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Second Harmonic Control on the Helicopter Rotor

By

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Summary.—The application of second harmonic control on a helicopter rotor causes a redistribution of the loading over the disc. This can be utilised to postpone the forward speed limitations imposed by stalling of the retreating blade.

This report develops the theory for second harmonic control. The resultant flapping motion and subsequent incidence distribution depend mainly on blade inertia number. A practical check on the flapping with a full-scale rotor on a testing tower gave excellent agreement with the theory.

1. *Introduction*.—The ordinary rotor with flapping hinges and cyclic pitch control maintains a roughly uniform lift round the disc. As the helicopter forward speed increases, the resultant airflow over the retreating blade decreases and a higher incidence is required. However, stalling incidence is soon reached and, as a consequence of the vibration which blade stalling produces, a limitation is imposed on the forward speed of the helicopter. The stage in helicopter development has now been reached where this limitation gives a lower speed than that otherwise attainable with the available engine power.

One of the possible methods for postponing this limitation is the use of second harmonic control to redistribute the loading over the disc. In this scheme, the loading on the retreating and advancing sides could be reduced and compensated by an increased loading on the fore and aft sectors. By this means, a much more even distribution of incidence on the retreating side is achieved and the stalling limitation of forward speed is postponed to a higher tip speed ratio.

It is fairly easy to devise a mechanism for applying this second harmonic cyclic pitch to the blades. Any form of swashplate, rotating at 3Ω in the same direction or at Ω in the opposite direction in relation to the rotor speed Ω, could give the appropriate application of control. This, of course, means additional complication in the rotor head; further gears, bearings, etc., would be required, adding to a system which is already regarded by many as too complicated mechanically. Also, the second harmonic pitch has to be operated by a fluctuating torque (since the control is not at the fundamental rotor frequency) of amplitude $3\sqrt{(A_2^2 + B_2^2)}I\Omega^2$, where I is the moment of inertia of the blade about its longitudinal axis. Taking a typical example with a control amplitude of 6 deg, the fluctuating torque is equivalent to a pitching-moment coefficient on the blade of $\pm 0{\cdot}01$. In addition to this loading on the swashplate and control links, there will also be a tendency to cause a twisting of the blade. The purpose of the present report is to develop the theory and to investigate the advantages to be gained.

2. *Theory*.—A rotor of radius R is assumed to have a forward velocity V and rotational velocity Ω. The axes of reference are taken, through the rotor centre, parallel and perpendicular to the mean tip-path plane. This definition is similar to that used in general helicopter work, but the mean tip-path plane is taken to allow for the deviations from this due to the second harmonic flapping motion.

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The component of the forward velocity parallel to the mean tip-path plane is

$$\mu \Omega R\,.$$

The velocity through the disc, perpendicular to the mean tip-path plane, is

$$\lambda \Omega R\,.$$

At the present stage of the theory, the usual assumption is made that λ is constant over the disc.

The velocities of the air with respect to a blade element distant $r = xR$ from the rotor axis are as follows.

The velocity parallel to the mean tip-path plane and perpendicular to the blade (chordwise) is

$$(x + \mu \sin \psi)\Omega R\,. \qquad \qquad (1)$$

The velocity perpendicular to the mean tip-path plane and to the blade (through the disc) is

$$(\lambda + \beta\mu \cos \psi + \frac{\dot{\beta}}{\Omega} x)\Omega R \qquad \qquad (2)$$

where β is assumed to be small, so that $\sin \beta$ can be replaced by β and $\cos \beta$ by unity.

Defining flapping angle in the usual way and considering terms only as far as the second harmonic

$$\beta = a_0 - a_1 \cos \psi - b_1 \sin \psi - a_2 \cos 2\psi - b_2 \sin 2\psi \qquad \qquad (3)$$

where, by definition of the axes as above, a_1 and b_1 are zero but in order to make the theoretical treatment more general they may be retained in the form given. Differentiating,

$$\frac{\dot{\beta}}{\Omega} = + a_1 \sin \psi - b_1 \cos \psi + 2a_2 \sin 2\psi - 2b_2 \cos 2\psi \qquad \qquad (4)$$

and

$$\frac{\ddot{\beta}}{\Omega^2} = + a_1 \cos \psi + b_1 \sin \psi + 4a_2 \cos 2\psi + 4b_2 \sin 2\psi\,. \qquad \qquad (5)$$

Hence, the velocity through the disc, by combining equations (2), (3) and (4) is given by

$$[\lambda + \mu \cos \psi\,(a_0 - a_1 \cos \psi - b_1 \sin \psi - a_2 \cos 2\psi - b_2 \sin 2\psi)$$
$$+ x(a_1 \sin \psi - b_1 \cos \psi + 2a_2 \sin 2\psi - 2b_2 \cos 2\psi)]\Omega R\,. \qquad \qquad (6)$$

The blade pitch setting at any azimuth position can be expressed in the form

$$\vartheta = \vartheta_0 - A_1 \cos \psi - B_1 \sin \psi - A_2 \cos 2\psi - B_2 \sin 2\psi\,. \qquad \qquad (7)$$

The effective incidence of the blade element is

$$\alpha = \vartheta_0 - A_1 \cos \psi - B_1 \sin \psi - A_2 \cos 2\psi - B_2 \sin 2\psi - \frac{\lambda + \beta\mu \cos \psi + \frac{\dot{\beta}}{\Omega} x}{x + \mu \sin \psi} \qquad \qquad (8)$$

or, using the expansion given by equation (6):

$$\alpha = \vartheta_0 - A_1 \cos \psi - B_1 \sin \psi - A_2 \cos 2\psi - B_2 \sin 2\psi$$
$$- [\lambda + \mu \cos \psi \, (a_0 - a_1 \cos \psi - b_1 \sin \psi - a_2 \cos 2\psi - b_2 \sin 2\psi)$$
$$+ x(a_1 \sin \psi - b_1 \cos \psi + 2a_2 \sin 2\psi - 2b_2 \cos 2\psi)]/[x + \mu \sin \psi] . \quad .. \quad (9)$$

The aerodynamic force acting on the blade element $cR\,dx$ is

$$dF = \tfrac{1}{2}\rho ac\Omega^2 R^3 (x + \mu \sin \psi)^2 \alpha \, dx . \quad .. \quad (10)$$

The corresponding moment with respect to the flapping hinge is

$$dM = xRdF$$
$$= \tfrac{1}{2}\rho ac\Omega^2 R^4 (x + \mu \sin \psi)^2 \alpha x \, dx . \quad .. \quad (11)$$

Integrating along the blade and taking limits $x = 0$ and B to allow for the tip loss

$$M = \int_0^B \tfrac{1}{2}\rho ac\Omega^2 R^4 x (x + \mu \sin \psi)^2 \alpha \, dx \quad .. \quad (12)$$

$$\frac{M}{\frac{1}{2}\rho ac\Omega^2 R^4} = \int_0^B (x^3 + 2x^2\mu \sin \psi + x\mu^2 \sin^2 \psi)\alpha \, dx . \quad .. \quad (13)$$

Using the evaluation of α from equation (9)

$$\frac{M}{\frac{1}{2}\rho ac\Omega^2 R^4} = \int_0^B \Big[(x^3 + 2x^2\mu \sin \psi + x\mu^2 \sin^2 \psi)\vartheta_0$$
$$+ (x^3 + 2x^2\mu \sin \psi + x\mu^2 \sin^2 \psi)(- A_1 \cos \psi)$$
$$+ (x^3 + 2x^2\mu \sin \psi + x\mu^2 \sin^2 \psi)(- B_1 \sin \psi)$$
$$+ (x^3 + 2x^2\mu \sin \psi + x\mu^2 \sin^2 \psi)(- A_2 \cos 2\psi)$$
$$+ (x^3 + 2x^2\mu \sin \psi + x\mu^2 \sin^2 \psi)(- B_2 \sin 2\psi)$$
$$- (x^2 + x\mu \sin \psi)\lambda$$
$$- (x^2 + x\mu \sin \psi)(\mu \cos \psi)(a_0 - a_1 \cos \psi - b_1 \sin \psi - a_2 \cos 2\psi - b_2 \sin 2\psi)$$
$$- (x^3 + x^2\mu \sin \psi)(a_1 \sin \psi - b_1 \cos \psi + 2a_2 \sin 2\psi - 2b_2 \cos 2\psi) \Big] dx . \quad .. \quad (14)$$

Integrating equation (14) and expressing trigonometrical products in harmonic angles, the aerodynamic moment becomes

$$\frac{M}{\frac{1}{2}\rho ac\Omega^2 R^4} = \left(\frac{B^4}{4} + \frac{2}{3} B^3\mu \sin \psi + \frac{B^2\mu^2}{4} - \frac{B^2\mu^2}{4} \cos 2\psi\right)\vartheta_0$$
$$+ \left(\frac{B^4}{4} \cos \psi + \frac{B^3\mu}{3} \sin 2\psi + \frac{B^2\mu^2}{8} \cos \psi - \frac{B^2\mu^2}{8} \cos 3\psi\right)(- A_1)$$
$$+ \left(\frac{B^4}{4} \sin \psi + \frac{B^3\mu}{3} - \frac{B^3\mu}{3} \cos 2\psi + \frac{3}{8} B^2\mu^2 \sin \psi - \frac{B^2\mu^2}{8} \sin 3\psi\right)(- B_1)$$
$$+ \Big(\frac{B^4}{4} \cos 2\psi + \frac{B^3\mu}{3} \sin 3\psi - \frac{B^3\mu}{3} \sin \psi + \frac{B^2\mu^2}{4} \cos 2\psi$$
$$- \frac{B^2\mu^2}{8} \cos 4\psi - \frac{B^2\mu^2}{8}\Big)(- A_2)$$

$$+\left(\frac{B^4}{4}\sin 2\psi + \frac{B^3\mu}{3}\cos\psi - \frac{B^3\mu}{3}\cos 3\psi + \frac{B^2\mu^2}{4}\sin 2\psi - \frac{B^2\mu^2}{8}\sin 4\psi\right)(-B_2)$$

$$-\left(\frac{B^3}{3} + \frac{B^2\mu}{2}\sin\psi\right)\lambda$$

$$-\left(\frac{B^3\mu}{3}\right)\left(a_0\cos\psi - \frac{a_1}{2}\cos 2\psi - \frac{a_1}{2} - \frac{b_1}{2}\sin 2\psi - \frac{a_2}{2}\cos 3\psi - \frac{a_2}{2}\cos\psi - \frac{b_2}{2}\sin 3\psi - \frac{b_2}{2}\sin\psi\right)$$

$$-\left(\frac{B^2\mu^2}{2}\right)\left(\frac{a_0}{2}\sin 2\psi - \frac{a_1}{4}\sin 3\psi - \frac{a_1}{4}\sin\psi + \frac{b_1}{4}\cos 3\psi - \frac{b_1}{4}\cos\psi - \frac{a_2}{4}\sin 4\psi - \frac{b_2}{4} + \frac{b_2}{4}\cos 4\psi\right)$$

$$-\left(\frac{B^4}{4}\right)(a_1\sin\psi - b_1\cos\psi + 2a_2\sin 2\psi - 2b_2\cos 2\psi)$$

$$-\left(\frac{B^3\mu}{3}\right)\left(\frac{a_1}{2} - \frac{a_1}{2}\cos 2\psi - \frac{b_1}{2}\sin 2\psi - a_2\cos 3\psi + a_2\cos\psi - b_2\sin 3\psi + b_2\sin\psi\right). \quad \ldots \quad (15)$$

For a blade element dx of unit mass m, distant xR from the rotor axis, the increment in centrifugal force and the corresponding moment about the flapping hinge are as follows:

$$d(\text{C.F.}) = m\Omega^2 xR\,dx \quad \ldots \quad (16)$$

$$d(\text{C.F. Mom.}) = m\Omega^2 x^2 R^2 \beta\,dx \quad \ldots \quad (17)$$

Therefore

$$\text{C.F. Mom.} = \int_0^1 mx^2R^2\Omega^2\beta\,dx = I_1\Omega^2\beta \quad \ldots \quad (18)$$

Taking moments about the flapping hinge and neglecting the moment due to the weight of the blade

$$M = I_1\Omega^2\beta + I_1\ddot{\beta} = I_1\Omega^2\left(\beta + \frac{\ddot{\beta}}{\Omega^2}\right). \quad \ldots \quad (19)$$

Rewriting to give the form of the left-hand side of equation (15)

$$\frac{M}{\frac{1}{2}\rho ac\Omega^2R^4} = \frac{I_1\Omega^2}{\frac{1}{2}\rho ac\Omega^2R^4}\left(\beta + \frac{\ddot{\beta}}{\Omega^2}\right) = \frac{2}{\gamma}\left(\beta + \frac{\ddot{\beta}}{\Omega^2}\right) \quad \ldots \quad (20)$$

where γ is the blade inertia number $= \rho acR^4/I_1$.

Evaluating by substituting for β and $\ddot{\beta}$ from equations (3) and (5)

$$\frac{M}{\frac{1}{2}\rho ac\Omega^2R^4} = \frac{2}{\gamma}(a_0 + 3a_2\cos 2\psi + 3b_2\sin 2\psi). \quad \ldots \quad (21)$$

Comparing the equations (15) and (21) and equating the corresponding coefficients, the following equations are obtained.

$$\frac{B^2}{4}(B^2+\mu^2)\vartheta_0 - \frac{B^3}{3}\lambda - \frac{B^3}{3}\mu B_1 + \frac{B^2\mu^2}{8}A_2 \quad - \frac{2}{\gamma}a_0 \qquad + \frac{B^2\mu^2}{8}b_2 = 0 \quad .. \quad (22)$$

$$- \frac{B^2}{4}(B^2 + \tfrac{1}{2}\mu^2)A_1 \quad - \frac{B^3\mu}{3}B_2 - \frac{B^3\mu}{3}a_0 + \frac{B^2}{4}(B^2 + \tfrac{1}{2}\mu^2)b_1 - \frac{B^3\mu}{6}a_2 = 0 \quad .. \quad (23)$$

$$\frac{2}{3}B^3\mu\vartheta_0 - \frac{B^2\mu}{2}\lambda \quad - \frac{B^2}{4}(B^2 + \frac{3}{2}\mu^2)B_1 + \frac{B^3\mu}{3}A_2 - \frac{B^2}{4}(B^2 - \tfrac{1}{2}\mu^2)a_1 \quad - \frac{B^3\mu}{6}b_2 = 0 \quad .. \quad (24)$$

$$-\frac{B^2\mu^2}{4}\vartheta_0 \quad + \frac{B^3\mu}{3}B_1 - \frac{B^2}{4}(B^2+\mu^2)A_2 \quad + \frac{B^3\mu}{3}a_1 \quad - \frac{6}{\gamma}a_2 + \frac{B^4}{2}b_2 = 0 \quad .. \quad (25)$$

$$-\frac{B^3\mu}{3}A_1 \quad - \frac{B^2}{4}(B^2+\mu^2)B_2 - \frac{B^2\mu^2}{4}a_0 + \frac{B^3\mu}{3}b_1 - \frac{B^2}{2}a_2 - \frac{6}{\gamma}b_2 = 0 \quad .. \quad (26)$$

These equations are similar to equations (22) to (26) of Ref. 1, except for the additional terms due to the second harmonic pitch control and the omission of the flapping harmonics above the second. The work could be extended to include the effects on the higher harmonics of flapping if necessary. The equations so derived would be identical with equations (22) to (34) of Ref. 1, except that the additional coefficients would appear.

Reference term	*Equation*	*Additional coefficient*
Constant	22	$+\frac{B^2\mu^2}{8}A_2$
$\cos\psi$	23	$-\frac{B^3\mu}{3}B_2$
$\sin\psi$	24	$+\frac{B^3\mu}{3}A_2$
$\cos 2\psi$	25	$-\frac{B^2}{4}(B^2+\mu^2)A_2$
$\sin 2\psi$	26	$-\frac{B^2}{4}(B^2+\mu^2)B_2$
$\cos 3\psi$	27	$\frac{B^3\mu}{3}B_2$
$\sin 3\psi$	28	$-\frac{B^3\mu}{3}A_2$
$\cos 4\psi$	29	$\frac{B^2\mu^2}{8}A_2$
$\sin 4\psi$	30	$\frac{B^2\mu^2}{8}B_2$
Higher harmonics	31 to 34	—

λ can be eliminated from equations (22) and (24). From equation (22), the value for λ is

$$\lambda = \frac{3}{B^3}\left[\frac{B^2}{4}(B^2 + \mu^2)\vartheta_0 - \frac{B^3\mu}{3}B_1 + \frac{B^2\mu^2}{8}A_2 - \frac{2}{\gamma}a_0 + \frac{B^2\mu^2}{8}b_2\right]. \quad \ldots \quad (27)$$

Substituting in equation (24)

$$\frac{B\mu}{24}(7B^2 - 9\mu^2)\vartheta_0 - \frac{B^2}{4}(B^2 - \tfrac{1}{2}\mu^2)B_1 + B\mu\left(\frac{B^2}{3} - \frac{3}{16}\mu^2\right)A_2 + \frac{3\mu}{B\gamma}a_0$$
$$- \frac{B^2}{4}(B^2 - \tfrac{1}{2}\mu^2)a_1 - B\mu\left(\frac{B^2}{6} + \frac{3}{16}\mu^2\right)b_2 = 0. \quad \ldots \quad (28)$$

From equations (28) and (23) we obtained the evaluation of the first harmonics of control and flapping

$$B_1 + a_1 = \frac{\mu}{6B}\left(\frac{7B^2 - 9\mu^2}{B^2 - \frac{1}{2}\mu^2}\right)\vartheta_0 + \frac{4\mu}{3B}\left(\frac{B^2 - \frac{9}{16}\mu^2}{B^2 - \frac{1}{2}\mu^2}\right)A_2 + \frac{12\mu a_0}{B^3\gamma(B^2 - \frac{1}{2}\mu^2)}$$
$$- \frac{2\mu}{3B}\left(\frac{B^2 + \frac{9}{8}\mu^2}{B^2 - \frac{1}{2}\mu^2}\right)b_2 \quad \ldots \quad (29)$$

$$-A_1 + b_1 = \frac{4}{3}\frac{B\mu}{B^2 + \frac{1}{2}\mu^2}a_0 + \frac{4}{3}\frac{B\mu}{B^2 + \frac{1}{2}\mu^2}B_2 + \frac{2}{3}\frac{B\mu}{B^2 + \frac{1}{2}\mu^2}a_2. \quad \ldots \quad (30)$$

Substituting these values from equations (29) and (30) in equations (25) and (26) respectively

$$\frac{B^2\mu^2}{B^2 - \frac{1}{2}\mu^2}(\tfrac{5}{36}B^2 - \tfrac{3}{8}\mu^2)\vartheta_0 - \frac{B^2}{4}\left(\frac{B^4 - \frac{23}{18}B^2\mu^2 + \frac{1}{2}\mu^4}{B^2 - \frac{1}{2}\mu^2}\right)A_2 + \frac{4\mu^2 a_0}{\gamma(B^2 - \frac{1}{2}\mu^2)}$$
$$- \frac{6}{\gamma}a_2 + \frac{B^2}{2}\left(\frac{B^4 - \frac{17}{18}B^2\mu^2 - \frac{1}{2}\mu^4}{B^2 - \frac{1}{2}\mu^2}\right)b_2 = 0 \quad \ldots \quad (31)$$

$$\frac{B^2\mu^2}{B^2 + \frac{1}{2}\mu^2}(\tfrac{7}{36}B^2 - \tfrac{1}{8}\mu^2)a_0 - \frac{B^2}{4}\left(\frac{B^4 - \frac{5}{18}B^2\mu^2 + \frac{1}{2}\mu^4}{B^2 + \frac{1}{2}\mu^2}\right)B_2$$
$$- \frac{B^4}{2}\left(\frac{B^2 + \frac{1}{18}\mu^2}{B^2 + \frac{1}{2}\mu^2}\right)a_2 - \frac{6}{\gamma}b_2 = 0. \quad \ldots \quad (32)$$

Equations (31) and (32) are linear in form and they can be split into parts dealing with the flapping due to the forward speed conditions and the flapping due to the second harmonic control. Without second harmonic control, *i.e.*, $A_2 = B_2 = 0$, the equations become

$$\frac{B^2\mu^2}{B^2 - \frac{1}{2}\mu^2}(\tfrac{5}{36}B^2 - \tfrac{3}{8}\mu^2)\vartheta_0 + \frac{4\mu^2}{\gamma(B^2 - \frac{1}{2}\mu^2)}a_0 - \frac{6}{\gamma}a_2 + \frac{B^2}{2}\left(\frac{B^4 - \frac{17}{18}B^2\mu^2 - \frac{1}{2}\mu^4}{B^2 - \frac{1}{2}\mu^2}\right)b_2 = 0 \quad (33)$$

and

$$\frac{B^2\mu^2}{B^2 + \frac{1}{2}\mu^2}(\tfrac{7}{36}B^2 - \tfrac{1}{8}\mu^2)a_0 - \frac{B^4}{2}\left(\frac{B^2 + \frac{1}{18}\mu^2}{B^2 + \frac{1}{2}\mu^2}\right)a_2 - \frac{6}{\gamma}b_2 = 0. \quad \ldots \quad (34)$$

Equations (33) and (34) represent the second harmonic flapping produced by the forward speed conditions (without second harmonic control). They are identical with equations (42) and (43) of Ref. 1 except for the omission of the small terms due to higher harmonics. The generalised solutions for this flapping motion can be obtained from Ref. 1, where the numerical evaluations are also included in graphical form.

The flapping due to the second harmonic control is given by the following equations:

$$-\frac{B^2}{4}\left(\frac{B^4-\frac{23}{18}B^2\mu^2-\frac{1}{2}\mu^4}{B^2-\frac{1}{2}\mu^2}\right)A_2-\frac{6}{\gamma}a_2+\frac{B^2}{2}\left(\frac{B^4-\frac{17}{18}B^2\mu^2-\frac{1}{2}\mu^4}{B^2-\frac{1}{2}\mu^2}\right)b_2=0 \quad \ldots \quad (35)$$

and

$$-\frac{B^2}{4}\left(\frac{B^4-\frac{5}{18}B^2\mu^2-\frac{1}{2}\mu^4}{B^2-\frac{1}{2}\mu^2}\right)B_2-\frac{B^4}{2}\left(\frac{B^2+\frac{1}{18}\mu^2}{B^2+\frac{1}{2}\mu^2}\right)a_2-\frac{6}{\gamma}b_2=0 \ldots \quad \ldots \quad (36)$$

Hence,

$$\left[\frac{12}{B^2\gamma}\left(\frac{B^2-\frac{1}{2}\mu^2}{B^4-\frac{17}{18}B^2\mu^2-\frac{1}{2}\mu^4}\right)+\frac{B^4\gamma}{12}\left(\frac{B^2+\frac{1}{18}\mu^2}{B^2+\frac{1}{2}\mu^2}\right)\right]a_2$$

$$=-\frac{1}{2}\left(\frac{B^4-\frac{23}{18}B^2\mu^2-\frac{1}{2}\mu^4}{B^4-\frac{17}{18}B^2\mu^2-\frac{1}{2}\mu^4}\right)A_2-\frac{B^2\gamma}{24}\left(\frac{B^4-\frac{5}{18}B^2\mu^2+\frac{1}{2}\mu^4}{B^2+\frac{1}{2}\mu^2}\right)B_2 \quad \ldots \quad (37)$$

$$\left[\frac{12}{B^4\gamma}\left(\frac{B^2+\frac{1}{2}\mu^2}{B^2+\frac{1}{18}\mu^2}\right)+\frac{B^2\gamma}{12}\left(\frac{B^4-\frac{17}{18}B^2\mu^2-\frac{1}{2}\mu^4}{B^2-\frac{1}{2}\mu^2}\right)\right]b_2$$

$$=\frac{B^2\gamma}{24}\left(\frac{B^4-\frac{23}{18}B^2\mu^2-\frac{1}{2}\mu^4}{B^2-\frac{1}{2}\mu^2}\right)A_2-\frac{1}{2B^2}\left(\frac{B^4-\frac{5}{18}B^2\mu^2+\frac{1}{2}\mu^4}{B^2+\frac{1}{18}\mu^2}\right)B_2\,. \quad \ldots \quad (38)$$

Dividing out the fractions and neglecting powers of μ^4 and higher, these equations simplify to the following form :

$$\left[\frac{12}{B^4\gamma}\left(1+\frac{4}{9}\frac{\mu^2}{B^2}\right)+\frac{B^4\gamma}{12}\left(\frac{1}{1+\frac{4}{9}\frac{\mu^2}{B^2}}\right)\right]a_2=-\frac{1}{2}\left(1-\frac{1}{3}\frac{\mu^2}{B^2}\right)A_2-\frac{B^4\gamma}{24}\left(1-\frac{7}{9}\frac{\mu^2}{B^2}\right)B_2 \quad \ldots \quad (39)$$

$$\left[\frac{12}{B^4\gamma}\left(1+\frac{4}{9}\frac{\mu^2}{B^2}\right)+\frac{B^4\gamma}{12}\left(\frac{1}{1+\frac{4}{9}\frac{\mu^2}{B^2}}\right)\right]b_2=\frac{B^4\gamma}{24}\left(1-\frac{7}{9}\frac{\mu^2}{B^2}\right)A_2-\frac{1}{2}\left(1-\frac{1}{3}\frac{\mu^2}{B^2}\right)B_2\,. \quad \ldots \quad (40)$$

The flapping coefficients a_2 and b_2 can be evaluated independently for any given second harmonic control application or alternatively a generalised form can be derived by combining equations (39) and (40) to give the ratio of the flapping amplitude to the applied second harmonic control amplitude together with the corresponding phase relationship.

The amplitude ratio is given by

$$\sqrt{\left\{\frac{a_2^2+b_2^2}{A_2^2+B_2^2}\right\}}=\frac{1}{2}\frac{\sqrt{\left\{\left(1-\frac{1}{3}\frac{\mu^2}{B^2}\right)^2+\left(\frac{B^4\gamma}{12}\right)^2\left(1-\frac{7}{9}\frac{\mu^2}{B^2}\right)^2\right\}}}{\frac{12}{B^4\gamma}\left(1+\frac{4}{9}\frac{\mu^2}{B^2}\right)+\frac{B^4\gamma}{12}\left(\frac{1}{1+\frac{4}{9}\frac{\mu^2}{B^2}}\right)}\,. \quad \ldots \quad (41)$$

If the azimuth phase angle of the flapping is denoted by ψ_f and the corresponding phase of the second harmonic control by ψ_c, where these angles are the actual azimuth positions on the disc

$$\tan 2\psi_f = \frac{b_2}{a_2} \quad \ldots \quad (42)$$

$$\tan 2\psi_c = \frac{B_2}{A_2}. \quad \ldots \quad (43)$$

Evaluating from equations (39) and (40)

$$\psi_f - \psi_c = \tfrac{1}{2}\tan^{-1}\left[-\frac{B^4\gamma}{12}\left(\frac{1-\frac{7}{9}\frac{\mu^2}{B^2}}{1-\frac{1}{3}\frac{\mu^2}{B^2}}\right)\right]. \quad \ldots \quad (44)$$

The amplitude and phase angle of the flapping depend mainly on the blade inertia number. The tip speed ratio and the tip loss have minor effects. It is perhaps useful to obtain a physical picture of the main results by putting $\mu = 0$ and $B = 1$.

Then

$$\sqrt{\left\{\frac{a_2^2 + b_2^2}{A_2^2 + B_2^2}\right\}} \simeq \frac{1}{2}\frac{1}{\sqrt{\left\{\left(\frac{12}{\gamma}\right)^2 + 1\right\}}} \quad \ldots \quad (45)$$

$$\psi_f - \psi_c \simeq \frac{1}{2}\tan^{-1}\left(-\frac{\gamma}{12}\right). \quad \ldots \quad (46)$$

Considering two limiting cases for the inertia number, (i) as $\gamma \to 0$ the amplitude ratio $\to 0$ and $\psi_f - \psi_c \to \pi/2$ and (ii) as $\gamma \to \infty$ the amplitude ratio $\to 0{\cdot}5$ and $\psi_f - \psi_c \to \pi/4$. For $0 < \gamma < \infty$, $\pi/2 > \psi_f - \psi_c > \pi/4$, *i.e.*, the flapping follows the second harmonic control. The example discussed later illustrates this in greater detail.

The incidence at any point along the blade is given by equation (9). The change in incidence distribution due to the application of the second harmonic control is given by

$$\begin{aligned}\alpha_{2H} = &- A_2\cos 2\psi - B_2\sin 2\psi \\ &- \frac{\mu\cos\psi}{x+\mu\sin\psi}(-a_2\cos 2\psi - b_2\sin 2\psi) \\ &- \frac{x}{x+\mu\sin\psi}(2a_2\sin 2\psi - 2b_2\cos 2\psi)\end{aligned} \quad \ldots \quad (47)$$

where a_2 and b_2 are the coefficients of the flapping motion due to the application of A_2 and B_2 and are given by equations (39) and (40). The distribution varies along the blade as well as with azimuth position. The general solution can only be obtained by plotting incidence contours over the disc for given values of μ and second harmonic control coefficients.

However, while any particular example must be evaluated as above, it is possible by suitable approximation to obtain a simple expression giving the general effect. The value of μ does not have a serious influence in equation (47) (except near the root of the blade) and the general order of the incidence is given by

$$\begin{aligned}\alpha_{2H} \simeq &- A_2\cos 2\psi - B_2\sin 2\psi \\ &- 2a_2\sin 2\psi + 2b_2\cos 2\psi\,.\end{aligned} \quad \ldots \quad (48)$$

Again, we use approximations of the expressions for a_2 and b_2 given in equations (39) and (40) viz. :

$$a_2 \simeq \frac{-\frac{1}{2}A_2 - \frac{\gamma}{24}B_2}{\frac{12}{\gamma} + \frac{\gamma}{12}} \quad \ldots \quad (49)$$

and

$$b_2 \simeq \frac{\frac{\gamma}{24}A_2 - \frac{1}{2}B_2}{\frac{12}{\gamma} - \frac{\gamma}{12}}. \quad \ldots \quad (50)$$

Evaluating in terms of the ratio of incidence amplitude to feathering amplitude and the corresponding phase angle

$$\frac{\alpha_{2H}}{\sqrt{\{A_2^2 + B_2^2\}}} \simeq \frac{1}{\sqrt{\left\{1 + \left(\frac{\gamma}{12}\right)^2\right\}}} \quad \ldots \quad (51)$$

$$\psi_\alpha - \psi_c \simeq \tfrac{1}{2}\tan^{-1}\left(-\frac{\gamma}{12}\right). \quad \ldots \quad (52)$$

Thus, for ordinary blades with an inertia number of about 12, the incidence to feathering amplitude is about $1/\sqrt{2}$, *i.e.*, about 70 per cent of the control pitch becomes useful incidence.

If we consider the limiting cases, heavy blades ($\gamma \to 0$) have practically no flapping and the total control input appears as incidence whereas for very light blades ($\gamma \to \infty$) the flapping motion is such as to cancel out the control input and results in no change of incidence. The approximate values of incidence and flapping amplitude are plotted in Fig. 1.

3. *Numerical Evaluation*.—The expressions for a_2 and b_2 due to the second harmonic control A_2 and B_2 are given in equations (39) and (40) and are functions of the blade inertia number γ, the tip speed ratio μ, A_2 and B_2. Instead of presenting evaluations of a_2 and b_2 independently, it is much more convenient to express the resultant flapping in terms of the amplitude ratio as given by equation (41) and the phase angle between the flapping and applied second harmonic control as given by equation (44). These parameters are plotted against the blade inertia number for a range of tip speed ratios in Figs. 2 and 3, taking $B = 0{\cdot}97$.

An extensive range of blade inertia numbers is taken to assist in the general interpretation of the control effects but for most helicopters the blade inertia number will lie in the range 8 to 12. A range of tip speed ratio from 0 to 0·6 is taken but it must be remembered that blade stalling is not taken into account, so that for many practical examples the higher part of the range may not be strictly applicable.

In the case of the incidence distribution, there is a variation along the blade in addition to the other variables which occur in the flapping motion. It is therefore impossible to give any generalised evaluations and each particular case must be considered independently and estimated from equation (9) or, if only the difference due to the second harmonic control is required, equation (47) may be used.

To give some idea of the order of the incidence variation produced by the second harmonic control, equation (51) has been evaluated and the amplitude has been plotted in Fig. 1, together with the corresponding flapping amplitude. It will be seen from Fig. 1 that, for very heavy blades ($\gamma \to 0$), there is practically no flapping and the incidence change is almost equivalent to the applied feathering. For very light blades ($\gamma \to \infty$), the flapping virtually cancels out the feathering and no change in incidence is produced. For normal helicopter blades (γ about 12) the resultant incidence produced is about 70 per cent of the feathering input. The value for phase angle given in equation (52) is the same as for the flapping in equation (46) but in this case the applicable range is from 0 to -45 deg, *i.e.*, the incidence follows 90 deg after the flapping.

In order to give some idea of the influence of the second harmonic control in suppressing blade stalling, a particular example has been evaluated. A representative rotor is considered at a tip speed ratio of 0·4. The incidence distribution for this rotor is shown in Fig. 4, and the conditions have been chosen such that the incidence over large area on the retreating side exceeds 16 deg, a value which is generally accepted as a limiting condition for flight. Fig. 5 gives the incidence distribution for the same rotor under similar conditions but with the second harmonic control applied in such a way as to reduce the large incidences on the retreating side, the second harmonic control amplitude being 6 deg. Fig. 6 shows, on a unit basis, the incidence changes which are obtained for second harmonic control applied at the appropriate phase angle to provide the resultant effects as given by Fig. 5.

Since the incidence distributions in Figs. 4, 5 and 6 are somewhat complicated, it may be easier to appreciate the general effects of the second harmonic control by taking the approximate solutions as illustrated in Fig. 7. The second harmonic control with unit amplitude is plotted at the appropriate phase angle to give maximum reduction of incidence in the 270 to 300 deg region. The maximum flapping angle amplitude is about 30 per cent of the control angle, the flapping occurring about 70 deg later. The flapping angular velocity has been divided by Ω and is therefore in the form of the incidence changes due to flapping. Subtracting this from the control pitch application gives the resultant incidence, *viz.*, about 75 per cent of the control input at a phase angle of about 20 deg before the control. These approximate values in Fig. 7 compare reasonably well with the distribution for the outer sections of the blade in Fig. 6.

4. *Rotor Tower Test.*—In his paper on the Sikorsky rotor tower[2], Jensen describes a test on an S–52 metal blade where second harmonic control was applied in an attempt to introduce vibratory stresses similar to the flight stresses. This idea was discussed in more detail in an earlier paper by Winson on rotor fatigue life[3]. An example of the records of flapping due to the second harmonic control is given in Fig. 12 of Ref. 2 and this record has been analysed as a check on the theory presented in this report.

The rotor tower test was analysed into the usual form of a Fourier series, the terms showing the usual coning angle, a first harmonic, *i.e.*, tilt due to wind speed on the tower, the predominant second harmonic flapping and negligible amplitudes for the higher harmonics. The blades under test were the S–52 metal main rotor blades which have an inertia number of 9·3, based on a lift slope of 5·6.

The tower test gives an amplitude ratio of flapping of 0·27 at a phase angle of 74 deg. Using the curves of Figs. 1 and 2, the corresponding theoretical estimates are 0.28 and 73 deg giving exceptionally good agreement with the practical test.

5. *Discussion.*—The calculations of the blade flapping motion due to an imposed second harmonic control show that the resultant amplitude and phase angle are mainly a function of blade inertia number. This is in contrast to the effect of first harmonic control which imposes a flapping motion with amplitude and phase angle independent of inertia number. If we restrict our attention in Figs. 2 and 3 to the range of inertia numbers for the ordinary helicopter blades, *viz.*, from 8 to 12, the amplitude ratio only varies from 0·25 to 0·33 and the phase angle from 75 to 70 deg. Thus, for ordinary helicopter blades the results do not depend critically on inertia number. On the practical side, we have the rotor tower test which verifies the calculations, giving agreement within the experimental accuracy of the tests.

Considering now the variations with tip speed ratio, we find these are very small. Thus, for most purposes, the positioning of the second harmonic control in relation to the rotor azimuth need not be altered with the flight conditions. There is no direct experimental evidence available on this point but, remembering that the calculations of the second harmonic of flapping due to forward speed[1] show general agreement with experimental results, there is reason to suppose that similar results can be expected in the present conditions.

The use of a second harmonic control could have several applications :

(*a*) the elimination of the inherent second-order flapping due to forward speed

(*b*) the reproduction during ground-testing of stresses measured in forward flight cases

(*c*) the redistribution of loading on the disc to avoid the blade stalling conditions limiting forward speed.

With regard to (*a*), such a control might be of some use on a two-bladed rotor but for rotors with three or more blades there is no indication that the inherent second-order flapping has any serious effects on the helicopter characteristics.

With regard to (*b*), this form of control would be of very limited use, since the second harmonic does not appear to introduce any serious stress variations[2] and the natural blade bending frequency seems to be a much more critical condition.

The redistribution of loading over the disc could be an important factor in relation to the forward speed limitations imposed by stalling of the retreating blade.

The example given in Figs. 4 and 5 illustrates the marked effect that such a control can have in reducing the high incidences over the retreating sector of the disc. This is, of course, only a temporary remedy but it does indicate that forward speeds of the helicopter could be increased by the equivalent of about $0 \cdot 1$ on tip speed ratio, as far as blade stalling limitations are concerned, if there is sufficient engine power available to meet this increase. On the other hand, it does imply further complication to the rotor head, which may be an undesirable feature at the present stage of helicopter development as the mechanical reliability is still somewhat suspect.

As mentioned earlier, oscillating forces are required to apply the second harmonic pitch angles to the blade and the stressing conditions, which these forces apply to the swash plate, bearings, etc. and their tendency to twist the blade, will require some analysis. Also, the larger flapping angles in the higher harmonics may introduce appreciable Coreolis forces which might cause some effects on the in-plane motion of the blades.

There is practically no effect on the general rotor performance with the second harmonic control applied. The reduction of the large incidences on the retreating side gives some reduction in profile drag in that sector. The torque required is decreased to a small extent but this is partly offset by an increase in the rotor H-force. The net result is that the performance remains about the same and the main influence of the control is to allow higher forward speeds by suppressing the high incidences on the retreating side.

6. *Conclusions.*—6.1. A theory has been developed to calculate the flapping and incidence distribution imposed on a rotor by a second harmonic control.

6.2. The flapping amplitude and phase angle depend mainly on blade inertia number and to a small extent on tip speed ratio. For the range of ordinary blades the variation in the results is not large.

6.3. A rotor tower test shows excellent agreement with this theory.

6.4. The second harmonic control is a means for redistributing the loading on a rotor disc and could be utilised to postpone the forward speed limitations due to stalling of the retreating blade.

LIST OF SYMBOLS

a		Lift slope for blade
a_0		Coning angle
a_1, a_2 b_1, b_2		Coefficients in Fourier series for flapping angle
A_1, A_2 B_1, B_2		Coefficients in Fourier series for pitch angle
B		Factor to allow for tip loss, taken as $0 \cdot 97$
c		Blade chord
I_1		Moment of inertia of blade about flapping hinge
M		Aerodynamic moment about flapping hinge
R		Blade radius
x	=	r/R, fraction of blade radius
α		Blade incidence
α_{2H}		Incidence due to second harmonic control
β		Flapping angle
γ	=	$\frac{\rho a c R^4}{I_1}$, blade inertia number
ψ		Blade azimuth angle
$\psi_c, \psi_f, \psi_\alpha$		Phase angles for second harmonic control and resulting flapping and incidence
ϑ		Blade pitch angle
λ		Coefficient of flow through the disc
μ		Tip speed ratio
Ω		Rotor angular velocity

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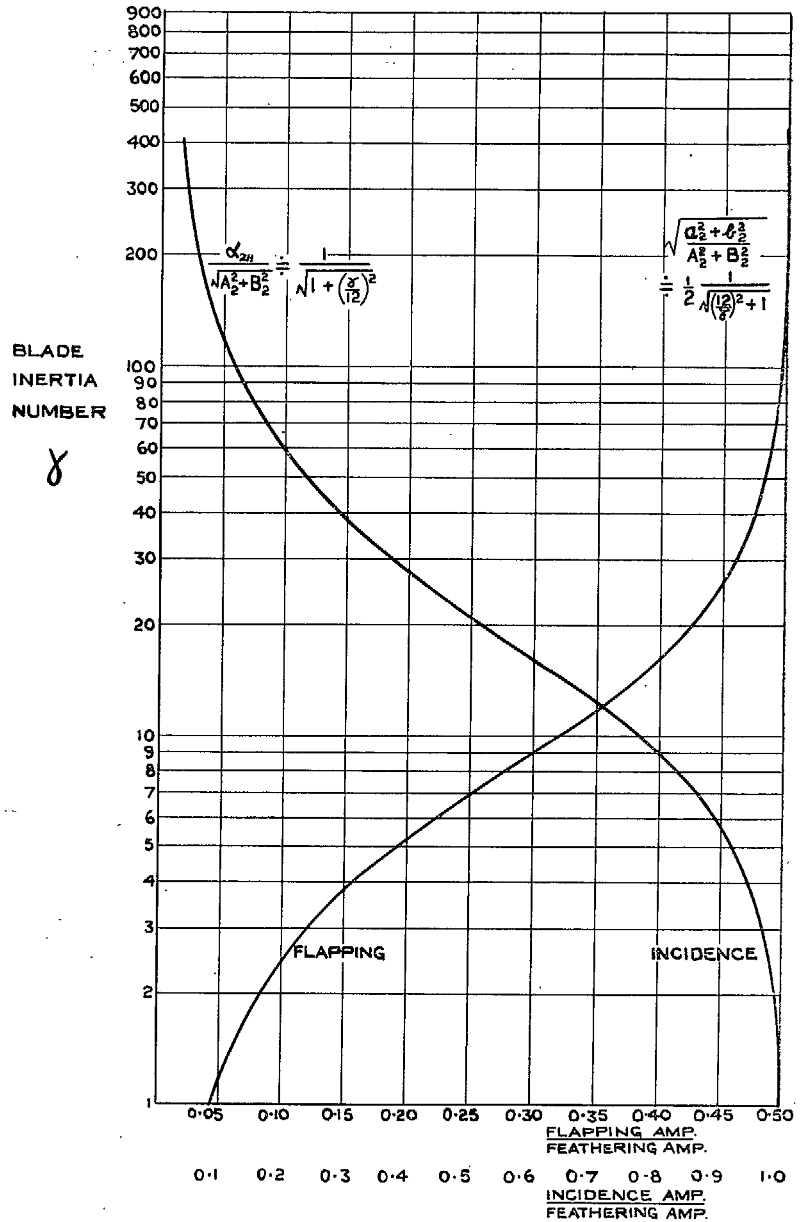

Fig. 1. Approximate flapping and incidence amplitudes.

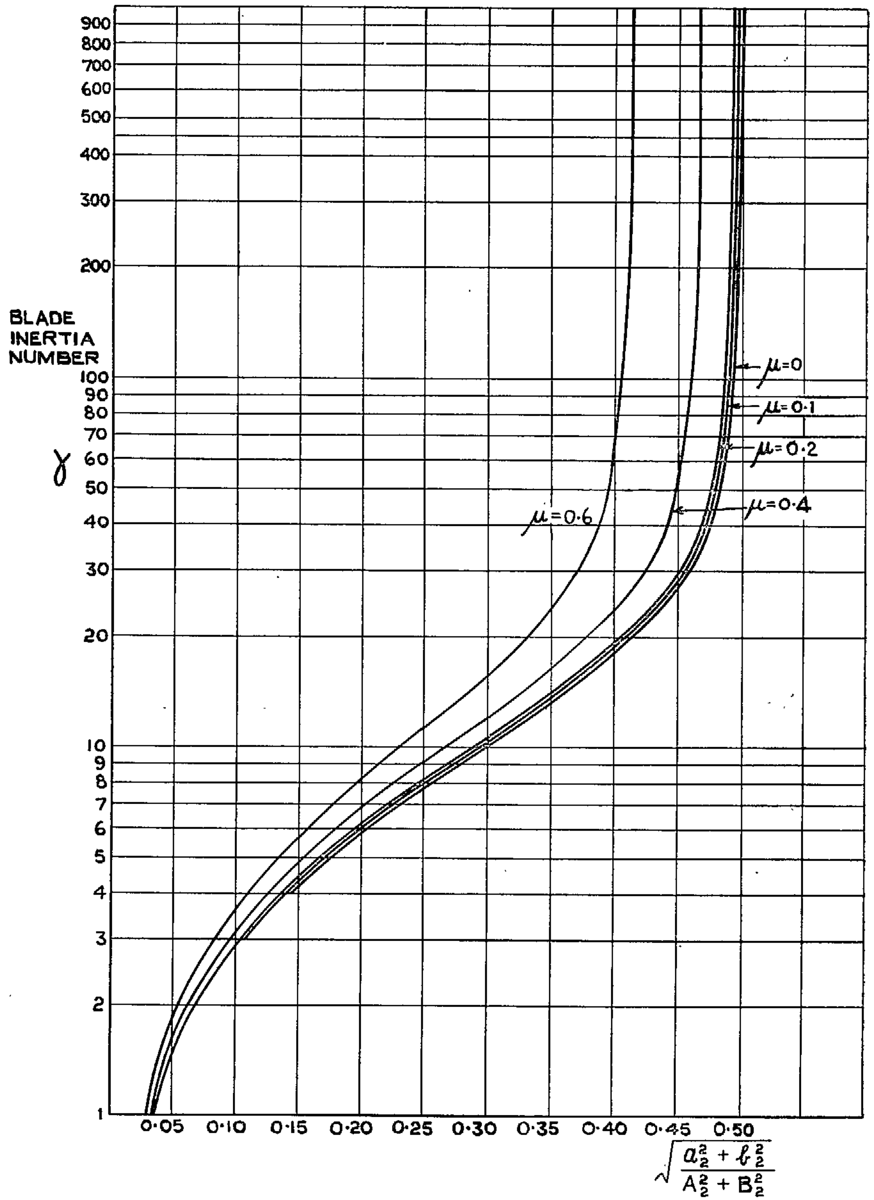

Fig. 2. Ratio of flapping amplitude to feathering amplitude.

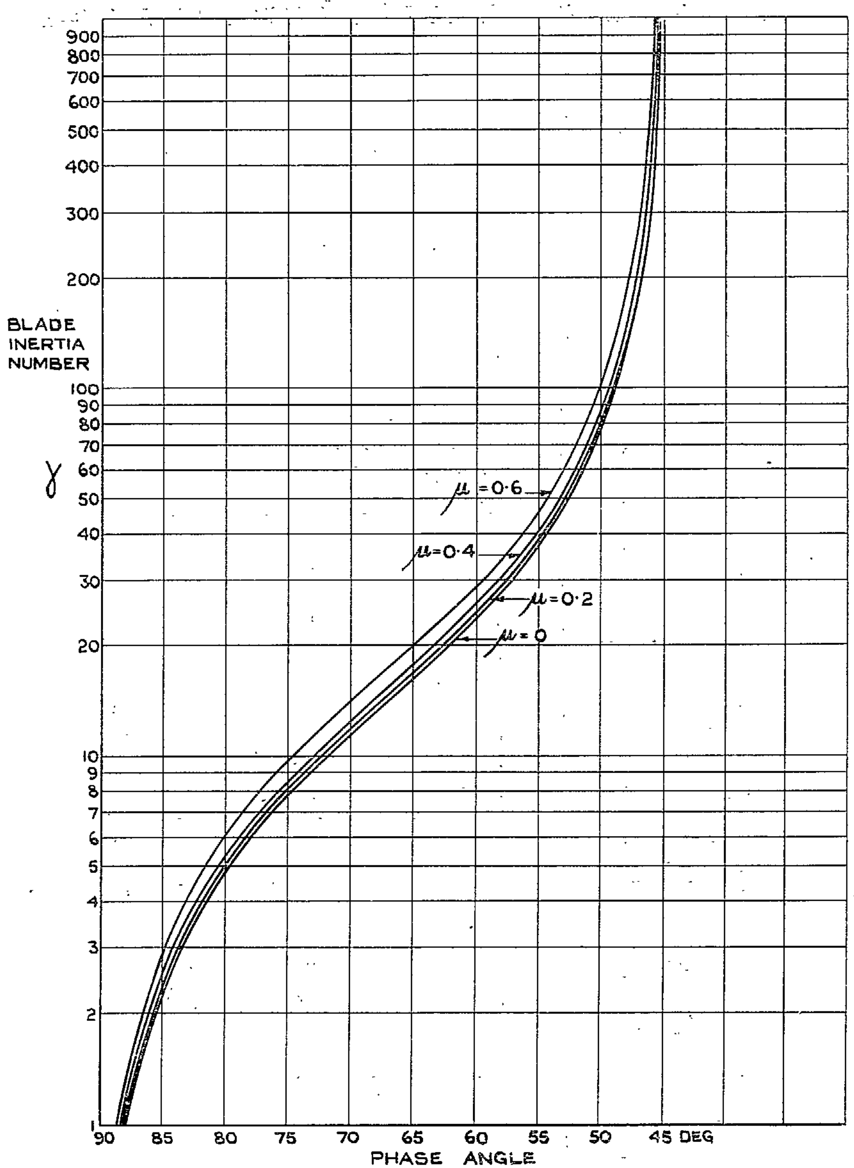

FIG. 3. Phase angle of flapping relative to second harmonic pitch.

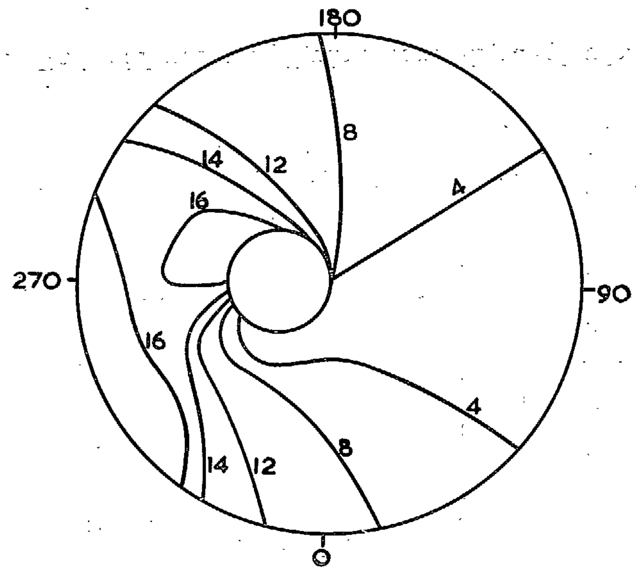

FIG. 4. Incidence contours. Ordinary rotor. $\mu = 0{\cdot}4$.

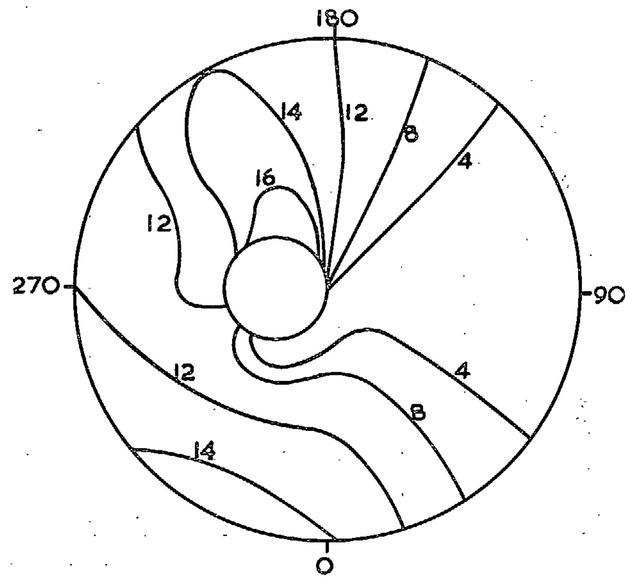

FIG. 5. Incidence contours. Second harmonic control applied.
$\sqrt{(A_2^2 + B_2^2)} = 6$ deg. $\mu = 0{\cdot}4$.

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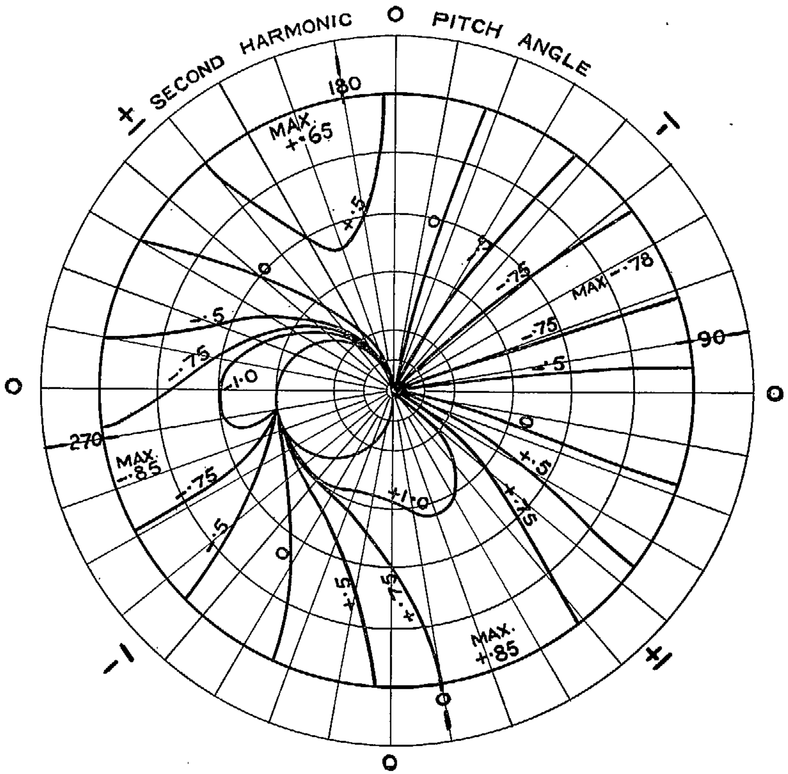

FIG. 6. Incidence distribution due to second harmonic control. $\sqrt{(A_2^2 + B_2^2)} = 1$. $\mu = 0{\cdot}4$.

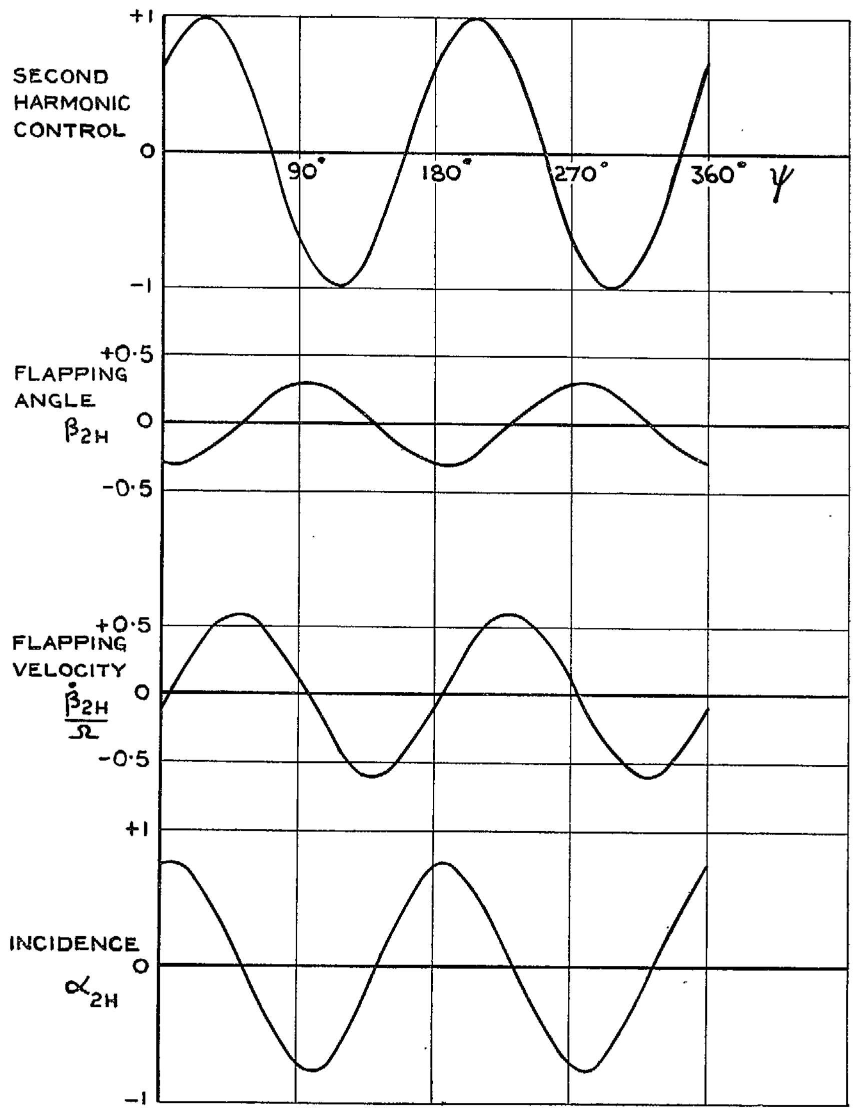

FIG. 7. Flapping and incidence due to second harmonic control. (Approximate solution : *cf*. Fig. 6.)

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