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The Stress Distribution in Panels Bounded by Constant-Stress Edge Members

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Summary.—Exact solutions are given for the stress distributions in long panels bounded by constant-stress edge members. The influence of closely spaced stringers and ribs on the peak shear stresses is investigated.

1. *Introduction.*—The stress distributions in panels bounded by constant-stress and constant-area edge members have been considered by a number of writers[1,2,3] by assuming that the transverse strains may be neglected. This assumption is justifiable in that the longitudinal direct stresses are then determined sufficiently accurately although the peak shear stresses are in error. In this report it is shown that if the longitudinal edge members are tapered so that their stress will not vary along their length it is possible to obtain simple expressions for the stresses in an unreinforced panel without recourse to more drastic simplifying assumptions. If the panel is reinforced by stringers and ribs, simple expressions for the stresses are determined on the assumption that the panel has orthotropic properties.

2. *List of Symbols* (*see Fig.* 1)

Structure properties

$2b$	Width of panel
t	Thickness of sheet
S	Relative stiffness of stringers to sheet (*i.e.*, stringer area/t $\times$ stringer pitch)
R	Relative stiffness of ribs to sheet (*i.e.*, rib area/t $\times$ rib pitch)
F	Section area of longitudinal edge member
ν	Poisson's ratio

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Axes

Ox, Oy		Cartesian co-ordinates, Ox measured longitudinally
ξ	=	$\pi x/2b$
η	=	$\pi y/2b$

Stresses

σ_x, σ_y, τ_{xy}	Stresses in the sheet
σ_e	Stress in the longitudinal edge members
σ_S, σ_R	Stresses in the stringers and ribs
$\bar{\sigma}_x$, $\bar{\sigma}_y$	Stress resultants in the reinforced panel

Non-dimensional parameters

$$K = 1 + S + R + SR(1 - \nu^2)$$

$$\alpha = 1 + S(1 - \nu^2)$$

$$\gamma = 1 + (1 + \nu)\{S + R + SR(1 - \nu^2)\}$$

$$\varepsilon = 1 + R(1 - \nu^2)$$

$$n_1 = \sqrt{\left\{\frac{\gamma + \sqrt{(\gamma^2 - \alpha\varepsilon)}}{\varepsilon}\right\}}$$

$$n_2 = \sqrt{\left\{\frac{\gamma - \sqrt{(\gamma^2 - \alpha\varepsilon)}}{\varepsilon}\right\}}$$

$$\psi = n_1 - n_2$$

$$= \sqrt{\left\{\frac{2}{\varepsilon}[\gamma - \sqrt{(\alpha\varepsilon)}]\right\}}$$

$$\mu = n_1 + n_2$$

$$= \sqrt{\left\{\frac{2}{\varepsilon}[\gamma + \sqrt{(\alpha\varepsilon)}]\right\}}$$

3. *Stress Distribution in a Long Panel Bounded by Constant-Stress Edge Members.*—In this section expressions are given in closed form for the stresses in a long panel bounded by constant-stress edge members. The analysis is given in Appendix I and is based on a series expansion for the stress function ; the resulting series for the stresses are shown to be summable in terms of known functions. The boundary conditions considered along the transverse edge are either that the edge is free or that it is supported by an inextensional but flexible member.

3.1 *Transverse Edge Free.*—The boundary conditions considered here are that along the longitudinal edges

and
$$\left.\begin{aligned} \sigma_x - \nu\sigma_y &= \sigma_e \\ \bar{\sigma}_y &= 0 \end{aligned}\right\} \qquad (1)$$

so that there are no transverse loads ; and along the transverse edge

$$\left.\begin{array}{l}\bar{\sigma}_x = 0 \\ \tau_{xy} = 0\end{array}\right\} \quad \text{and} \qquad (2)$$

so that this edge is free.

3.1.1. *Plain sheet.*—It is shown in Appendix I that the stresses in the panel are given by

$$\frac{\sigma_x}{\sigma_e} = 1 - \frac{2}{\pi}\left\{\frac{\xi \cosh \xi \cos \eta}{\cosh^2 \xi - \sin^2 \eta} + \tan^{-1}\left(\frac{\cos \eta}{\sinh \xi}\right)\right\} \qquad (3)$$

$$\frac{\sigma_y}{\sigma_e} = \frac{2}{\pi}\left\{\frac{\xi \cosh \xi \cos \eta}{\cosh^2 \xi - \sin^2 \eta} - \tan^{-1}\left(\frac{\cos \eta}{\sinh \xi}\right)\right\} \qquad (4)$$

$$\frac{\tau_{xy}}{\sigma_e} = -\frac{2}{\pi}\left\{\frac{\xi \sinh \xi \sin \eta}{\cosh^2 \xi - \sin^2 \eta}\right\}. \qquad (5)$$

These stresses have been plotted as contours over the panel in Figs. 2, 3 and 4.

The maximum value of σ_y is $-\sigma_e$ and it occurs along the length of the free edge. The maximum value of τ_{xy} is $2\sigma_e/\pi$ and it occurs at the corners of the panel. The variation of τ_{xy} along the longitudinal edges of the panel assumes a particularly simple form :

$$\frac{(\tau_{xy})_e}{\sigma_e} = \frac{2\xi}{\pi \sinh \xi} \qquad (6)$$

and this may be integrated to give the required variation of the section area of the constant-stress edge members :

$$F = F_0 - \frac{4bt}{\pi^2}\int_0^{\xi} \frac{\xi \, d\xi}{\sinh \xi}. \qquad (7)$$

3.1.2. *Reinforced sheet.*—It is shown in Appendix I that the stress resultants* in the panel are given by

$$\frac{\bar{\sigma}_x}{\sigma_e} = \frac{K}{\varepsilon} - \frac{2K}{\pi\psi\varepsilon}\left\{n_1 \tan^{-1}\left(\frac{\cos \eta}{\sinh (\xi/n_1)}\right) - n_2 \tan^{-1}\left(\frac{\cos \eta}{\sinh (\xi/n_2)}\right)\right\} \qquad (8)$$

$$\frac{\bar{\sigma}_y}{\sigma_e} = \frac{2K}{\pi\psi\varepsilon}\left\{\frac{1}{n_1} \tan^{-1}\left(\frac{\cos \eta}{\sinh (\xi/n_1)}\right) - \frac{1}{n_2} \tan^{-1}\left(\frac{\cos \eta}{\sinh (\xi/n_2)}\right)\right\} \qquad (9)$$

$$\frac{\tau_{xy}}{\sigma_e} = \frac{K}{\pi\psi\varepsilon} \log \left[\frac{\{\cosh (\xi/n_1) - \sin \eta\}\{\cosh (\xi/n_2) + \sin \eta\}}{\{\cosh (\xi/n_1) + \sin \eta\}\{\cosh (\xi/n_2) - \sin \eta\}}\right]. \qquad (10)$$

The maximum value of $\bar{\sigma}_y$ occurs along the length of the free edge and is given by

$$\frac{(\bar{\sigma}_y)_{\max}}{\sigma_e} = -\frac{K}{\sqrt{(\alpha\varepsilon)}}. \qquad (11)$$

*Stress resultants are here defined as (the resultant force in the stiffened sheet per unit length)/t. They therefore have the dimensions of a stress, and when there is no reinforcement in a particular direction the stress resultants are the actual stresses in the sheet.

The maximum value of τ_{xy} occurs at the corner of the panel and is given by

$$\frac{(\tau_{xy})_{\max}}{\sigma_e} = \left(\frac{2K}{\pi\psi\varepsilon}\right) \log\left(\frac{n_1}{n_2}\right) \qquad (12)$$

and this has been plotted in Fig. 5 for varying values of the stringer and rib stiffness. The variation of τ_{xy} along the longitudinal edges of the panel may be written in the form :

$$\frac{(\tau_{xy})_e}{\sigma_e} = \left(\frac{2K}{\pi\psi\varepsilon}\right) \log\left\{\frac{\tanh(\xi/2n_1)}{\tanh(\xi/2n_2)}\right\} \qquad (13)$$

and this may be integrated to give the required variation of the section area of the constant stress edge members :

$$F = F_0 - \left(\frac{4Kbt}{\pi^2\psi\varepsilon}\right) \int_0^{\xi} \log\left\{\frac{\tanh(\xi/2n_1)}{\tanh(\xi/2n_2)}\right\} d\xi \,. \qquad (14)$$

3.1.3. *Direct stresses in the sheet, stringers and ribs.*—When the panel is reinforced the direct stresses in the sheet, stringers and ribs are related to the stress resultants by the equations[4]

$$\sigma_x = \left(\frac{1+R}{K}\right)\bar{\sigma}_x + \frac{\nu S}{K}\bar{\sigma}_y \qquad (15)$$

$$\sigma_y = \left(\frac{1+S}{K}\right)\bar{\sigma}_y + \frac{\nu R}{K}\bar{\sigma}_x \qquad (16)$$

$$\sigma_S = \frac{\varepsilon}{K}\bar{\sigma}_x - \frac{\nu}{K}\bar{\sigma}_y \qquad (17)$$

and

$$\sigma_R = \frac{\alpha}{K}\bar{\sigma}_y - \frac{\nu}{K}\bar{\sigma}_x \,. \qquad (18)$$

3.2. *Transverse Edge Supported.*—If the transverse edge is supported by an inextensional but flexible member the second part of equation (2) becomes

$$\sigma_R = 0 \qquad (19)$$

and the other boundary conditions are unaltered.

3.2.1. *Plain sheet.*—It is shown in Appendix I that the stresses in the panel are given by

$$\frac{\sigma_x}{\sigma_e} = 1 - \frac{1}{\pi}\left\{\frac{\xi\cosh\xi\cos\eta}{\cosh^2\xi - \sin^2\eta} + 2\tan^{-1}\left(\frac{\cos\eta}{\sinh\xi}\right)\right\} \qquad (20)$$

$$\frac{\sigma_y}{\sigma_e} = \frac{1}{\pi}\left\{\frac{\xi\cosh\xi\cos\eta}{\cosh^2\xi - \sin^2\eta}\right\} \qquad (21)$$

$$\frac{\tau_{xy}}{\sigma_e} = -\frac{1}{2\pi}\left\{\frac{2\xi\sinh\xi\sin\eta}{\cosh^2\xi - \sin^2\eta} + \log\left(\frac{\cosh\xi + \sin\eta}{\cosh\xi - \sin\eta}\right)\right\} \qquad (22)$$

and the shear stress becomes infinite at the corners because of the logarithmic term.

3.2.2. *Reinforced sheet.*—It is shown in Appendix I that the stress resultants in the panel are given by

$$\frac{\bar{\sigma}_x}{\sigma_e} = \frac{K}{\varepsilon} - \frac{2K}{\pi\mu\varepsilon\psi}\left\{n_1^2 \tan^{-1}\left(\frac{\cos\eta}{\sinh(\xi/n_1)}\right) - n_2^2 \tan^{-1}\left(\frac{\cos\eta}{\sinh(\xi/n_2)}\right)\right\} \quad .. \quad (23)$$

$$\frac{\bar{\sigma}_y}{\sigma_e} = \frac{2K}{\pi\mu\varepsilon\psi}\left\{\tan^{-1}\left(\frac{\cos\eta}{\sinh(\xi/n_1)}\right) - \tan^{-1}\left(\frac{\cos\eta}{\sinh(\xi/n_2)}\right)\right\} \quad .. \quad .. \quad (24)$$

$$\frac{\tau_{xy}}{\sigma_e} = \frac{K}{\pi\mu\varepsilon\psi}\left\{n_1 \log\left(\frac{\cosh(\xi/n_1) - \sin\eta}{\cosh(\xi/n_1) + \sin\eta}\right) - n_2 \log\left(\frac{\cosh(\xi/n_2) - \sin\eta}{\cosh(\xi/n_2) + \sin\eta}\right)\right\}. \quad (25)$$

4. *Discussion of Results.*—From the analysis in the appendices it appears that the exact solutions given in section 3 are the only ones capable of expression in closed form. The case of a short panel is considered in Appendix II. The expressions for the stresses are complicated but are unlikely to differ significantly from those for a long panel unless the panel length is less than three times the panel width. The stress distribution in a long panel bounded by constant-area edge members loaded at their ends is considered in Appendix III. Contours of constant $\sigma_x/\sigma_{e,0}$ in an unreinforced panel with a free edge have been drawn in Figs. 6, 7 and 8 for values of F/bt equal to $\frac{1}{2}$, 1, 2. These contours differ appreciably near the longitudinal edges from those shown in Fig. 2 which correspond to infinite F/bt. The peak value of the shear stress is independent of F and is $2\sigma_{e,0}/\pi$.

5. *Conclusions.*—The stress distributions in long panels bounded by constant stress edge members are considered theoretically using the exact equations of elasticity. The stresses in the panel are expressed in closed form, and may therefore be readily determined. Contours of stress in the panel are shown and the influence of closely spaced stringers and ribs on the peak shear stresses is investigated.

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2	M. Fine and H. G. Hopkins	Stress diffusion adjacent to gaps in the interspar skin of a stressed skin wing. R. & M. 2618. May, 1942.
3	J. Hadji-Argyris	Diffusion of symmetrical loads into stiffened parallel panels with constant area edge members. R. & M. 2038. November, 1944.
4	E. H. Mansfield	Elasticity of a sheet reinforced by stringers and skew ribs, with application to swept wings. R. & M. 2758. December, 1949.

Additional Symbols Used Only in the Appendices

ϕ		Airy stress function
B, C_n, C_n'		Constants in a summation
$\sum_n$		Summation for $n = 0, 1, 2, \ldots\ldots \infty$
S_1, S_2		Summation introduced in equation (43)
ξ'	=	ξ/n_1 or ξ/n_2
S_0	=	$S_1 + iS_2$
$S_{1,1}, S_{2,1}$		Values of S_1, S_2 with $\xi' = \xi/n_1$
$S_{1,2}, S_{2,2}$		Values of S_1, S_2 with $\xi' = \xi/n_2$
λ	=	$\pi \times$ (length of panel)$/4b$
ϱ	=	$Kbt/F\varepsilon$
r_n		Positive root of the equation : $r + \varrho \tan r = 0$

APPENDIX I

Stress Distribution in an Infinitely Long Panel Bounded by Constant-Stress Edge Members

In determining the stress distribution in the reinforced panel it is convenient to introduce the stress function ϕ, such that the stress resultants are given by

$$\left.\begin{aligned} \bar{\sigma}_x &= \frac{\partial^2\phi}{\partial y^2} \\ \bar{\sigma}_y &= \frac{\partial^2\phi}{\partial x^2} \\ \tau_{xy} &= -\frac{\partial^2\phi}{\partial x\,\partial y} \end{aligned}\right\} \qquad (26)$$

The equilibrium conditions are then automatically satisfied, and the condition of compatability is satisfied if ϕ satisfies the differential equation[4] :

$$\alpha\frac{\partial^4\phi}{\partial x^4} + 2\gamma\frac{\partial^4\phi}{\partial x^2\partial y^2} + \varepsilon\frac{\partial^4\phi}{\partial y^4} = 0. \qquad (27)$$

A suitable form for the stress function, which is a solution for this equation, is

$$\phi = By^2 + \frac{4b^2}{\pi^2}\sum_n \frac{1}{(2n+1)^2}\left\{C_n \exp\left[-\left(\frac{(2n+1)\pi x}{2bn_1}\right)\right] + C_n' \exp\left[-\left(\frac{(2n+1)\pi x}{2bn_2}\right)\right]\right\}\cos\left(\frac{(2n+1)\pi y}{2b}\right) \qquad (28)$$

where n_1 and n_2 are the positive roots of the equation

$$\alpha - 2\gamma n^2 + \varepsilon n^4 = 0. \qquad (29)$$

The stress resultants, obtained from equations (26) and (28), are written more conveniently in terms of ξ and η :

$$\bar{\sigma}_x = 2B - \sum_n \{C_n e^{-(2n+1)\xi/n_1} + C_n' e^{-(2n+1)\xi/n_2}\} \cos (2n + 1)\eta. \quad (30)$$

$$\bar{\sigma}_y = \sum_n \left\{\frac{C_n}{n_1^2} e^{-(2n+1)\xi/n_1} + \frac{C_n'}{n_2^2} e^{-(2n+1)\xi/n_2}\right\} \cos (2n + 1)\eta \quad (31)$$

$$\tau_{xy} = - \sum_n \left\{\frac{C_n}{n_1} e^{-(2n+1)\xi/n_1} + \frac{C_n'}{n_2} e^{-(2n+1)\xi/n_2}\right\} \sin (2n + 1)\eta \quad (32)$$

and the actual direct stresses in the sheet, stringers and ribs are given by equations (15) to (18). The constant B is determined from the condition that as ξ tends to infinity,

$$\sigma_S = \sigma_e \quad (33)$$

so that,

$$B = \frac{K\sigma_e}{2\varepsilon}. \quad (34)$$

Along the longitudinal edges $\eta = \pm \frac{1}{2}\pi$, so that $\cos (2n + 1)\eta$ vanishes and therefore the boundary conditions represented by equation (1) are satisfied. (Note that $\sigma_S \equiv \sigma_x - \nu\sigma_y$.)

Transverse Edge Free.—Along the transverse edge, $\xi = 0$ and the boundary conditions represented by equation (2) are

$$\frac{K\sigma_e}{\varepsilon} - \sum_n (C_n + C_n') \cos (2n + 1)\eta = 0 \quad (35)$$

and

$$\sum_n \left(\frac{C_n}{n_1} + \frac{C_n'}{n_2}\right) \sin (2n + 1)\eta = 0. \quad (36)$$

Now from Fourier analysis

$$\frac{K\sigma_e}{\varepsilon} \equiv \frac{4K\sigma_e}{\pi\varepsilon} \sum_n \frac{(-1)^n \cos (2n + 1)\eta}{2n + 1} \quad (37)$$

so that

$$C_n + C_n' = \frac{(-1)^n 4K\sigma_e}{(2n + 1)\pi\varepsilon} \quad (38)$$

and

$$\frac{C_n}{n_1} + \frac{C_n'}{n_2} = 0. \quad (39)$$

The solution of equations (38) and (39) is

$$\left.\begin{aligned} C_n &= \frac{(-1)^n 4K n_1 \sigma_e}{(2n + 1)\pi\varepsilon\psi} \\ C_n' &= \frac{-(-1)^n 4K n_2 \sigma_e}{(2n + 1)\pi\varepsilon\psi} \end{aligned}\right\}. \quad (40)$$

Transverse Edge Supported.—When the second part of equation (2) is replaced by equation (19), it will be found that equation (36) is replaced by

$$\frac{\nu K\sigma_e}{\varepsilon} - \sum_n \left\{C_n\left(\nu + \frac{\alpha}{n_1^2}\right) + C_n'\left(\nu + \frac{\alpha}{n_2^2}\right)\right\} \cos (2n + 1)\,\eta = 0 \quad (41)$$

and C_n and C_n' are given by

$$\left.\begin{aligned} C_n &= \frac{(-1)^n 4K n_1^2 \sigma_e}{(2n+1)\pi\varepsilon\mu\psi} \\ C_n' &= \frac{-(-1)^n 4K n_2^2 \sigma_e}{(2n+1)\pi\varepsilon\mu\psi} \end{aligned}\right\} \qquad (42)$$

Solution in Closed Form.—It will be seen by comparing equations (28), (40) and (42) that two distinct summations occur in the stress resultants, and these may be written as

$$\left.\begin{aligned} S_1 &= \sum_n \frac{(-1)^n e^{-(2n+1)\xi'} \cos(2n+1)\eta}{2n+1} \\ S_2 &= \sum_n \frac{(-1)^n e^{-(2n+1)\xi'} \sin(2n+1)\eta}{2n+1} \end{aligned}\right\} \qquad (43)$$

and it will now be seen that S_1 and S_2 are respectively the real and imaginary parts of

$$\left.\begin{aligned} S_0 &= \sum_n \frac{(-1)^n e^{-(2n+1)(\xi'-i\eta)}}{2n+1} \\ &= \frac{1}{2i}\log\left(\frac{1+ie^{-\xi'+i\eta}}{1-ie^{-\xi'+i\eta}}\right) \\ &= \frac{1}{2i}\log\left(\frac{\sinh\xi' + i\cos\eta}{\cosh\xi' + \sin\eta}\right) \\ &= \tfrac{1}{2}\tan^{-1}\left(\frac{\cos\eta}{\sinh\xi'}\right) + \frac{i}{4}\log\left(\frac{\cosh\xi' + \sin\eta}{\cosh\xi' - \sin\eta}\right) \end{aligned}\right\} \qquad (44)$$

so that $$S_1 = \tfrac{1}{2}\tan^{-1}\left(\frac{\cos\eta}{\sinh\xi'}\right) \qquad (45)$$

and $$S_2 = \tfrac{1}{4}\log\left(\frac{\cosh\xi' + \sin\eta}{\cosh\xi' - \sin\eta}\right). \qquad (46)$$

The stress resultants are to be determined from equations (30), (31), (32) and (40), (42), (43). If the transverse edge is free :

$$\frac{\bar{\sigma}_x}{\sigma_e} = \frac{K}{\varepsilon} - \frac{4K}{\pi\varepsilon\psi}\{n_1 S_{1,1} - n_2 S_{1,2}\} \qquad (47)$$

$$\frac{\bar{\sigma}_y}{\sigma_e} = \frac{4K}{\pi\varepsilon\psi}\left\{\left(\frac{1}{n_1}\right)S_{1,1} - \left(\frac{1}{n_2}\right)S_{1,2}\right\} \qquad (48)$$

$$\frac{\tau_{xy}}{\sigma_e} = -\frac{4K}{\pi\varepsilon\psi}\{S_{2,1} - S_{2,2}\} \qquad (49)$$

and these equations correspond to equations (8), (9) and (10) of the main text. If the transverse edge is supported :

$$\frac{\bar{\sigma}_x}{\sigma_e} = \frac{K}{\varepsilon} - \frac{4K}{\pi\varepsilon\mu\psi}\{n_1^2 S_{1,1} - n_2^2 S_{1,2}\} \qquad (50)$$

$$\frac{\bar{\sigma}_y}{\sigma_e} = \frac{4K}{\pi\varepsilon\mu\psi}\{S_{1,1} - S_{1,2}\} \qquad (51)$$

$$\frac{\tau_{xy}}{\sigma_e} = \frac{-4K}{\pi\varepsilon\mu\psi}\{n_1 S_{2,1} - n_2 S_{2,2}\} \qquad (52)$$

and these equations correspond to equations (23), (24) and (25) of the main text.

Plain Sheet.—If the panel is unreinforced the coefficients n_1 and n_2 are each equal to unity and the expressions derived above for the stresses assume an indeterminate form. The limiting values as n_1 and n_2 tend to unity may be readily found by observing that, for example in equation (47),

$$\underset{n_1 \to n_2 \to 1}{\text{Limit}} \left\{ \frac{n_1 S_{1,1} - n_2 S_{1,2}}{\psi} \right\} = \left[\frac{\partial}{\partial n_1} \{ n_1 S_{1,1} \} \right]_{n_1 = 1} \quad \ldots \quad (53)$$

with similar relations for the indeterminate forms occurring in equations (48) to (52).

Now,
$$\frac{\partial}{\partial n_1} S_{1,1} = \frac{\xi \cosh \xi \cos \eta}{2(\cosh^2 \xi - \sin^2 \eta)} \quad \ldots \quad (54)$$

and
$$\frac{\partial}{\partial n_1} S_{2,1} = \frac{\xi \sinh \xi \sin \eta}{2(\cosh^2 \xi - \sin^2 \eta)} \quad \ldots \quad (55)$$

so that the derivation of equations (3), (4), (5), (20), (21) and (22) is now straightforward.

APPENDIX II

Stress Distribution in a Finite Panel Bounded by Constant-Stress Edge Members

The stress function is symmetrical about the line $\xi = \lambda$, and in the expansion for ϕ (*see* equation (28)) the term

$$e^{-(2n+1)\xi/n_1} \text{ is therefore replaced by } \frac{\cosh \{(2n+1)(\lambda - \xi)/n_1\}}{\cosh \{(2n+1)\lambda/n_1\}} \quad \ldots \quad (56)$$

and there is a similar replacement with n_2 instead of n_1.

The stress resultants are then given by

$$\bar{\sigma}_x = 2B - \sum_n \left\{ C_n \frac{\cosh \{(2n+1)(\lambda - \xi)/n_1\}}{\cosh \{(2n+1)\lambda/n_1\}} + C_n' \frac{\cosh \{(2n+1)(\lambda - \xi)/n_2\}}{\cosh \{(2n+1)\lambda/n_2\}} \right\} \cos (2n+1)\eta \quad \ldots \quad (57)$$

$$\bar{\sigma}_y = \sum_n \left\{ C_n \frac{\cosh \{(2n+1)(\lambda - \xi)/n_1\}}{n_1^2 \cosh \{(2n+1)\lambda/n_1)\}} + C_n' \frac{\cosh \{(2n+1)(\lambda - \xi)/n_2\}}{n_2^2 \cosh \{(2n+1)\lambda/n_2\}} \right\} \cos (2n+1)\eta \quad \ldots \quad (58)$$

$$\tau_{xy} = - \sum_n \left\{ C_n \frac{\sinh \{(2n+1)(\lambda - \xi)/n_1\}}{n_1 \cosh \{(2n+1)\lambda/n_1\}} + C_n' \frac{\sinh \{(2n+1)(\lambda - \xi)/n_2\}}{n_2 \cosh \{(2n+1)\lambda/n_2\}} \right\} \sin (2n+1)\eta. \quad \ldots \quad (59)$$

Transverse Edge Free.—It is found that

$$\left.\begin{aligned} C_n &= \frac{(-1)^n 4K\sigma_e}{(2n+1)\pi\varepsilon}\left(\frac{n_1 \tanh\{(2n+1)\lambda/n_2\}}{n_1 \tanh\{(2n+1)\lambda/n_2\} - n_2 \tanh\{(2n+1)\lambda/n_1\}}\right) \\ C_n' &= \frac{-(-1)^n 4K\sigma_e}{(2n+1)\pi\varepsilon}\left(\frac{n_2 \tanh\{(2n+1)\lambda/n_1\}}{n_1 \tanh\{(2n+1)\lambda/n_2\} - n_2 \tanh\{(2n+1)\lambda/n_1\}}\right) \end{aligned}\right\} . \quad (60)$$

Transverse Edge Supported.—It is found that C_n and C_n' are given by equation (42). It does not appear possible to obtain closed forms for either of these cases.

APPENDIX III

Stress Distribution in an Infinitely Long Panel Bounded by Constant-Area Edge Members

If the panel is bounded by constant-area edge members loaded only at their ends the boundary condition along the longitudinal edges corresponding to the first part of equation (1) is replaced by the equilibrium condition

$$t\tau_{xy} \pm F\frac{\partial \sigma_S}{\partial x} = 0. \quad (61)$$

This condition will be satisfied by introducing a stress function similar to that of equation (28) with $(2n + 1)\pi/2$ replaced by r_n, for this gives the stress resultants in the form :

$$\bar{\sigma}_x = 2B - \sum_n \{C_n e^{-r_n x/bn_1} + C_n' e^{-r_n x/bn_2}\} \cos r_n y/b \quad (62)$$

$$\bar{\sigma}_y = \sum_n \left\{\frac{C_n}{n_1^2} e^{-r_n x/bn_1} + \frac{C_n'}{n_2^2} e^{-r_n x/bn_2}\right\} \cos r_n y/b \quad (63)$$

$$\tau_{xy} = -\sum_n \left\{\frac{C_n}{n_1} e^{-r_n x/bn_1} + \frac{C_n'}{n_2} e^{-r_n x/bn_2}\right\} \sin r_n y/b \quad (64)$$

and equation (61) becomes, on dividing by $\left\{\frac{C_n}{n_1} e^{-r_n x/bn_1} + \frac{C_n'}{n_2} e^{-r_n x/bn_2}\right\}$:

$$t \sin r_n + \frac{F\varepsilon r_n}{Kb} \cos r_n = 0 \quad (65)$$

which is satisfied because of the definition of the r_n terms. The boundary condition represented by the second part of equation (1) will not now be completely satisfied, but the effect on the stress distribution is negligible.

From generalised Fourier analysis

$$\sum_n \left(\frac{-2(1+\varrho)\cos r_n}{\varrho + \cos^2 r_n}\right) \cos\frac{r_n y}{b} \equiv 1 \quad (66)$$

so that the condition that $\bar{\sigma}_x$ vanishes along the transverse edge is :

$$C_n + C_n' = \frac{-2K\sigma_{e,0}(1+\varrho)\cos r_n}{\varepsilon(\varrho + \cos^2 r_n)} . \quad (67)$$

If the transverse edge is free

$$\frac{C_n}{n_1} + \frac{C_n'}{n_2} = 0 \quad \ldots \tag{68}$$

and if the transverse edge is supported

$$\frac{C_n}{n_1^2} + \frac{C_n'}{n_2^2} = 0. \tag{69}$$

If C_n and C_n' are solved for equations (67) and (68), or (67) and (69), and substituted in equations (62) to (64) the problem is formally solved.

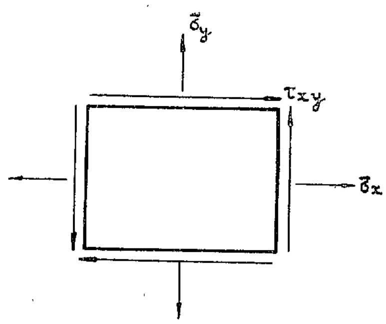
Plain Sheet.—The case when the sheet is unreinforced and the transverse edge is free is of interest. It is found that the stresses are then given by :

$$\frac{\sigma_x}{\sigma_{e,0}} = \frac{1}{1+\varrho} - 2\varrho \sum_n \frac{\sin r_n\,(1 + r_n x/b)e^{-r_n x/b} \cos (r_n y/b)}{r_n(\varrho + \cos^2 r_n)} \tag{70}$$

$$\frac{\sigma_y}{\sigma_{e,0}} = -\,2\varrho \sum_n \frac{\sin r_n\,(1 - r_n x/b)e^{-r_n x/b} \cos (r_n y/b)}{r_n(\varrho + \cos^2 r_n)} \tag{71}$$

$$\frac{\tau_{xy}}{\sigma_{e,0}} = \frac{2\varrho x}{b} \sum_n \frac{\sin r_n e^{-r_n x/b} \sin (r_n y/b)}{\varrho + \cos^2 r_n}\,. \tag{72}$$

Contours of constant $\sigma_x/\sigma_{e,0}$ are plotted in Figs. 6, 7 and 8 for values of $1/\varrho$ equal to $\frac{1}{2}$, 1 and 2. The maximum value of τ_{xy} is $2\sigma_{e,0}/\pi$.



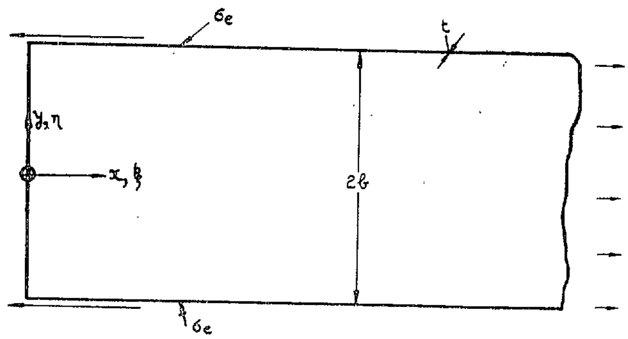


FIG. 1. Figure showing notation.

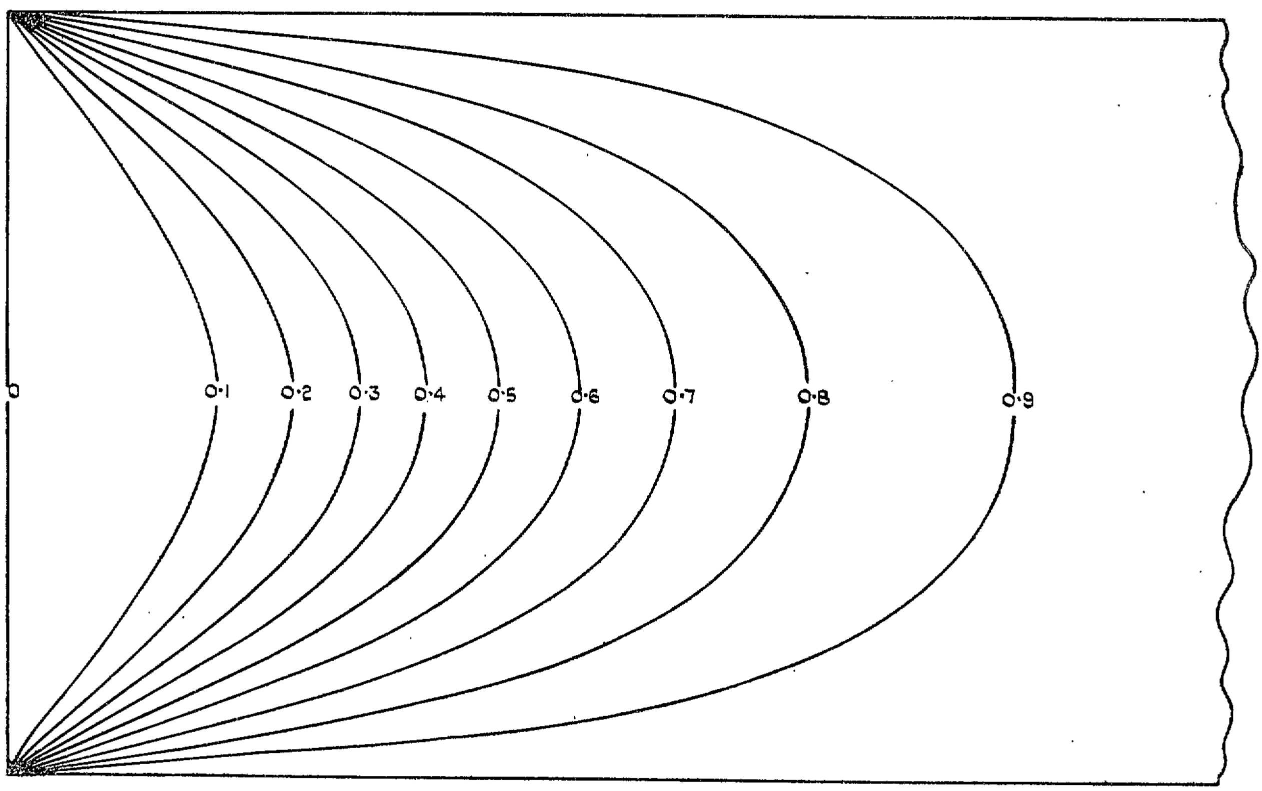


FIG. 2. Contours of constant σ_x/σ_e.

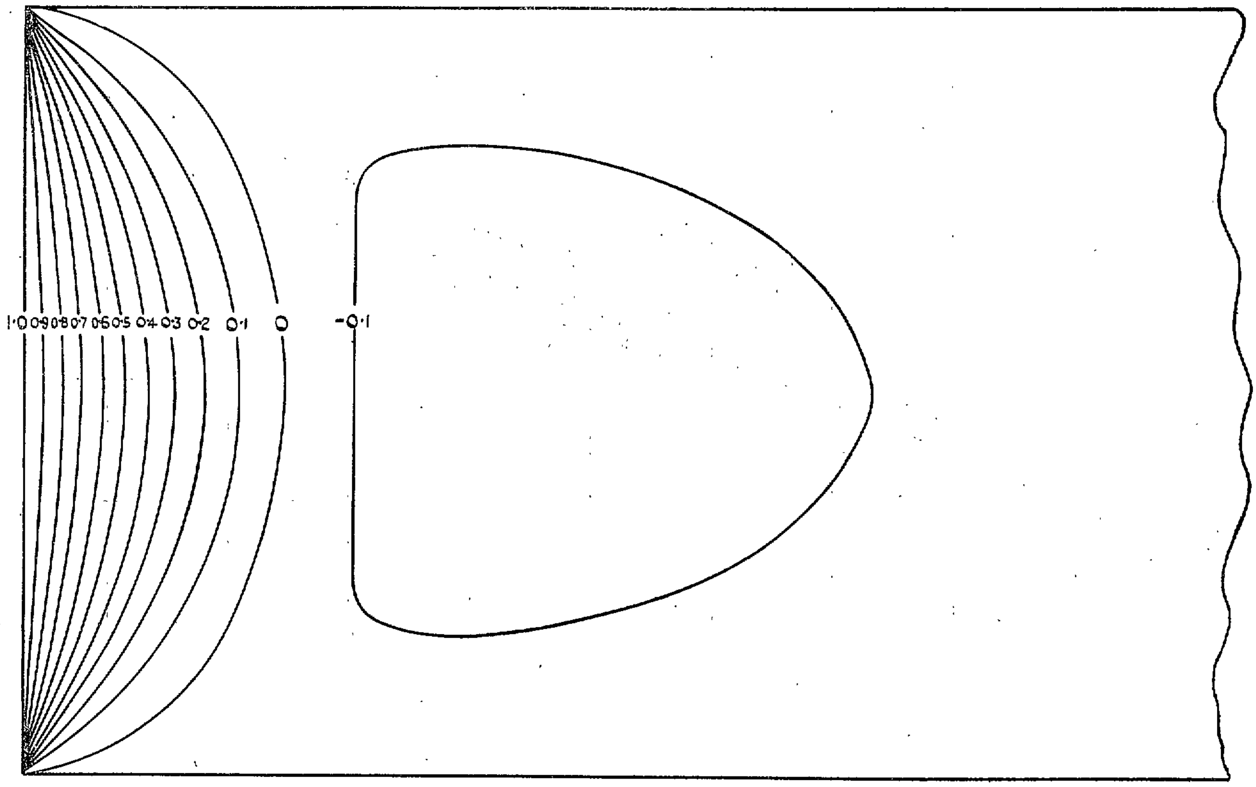


FIG. 3. Contours of constant σ_y/σ_e.

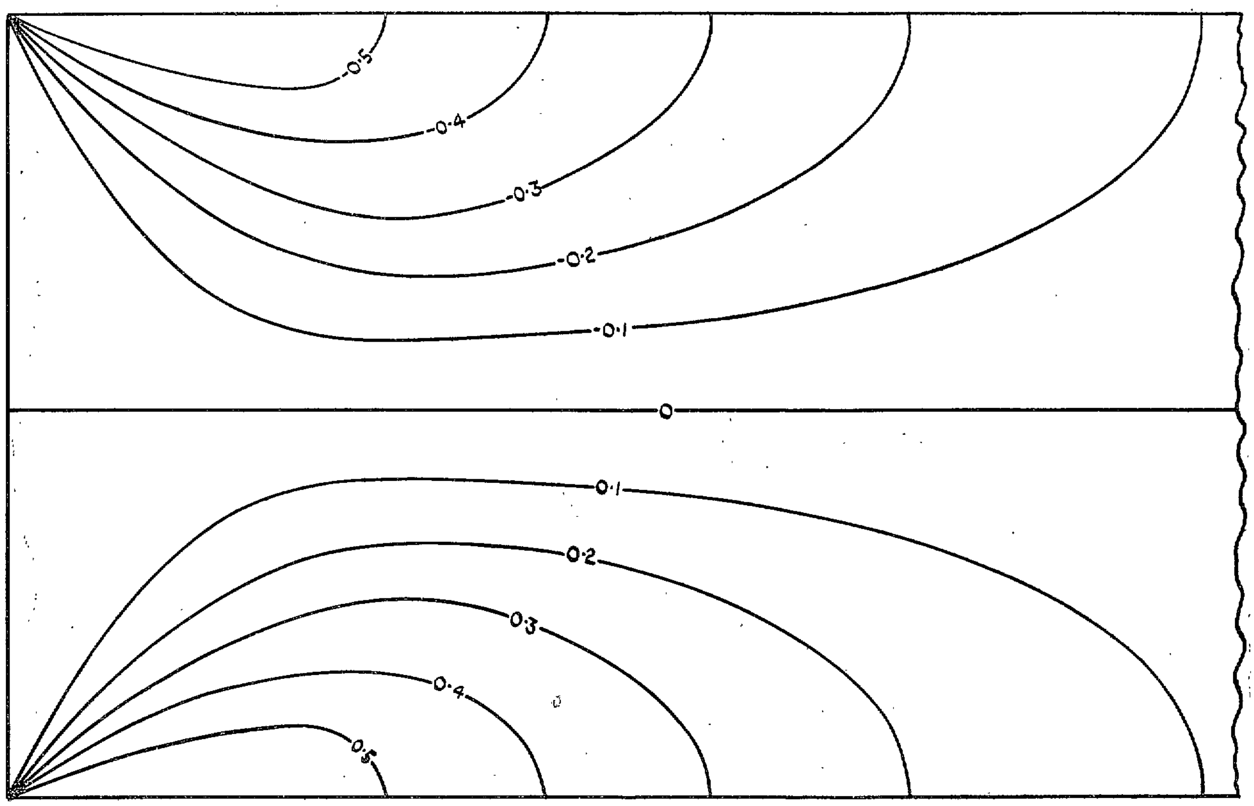


FIG. 4. Contours of constant τ_{xy}/σ_e.

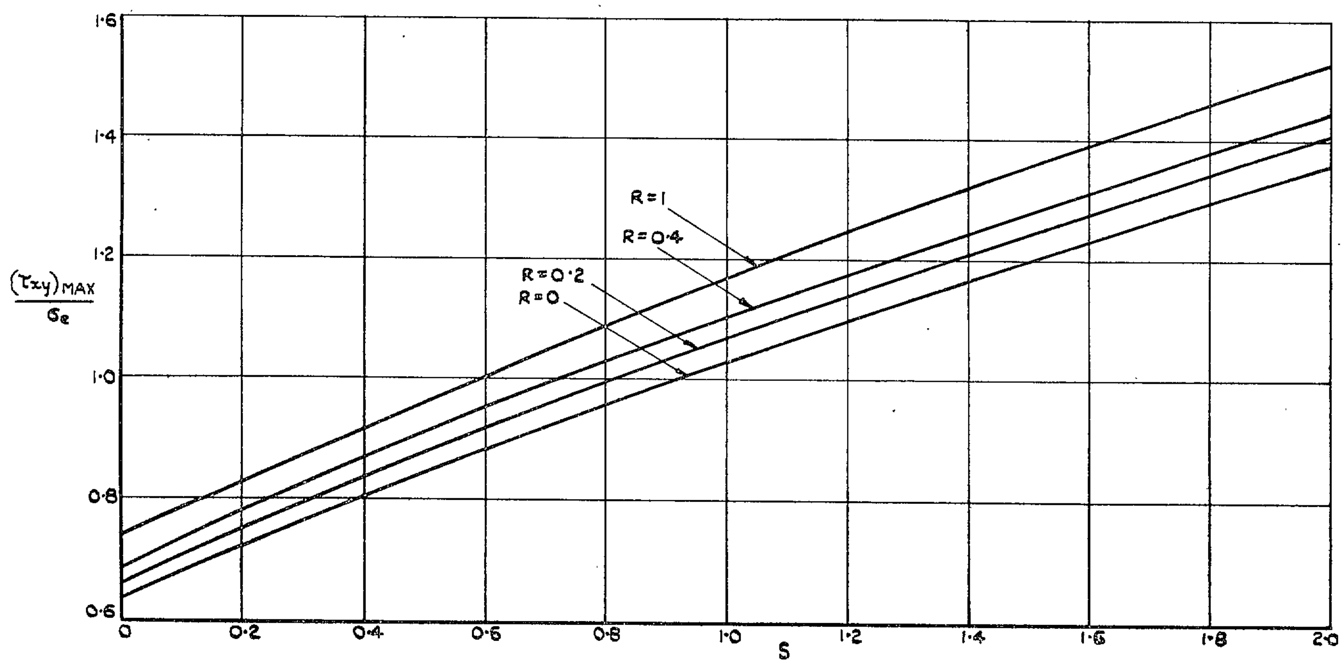


FIG. 5. Peak shear stresses in reinforced sheet.

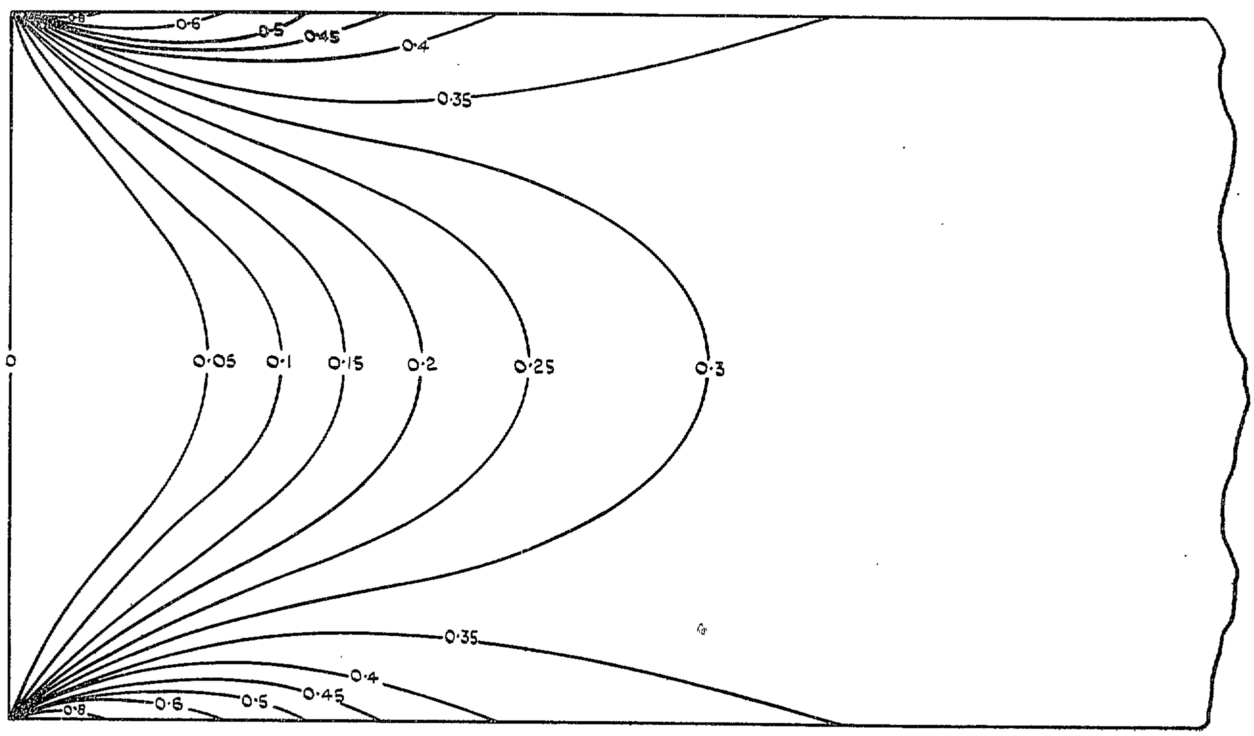


FIG. 6. Contours of constant $\sigma_x/\sigma_{x,0}$: $F = \frac{1}{4}bt$.

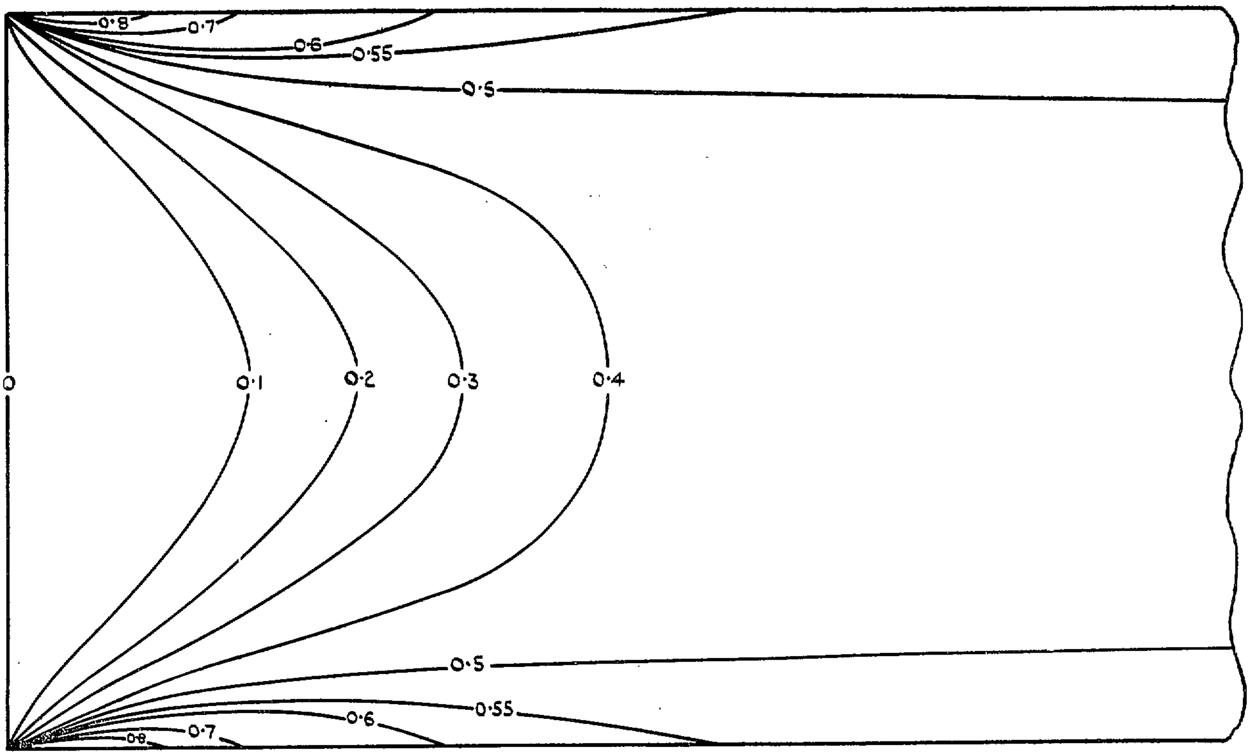


FIG. 7. Contours of constant $\sigma_x/\sigma_{e,0}$: $F = bt$.

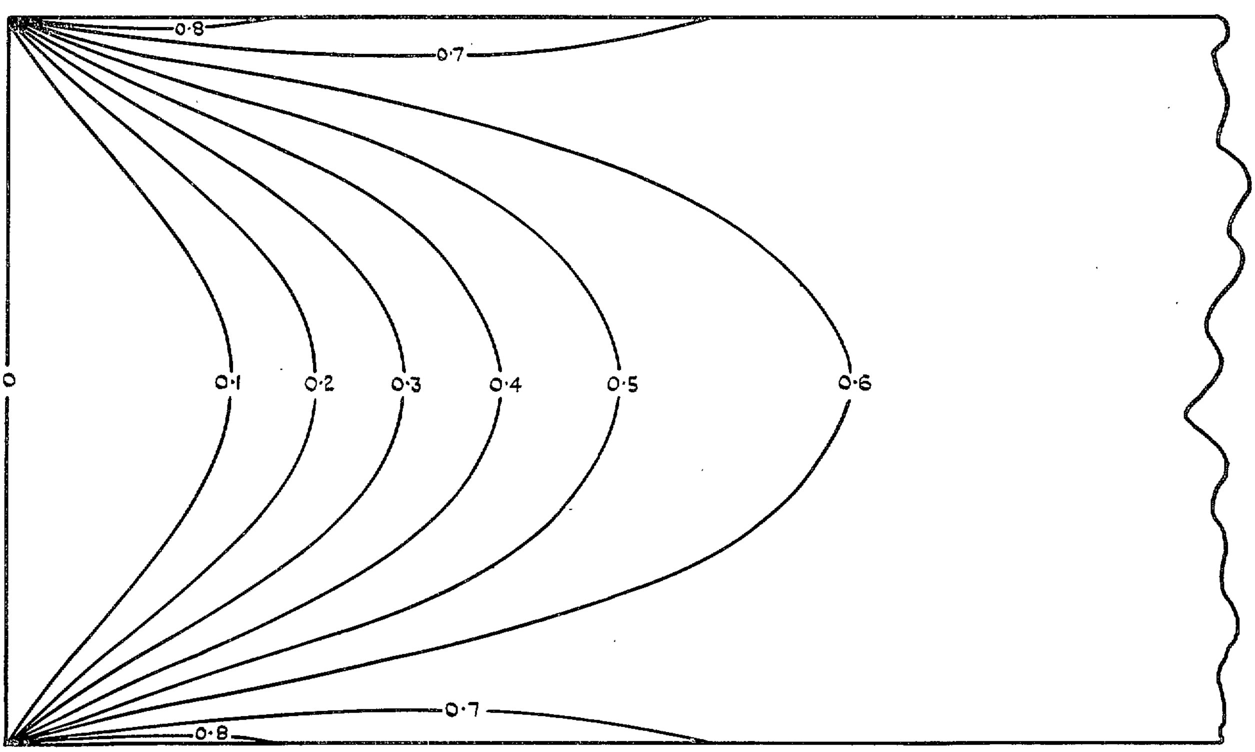


FIG. 8. Contours of constant $\sigma_x/\sigma_{e,0}$: $F = 2bt$.

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