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Note on the Lift Slope, and some other Properties, of Delta and Swept-Back Wings

By

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June, 1952

In studying and comparing various theories for the determination of the distribution of loading on wings, Garner[1] has given values for the lift slope of several families of swept-back and delta wings deduced from several different lifting-surface theories. In Fig. 8 of Ref. 1, Garner has plotted these lift slopes as functions of the aspect ratio A, for different values of the angle of sweep. It occurred to the writer to try plotting the ratio of the lift slope to that for elliptic loading instead of the lift slope itself, and when this was done it was noticed that the above ratio was very nearly independent of aspect ratio A, and gave a unique curve for all the available results when plotted against sweepback angle, Λ. The curve is shown in Fig. 2 and it will be seen that none of the points is more than 3 per cent from the mean curve and most are much closer than this. The cases given by Garner cover an aspect-ratio range from 2 to 8 and a sweep range from 20 to 70 deg, as will be seen from Fig. 1, reproduced from his report. The value of the two-dimensional lift slope used in deducing that for elliptic loading at any given aspect ratio was, of course, 2π, since comparison is with potential calculations on wings of zero thickness. In using the mean curve to predict a lift slope for practical purposes it might be more logical to use the most probable value of the two-dimensional lift slope for the case in question rather than the value for an ideal fluid and zero aerofoil thickness.

It has been possible to make some comparisons of the above theoretical deductions with measurement in the Compressed Air Tunnel at high Reynolds numbers, where one could expect a close approximation to potential theory. The cases available are four delta wings and one swept wing in report R. & M. 2871 (Ref. 2), a tapered swept wing on a body, reported in R. & M. 2738 (Ref. 3), and two untapered swept wings of thickness/chord ratios 12 per cent and 9 per cent on a body.

These cases are collected in the Table, the elliptic loading slope having been calculated from a value 2π for two-dimensions and also for a value $5{\cdot}9$, which is about the least value found from C.A.T. tests on straight wings of about 10 per cent thickness at the higher Reynolds numbers. The results are plotted in Fig. 3 for comparison with the mean curve found in Fig. 2, and it will be seen that they lie close to the curve and confirm very well the rate of change with angle of sweep over the range covered. It would therefore appear that this mean curve can be used with some confidence to predict lift slope for a wide range of plan-forms.

While analyzing the C.A.T. results the opportunity was taken to collect also values of the quantity K in the formula

$$C_D = C_{D0} + \frac{K}{\pi A} C_L^2.$$

Most of the results yield very good straight lines when C_D is plotted against C_L^2 as long as the Reynolds number is above, say 5×10^6, but below this Reynolds number it is sometimes impossible to obtain a reasonable slope, as the plot is often a curve, concave upwards, right from $C_L^2 = 0$.

* Previously published as C.P. 127.

The values obtained are also given in the Table, and it will be seen that for most of the cases considered K is about $1 \cdot 10$ for the wings alone and a little higher for wings with body. It appears, therefore, that very little induced-drag penalty is paid apart from that inherent in the low aspect ratio.

Lastly, in making these analyses it was noted that in some cases there were marked scale effects in the C.A.T. tests at the lower Reynolds numbers. This was particularly the case in the tests of the tapered wing on a body, and the curves of Figs. 4, 5 and 6 have been prepared to show how large such scale effects may be in some cases. It will be seen that while the lift curve is substantially the same at all Reynolds numbers, those of drag and pitching moment exhibit considerable variations and the values do not settle down until the Reynolds number is of the order of 5×10^6. The ' straightening ' of the curve of C_D against $C_L{}^2$ as Reynolds number increases is well brought out in Fig. 6. The results on the two untapered wings do not show any such marked scale effects. Unfortunately, the tests on delta wings in Ref. 2 were not carried to low enough Reynolds numbers for the scale effects to be studied in the same way. It was, however, considered worth while to draw attention to the marked scale effects on the tapered wing, because many tests have been made and are being made on wings with considerable sweep and taper and at Reynolds numbers of the order of a million. It is evident that the results must be viewed with some suspicion unless there is evidence that the scale effects are not important.

REFERENCES

No.	Author	Title, etc.
1	H. C. Garner ..	Swept-wing loading. A critical comparison of four subsonic vortex-sheet theories. C.P. 102. July, 1951.
2	R. Jones, C. J. W. Miles and P. S. Pusey	Experiments in the Compressed Air Tunnel on swept-back wings including two delta wings. R. & M. 2871. March, 1948.
3	C. Salter, C. J. W. Miles and Miss H. M. Lee	Tests on a swept-back wing and body in the Compressed Air Tunnel. R. & M. 2738. May, 1950.

TABLE

Case	Aspect ratio (A)	Sweep $\Lambda°$	Experimental lift slope	Ratio $a_0 = 2\pi$	Ratio $a_0 = 5\cdot9$	K from $C_{Di} = \frac{K}{\pi A} C_L^2$
90° deltas (Ref. 2) ..	2·38	37	2·77	0·808	0·839	1·10
	3·04	37	3·06	0·806	0·840	1·08
	3·87	37	3·26	0·787	0·821	1·11
60° delta (Ref. 2) ..	2·31	52·5	2·40	0·712	0·737	1·12
Swept wing (Ref. 2) ..	3·07	45	2·92	0·769	0·798	1·07
Swept and tapered wing on body (Ref. 3)	3·29	42·5	3·10	0·792	0·825	1·18
Swept untapered wings on body						
(a) $t/c = 12\%$ ..	3·04	45	2·93	0·771	0·802	1·15
(b) $t/c = 9\%$ ..	3·04	45	2·88	0·759	0·790	1·15

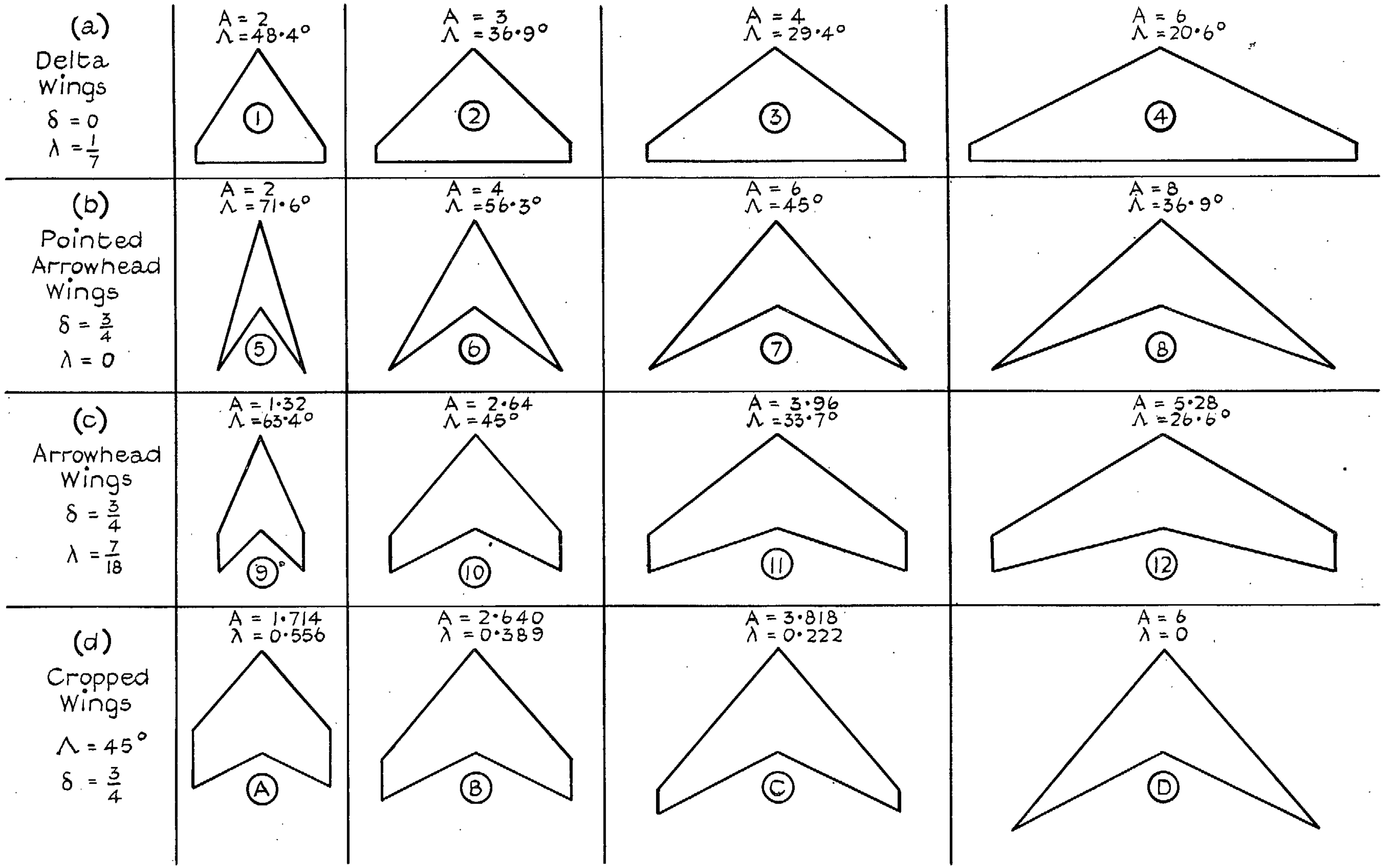
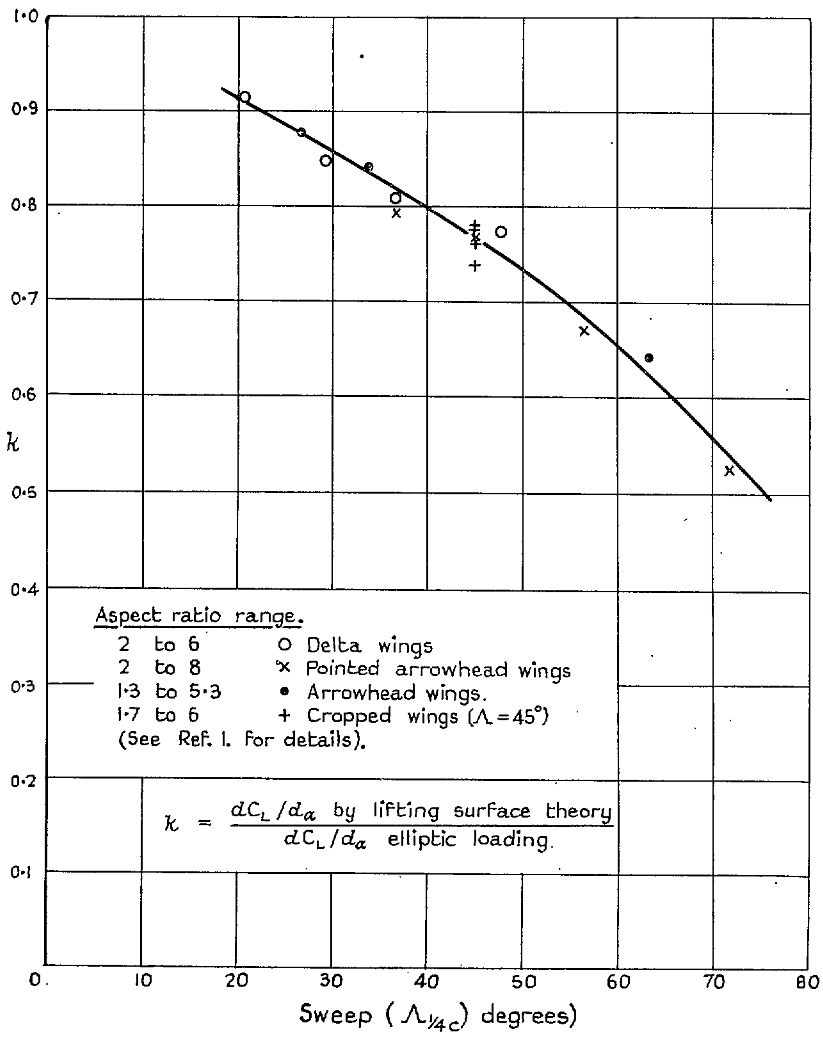
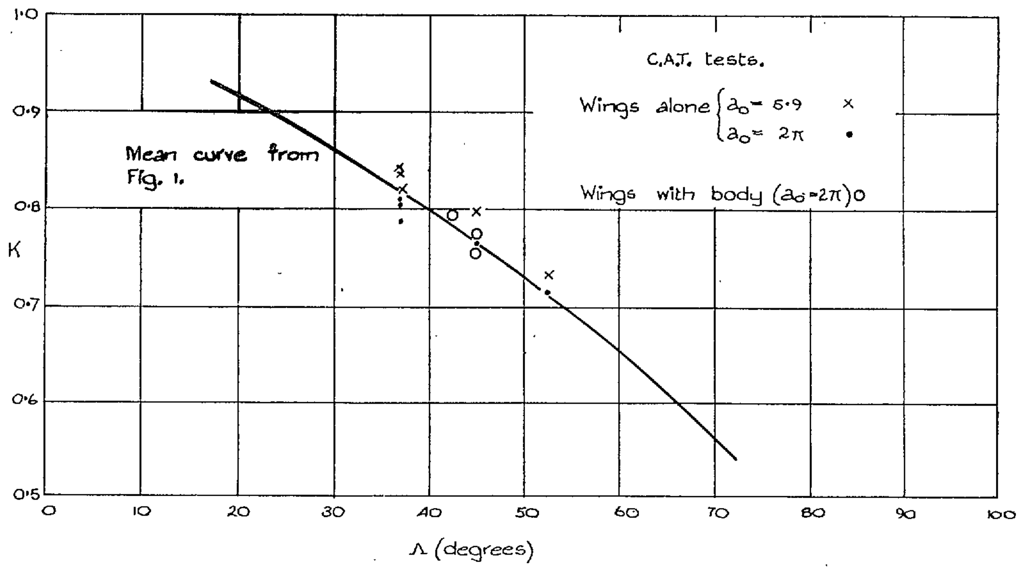


FIG. 1. Four series of swept plan-forms.



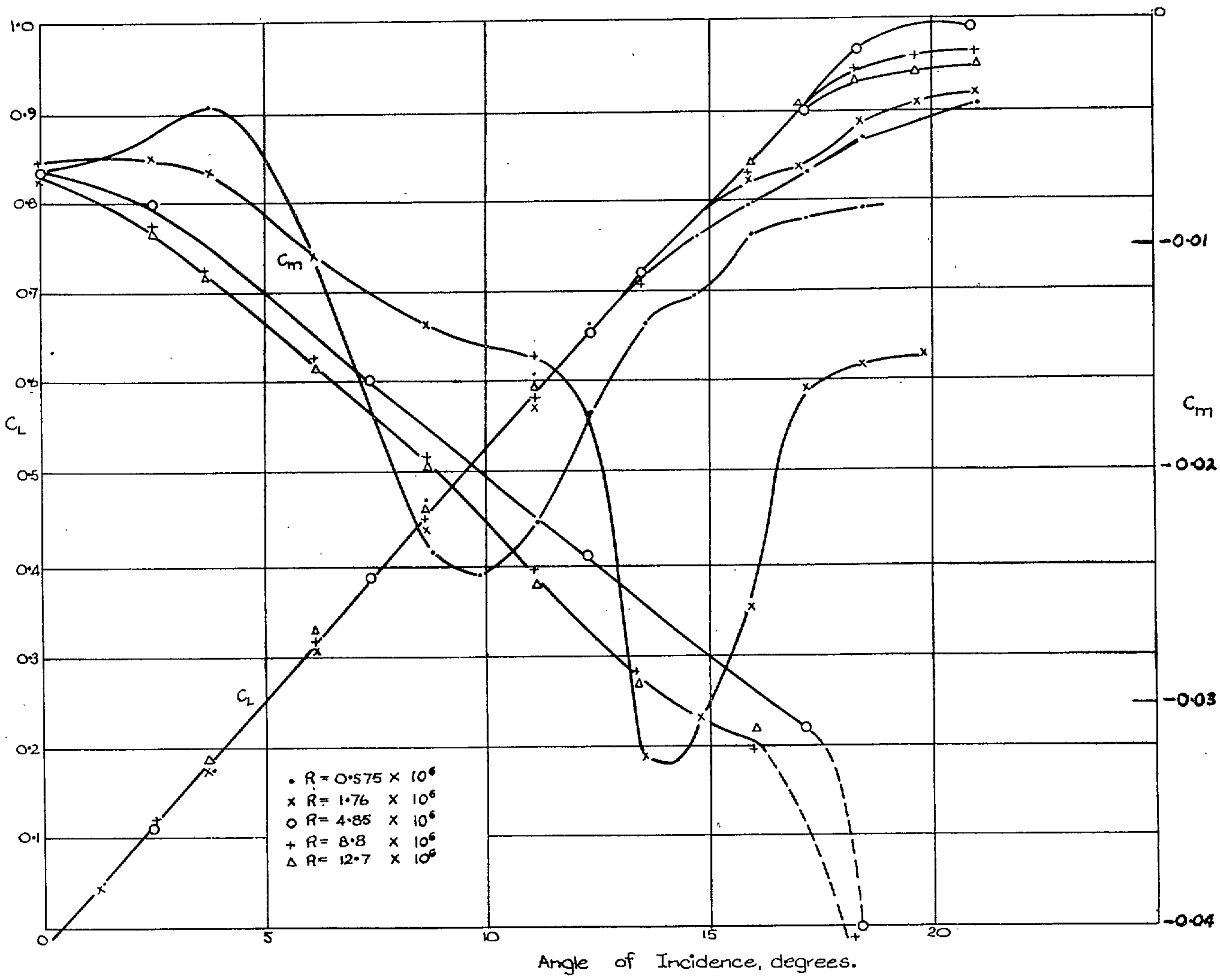
Aspect ratio range.
2 to 6 o Delta wings
2 to 8 × Pointed arrowhead wings
1·3 to 5·3 • Arrowhead wings.
1·7 to 6 + Cropped wings (Λ = 45°)
(See Ref. 1. For details).
k = dC_L/dα by lifting surface theory / dC_L/dα elliptic loading
k
Sweep (Λ_{1/4c}) degrees)

FIG 2.



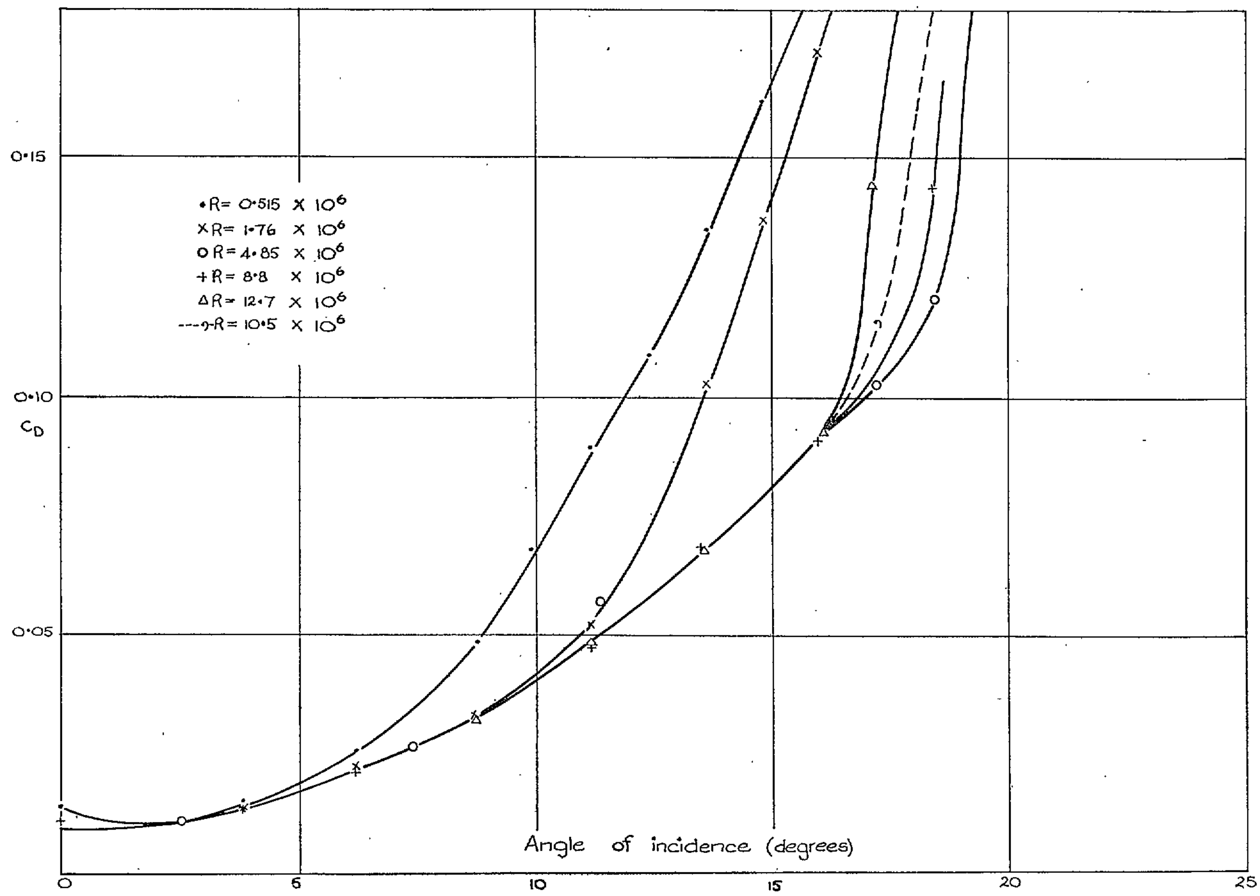
C.A.T. tests.
Wings alone a₀ = 5·9 ×
a₀ = 2π •
Wings with body (a₀ = 2π) o
Mean curve from Fig. 1.
K
Λ (degrees)

FIG. 3.



1·0
0·9
0·8
0·7
0·6
0·5
0·4
0·3
0·2
0·1
C_L
C_m
0
-0·01
-0·02
-0·03
-0·04
0
5
10
15
20
Angle of Incidence, degrees.
• R = 0·575 × 10^6
× R = 1·76 × 10^6
○ R = 4·85 × 10^6
+ R = 8·8 × 10^6
Δ R = 12·7 × 10^6

FIG. 4.



0·15
0·10
C_D
0·05
• R= 0·515 × 10^6
× R= 1·76 × 10^6
o R= 4·85 × 10^6
+ R= 8·8 × 10^6
Δ R= 12·7 × 10^6
---ρ R= 10·5 × 10^6
Angle of incidence (degrees)
0
5
10
15
20
25

FIG. 5.

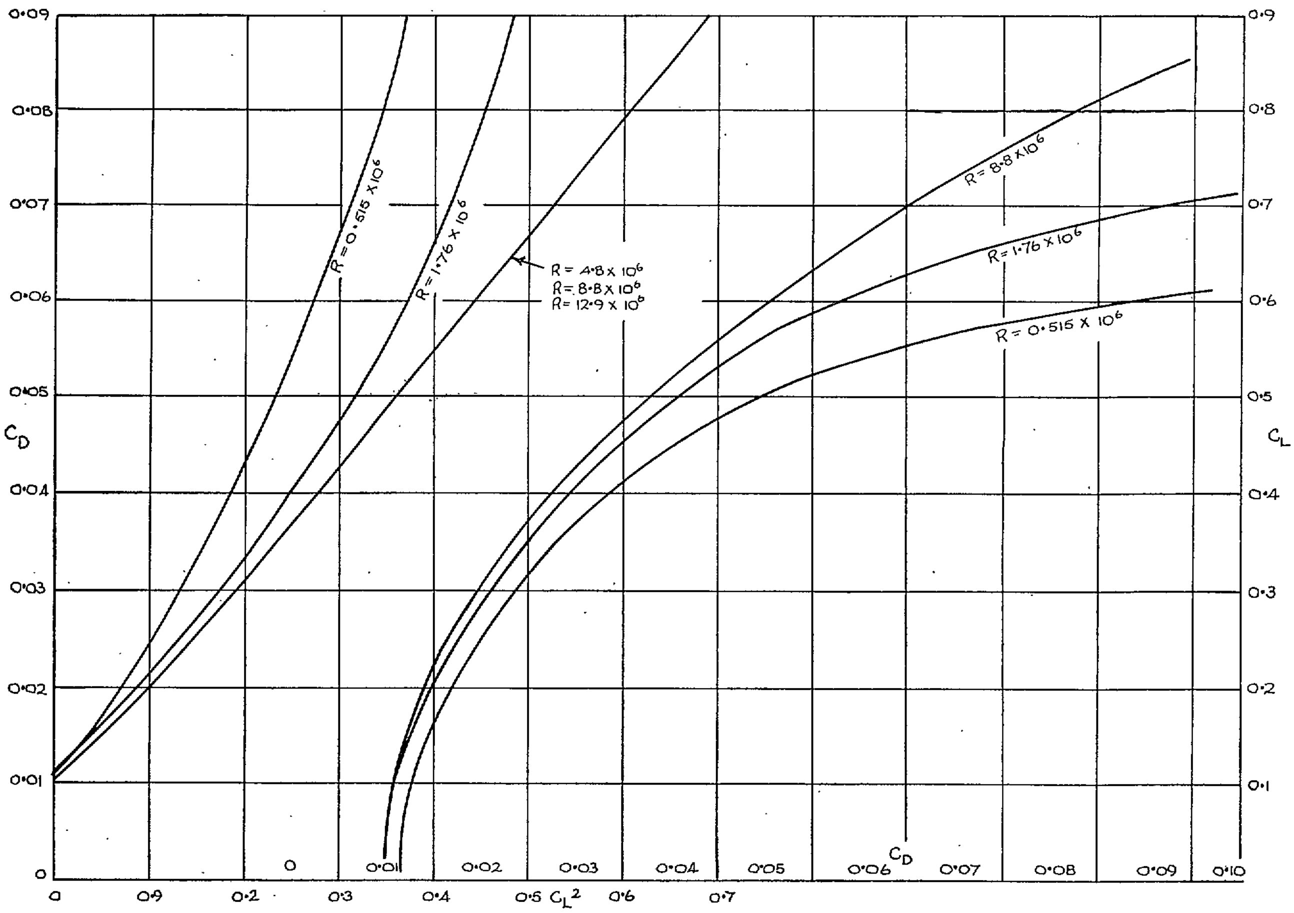


FIG. 6.

(74589) Wt. 53/2293 K.7 12/58 Hw.

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