, show little difference. This point is clarified in the following section where the case of small d/L is reviewed in more detail.

4.3 Asymptotic results for small d/L

In the case of gradients over distances d small compared with the scale length of the turbulence the variance of Δu becomes proportional to d and depends only on the parameter σ^2/L (equation (8)). This result is illustrated in Fig. 7 and is particularly useful for describing the gradient behaviour of samples of turbulence whose sample spectrum can be approximated by equation (2). This property holds in the case of many samples of atmospheric turbulence whose spectra have been measured: the scale length L, which is associated with a departure from linearity in the logarithmic plot of the spectrum, if often non-existent, or at least ill-defined.

5 APPLICATIONS

5.1 Airspeed response of aircraft with autothrottle

We consider an aircraft, with airspeed controlled by an autothrottle system, flying at 150 kt at an altitude of the order of 1000 ft under moderately severe turbulence conditions. Existing experimental data suggests that under these circumstances we can use the spectral model of turbulence, equation (1), with parameters for the horizontal component:

$$\left.\begin{aligned} \sigma &= 8 \text{ ft/sec} \\ L &= 1200 \text{ ft} \end{aligned}\right\} . \qquad (9)$$

Typical present day autothrottle systems have a response time of about 10 seconds. Assuming that the speed stability of the basic aircraft is low, it follows that fluctuations in horizontal gust velocity over a time interval of the order of 5 seconds, for example, will largely appear as airspeed fluctuations (though they will not necessarily be apparent to the pilot because of instrument lags). It is of interest, therefore, to estimate the probability density for changes in horizontal gust velocity in 5 seconds. Using Taylor's hypothesis (equation (5)) we relate the turbulence fluctuations in time to the spatial distribution used in the turbulence model. At 150 kt the distance travelled by the aircraft in 5 seconds in approximately d = 1250 ft. Expressed as a ratio of turbulence length (equation (9)) we have

$$d/L \doteqdot 1 \quad . \tag{10}$$

Starting from an arbitrary instant of time it is appropriate to use the histograms for random initial turbulence velocity, and the corresponding case $d/L = 1$ is illustrated in Fig.6c. Taking $\sigma = 8$ ft/sec (equation (9)) this histogram (Fig.6c) can thus be used to describe the probability distribution of airspeed fluctuations over arbitrary intervals of 5 seconds. For example, considering the case of large fluctuations, changes in airspeed $(\Delta u)_{5\ \text{sec}}$ greater than 16 ft/sec ($9\frac{1}{2}$ kt) correspond to $\Delta u/\sigma > 2$, and occur with a probability of about $2\frac{1}{2}\%$, or on average once every 200 seconds.

All this assumes, of course, that the spectral model, based on Gaussian process assumptions, gives an adequate description of turbulence velocity gradients over the corresponding spatial distance. This point is taken up further in the following section.

5.2 Proposed comparison with gradient properties of measured turbulence samples

As pointed out in the Introduction, it is advisable to check how well the probability distribution of changes in turbulence velocity over given distances as predicted by the model agrees with the corresponding distribution for measured turbulence samples. Good agreement would not necessarily follow even if the spectrum given by equation (1) (with suitable choice of parameters) fitted the measured spectrum well, and the first order amplitude distribution of the sample were Gaussian. This is because the measured gradient properties depend on the joint probability of turbulence velocity at two points, and this need not be Gaussian under the above conditions. For instance, in the case of turbulence generated by a grid it has been experimentally established that the first order amplitude distribution of the components of turbulence velocity are normal to a high degree of accuracy, but on the other hand measurements of the joint probability distribution of velocities at two different points are in general not normal. Our general understanding of the nature of turbulence, in fact, suggests that the Gaussian random process will be a poor model for describing gradients of turbulence velocity owing to the tendency for discrete shear layers to form.

There is an increasing tendency for the spectral approach to be used in the estimation of aircraft loads, etc., and there is clearly a case for checking just how well the model predicts velocity gradients. The most straightforward way to go about this is to measure gradients starting from each crossing of the estimated mean (this is to ensure, as far as possible, statistical independence of consecutive samples). Probability densities for the change in turbulence velocity should be estimated for a range of gradient distances. By estimating, in addition, the values of σ and L which give the best fit of the spectral model, equation (1), the histograms presented in the present Report can be used to give a direct comparison with the model gradient properties.

6 CONCLUSIONS

Aircraft responses (loads, etc.) due to turbulence depend to a large extent on the change in turbulence velocity in an interval of the order of the aircraft response time. There is an increasing tendency for the power spectral approach to be used in the estimation of aircraft response and this spectral approach depends on the assumption that samples of turbulence velocity can be adequately approximated by a stationary Gaussian process. In particular, the joint probability distribution at two points, which determines the gradient properties of the process, is assumed to be Gaussian. On the other hand there is strong experimental evidence in the case of turbulence behind a grid that the joint probability distributions of turbulence velocity at two points are not Gaussian. There is a tendency for a small number of strong gradients to form rather than a uniform distribution of smaller gradients. In view of this it appears that there is a strong case for comparing the gradient properties of measured samples of atmospheric turbulence with the gradient properties of the spectral model. To further this end, histograms for changes in turbulence velocity over given distances are presented in this Report, based on a standard power spectral model, equation (1). It is intended to use these histograms in the analysis of experimental turbulence data.

Assuming that the power spectral model proves adequate, at least for some purposes, the histograms can be used to estimate orders of magnitude of some aircraft responses without resort to a full computation of the power spectrum of the response variables. As an illustration, the airspeed fluctuations of an aircraft with an autothrottle flying in moderately severe turbulence have been considered.

Appendix A

ETKIN'S CONVERSION FORMULA

Fig. 1 is quoted from Etkin[3] and illustrates the similarity between the two types of Dryden spectra when suitably scaled. The two forms illustrated are:

$$\Phi(\Omega) = \sigma^2 \frac{L}{2\pi} \frac{1 + 3\,(L\Omega)^2}{\{1 + (L\Omega)^2\}^2} , \qquad \text{(A-1)}$$

which is identical with equation (3), and

$$\Phi(\Omega) = \frac{3}{2}\sigma^2 \frac{L}{\pi} \frac{1}{2.25 + (L\Omega)^2} , \qquad \text{(A-2)}$$

which is a scaled version of equation (1). The relation between equations (A-2) and (1) is given by the condition:

$$\frac{3}{2}\sigma^2 \frac{L}{\pi} \frac{1}{2.25 + (L\Omega)^2} \equiv \sigma'^2 \frac{L'}{\pi} \frac{1}{1 + (L'\Omega)^2}$$

if

$$L' = \sqrt{\frac{1}{2.25}}\, L$$

and

$$\sigma'^2 = \frac{3}{2}\sqrt{\frac{1}{2.25}}\, \sigma^2 .$$

Appendix B

CONDITIONAL VARIANCE FOR ZERO INITIAL VALUE

We consider the Gaussian process u(s) defined by the power spectrum

$$\Phi(\Omega) = \sigma^2 \frac{L}{\pi} \frac{1}{1 + (L\Omega)^2} \tag{B-1}$$

and we are given that $u(s_1) = 0$. We wish to find the variance σ_d^2 of $u(s_2)$, where $d = s_2 - s_1$. The Gaussian process with spectrum given by equation (B-1) can be generated by passing white noise with autocorrelation function $R(\tau) = \frac{2\sigma^2}{L} \delta(\tau)$ through a shaping filter with transfer function $\frac{L}{Ls + 1}$, as illustrated in Fig. 8 (where we have used the conventional independent variable τ in the autocorrelation function and impulse response, even though it here represents a distance rather than time).

In order to obtain the variance σ_d^2 subsequent to the initial condition $u(s_1) = 0$ we add a switch to the filter system as illustrated in Fig. 9. At $s = s_1$, the white noise input to the shaping filter is switched on. Thus the condition $u(s_1) = 0$ is satisfied. It is shown in Ref. 9 that with an initial constraint of this kind the variance σ_d^2 of u(s) is given by the equation

$$\sigma_d^2 = 2 \int_0^d F(\tau)\, h(\tau)\, d\tau \tag{B-2}$$

where $h(\tau)$ is the impulse response of the shaping filter.

In the case $R(\tau) = \frac{2\sigma^2}{L} \delta(\tau)$, we have[9]

$$F(\tau) = \int_0^\tau \frac{\sigma^2}{L} \delta(\tau - \tau')\, h(\tau')\, d\tau' = \frac{\sigma^2}{L} h(\tau) \quad . \tag{B-3}$$

Substituting $h(\tau) = e^{-\tau/L}$, which corresponds to our shaping filter $\frac{1}{Ls + 1}$, we obtain the required result

$$\frac{\sigma_d^2}{\sigma^2} = 1 - e^{-\frac{2d}{L}} \quad . \tag{B-4}$$

Appendix C

CONDITIONAL VARIANCE FOR RANDOM INITIAL VALUE

In this case we consider the Gaussian process $u(s)$, which can be generated with a shaping filter as illustrated in Fig.8, with a random initial value $u(s_1)$. We wish to find the variance $\bar{\sigma}_d^2$ of the related process $\Delta u(s) = u(s) - u(s_1)$, where $d = s - s_1$. Since the initial position s_1 is chosen at random, we have

$$\bar{\sigma}_d^2 = \overline{\{u(s + d) - u(s)\}^2} \quad . \tag{C-1}$$

This function is known as the structure function of $u(s)$.

Expanding equation (C-1):

$$\bar{\sigma}_d^2 = \overline{\{u(s + d)\}^2} + \overline{\{u(s)\}^2} - 2\,\overline{u(s)\ u(s + d)} = \sigma^2 + \sigma^2 - 2R(d) \quad , \tag{C-2}$$

where

$$R(d) = \sigma^2 e^{-d/L} \quad . \tag{C-3}$$

Thus

$$\frac{\bar{\sigma}_d^2}{\sigma^2} = 2\left(1 - e^{-d/L}\right) \quad . \tag{C-4}$$

SYMBOLS

$d = s_2 - s_1$	distance (ft)
L	turbulence scale length
s	distance feet Laplace transform variable
u(s)	turbulence velocity
V	aircraft velocity
$\Phi(\Omega)$	power spectral density of u(s)
$\sigma^2 = \int_{-\infty}^{\infty} \Phi(\Omega)\, d\Omega$	mean square value of u(s)
σ_d	conditional variance of $u(s_2)$ when $u(s_1) = 0$
$\bar{\sigma}_d$	conditional variance of $\Delta u(s_2)$ when $u(s_1)$ is random
ω	frequency (rad/sec)
Ω	spatial frequency (rad/ft)

REFERENCES

No.	Author	Title, etc.
1	J.K. Zbrozek	Atmospheric gusts - present state of the art and further research. J. Roy. Aero. Soc. Vol.69 No.649 (1965)
2	J.K. Zbrozek D.M. Ridland	A measured power spectrum of the vertical component of atmospheric turbulence. A.R.C. CP 522 (1960)
3	B. Etkin	A simple method for the analogue computation of the mean-square response of airplanes to atmospheric turbulence. University of Toronto UTIA Technical Note 32 (1960)
4	J.C. Houbolt R. Steiner K.G. Pratt	Flight data and considerations of the dynamic response of airplanes to atmospheric turbulence. AGARD Flight Mechanics Panel (1962)
5	J.L. Lumley H.A. Panofsky	The structure of atmospheric turbulence. Interscience, (1964)
6	J.K. Zbrozek	The relationship between the discrete gust and power spectra presentation of atmospheric turbulence, with a suggested model of low-altitude turbulence. A.R.C. R & M 3216 (1960)
7	F.E. Pritchard	A statistical model of atmospheric turbulence and a review of the assumptions necessary for its use. AGARD Stab. & Control Panel (1966)
8	G.K. Batchelor	The theory of homogeneous turbulence. Camb. Univ. Press (1953)
9	J.G. Jones	Unpublished Mintech Report

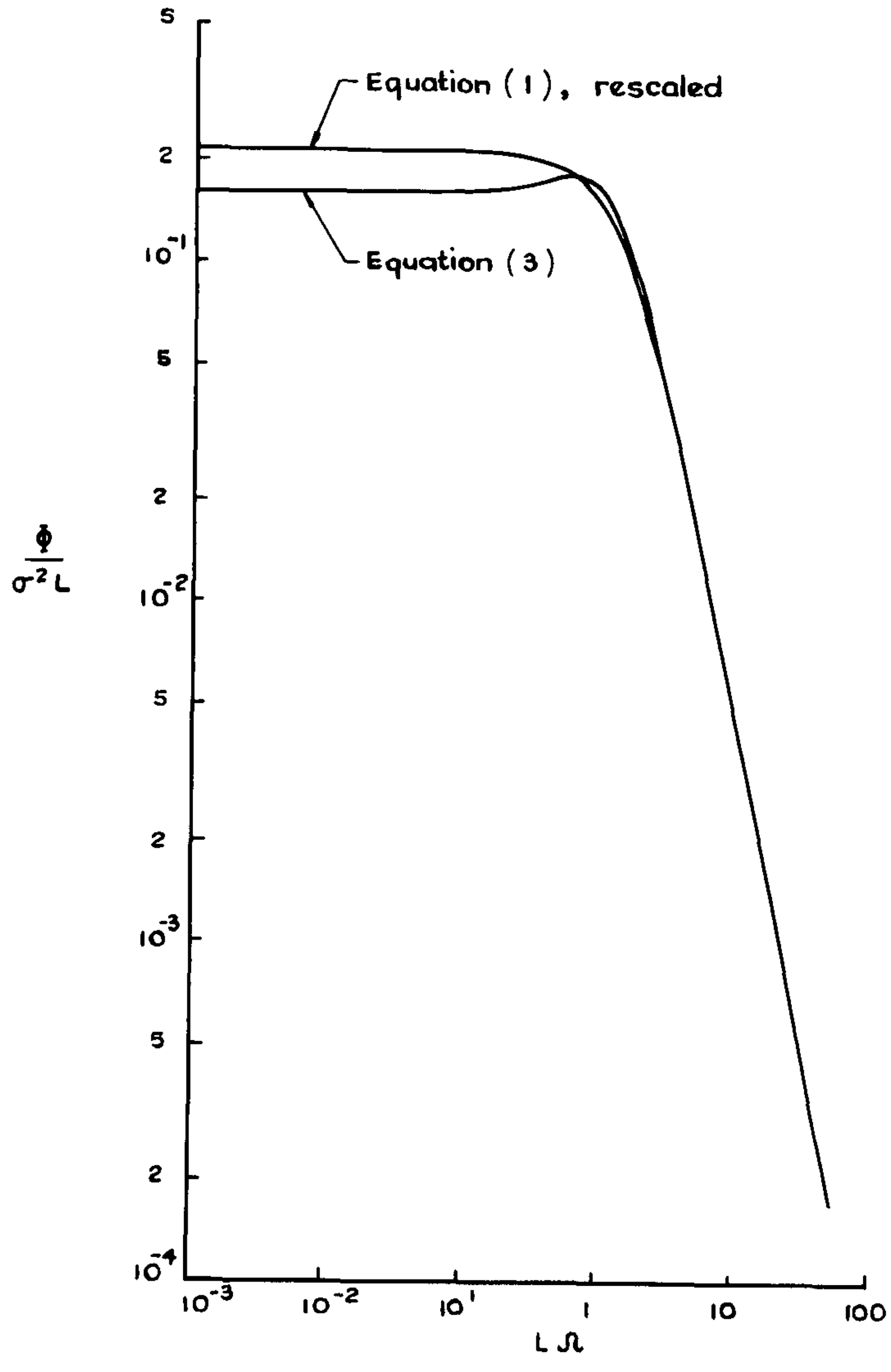


Fig. 1 Comparison of power spectra (from Etkin[3])

Fig. 2 Illustration of Δu for zero initial value

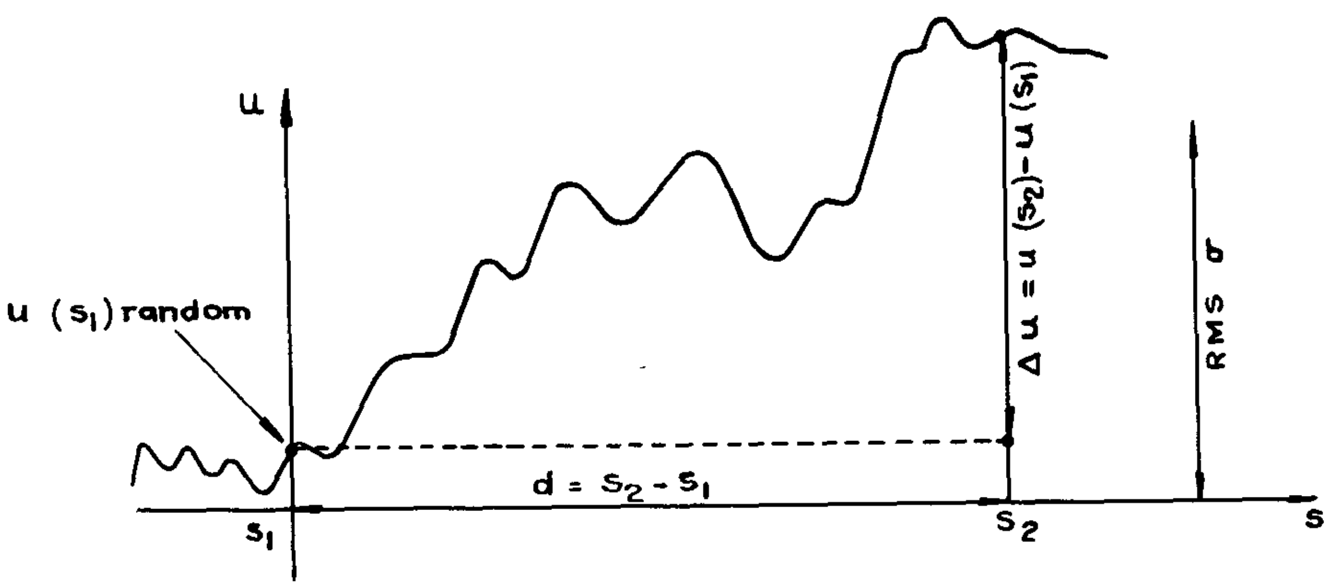


Fig. 3 Illustration of Δu for random initial value

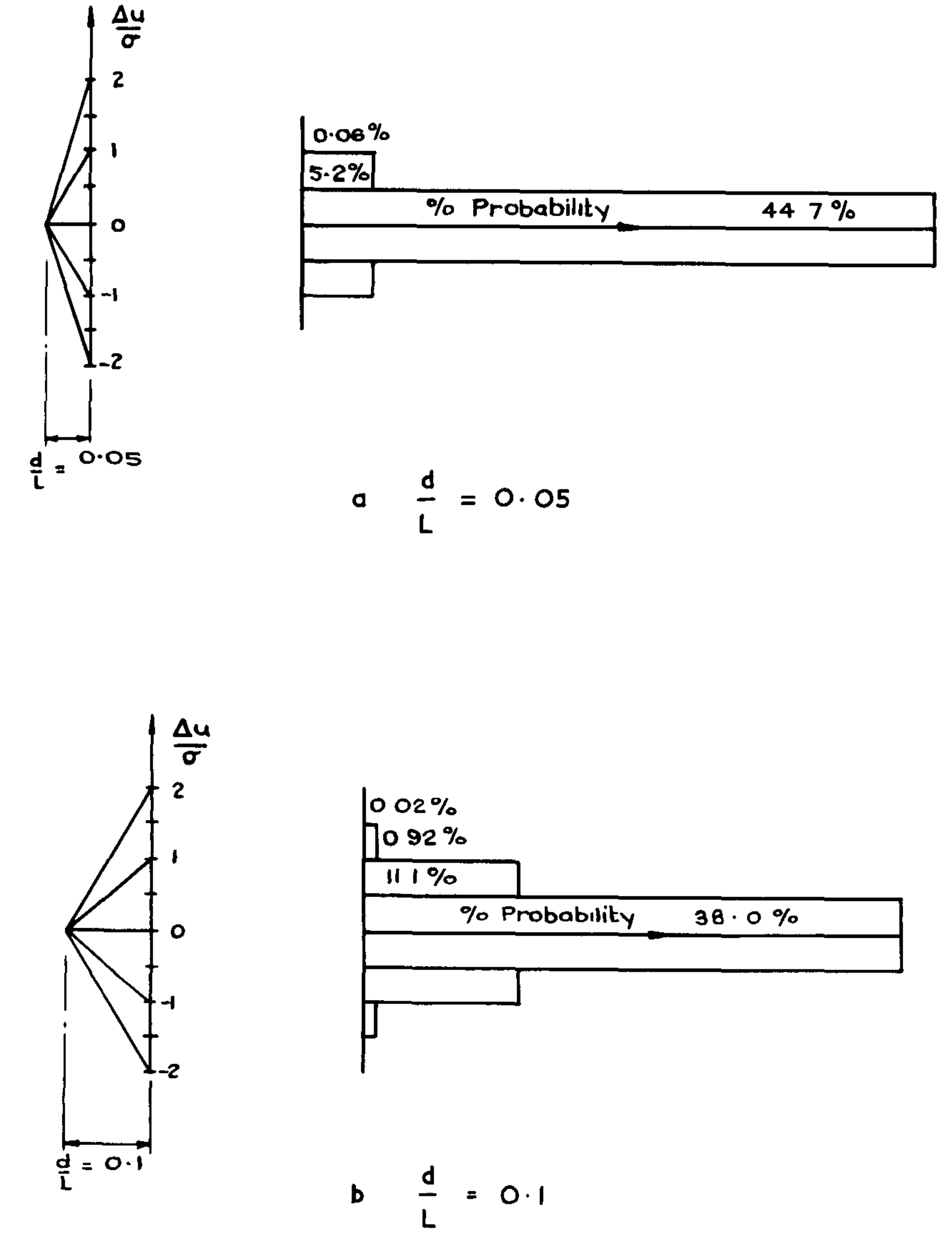
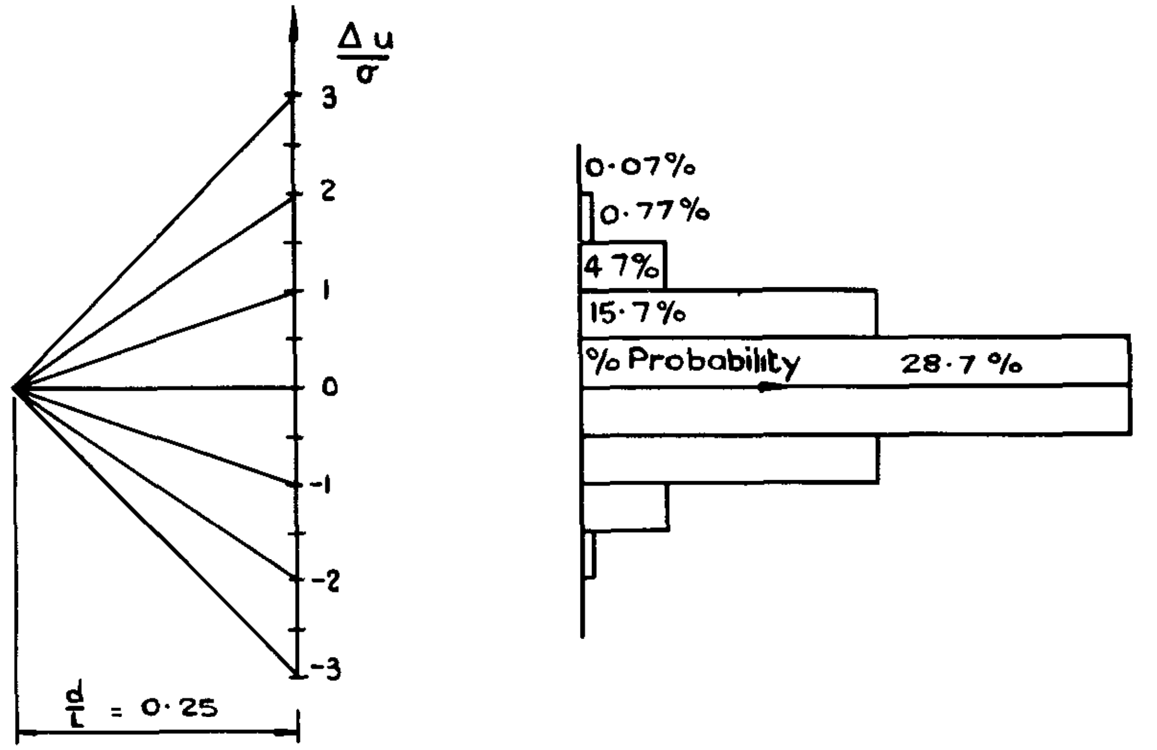
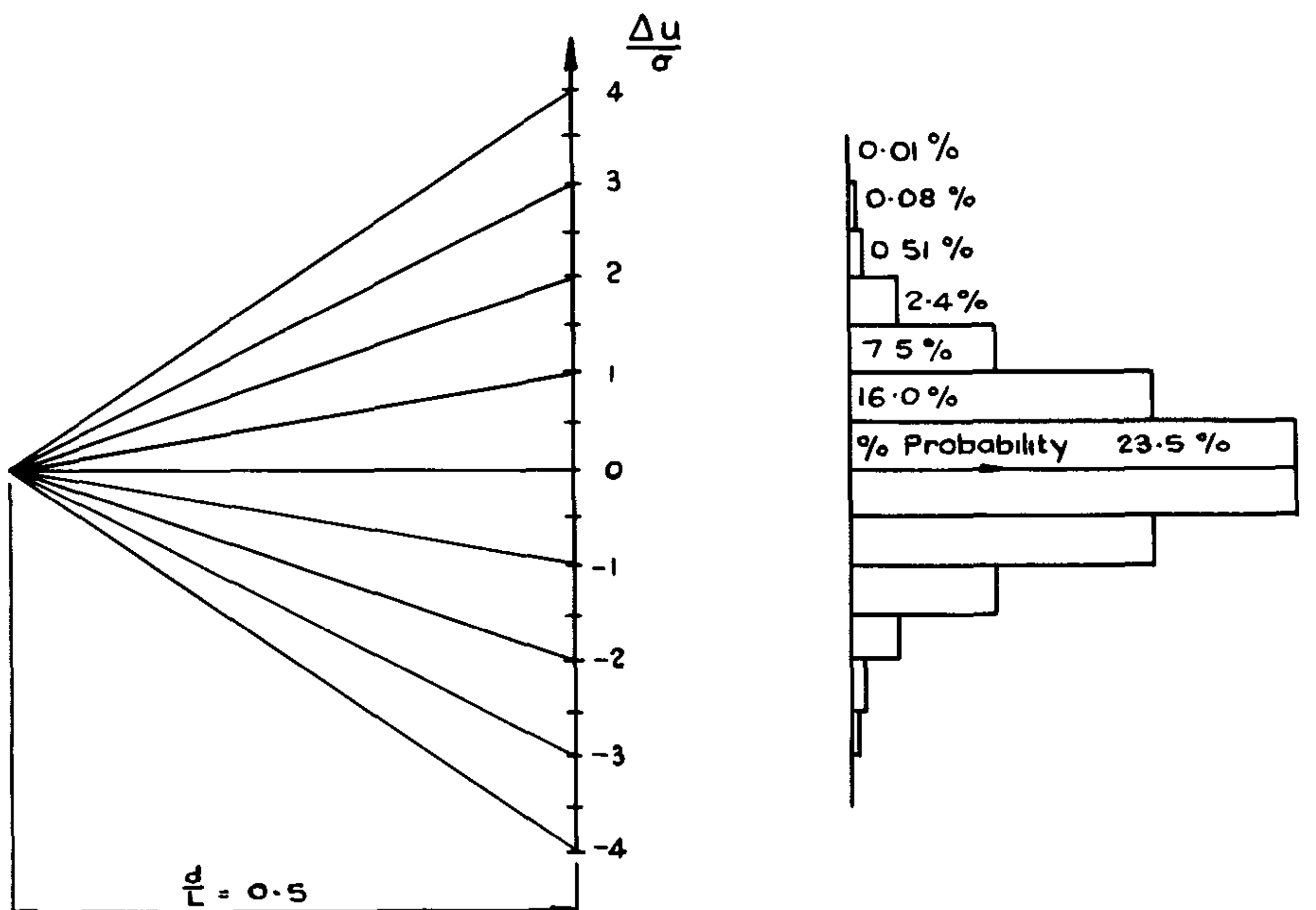


Fig. 4 a & b Histograms for change in gust velocity Δu in distance d when initial gust velocity is zero

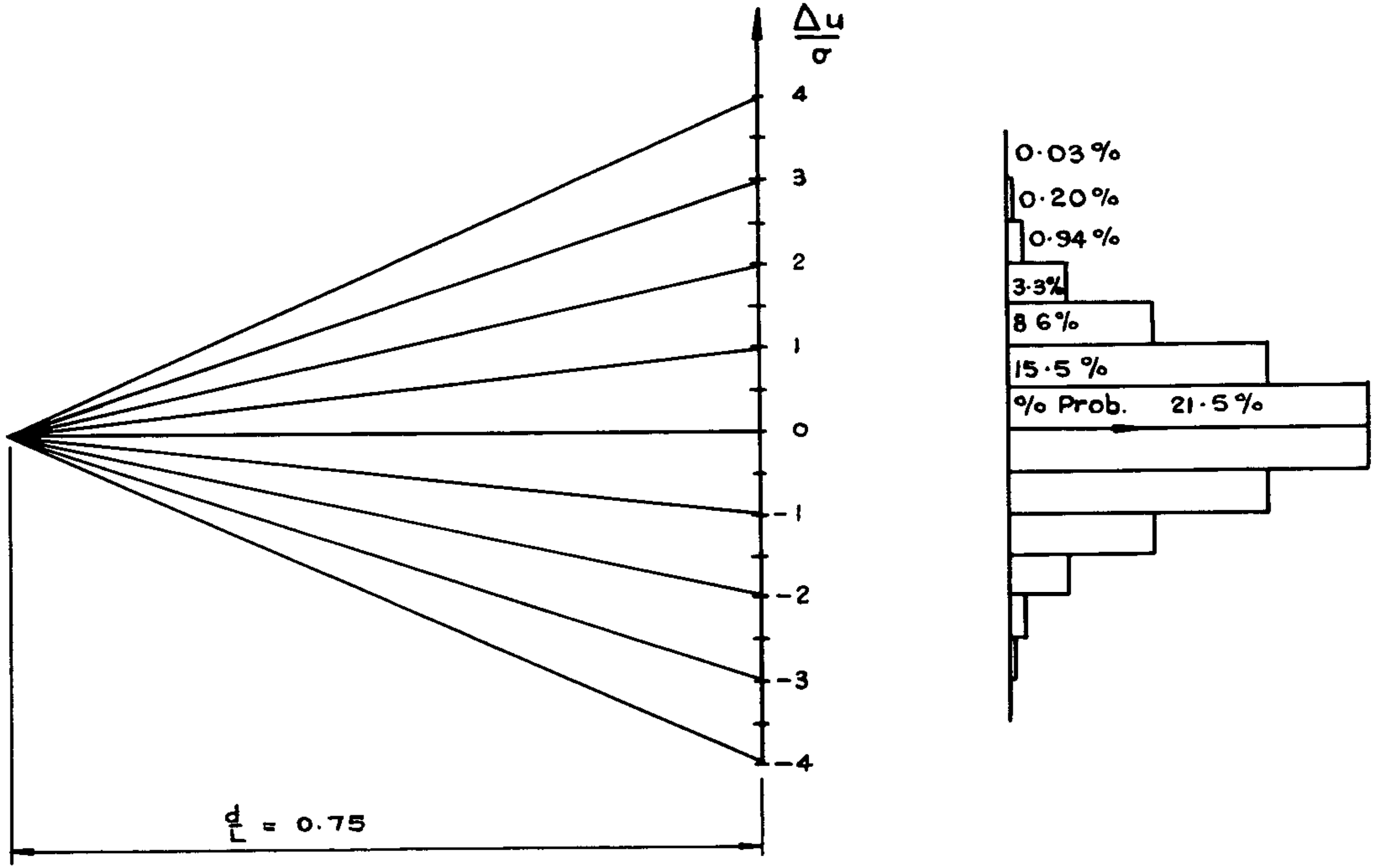


c $\frac{d}{L} = 0 \cdot 25$



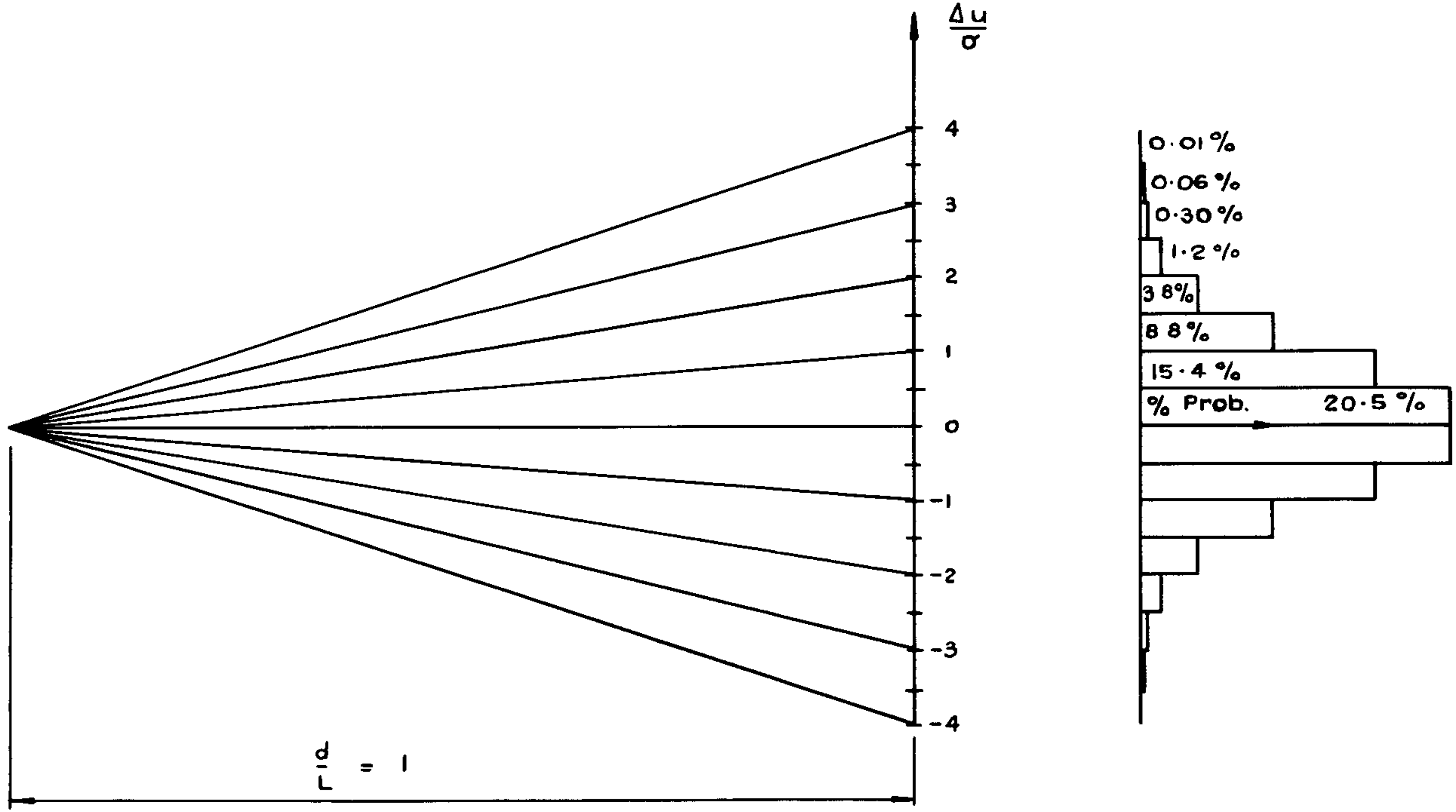
d $\frac{d}{L} = 0 \cdot 5$

Fig. 4 c & d Histograms for change in gust velocity Δu in distance d when initial gust velocity is zero



e. $\frac{d}{L} = 0 \cdot 75$

Fig. 4 e Histograms for change in gust velocity Δu in distance d when initial gust velocity is zero



f $\frac{d}{L} = 1$

Fig. 4 f Histograms for change in gust velocity Δu in distance d when initial gust velocity is zero

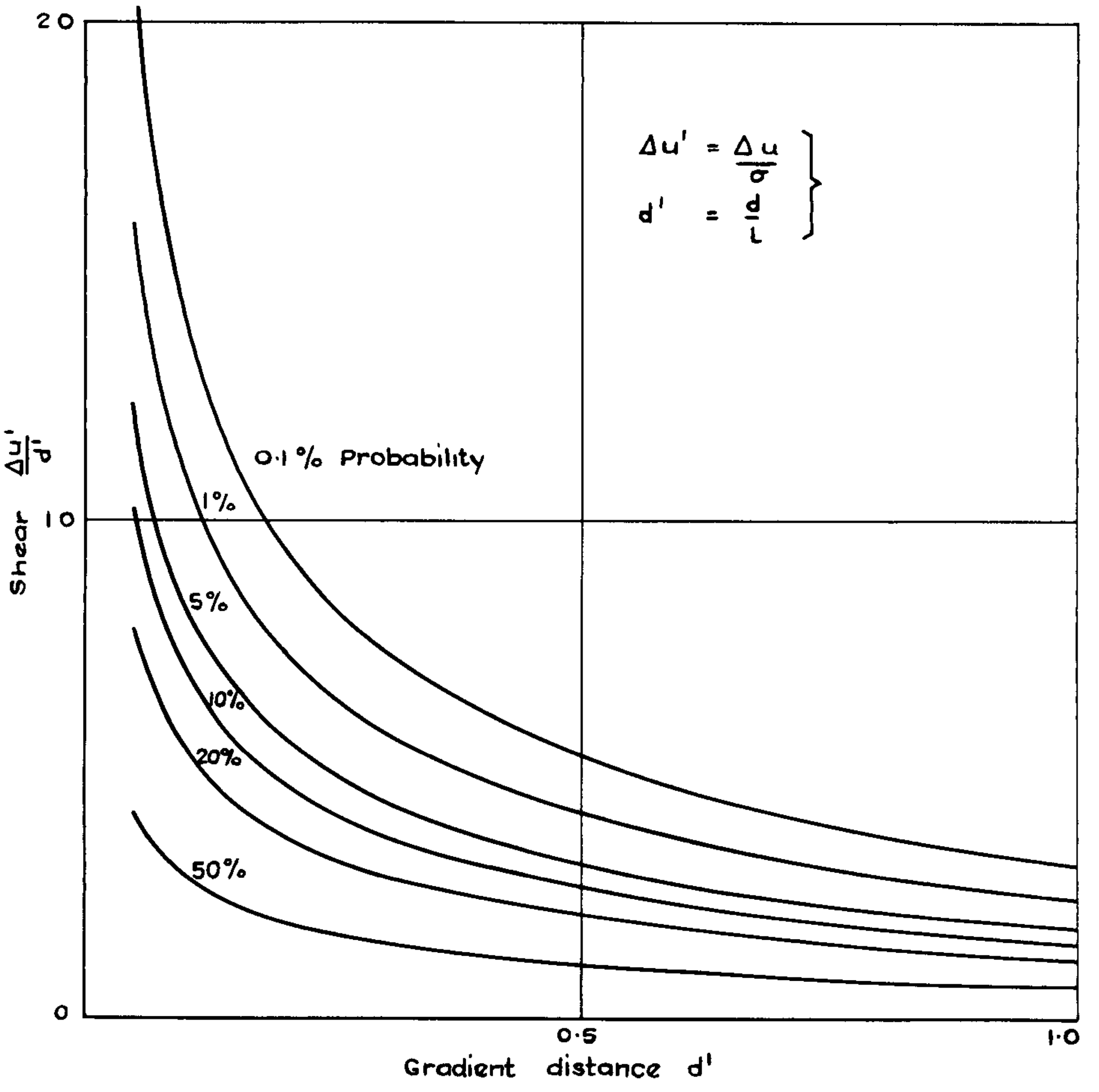
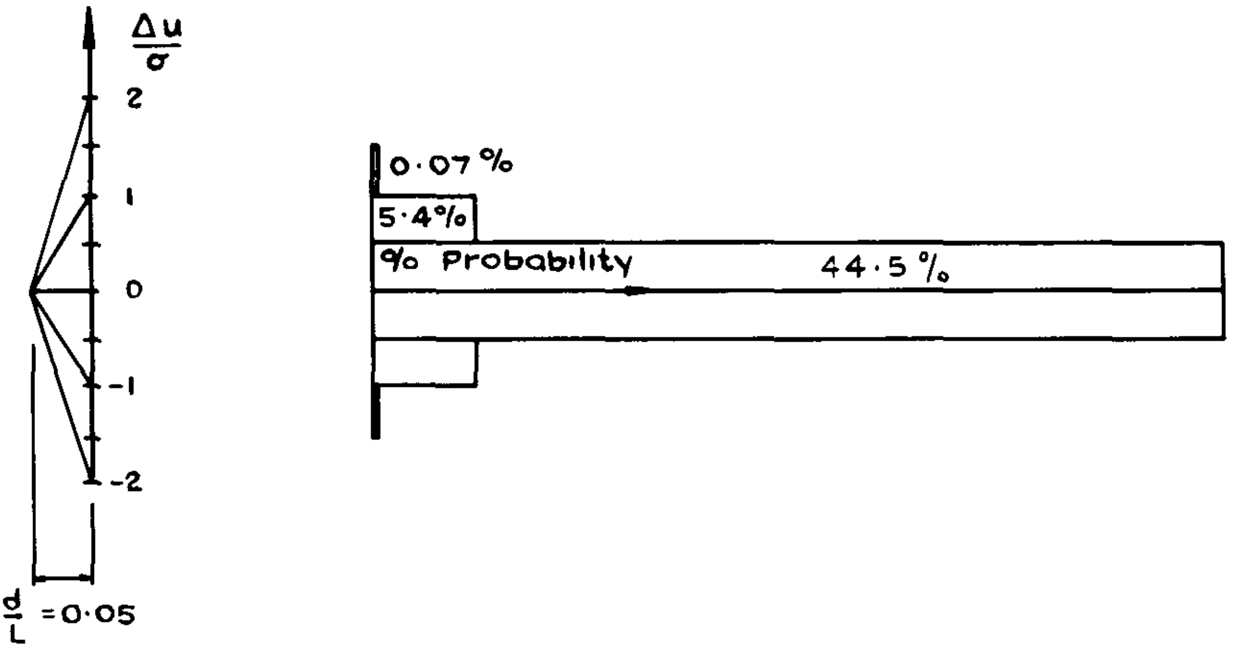
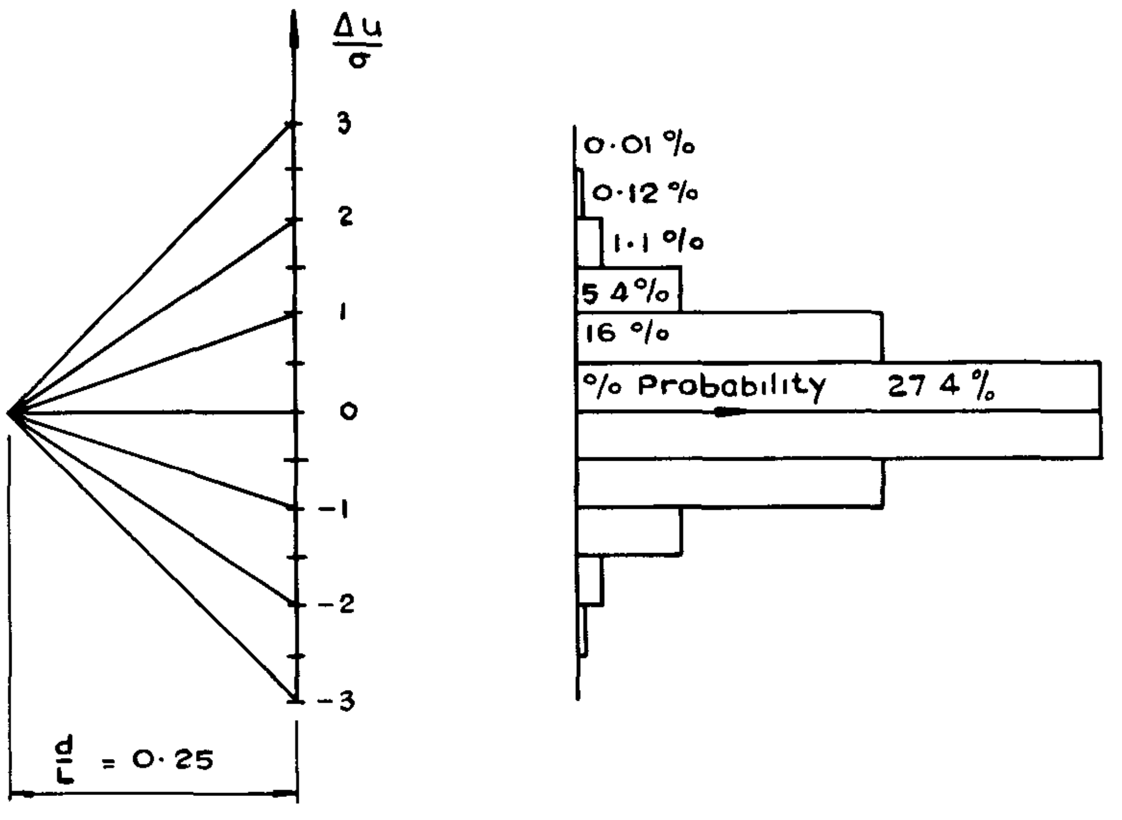


Fig. 5 Probability boundaries for shear in given gradient distance with zero initial gust velocity



a $\frac{d}{L} = 0 \cdot 05$



b $\frac{d}{L} = 0 \cdot 25$

Fig. 6 a & b Histograms for change in gust velocity Δu in distance d when initial gust velocity is random

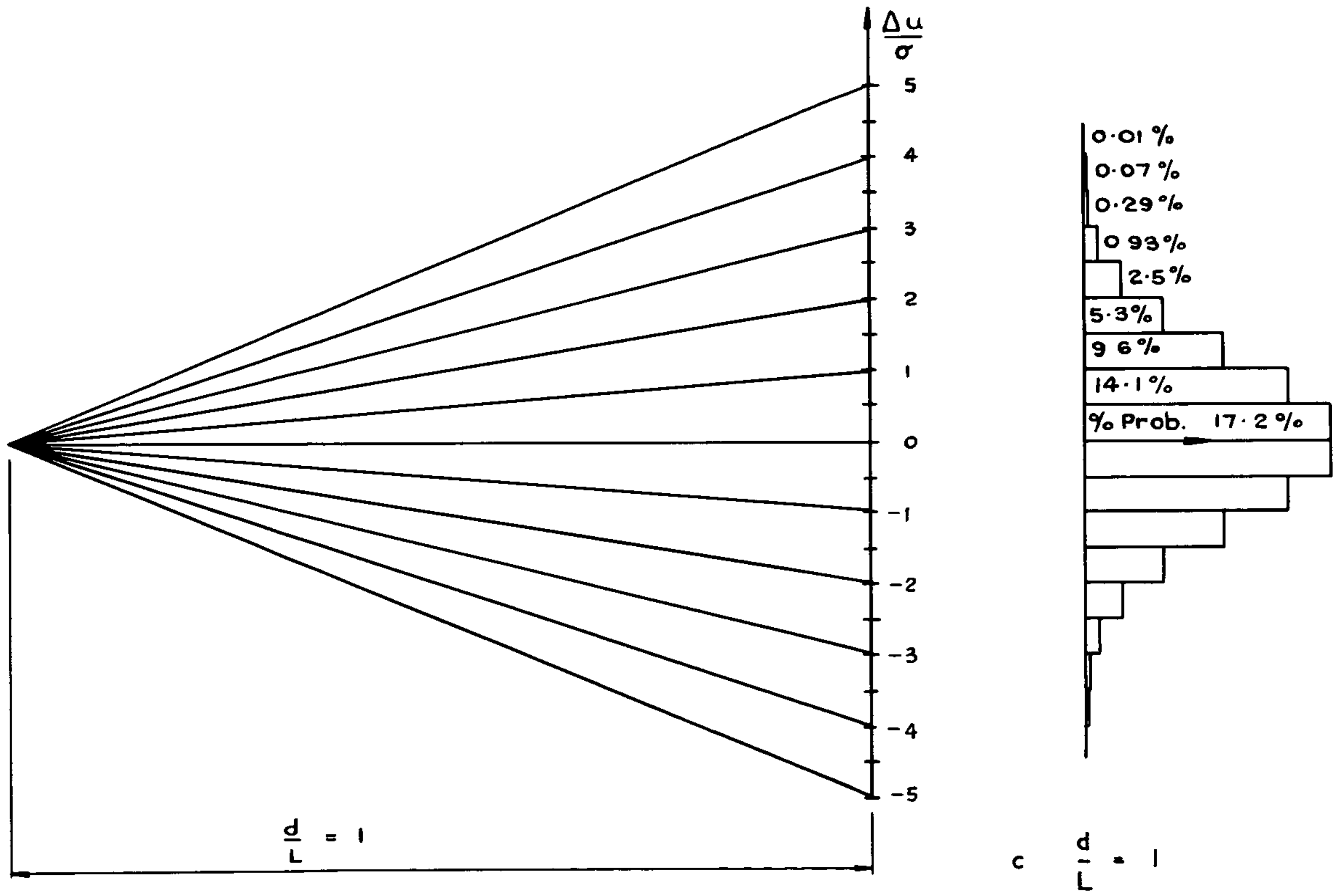


Fig. 6 c Histograms for change in gust velocity Δu in distance d when initial gust velocity is random

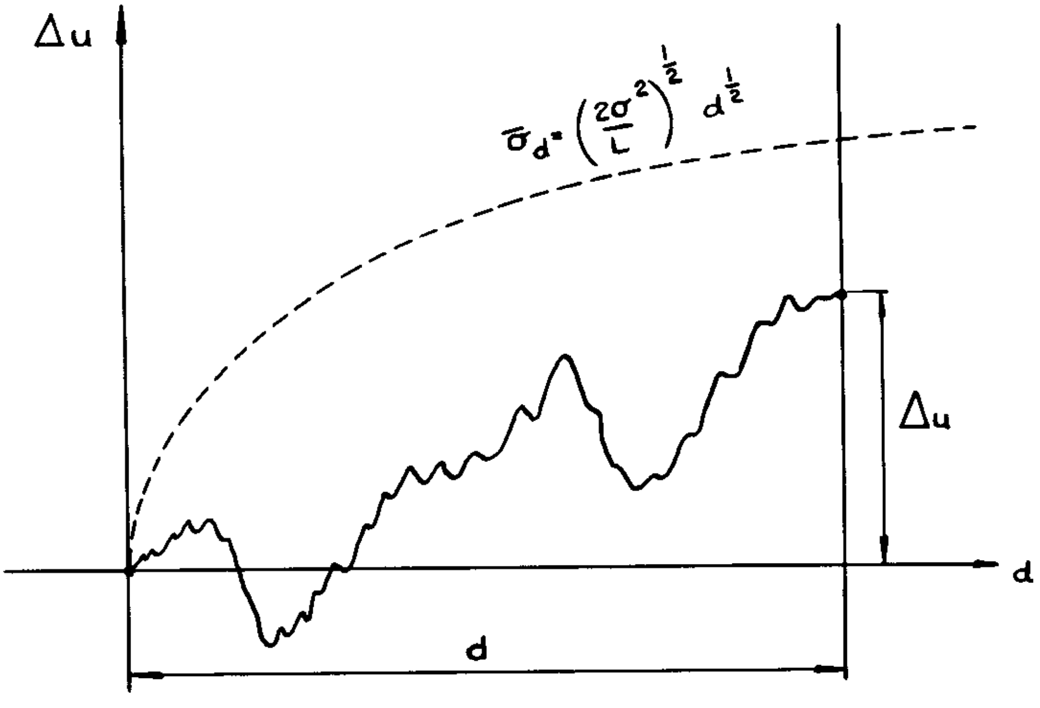


Fig. 7 Illustration of Δu for asymptotic limit of large L

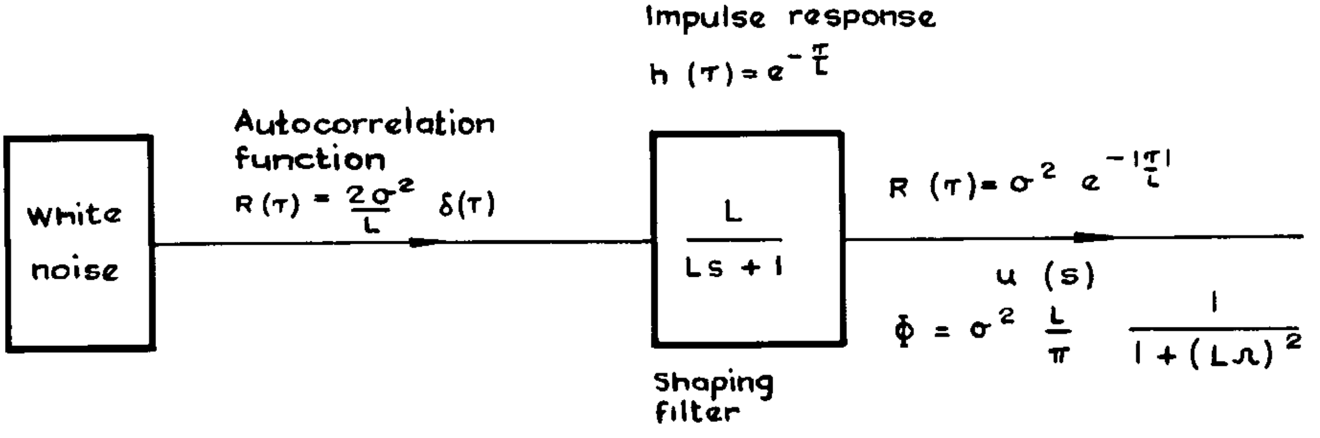


Fig 8 Shaping filter and turbulence model

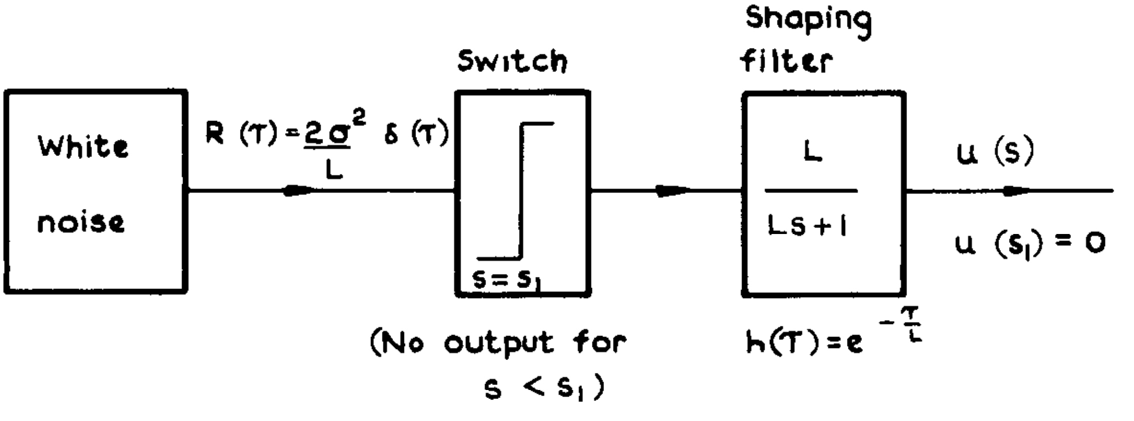


Fig. 9 Shaping filter with switch to give zero initial condition $u(s_1)=0$

Printed in England for Her Majesty's Stationery Office by the Royal Aircraft Establishment, Farnborough. Dd.135645. K.3.

A.R.C. C.P. No.998 551.551 :
June 1967 519.217 :
519.242

Jones, J.G.

GRADIENT PROPERTIES OF A MODEL OF STATIONARY RANDOM TURBULENCE

Histograms for changes in turbulence velocity over given distances are presented, based on a standard power spectral model. It is proposed that these be used as the basis of a comparison between the gradient properties of the model and the measured gradient properties of samples of atmospheric turbulence. An application of the histograms to the gust response of an aircraft with an autothrottle is described.

A.R.C. C.P. No.998 551.551 :
June 1967 519.217 :
519.242

Jones, J.G.

GRADIENT PROPERTIES OF A MODEL OF STATIONARY RANDOM TURBULENCE

Histograms for changes in turbulence velocity over given distances are presented, based on a standard power spectral model. It is proposed that these be used as the basis of a comparison between the gradient properties of the model and the measured gradient properties of samples of atmospheric turbulence. An application of the histograms to the gust response of an aircraft with an autothrottle is described.

A.R.C. C.P. No.998 551.551 :
June 1967 519.217 :
519.242

Jones, J.G.

GRADIENT PROPERTIES OF A MODEL OF STATIONARY RANDOM TURBULENCE

Histograms for changes in turbulence velocity over given distances are presented, based on a standard power spectral model. It is proposed that these be used as the basis of a comparison between the gradient properties of the model and the measured gradient properties of samples of atmospheric turbulence. An application of the histograms to the gust response of an aircraft with an autothrottle is described.

DETACHABLE ABSTRACT CARDS

C.P. No. 998

Published by
HER MAJESTY'S STATIONERY OFFICE

To be purchased from
49 High Holborn, London W C 1
423 Oxford Street, London W 1
13A Castle Street, Edinburgh 2
109 St Mary Street, Cardiff
Brazennose Street, Manchester 2
50 Fairfax Street, Bristol 1
258-259 Broad Street, Birmingham 1
7-11 Linenhall Street, Belfast 2
or through any bookseller

C.P. No. 998

S.O. CODE No. 23-9017-98