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The Effect of the Ground on a Helicopter Rotor in Forward Flight

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Summary.—An approximate method of estimating the effect of the ground on the lift of a rotor at any forward speed is described. Flight tests on several different helicopters show reasonable agreement with the theory. Curves are given showing the relation between thrust, height, speed and power.

The theory has been extended to include the effect of a variation in blade loading coefficient and shows that, within the range that this parameter takes on present single-rotor helicopters, the effect is small.

The effect of fitting flat surfaces beneath the rotor is also considered and it is shown that the ratio of the thrust in the ground cushion to the thrust clear of the ground, at constant power, is greater for the winged helicopter, but that for the same rotor operating at constant power, the clean helicopter will have the higher net thrust for speeds less than the stalling speeds of the attached surfaces.

1. *Introduction.*—The thrust of a helicopter rotor, operating at constant power, increases as it approaches the ground. This has been determined experimentally by Zbrozek (Ref. 1) on a helicopter at zero forward speed. Helicopter pilots have reported that the effect of the ground decreases rapidly as air-speed is increased. Such an effect is of considerable importance when determining the performance of a helicopter operating at zero ground speed in different wind speeds, a flight condition which is becoming of increasing importance in some operational roles.

In this report a simple theoretical analysis of the variation of the ground effect with air speed has been made and compared with the results of flight tests on several single-rotor helicopters. Empirical charts have been developed to enable this effect to be determined for a variety of speeds and heights near to the ground. The theory has been extended to show the effect of blade loading coefficient, and the variation produced when flat surfaces are fitted beneath the rotor.

2. *Theoretical Analysis.*—2.1. *Ground Effect at Zero Air Speed.*—The analysis is based on the method of images. Betz (Ref. 2) replaced the rotor by a sink and the ground by an image sink of equal strength placed below the rotor at a distance equal to twice the height of the rotor. This, however, led to results which were in direct opposition to experimental evidence, since the flow pattern beneath a lifting rotor resembles a source rather than a sink. In the present theory the rotor and its image have been represented by sources of strength $Av_i/4\pi$. The induced velocity v_i at the rotor is therefore reduced by the presence of the image; this change is a measure of the ground effect.

The rotor power equation for a helicopter flying at zero air speed may be written as $(EP - P_R)\sqrt{\sigma} = Tv_i$, where T is the thrust and $(EP - P_R)$ is the effective power at the rotor.

* A.A.E.E. Report Res./288, received 14th September, 1955. A.A.E.E. Airborne Experimental Division Note 18, received 15th October, 1955.

In this work equivalent air speeds and powers have been used so that estimates of the ground effect in various atmospheric conditions can be made directly.

Therefore at constant power:

$$\frac{T_g}{T_\infty} = \frac{v_{i\infty}}{v_{ig}}, \quad \dots \quad (1)$$

where T_g, v_{ig} , and $T_\infty, v_{i\infty}$, refer to conditions inside and outside the ground cushion respectively.

The velocity induced at the centre of the rotor by its image is δv_i , where $\delta v_i = (Av_i)/(16\pi Z^2)$, A is the area of the rotor and Z the height of the rotor above the ground.

Assuming that v_i and δv_i are constant over the disc, then $v_{ig} = v_i - \delta v_i$, and equation (1) becomes:

$$\frac{T_g}{T_\infty} = \frac{1}{1 - \frac{R^2}{16Z^2}}, \quad \dots \quad (2)$$

where R is the radius of the rotor. Plotting this result (Fig. 4. $V_i/v_T = 0$) for different values of Z/R , the variation of T_g/T_∞ with height is found for the case of zero forward speed. Comparing this with results given in Ref. 1, general agreement as shown by Fig. 4 is found. No indication of the effect of blade loading coefficient is given by this formula but the method has been extended by use of blade-element theory to cover this case (section 3).

2.2. Variation of Ground Effect with Forward Speed at Constant Power.—When the helicopter develops forward air speed the flow through the rotor does not resemble the uniform distribution of a simple source but is markedly directional, the mean direction of the flow being given approximately by $\theta = \tan^{-1} V_i/v_i$, the flow normal to this direction being approximately zero. An approximation to the flow through the rotor in forward flight has been made by replacing the rotor with a source, the strength of which has the form $(Au_i/4\pi^2) \cos^n \alpha$ where α is measured relative to the resultant flow and $n \geq 1$ (Fig. 1).

The strength of the source is determined by equating the flow over a sphere of unit radius to the output of the source in unit time. However, the equivalent figure for the rotor corresponding to this source is not known, so it is assumed to be of the form $kAu_{i\infty}$, where k is an arbitrary constant which is adjusted to give equivalence in the case $V_i = 0$ with the results of section 2.1. Performing the integration over unit sphere it is found that the lowest value of n for which the strength is not zero is $n = 2$. The change in downwash at the rotor due to the image is then:

$$\delta u_i = \frac{3kAu_i \cos^2(\theta + i)}{16\pi Z^2} \quad \dots \quad (3)$$

The rotor-power equation for a helicopter flying at speed V_i with a disc incidence i can be written $(EP - P_R)\sqrt{\sigma} = T_\infty u_{i\infty} = T_g u_{ig}$, where $u_{i\infty} = V_i \sin i + v_{i\infty}$. Writing $u_{ig} = u_{i\infty} - \delta u_i$ and neglecting i in comparison to θ , the equation corresponding to equation (2) is:

$$\frac{T_g}{T_\infty} = \frac{1}{1 - \frac{1}{16} \left(\frac{R}{Z}\right)^2 \left\{ \frac{1}{1 + \left(\frac{V_i}{v_i}\right)^2} \right\}}, \quad \dots \quad (4)$$

in which it is assumed that $k = 1/3$ to obtain agreement with equation (2) for $V_i = 0$.

For very small values of Z/R large errors are introduced by replacing the rotor with a point source, but this region is of no practical interest due to the height of the fuselage. Equation (4) is plotted in Fig. 4 and shows the rapid fall off in ground effect with increase in airspeed at all heights. The overall picture of T_g/T_∞ against speed and height is given in Fig. 5.

2.3. *Ground Effect on Power Required to Maintain Constant Thrust.*—Let the power $(EP - P_R)_1$ be required at the rotor to maintain a helicopter of weight W in a given flight condition outside the ground cushion. If this helicopter is now moved into the ground cushion, then the weight of the helicopter can be increased to $(T_g/T_\infty)W$ while still maintaining the same flight condition. If $(EP - P_R)_2$ is the power required to support the helicopter at a weight of $(T_g/T_\infty)W$ outside the ground cushion, then the ground effect on power at constant thrust is given by $(EP - P_R)_1/(EP - P_R)_2$. Using the values of (T_g/T_∞) given by equation (4), this expression can be calculated by any performance estimation method (e.g., Ref. 3). Typical results are plotted in Fig. 2. This figure shows that the ground effect on power decreases with increase in forward speed at a constant height, resulting in a demand for more power to maintain height. The overall effect on a typical curve of power required to maintain height with increase of speed is shown in Fig. 3. From this figure it is seen that at a very low height ($Z/R \approx 0.6$) the power required to maintain level flight increases up to approximately 10 knots since the ground effect decreases more rapidly than translational lift develops. Helicopter pilots have noted that at heights and low speeds of this order the helicopter tends to sink to the ground if forward speed is developed while still maintaining hovering power.

3. *Estimation of the Effect of Blade Loading Coefficient on the Ground Effect with Forward Speed at Constant Power.*—An indication of the effect of blade loading coefficient on the ground effect can be obtained by starting from the blade-element formula for the thrust of a rotor with b blades set at a collective pitch of θ and rotating at an angular velocity of Ω , then:

$$T_\infty = \frac{1}{2} \rho abc \Omega^2 R^3 \left(\frac{\theta}{3} - \frac{\lambda}{2} \right), \quad \dots \quad (5)$$

where a is the slope of the blade lift curve, c the mean chord and λ the inflow ratio, $u/\Omega R$.

Using the induced-flow model described in section 2.2 the changes in induced velocity will appear in equation (5) as a change in λ of $\delta\lambda = \delta u/\Omega R$. Thus, using the suffices in their usual sense it is found, substituting for $\delta\lambda$ from equation (3), that:

$$T_g/T_\infty = 1 + \frac{1}{4} \frac{\rho abc \Omega^2 R^3}{T_\infty} \frac{\lambda}{16 \left(\frac{Z}{R} \right)^2 \left\{ 1 + \left(\frac{V_i}{v_i} \right)^2 \right\}} \quad \dots \quad (6)$$

An expression for λ can be obtained either from momentum theory or more exactly in the low-speed region from Ref. 3 in the form $\lambda = \eta (V_T/\Omega R) (1/\sqrt{\sigma})$, where $v_T = \sqrt{\{T_\infty/(2\pi\rho_0 R^2)\}}$ and η is a non-dimensional parameter depending on forward speed V_i . Equation (6) can then be re-written as:

$$\frac{T_g}{T_\infty} = 1 + 0.25\eta a \sqrt{\sigma_{0.7}} / \sqrt{(C_T/\sigma_{0.7})} \left[\frac{1}{16 \left(\frac{Z}{R} \right)^2 \left\{ 1 + \left(\frac{V_i}{v_i} \right)^2 \right\}} \right] \quad \dots \quad (7)$$

The effect of varying $C_T/\sigma_{0.7}$ by varying T and Ω on one helicopter at a constant $\sigma_{0.7}$ has been shown in Figs. 6 and 7, from which it will be seen that an increase in $C_T/\sigma_{0.7}$ results in a decrease in the ground effect. This is in agreement with the effect found in Ref. 1, although the above equation indicates that T_g/T_∞ depends on $(C_T)^{1/2}/\sigma_{0.7}$ rather than $(C_T/\sigma_{0.7})^{1/2}$ as shown in that work. Equations (4) and (7) are the same if the second and higher terms in the expansion of $[1/\{16(Z/R)^2\}\{1 + (V_i/v_i)^2\}]$ can be neglected and if $0.25\eta a \sigma_{0.7} = \sqrt{C_T}$. For the *Hoverfly* as tested in Ref. 1 this equation corresponds to a $C_T/\sigma_{0.7}$ of 0.1276 which is greater than the value of 0.09 achieved during the tests. Hence the theoretical curves of equation (4) give a slightly optimistic answer for the *Hoverfly*; reference to Fig. 4 shows this to be the case (curve and points $V_i/v_T = 0$).

4. *The Variation of the Ground Effect when Flat Surfaces are Fitted Beneath the Rotor.*—When a helicopter is fitted with lifting surfaces, it is no longer possible to consider only the effect of the ground on the rotor and neglect the corresponding change in flow conditions over the body.

A wind-tunnel result for a model with these dimensions is in good agreement with the theoretical value at $V_i/V_T = 0$ and $Z/R = 0.45$. For comparison the corresponding curves from Fig. 4 have been plotted on Fig. 10, thus enabling the effect of changing S/A from 0 to 0.157 to be appreciated.

5. *Flight Tests.*—The helicopters used in the tests were a *Sikorsky S.51*, *Dragonfly Mk. 1*, and various marks of *Sycamore*. The performance in the ground cushion was measured by two distinct methods.

5.1. *Constant Weight.*—The helicopter was hovered at zero ground speed and the power increased in steps. The height of the rotor was measured photographically and the power required at this height was obtained from boost and engine-speed measurements using the manufacturer's power curves. In order to find the true air speed of the helicopter at all heights the wind gradient was obtained by flying at approximately constant air speed with and against the wind at a given height and measuring the air-speed by the aircraft instruments and the ground speed photographically. The aircraft instrument error was thus eliminated and the wind speed obtained accurately to within 1 ft/sec. This was repeated at several heights and was used in conjunction with a wind measurement made 6 ft above the ground to give the wind gradient. Further tests were conducted in zero wind conditions, flying the aircraft in the ground cushion at measured ground speeds, to obtain the effect of forward speed. The results obtained by the above method were plotted as a function of $(EP - P_R)_g / (EP - P_R)_\infty$ against V_i/v_T and Z/R . Mean curves were drawn, and then converted to the more usual form of T_g/T_∞ against V_i/v_T and Z/R by the method described in section 2.3.

5.2. *Constant Power.*—The helicopter was loaded to maximum weight with lead weights which could be jettisoned in flight. The aircraft was hovered at constant power and weights jettisoned, the new height being measured photographically. It was hoped by this method to make the machine hover at all heights up to 100 ft without altering the power setting, but due to movement of the centre of gravity as the weights were jettisoned, this was not possible. The climb was therefore performed in stages, each stage starting with the helicopter fully loaded and with a power setting which maintained a height just below that reached in the previous test. In this way heights up to 100 ft were obtained. The experiments were repeated with the helicopter in forward flight, the air speed being obtained as indicated in section 5.1.

The results obtained in these tests were plotted as T_g/T_∞ against V_i/v_T and Z/R on the same figure as the T_g/T_∞ curves obtained in section 4.1. Mean curves (Figs. 8 and 9) were then drawn through these points. These experimental points have also been plotted in Fig. 4 to give an indication of the accuracy of the theory.

In this test the blade loading of the helicopters varied as the ballast was jettisoned. The maximum variation in the blade loading coefficient during any one experiment corresponded to a decrease in $C_T/\sigma_{0.7}$, of the order of 4.5 per cent. This resulted in an error which can be estimated from Figs. 6 and 7 and from which it can be seen that the error is comparable to the experimental accuracy.

6. *Conclusions.*—The flight results agree fairly well with the theory at high and low values of V_i/v_T and for values of Z/R greater than 0.6, but there is an indication that initially the ground effect falls off more rapidly with increasing speed than predicted. An estimate of the ground effect for any single-rotor helicopter with a blade loading coefficient of the order of 0.12 can be obtained from the empirical curves given in Figs. 2, 8 and 9. The effect of changing the blade loading coefficient is shown in Figs. 6 and 7.

Fig. 3 shows the effect of the ground on a typical curve of power against level flight speed and demonstrates that for a critical height and for very low speeds the helicopter speed is almost independent of the power; below this height the helicopter may require increased power for increasing speed up to about 10 knots.

If lifting surfaces are fitted beneath the rotor of a helicopter, then the vertical and slow-speed performance outside the ground cushion is decreased due to negative lift, but the performance inside the ground cushion, measured relative to the machine's performance outside the ground cushion is appreciably improved. For direct comparison, however, it is found that at a given power the helicopter without lifting surfaces has the greater lifting capacity at all heights and speeds up to the stalling speed of the fixed surfaces.

Attention is now being given to the case of hovering over water and the work has been extended to include the ground effect on tandem-rotor helicopters.

LIST OF SYMBOLS

A	Disc area
b	Number of rotor blades
$c_{0.7}$	Chord at 0.7 radius
C_n	Normal force coefficient of a flat plate
$C_T =$	$T/\sigma\rho_0\pi R^2(\Omega R)^2$. (Thrust coefficient)
$C_T/\sigma_{0.7}$	Blade loading coefficient
E	Ratio of effective power at the rotor to total engine power
i	Rotor disc incidence to flight path, positive upward from the disc
k	Constant defined in section 2.2
P	Engine power
P_R	Power required to rotate the rotor
R	Rotor radius
S	Equivalent flat-plate area to surface placed beneath rotor
T	Rotor thrust
T_∞	Rotor thrust away from the ground
δT_∞	Negative lift of body away from the ground
$[T_\infty]$	Net lift of rotor-wing combination away from the ground
T_g	Rotor thrust in the ground cushion
δT_g	Negative lift of body inside the ground cushion
$[T_g]$	Net lift of rotor-wing combination inside ground cushion
u	Total velocity of flow normal to rotor disc = $(v + V \sin i)$
$u_i =$	$u\sqrt{\sigma}$
δu_i	Change in u_i due to presence of the ground
$u_{i\infty}$	Value of u_i away from the ground
u_{ig}	Value of u_i in ground cushion
v	Velocity of flow induced by the rotor normal to the rotor disc
$v =$	$v\sqrt{\sigma}$

LIST OF SYMBOLS—*continued*

$v_{i\infty}$	Value of v_i away from the ground
v_{ig}	Value of v_i inside the ground cushion
$v_T =$	$\sqrt{(T/2\pi\rho_0R^2)}$
V	Aircraft speed
$V_i =$	$V\sqrt{\sigma}$
Z	Height of rotor hub above the ground
α	General angle measured from mean direction of flow when in forward flight
η	Non-dimensional constant defined in section 3
$\theta =$	$\tan^{-1} V_i/v_i =$ mean direction of flow under rotor
θ_0	Mean blade pitch angle
λ	Inflow ratio $= u/\Omega R$
ρ	Air density
ρ_0	Air density at sea level for I.C.A.N. conditions
σ	Relative air density
$\sigma_{0.7} =$	$bc_{0.7}/1.4R =$ solidity taken at 0.7 radius
Ω	rotor angular velocity

REFERENCES

<i>No.</i>	<i>Author</i>	<i>Title, Etc.</i>
1	J. K. Zbrozek	Ground effect on the lifting rotor. R. & M. 2347. July, 1947.
2	A. Betz	The ground effect on lifting propellers. N.A.C.A. T.M. 836. April, 1937.
3	A. L. Oliver	The low speed performance of a helicopter. C.P. 122. May, 1952.
4	A. Fage and F. C. Johansen	On the flow of air behind an inclined flat plate. R. & M. 1104. February, 1927.

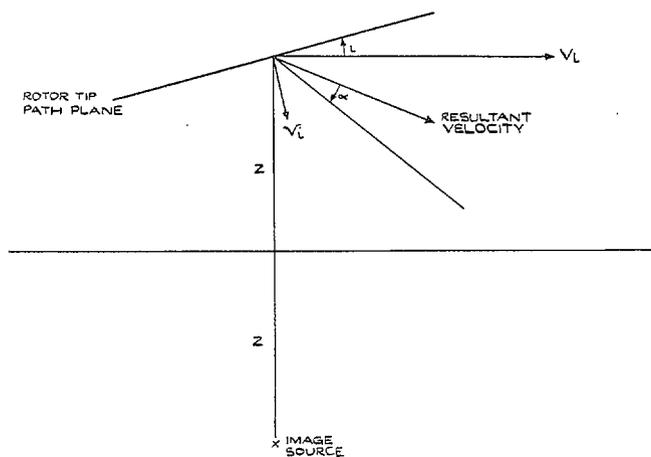


FIG. 1. Representation of the ground effect by an image source.

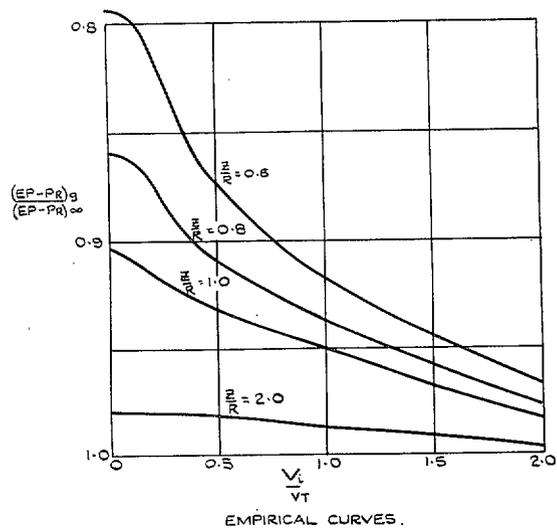


FIG. 2. Variation of power with forward speed at a given height.

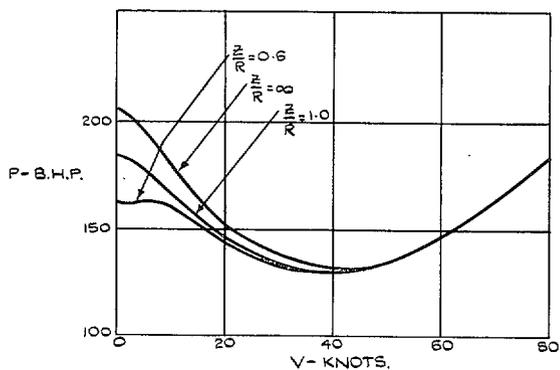


FIG. 3. Typical variation of power required to maintain a given height in forward flight.

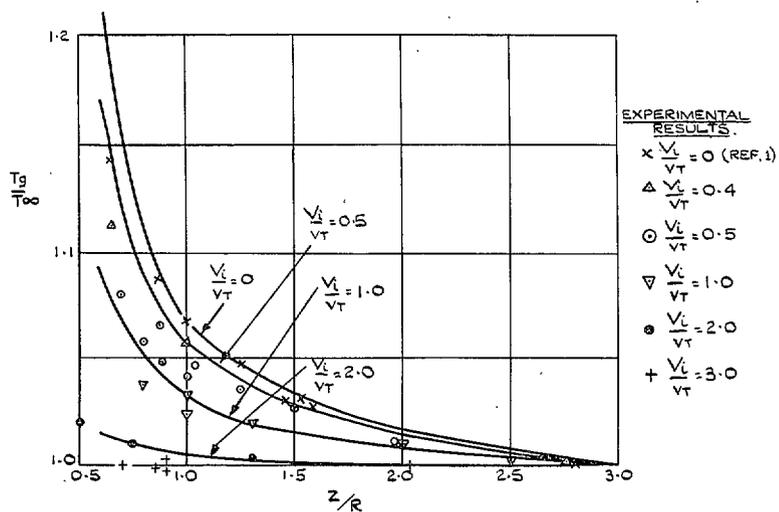


FIG. 4. Variation of thrust with height at a given air speed. (Theoretical curves.)

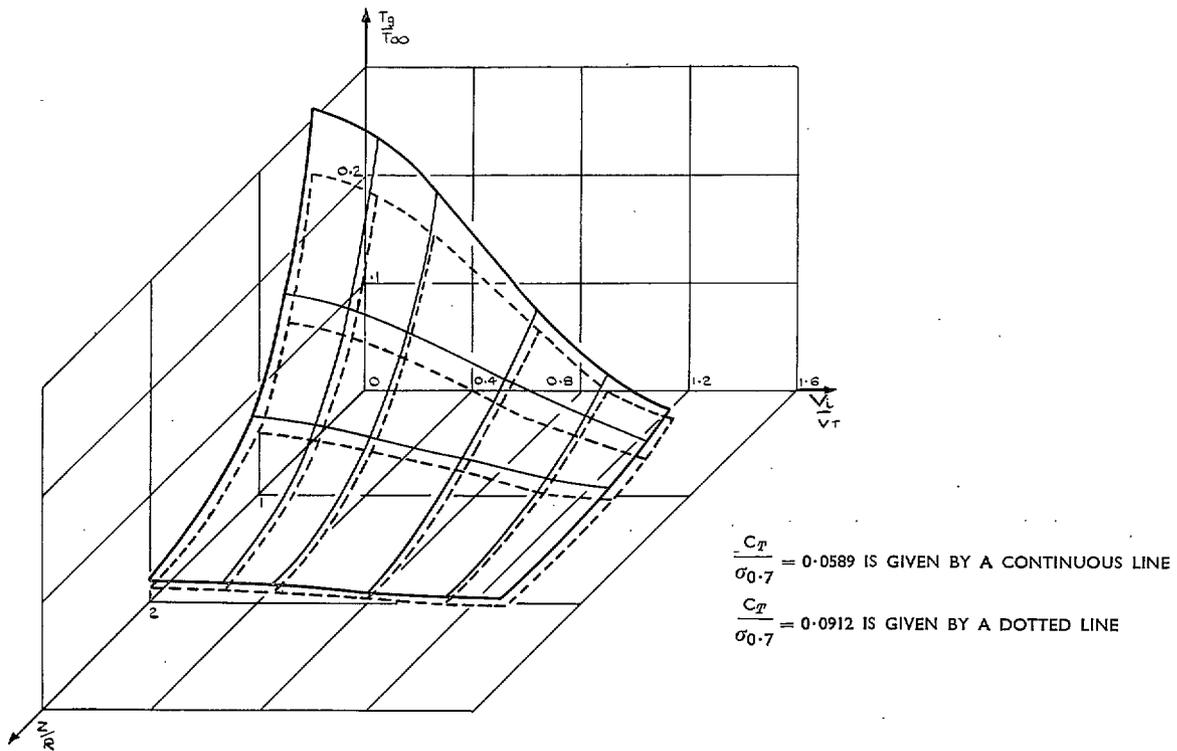


FIG. 5. Variation of thrust with height, speed and blade loading.

	SYCAMORE.			
$\frac{Cl}{C}$	0.0589	0.0692	0.0777	0.0912
W_c	4600	5400	4600	5400
z	287	287	250	250

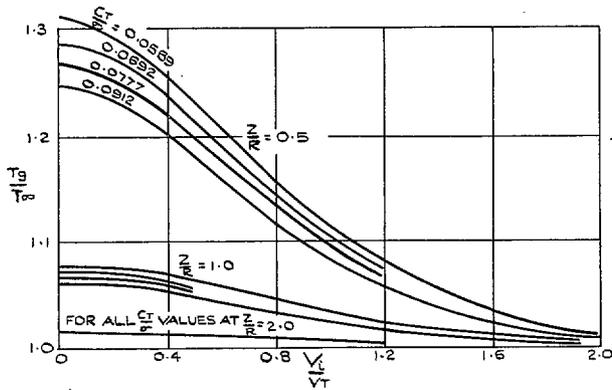


FIG. 6. Variation of T_g/T_∞ with speed and blade loading coefficient at a given height. (Theoretical curves.)

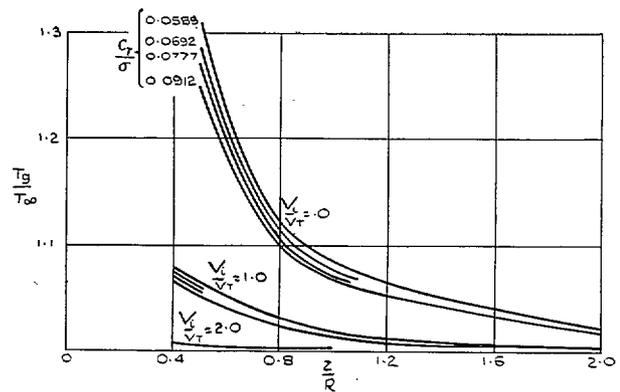


FIG. 7. Variation of T_g/T_∞ with height and blade loading coefficient at a given speed. (Theoretical curves.)

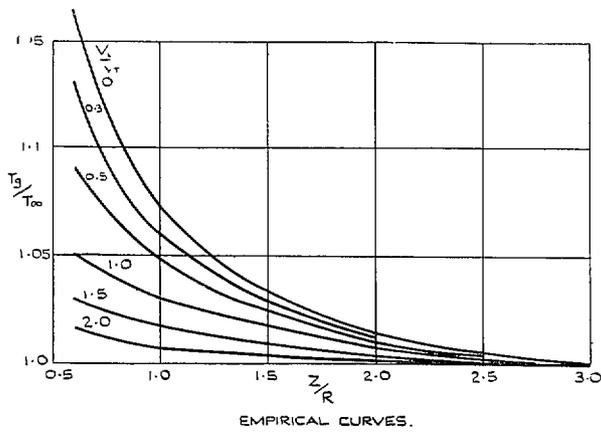


FIG. 8. Variation of thrust with height at a given forward speed.

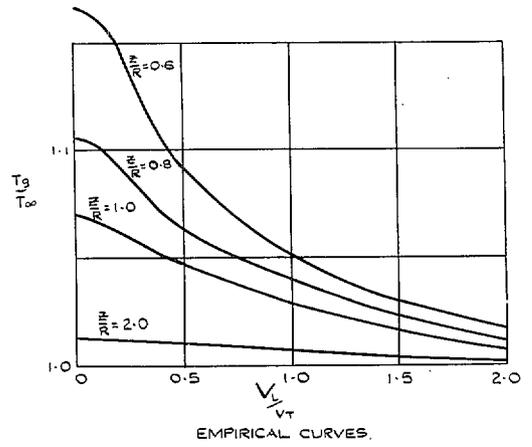


FIG. 9. Variation of thrust with forward speed at a given height.

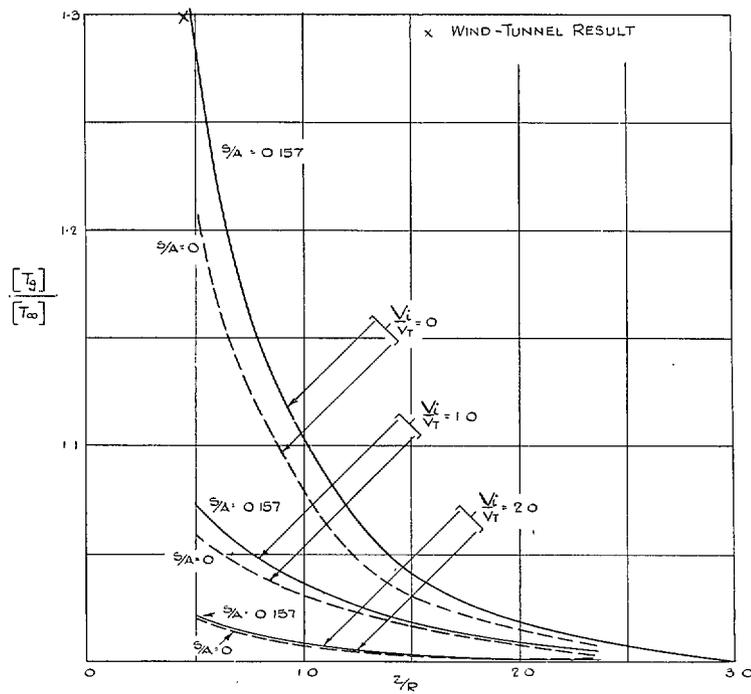


FIG. 10. The effect of the ground on a lifting rotor at constant power with and without stub wings.

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