R. & M. No. 3089
(19,345)
A.R.C. Technical Report

MINISTRY OF SUPPLY

AERONAUTICAL RESEARCH COUNCIL
REPORTS AND MEMORANDA

Some Factors Influencing the Speed of Response of Hydraulic Position Servomechanisms

By

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LONDON: HER MAJESTY'S STATIONERY OFFICE
1958
FIVE SHILLINGS NET

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Communicated by the Director-General of Scientific Research (Air), Ministry of Supply

Reports and Memoranda No. 3089*

August, 1955

Summary.—A study is made of the influence of working pressure, relay torque-arm radius and other design factors on the maximum output speed and velocity constant of a single-stage hydraulic servo for guided missile use. Methods are given for determining the best relay torque-arm radius, which may be applied to valves having non-linear flow and reaction characteristics.

1. *Introduction.*—The analysis given in this paper applies to typical guided-missile control-fin servos in which a hydraulic piston valve is driven by an electro-mechanical proportional relay through a torque arm and strut linkage (Figs. 1 and 2). In the type considered, the reaction or flow force experienced by the valve is used as a hydraulic spring and thus largely determines the characteristics of the combination of relay and valve.

The analysis studies the output oil flow produced by the valve for a given current flow through the relay. This is generally called the sensitivity of the relay and valve combination. The significance of this sensitivity and related parameters may be summarised as follows :

(*a*) The sensitivity as defined above is directly related to the open loop velocity constant of the servo.

(*b*) The analysis may be applied to determining the speed of response of a servo when the demand saturates the amplifier or relay, which is generally the case with step demands.

(*c*) The maximum velocity attainable by the servo sets an upper limit to its band width when subjected to a sinusoidal input.

It has been found experimentally that the length of the relay torque arm has a non-linear effect on the servo performance, and also that, paradoxically enough, in some cases an increase in oil supply pressure to the valve results in a reduced speed of transient response.

This paper attempts to explain these effects and to formulate conditions for achieving a specified sensitivity, only using assumptions which are fully justified by experimental evidence.

2. *Principle of Operation of Relay and Valve.*—Figs. 2 and 3 show the principles governing the action of the relay and valve combination. In Fig. 3 AB represents a force-displacement characteristic of the relay for a constant current, or for saturation conditions. OC is a flow force-displacement characteristic for a valve at a constant supply pressure. The relay and valve

*R.A.E. Tech. Note G.W.378, received 24th June, 1957.

forces are made in opposition, so that their intersection D represents a position of stable equilibrium. From the displacement x of the valve obtained in this way, the flow Q through the valve may be determined. The flow may be regarded as proportional to the response speed of the servo if the following assumptions are fulfilled :

(*a*) The stall torque of the servo is large compared with hinge-moment loads on the actuator shaft.

(*b*) The stall torque is large compared with frictional loads on the actuator shaft.

(*c*) The output acceleration is zero since the inertia of the output is small.

These assumptions are justified by experimental results in which it was found that the servo performance was only slightly modified by the addition of small hinge-moment loads.

Fig. 4 shows an experimental record in which three important points may be noted :

(*a*) The acceleration time of the fin is small.

(*b*) The valve reaches a position of equilibrium and stays there for a considerable part of the total transient time.

(*c*) The fin velocity is constant during this time.

During this constant velocity period, therefore, the servo is effectively working as if on open loop with a constant relay current. The same analysis will therefore apply as for finding the open-loop velocity constant.

3. *Factors Influencing Performance.*—3.1. *Equations for Relay Characteristics.*—From torque displacement curves plotted for typical servo relays, it is found that the relay torque T may be approximately expressed by

$$T = f(i) - k_2\theta \,, \qquad (1)$$

where i = relay control current

k_2 = a constant

θ = armature displacement in radians.

This may be expressed as a force-displacement characteristic, for small values of θ,

$$F = \frac{f(i)}{r} - \frac{k_2 x}{r^2} \,, \qquad (2)$$

where r = radius of relay torque arm

x = linear displacement of valve and linkage.

In equilibrium the relay force is equal and opposite to the valve reaction force R, i.e.

$$\frac{f(i)}{r} - \frac{k_2 x}{r^2} = - R. \qquad (3)$$

3.2. *Valve Reaction Forces Due to Oil Flow.*—In general, unless special measures have been taken to prevent it, the reaction force experienced by a valve tends to close it. Use is made here of what will be called a valve reaction characteristic, which expresses the relationship between valve reaction and displacement for a constant pressure drop across the valve. It may be shown theoretically (*see* Appendix), and demonstrated experimentally (Fig. 5), that at a constant valve opening, the reaction is proportional to the pressure drop. This relation and the reaction characteristic may therefore be combined in the form :

$$R = - Khf(x), \qquad (4)$$

where R = reaction force

K = a constant

h = head across valve

$f(x)$ = function of valve displacement.

In the case where the port area is proportional to the valve displacement, the reaction may be expressed :

$$R = - Bhx , \quad \ldots \quad (5)$$

where B = a constant.

These relationships, and the values of the constants, are developed in the Appendix.

3.3. *Frictional Effects.*—Friction in a hydraulic valve is manifested in several ways. The various effects may be listed as follows :

(*a*) Coulomb friction, independent of valve velocity and opposing its motion. It is generally attributed to the rubbing together of high spots on valve and liner, and sometimes to the presence of dirt in the oil.

(*b*) Viscous friction due to shearing of the thin oil film between the valve lands and the liner. This effect depends on the relative velocity of valve and liner.

(*c*) Viscous drag due to oil leakage past the valve lands. The effect is entirely negligible with the valves considered.

(*d*) 'Stiction' or valve lock[1], due to lateral unbalanced oil forces on the lands, which builds up whilst the valve is stationary, but vanishes as soon as the valve is moved. A similar effect is sometimes caused by dirt in the oil.

(*e*) Viscous drag due to flow of oil along the valve spindle. This effect is very small.

Of the factors governing coulomb friction in the valves considered, very little can be said. It has been found that the magnitude of the force is independent of the oil pressure (this work is described in a paper to be issued later). There is also evidence to suggest that the coulomb friction is related to the area of the valve lands in contact with the bore of the liner.

Viscous friction, under (*b*) above, does not affect the final equilibrium position of the valve, although the time to reach equilibrium may be altered.

Stiction is only encountered if the valve has been allowed some settling time, which in our experience is of the order of a few seconds. Although stiction will undoubtedly be present in a missile before the controls are operated, it is unlikely to occur subsequently during flight, owing to the amount of noise from various sources which keeps the valve continually in motion.

4. *Solution in the Case of Linear Valve Characteristics.*—In this section the pressure drop h is treated as a variable, and the friction is assumed zero.

4.1. *Valve displacement and Volume Flow.*—Using the relations developed in Section 3, it is now possible to find expressions for valve displacement and flow in terms of the system constants :

From equations (3) and (5) we have

$$\frac{f(i)}{r} - \frac{k_2 x}{r^2} = Bhx ,$$

whence

$$x = \frac{f(i)r}{Bhr^2 + k_2} .$$

A1

For a linear valve, the port area open

$$a = k_3 x$$

and the flow $Q = C_D a \sqrt{(2gh)}$ where C_D is the discharge coefficient of the port.

$$= C_D k_3 x \sqrt{(2gh)}$$

$$= \frac{C_D f(i) k_3 \sqrt{(2gh)} r}{Bhr^2 + k_2} .$$

In full, using the value of B obtained in the Appendix, the flow Q is given by

$$Q = \frac{C_D f(i) k_3 \sqrt{(2gh)} r}{(2\rho C_D k_3 \cos 69°) hr^2 + k_2} . \qquad (6)$$

This equation will subsequently be used in the form

$$Q = \frac{A h^{1/2} r}{Bhr^2 + k_2} , \qquad (7)$$

where $A = C_D f(i) k_3 \sqrt{(2g)}$.

We may now, using equation (7), examine the effect of variations in pressure drop and torque-arm radius on the flow.

4.2. *Condition for Maximum Speed of Response.*—Equation (7) may be treated as a surface of the form $Q = f(h, r)$ and examined for stationary points by putting

$$\left. \begin{aligned} \frac{\partial Q}{\partial h} &= 0 \\ \frac{\partial Q}{\partial r} &= 0 \end{aligned} \right\} .$$

These conditions are both satisfied when

$$Bhr^2 = k_2 , \qquad (8)$$

which indicates that the surface (7) has a line maximum, or ridge of constant altitude whose equation is (8) above. Thus, all combinations of h and r satisfying (8) will produce the same response.

The significance of equation (8) becomes evident when interpreted physically. Rewriting, we have $Bh = k_2/r^2$. Now Bh is the slope of the valve-reaction displacement line, and k_2/r^2 the slope of the relay force-displacement characteristic. The condition (8) therefore implies that these two slopes are equal.

It may be shown that since $\partial^2 Q/\partial h^2$ is negative in the neighbourhood of the maximum, for values of h in excess of k_2/Br^2, $\partial Q/\partial h$ becomes negative, and thus an increase in pressure beyond this value results in a reduced speed of response, an effect mentioned earlier.

The maximum value of Q in this case is $Q_{max} = (Arh^{1/2})/(2k_2)$ and it occurs when $r = \sqrt{\{k_2/(Bh)\}}$.

The response is not very sensitive to changes in h and r from the optimum. For example, if the torque-arm radius is made half or double the optimum, then the response is four fifths of the maximum.

4.3. *Conditions for Achieving a Specified Response.*—In practice it will most likely be specified that the maximum (*i.e.*, saturated) speed of the servo shall be not less than a certain value and in some cases a definite limit might be desirable on the maximum speed of response. If we then insert this value, Q_1, say, in equation (7), we have :

$$Q_1 = \frac{Ah^{1/2}r}{Bhr^2 + k_2}; \qquad \text{(7a)}$$

which is a quadratic in $h^{1/2}r$ and may be rewritten, putting $h^{1/2}r = X$:

$$Q_1BX^2 - AX + Q_1k_2 = 0 . \qquad (9)$$

This gives two solutions :

$$\left. \begin{aligned} h^{1/2}r &= X_1 \\ h^{1/2}r &= X_2 \end{aligned} \right\},$$

where X_1, X_2 have the values

$$\frac{A \pm \sqrt{(A^2 - 4Q_1^2Bk_2)}}{2Q_1B} \qquad (10)$$

or, in full :

$$\frac{C_Df(i)k_3\sqrt{(2g)} \pm \sqrt{[\{C_Df(i)k_3\sqrt{(2g)}\}^2 - 8Q_1^2\rho C_Dk_3 \cos 69° k_2]}}{4Q_1\rho C_Dk_3 \cos 69°}$$

These two relations represent contours of constant Q on the surface (7). If h is already fixed, the values of torque-arm radius may be determined.

If possible, it is preferable to use the smaller value of r for reasons discussed in Section 5.2.

Of the possible solutions to equation (9), only positive real values of $h^{1/2}r$ are admissible, since a complex solution implies that $4Q_1^2Bk_2 > A^2$, which means that the flow Q_1 is unattainable. A real negative solution is also meaningless since, as $4Q_1^2Bk_2$ is positive, $\sqrt{(A^2 - 4Q_1^2Bk_2)}$ cannot exceed A.

5. *Solutions for Non-linear Valve-Characteristics.*—5.1. *Solution for Fastest Response.*—In general the oil pressure supply to the servo will have been pre-determined by weight, space and torque requirements, so that having noted the effects described in 4.2 above, the pressure will now be treated as a constant. The valve opening will thus be sufficient to specify the delivery Q.

Equation (2) may be rewritten :

$$r^2F - rf(i) + k_2x = 0, \qquad (11)$$

which may be treated as a quadratic in r, representing a family of relay force-displacement lines. These lines have an envelope satisfied by the condition that equation (11) has equal roots, *i.e.*, by

$$[f(i)]^2 = 4Fk_2x. \qquad (12)$$

The generation of the envelope is shown in Fig. 6.

Referring to Fig. 7, if we erect an ordinate $x = x_1$ it will be seen that the height F_1 at which it meets the envelope is the highest value of F attainable at the opening x_1, and the value of r required is that corresponding to the relay force-displacement line touching the envelope at (x_1F_1).

If we now superimpose a valve reaction characteristic on the diagram, it is evident that the relay line r_1, touching the envelope at (x_1F_1) produces a greater displacement than any other relay line such as r_2.

The procedure, then, to find the value of r for maximum response speed, is as follows. The relay envelope and the reaction characteristic are plotted, and their point of intersection (x_1, F_1) is found. The values x_1, F_1 are then substituted in equation (11) which is then solved for r. The roots are equal, since (x_1, F_1) lies on the envelope, the solution being

$$r = \frac{2k_2x_1}{f(i)}. \qquad (13)$$

This is the value of r for maximum speed of response.

5.2. *Solution for a Specified Velocity Constant.*—The velocity constant is defined as the steady velocity output of the system when working on open loop, caused by unit input signal.

If a velocity constant is specified, then the procedure is slightly modified.

Suppose a flow Q_1 is required to be produced by a relay current i_1.

Referring to Fig. 8, x_2 represents the valve opening to produce a flow Q_1. The point $(x_2\, F_2)$ at which the ordinate $x = x_2$ cuts the valve reaction characteristic is noted, and r may be found by substituting the values $(x_2\, F_2)$ in equation (11) and solving. In this case two solutions r_1 and r_2 are obtained. It will be desirable if possible to use the smaller value of r, since the work available from the relay in moving out to a position x_2, neglecting magnetic effects, will be approximately given by

$$\text{Work done} = \int_0^{x_2} F\, dx$$

$$= \int_0^{x_2} \left(\frac{f(i)}{r} - \frac{k_2x}{r^2}\right) dx$$

$$W = \frac{f(i)x_2}{r} - \frac{k_2x_2^2}{2r^2}. \qquad (14)$$

Inertia does not appear in this expression since any work done in overcoming the valve and relay inertia is recovered when the valve again comes to rest.

The value of (14) decreases with increasing r, since dW/dr is negative for all $r > (k_2x_2)/f(i)$ (The case where $r < (k_2x_2)/f(i)$ does not arise since if this relation were true, x_2 would never be reached).

Thus a more rapid movement of the valve and relay to a position of equilibrium may be expected with the smaller value of r.

A further desirable feature is that the small radius gives a larger force at zero displacement, which is the position in which the valve is most likely to lock due to stiction before real operation starts, since the servo inputs are normally fixed at a datum level before operation, and the valve is centralised.

5.3. *Effect of Coulomb Friction on Response.*—Fig. 9 shows the effect of coulomb friction on the valve opening for a constant current. Since the friction is constant in magnitude and opposes the motion of the valve, and we are considering a displacement of the valve from centre, then the net force opposing motion is the sum of the friction and the reaction.

The valve displacement will thus be smaller in the presence of coulomb friction.

The total force required to move the valve is in this case $\{Khf(x) + F_c\}$, where F_c equals the coulomb friction and $K = 2\rho C_D \cos 69°$. Using the reaction-force-displacement curve the procedure is as before.

5.4. *Case Where Relay Movement is Limited by Stops.*—It is possible that, owing to choking, or flow-force reduction, the valve force cannot balance the saturation relay force. In this case the relay will move until it is limited by its stops, and the valve travel will then be $x = r\theta_{max}$, where θ_{max} is the limit of angular movement of the relay.

The flow in this case is found directly from a flow-displacement curve.

5.5. *Effect of Valve Lap.*—The shapes of the valve characteristics are not in general altered by changes in valve lap, but they are displaced horizontally by an amount δl, where δl is the change of lap. Referring to Fig. 10, A and B are reaction-displacement characteristics for valves which are identical except that the lap of B is greater than that of A by an amount δl. The difference between the displacements at equilibrium is δx.

Let dR/dx be the slope of A and B in the neighbourhood of their intersections with the relay line. The slope of the relay line is $-k_2/r^2$ from equation (2) so that

$$\delta l = \delta x + \delta x\left(\frac{k_2}{r^2}\frac{dx}{dR}\right).$$

Thus

$$\frac{\delta x}{\delta l} = \left(1 + \frac{k_2}{r^2}\frac{dx}{dR}\right)^{-1}.$$

Now as k_2/r^2, and dR/dx are positive, it follows that $\delta x/\delta l < 1$.

Now since the valve displacement is increased by δx, and the lap by δl, the effective change in valve opening is $\delta x - \delta l$.

Now $\quad \dfrac{\delta x - \delta l}{\delta l} = \dfrac{\delta x}{\delta l} - 1.$

But $\delta x/\delta l < 1$, so $(\delta x - \delta l/\delta l)$ is negative.

Therefore, since the flow Q is a function of valve opening, $\delta Q/\delta l$ is also negative, that is, an increase in valve lap results in a reduced speed of response.

5.6. *Low-Speed Performance.*—It is generally desirable that the servo sensitivity at small valve openings should be low, in order that the servo may 'creep' at low speed without oscillating between the limits of the valve lap. For this reason it is often convenient for a valve to have a reaction characteristic which is steeper at small valve openings than at large ones, thus making the sensitivity lower at small openings. It should be emphasized that if the relay and valve are matched to give a maximum response speed, this does not guarantee a satisfactory low-speed performance. In some cases, therefore, a compromise between good high-speed and low-speed performance is necessary.

6. *Discussion.*—With regard to the relative merits of designing for saturation conditions (*i.e.*, maximum valve displacement, as is required for step responses) or for a specified sensitivity at lower relay currents, there are several aspects to consider. In general, it is required of a servomechanism that it has a maximum speed of response which is not less than a specified figure, and that its frequency response should conform to certain limits. It is important to remember that the servo open-loop velocity constant, which to a large extent determines the frequency response, is dependent not only on the relay-valve sensitivity but on amplifier gain

as well, whereas the maximum response speed is solely determined by the valve and relay, since the system is then saturated. This argues in favour of designing for saturation conditions, since the velocity constant may still be altered within limits without changing the maximum speed. On the other hand, it may be argued that, in following the above procedure, too high a sensitivity may be obtained at small openings, thus leading to poor ' creep speed ' performance, especially if the valve lap is large.

A further factor which should not be ignored is the effect of the relay torque arm on the damping of the relay and valve. In general, a small torque arm gives a high damping factor of the relay and valve combination.

In estimating response speeds, the results are valid except in cases where the output load torque approaches the actuator stall torque and for systems where the gain around the feed-back path is low, in which case the system never saturates. If the system is sufficiently linear, the problem may then be solved by standard transient-response theory.

7. *Conclusions.*—By considering the equilibrium conditions for a valve and relay combination, relations have been derived in which the flow from the valve for a given relay current is expressed in terms of pressure drop, relay torque arm, and other system parameters. The flow is given by the equation

$$Q = \frac{C_D f(i) k_3 \sqrt{(2gh)} r}{(2\rho C_D k_3 \cos 69°)\, hr^2 + k_2} . \qquad (6)$$

The results are useful in estimating the velocity constant, step response and bandwidth of a servo.

In the case of a valve with linear reaction and flow characteristics, it is found that in order to achieve maximum sensitivity and output speed, the following relation between pressure and torque-arm radius must be satisfied :

$$Bhr^2 = k_2. \qquad (8)$$

This implies that in order to obtain maximum sensitivity the slopes of the relay force and reaction force-displacement curves must be made equal by adjustment of the torque-arm radius and supply pressure. The maximum sensitivity is obtained when

$$r = \sqrt{\left(\frac{k_2}{Bh}\right)} .$$

If, as is usually the case, the pressure is pre-determined by external factors, any required sensitivity is in principle attainable, provided it does not exceed the physical limits of the system such as choking in the valve. Thus, any given relay and valve may be matched to give a required sensitivity by adjustment of the torque arm only.

In order to study the effect of various parameters on the performance of non-linear valves, a graphical technique is used. If in this case it is required that the valve shall deliver a maximum flow Q_1 for a relay current i_1, the intersection of the envelope $[f(i_1)]^2 = 4Fk_2x$ and the reaction characteristic of the valve is determined graphically, and the required torque arm is defined by the point of intersection. A similar technique is used to find the torque arm for sensitivities less than the maximum.

The effects of coulomb friction and increased valve lap have been examined and in both cases the sensitivity is reduced. This is also borne out by experimental evidence.

Acknowledgement.—The author is indebted to Mr. G. T. Eynon for some of the experimental data on which the assumptions made in this analysis are based.

LIST OF SYMBOLS

a		Area of port opening
A	$=$	$C_D f(i) k_3 \sqrt{(2g)}$
B	$=$	$2\rho C_D k_3 \cos 69°$
C_D		Discharge coefficient of port
F		Force
F_c		Force to overcome coulomb friction of valve
g		Gravitational constant
h		Drop of pressure head through valve
i		Relay current
k_2		Slope of relay torque-angular displacement line
k_3		Port area exposed per unit valve displacement
l		Valve lap
M		Mass flow rate of fluid
Q		Volume flow rate of fluid
r		Radius of relay torque arm
R		Valve reaction force
T		Relay torque
V		Fluid velocity at port
W		Work done
x		Valve displacement
θ		Angular displacement of relay
ρ		Density of working fluid

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No.	Author	Title, etc.
1	D. C. Sweeney	Preliminary investigation of hydraulic lock. *Engineering.* p.513. 26th October, 1951. p.580. 9th November, 1951.
2	Shih-Ying Lee and J. F. Blackburn ..	Contributions to hydraulic control; I. Steady-state axial forces on control-valve pistons. *Trans. A.S.M.E.* p.1005. August, 1952.

APPENDIX

Derivation of Expression for Valve Reaction Force

It has been shown by Lee and Blackburn[2] that for a fixed valve opening, the reaction or flow force experienced by the valve is equal to the axial component of the rate of change of fluid momentum across the valve. If their assumption is accepted that the fluid stream leaving a square-edged port is at 69 deg to the valve axis, then the reaction force is given, neglecting inlet momentum, by

$$R = -MV \cos 69° \qquad \text{(A.1)}$$

(The negative sign is in agreement with the convention of equation (1)),

where M = mass flow rate of fluid

V = fluid velocity at port.

It is permissible to neglect inlet momentum since the fluid velocity along the inlet passages should, in a good design, be much less than that through the control port.

It should be noted that the flow régime does not necessarily remain the same over a large range of openings and that for high rates of flow the reaction may be less than that given by (A.1) above, since the outlet angle may no longer be 69 deg.

In the general case, the port area open at a valve displacement x may be written $a = f(x)$.

Assuming a coefficient of velocity of unity for flow through the port, the efflux velocity will be $V = \sqrt{(2gh)}$ and the mass flow M will then be $M = \rho C_D f(x)\sqrt{(2gh)}$.

From equation (A.1) the reaction is now given by

$$R = -\rho C_D f(x) 2gh \cos 69°. \qquad \text{(A.2)}$$

If the relation between port area and valve displacement is linear we may put $f(x) = k_3 x$, where k_3 is a constant with dimensions of length. The linearised expression for valve reaction is then

$$R = -\rho C_D k_3 x 2gh \cos 69°. \qquad \text{(A.3)}$$

Dividing by g to obtain units consistent with (2) and putting $2\rho C_D k_3 \cos 69° = B$, we obtain the convenient form

$$R = -Bhx. \qquad \text{(A.4)}$$

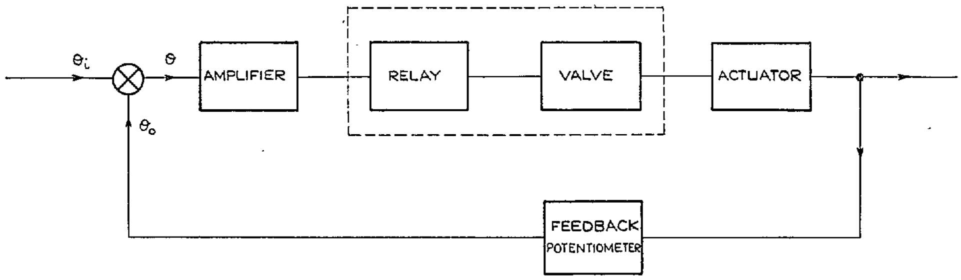


FIG. 1. Block diagram of hydraulic servomechanism.

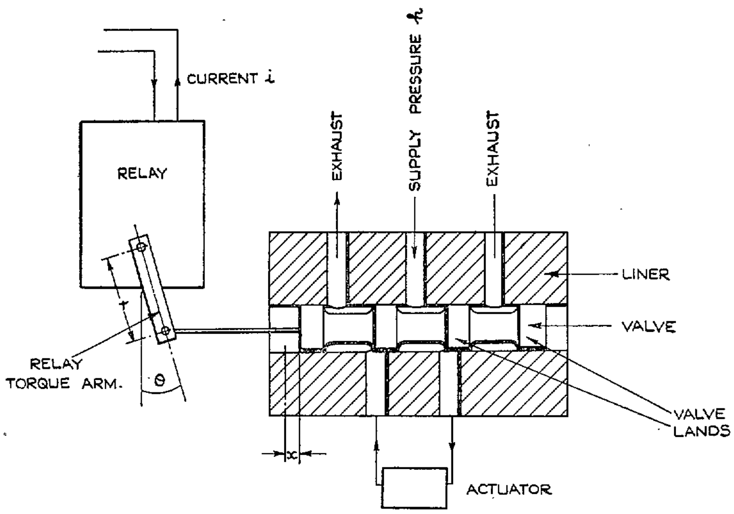


FIG. 2. Arrangement of valve and relay combination.

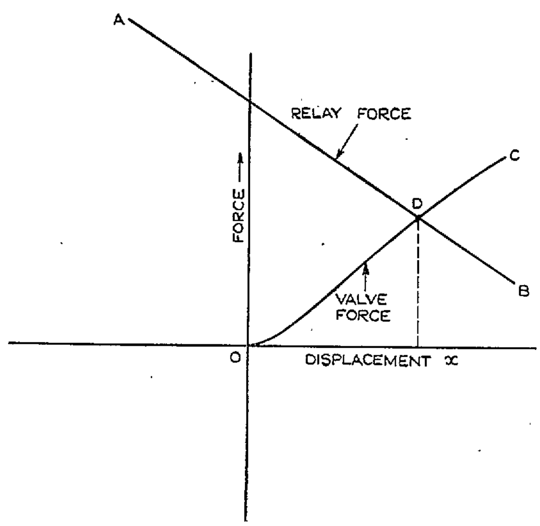


FIG. 3. Principle of equilibrium of relay and valve.

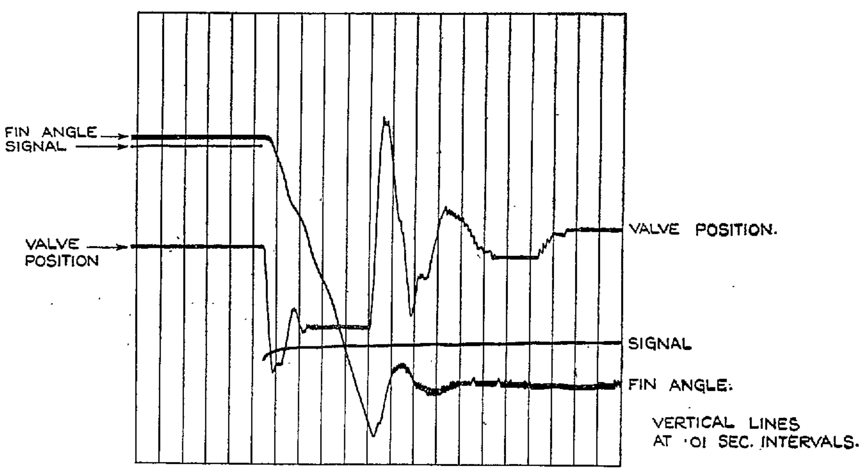


FIG. 4. Record showing constant-velocity condition during response to step function.

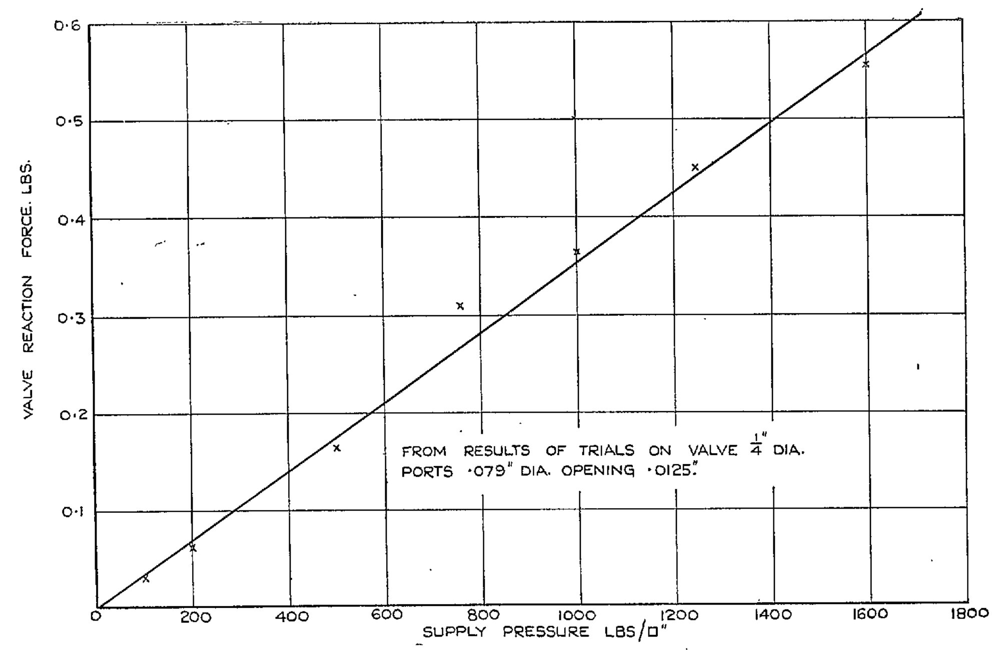
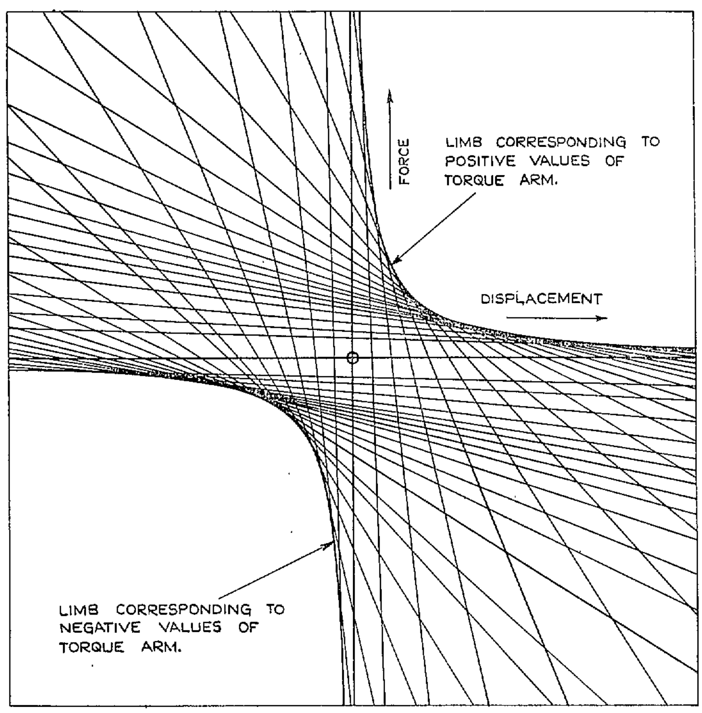


FIG. 5. Graph showing proportionality between valve reaction and supply pressure.



THE STRAIGHT LINES ARE CONSTANT CURRENT FORCE-DISPLACEMENT LINES FOR THE RELAY AND ARE OF THE FORM $\tau^2 F - \tau f(i) + k_2 x = 0$ WHERE τ = CONST.
THE ENVELOPE DESCRIBED BY VARYING τ HAS THE EQUATION $[f(i)]^2 = 4Fk_2x$

FIG. 6. Family of relay force-displacement characteristics for a constant current, showing envelope.

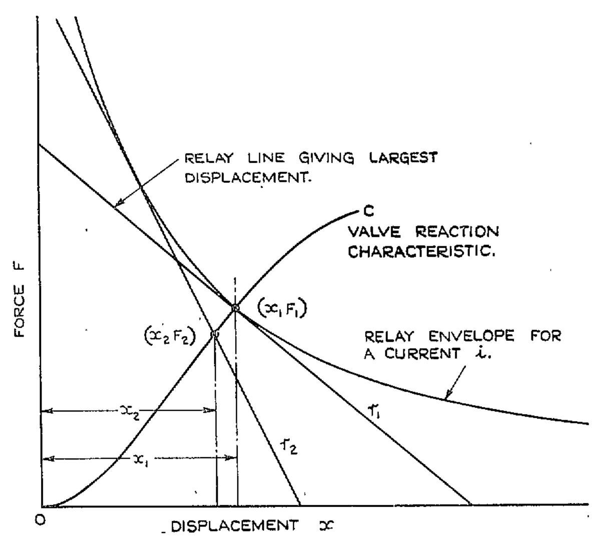


FIG. 7. Diagram illustrating condition for maximum displacement of valve.

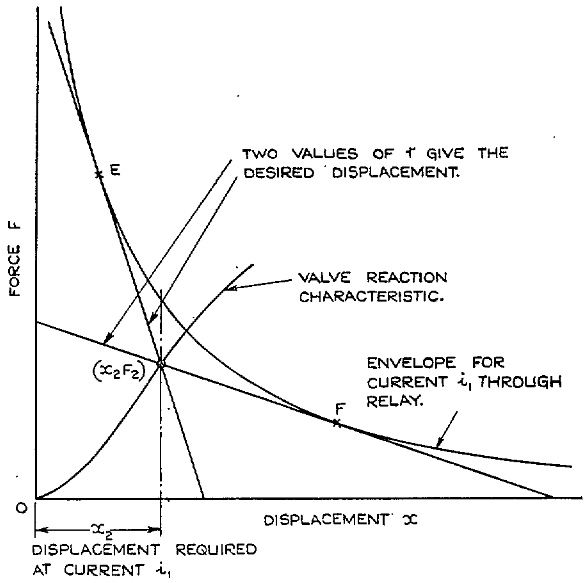


FIG. 8. Method of finding torque-arm radius when a velocity constant is specified.

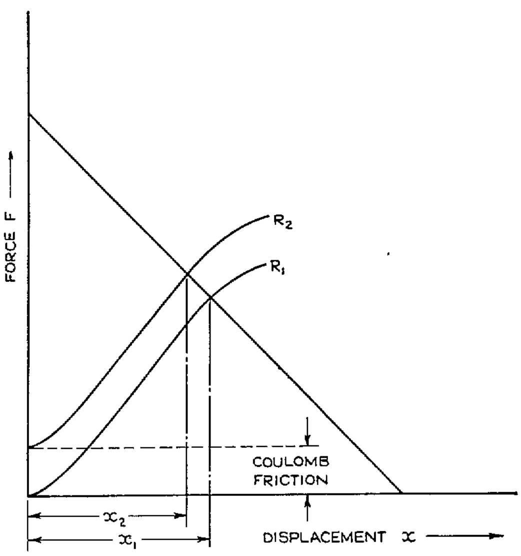


FIG. 9. Reduction in response due to coulomb friction.

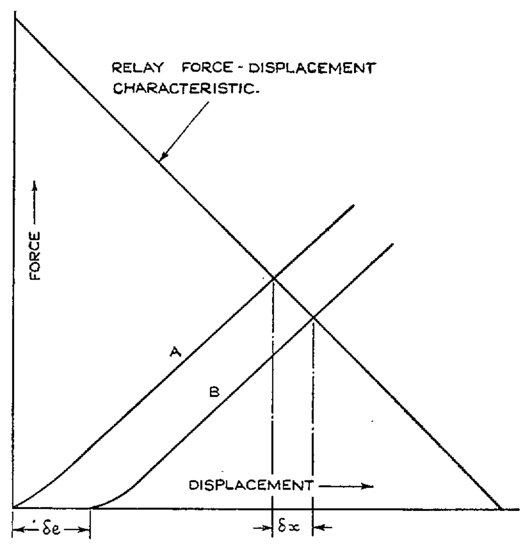


FIG. 10. Effect of increasing valve lap.

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