



MINISTRY OF SUPPLY

AERONAUTICAL RESEARCH COUNCIL  
REPORTS AND MEMORANDA

# The Effect of a Wind Gradient on the Rate of Climb of an Aircraft

*By*

W. A. MAIR,

Cambridge University Aeronautics Laboratory

*Crown Copyright. Reserved*

LONDON: HER MAJESTY'S STATIONERY OFFICE

1957

THREE SHILLINGS NET

# The Effect of a Wind Gradient on the Rate of Climb of an Aircraft

By

W. A. MAIR

Cambridge University Aeronautics Laboratory

---

*Reports and Memoranda No. 2953*

*January, 1954*

---

1. *Introduction and Summary.*—The behaviour of an aircraft climbing in the presence of a wind gradient is analysed by a method similar to that used in R. & M. 379<sup>1</sup>, but with fewer simplifying assumptions. It is shown that the result obtained in the earlier paper is only correct if the angle of climb is small and if the true air speed is constant during the climb. With the climb techniques that are now usual, however, the true air speed is not constant during the climb; with typical subsonic aircraft the effect of this is to change the correction due to a wind gradient by about 10 per cent. For aircraft climbing at supersonic speeds the effect of acceleration may be much greater than this.

Since the percentage change of rate of climb due to a given wind gradient is approximately proportional to the flight speed, the effect does *not* become less important as aircraft speeds increase. Moreover, wind gradients having a significant effect on the rate of climb are not confined to low altitudes. For a modern high-speed aircraft it is shown that the wind gradient may change the rate of climb by 20 per cent or more.

## 2. *Notation.*—

$a$	Velocity of sound
$D$	Drag
$E_K$	Kinetic energy of aircraft
$E_P$	Potential energy of aircraft
$g$	Acceleration due to gravity
$h$	Height
$H$	Energy height = $(E_P + E_K)/mg$
$M$	Mach number
$m$	Mass of aircraft
$T$	Thrust
$t$	Time
$u, v$	Horizontal and vertical components of velocity relative to ground
$V$	True air speed



It will now be assumed that, at a given height,  $V$  and  $C_L$  are unaffected by wind gradient\* ; then  $(T - D)$  is also independent of wind gradient and equations (6) and (7) give

$$\frac{\Delta v}{v} = \frac{\sin \theta - \sin \theta_0}{\sin \theta} = -\frac{Vw}{g} \cos \theta - \frac{\dot{V} - \dot{V}_0}{g \sin \theta}, \quad \dots \dots \dots (8)$$

where  $\Delta v$  is the amount by which the wind gradient increases the rate of climb.

Also, 
$$\frac{\dot{V}_0}{\dot{V}} = \frac{v_0}{v} = 1 - \frac{\Delta v}{v}.$$

Thus equation (8) becomes

$$\frac{\Delta v}{v} = -\frac{Vw}{g} \cos \theta - \frac{\dot{V}}{g \sin \theta} \frac{\Delta v}{v},$$

and hence 
$$\frac{\Delta v}{v} = -\frac{Vw \cos \theta}{g + \dot{V} \operatorname{cosec} \theta} \quad \dots \dots \dots (9)$$

In the special case where the true air speed is constant during the climb ( $\dot{V} = 0$ ) and where the angle of climb is small ( $\cos \theta = 1$ ), equation (9) reduces to the simple form given in Ref. 1, *viz.*,

$$\frac{\Delta v}{v} = -\frac{Vw}{g} \quad \dots \dots \dots (10)$$

It should be noted, however, that with the climb techniques that are now usual the true air speed is *not* constant during the climb. Thus equation (9) must be used in full unless it can be shown that  $\dot{V} \operatorname{cosec} \theta$  is small compared with  $g$ . This point is considered later.

4. *Numerical Values of Wind Gradient.*—It is of interest to consider the order of magnitude of the wind gradients occurring in the atmosphere, and hence estimate the importance of the correction given by equation (9). For this purpose it is sufficiently accurate to use the approximate form given in equation (10).

At very low altitudes the wind gradient is often large, and for heights up to about 100 ft the relations given by Deacon<sup>2</sup> may be used. In this range of height, Deacon's results suggest that the *maximum* wind gradient is given very roughly by

$$w = \frac{10}{h} \operatorname{sec}^{-1},$$

where  $h$  is the height in feet. Thus at heights up to about 100 ft the value of  $w$  may be  $0.1 \operatorname{sec}^{-1}$  or more. This conclusion is confirmed by some data quoted in R. & M. 1489<sup>3</sup>, showing values of  $w$  of  $0.12 \operatorname{sec}^{-1}$  at 50 ft. and  $0.23 \operatorname{sec}^{-1}$  at 20 ft.

Between about 3,000 and 7,500 ft, results given by Petterssen and Swinbank<sup>4</sup> show that the maximum values of  $w$  are about  $0.01 \operatorname{sec}^{-1}$  or greater. At heights between about 20,000 and 50,000 ft, statistics given by Bannon<sup>5</sup> show that wind gradients of order  $0.005 \operatorname{sec}^{-1}$  are fairly common over Britain, and that gradients up to about  $0.02 \operatorname{sec}^{-1}$  occur occasionally. It should be noted that these gradients are not confined to very thin layers ; they are mean gradients over depths of several thousand feet. The larger wind gradients are usually associated with jet streams. The wind gradients above and below the core of a jet stream are of opposite sign and are usually approximately equal in magnitude.

---

\* This assumption is considered further in sections 5 and 7.

Thus, for the purpose of estimating the importance of the effect of wind gradients on rate of climb, it may be assumed that the maximum values of  $w$  likely to be encountered are roughly as shown below :

$h$ (ft)	$w$ (sec <sup>-1</sup> )
Below 100	0·1 or more
100 to 1,000	0·05
1,000 to 50,000	0·01

Substituting the *lowest* of these values ( $w = 0\cdot01$  sec<sup>-1</sup>) in equation (10) gives

$$\frac{\Delta v}{v} = - \frac{V}{3220} \quad (\text{with } V \text{ in ft/sec}).$$

For a typical modern aircraft  $V$  may be about 600 ft/sec during the climb. Thus even a wind gradient as small as 0·01 sec<sup>-1</sup> can change the rate of climb by nearly 20 per cent. For aircraft climbing at higher speeds (including supersonic aircraft), the effect of a wind gradient will of course be greater. The larger wind gradients occurring at heights below about 1,000 ft will also have a greater effect ; this is considered below.

5. *Effect of Large Wind Gradients near the Ground.*—Immediately after take-off, the aircraft will normally be flying against the wind, so that  $w$  will be negative. If  $(-w)$  is sufficiently large, it is clearly possible for the term  $(-Vw/g) \cos \theta$  in equation (8) to be greater than 1. At first sight, this leads to an anomalous result, for if  $\dot{V} = \dot{V}_0 = 0$ ,  $\Delta v/v$  is then greater than 1, and if  $v$  is positive  $v_0$  must be negative. The explanation is that if  $(-Vw/g) \cos \theta > 1$ , the assumption that  $\dot{V} = 0$  necessarily implies  $T < D$ , if  $\theta$  is positive (from equation (6)), so that if  $\dot{V}_0$  is also zero,  $v_0$  must be negative. Thus with  $(-w)$  so large that  $(-Vw/g) \cos \theta > 1$  (and with  $\theta$  positive), equation (6) requires a positive value of  $V$  if  $T > D$ . Physically, this means that if the thrust is greater than the drag, the air speed must increase as the aircraft climbs into regions of higher wind velocity.

Thus, in regions of large wind gradient near the ground, the second term on the right-hand side of equation (8) is likely to be very important. Unfortunately, the assumption made in deriving equations (8) and (9), that at a given height  $V$  is independent of wind gradient, will not be correct in such cases. A further investigation of climbing flight near the ground would require a more detailed analysis, taking values of  $\dot{V}$  and  $\dot{V}_0$  appropriate to a particular take-off technique.

The case where  $w$  has a large *positive* value corresponds to an aircraft climbing down-wind near the ground. This is of less practical importance than the case considered above, but again the acceleration term in equation (8) is likely to be important. When  $w$  is positive the wind gradient reduces the rate of climb, but it should be noted that if  $v_0$  is positive  $v$  can never be reduced to zero.

6. *Energy Relations.*—Considering the usual low-altitude case, where the wind velocity increases with height, equation (9) shows that the wind gradient increases the rate of ascent of an aircraft flying against the wind, at a given speed. It is of interest to consider the source of the energy required for this increased rate of ascent, since the work done per unit time by the propulsion system of the aircraft is not affected by the wind gradient. For simplicity, it will be assumed here that  $V$  is constant ( $\dot{V} = 0$ ).

It is more convenient to consider an aircraft flying down-wind. Then, since the rate of climb is *reduced* as given by equation (9), the rate of increase of potential energy is *less* than it would be in the absence of a wind gradient by an amount

$$\Delta \left( \frac{dE_P}{dt} \right) = mV^2 w \sin \theta \cos \theta. \quad \dots \dots \dots (11)$$

The Kinetic Energy of the aircraft is

$$E_K = \frac{1}{2}m(w^2 + v^2) \\ = \frac{1}{2}m(W^2 + V^2 + 2WV \cos \theta).$$

Therefore  $\frac{dE_K}{dW} = m(W + V \cos \theta) = mu,$

and  $\frac{dE_K}{dt} = \frac{dE_K}{dW} \frac{dW}{dh} \frac{dh}{dt} = muwV \sin \theta$   
 $= mVw (W + V \cos \theta) \sin \theta. \quad \dots \dots \dots (12)$

Subtracting equations (11) and (12) now gives

$$\frac{dE_K}{dt} - \Delta \left( \frac{dE_P}{dt} \right) = mwWV \sin \theta = \frac{d}{dt} \left( \frac{1}{2}mW^2 \right). \quad \dots \dots \dots (13)$$

In the usual case where the wind velocity  $W$  is small compared with  $V \cos \theta$  (the horizontal component of the relative air velocity), equations (12) and (13) show that

$$\frac{d}{dt} \left( \frac{1}{2}mW^2 \right) < < \frac{dE_K}{dt},$$

and  $\frac{dE_K}{dt} \simeq \Delta \left( \frac{dE_P}{dt} \right).$

The wind gradient then reduces the rate of increase of potential energy by an amount equal to the rate of increase of kinetic energy.

It is convenient to introduce an 'Energy Height', similar to that suggested by Lush<sup>6</sup>. For the present purpose the energy height ( $H$ ) will be defined by

$$mgH = E_P + E_K,$$

*i.e.*, it is the height at which the potential energy of the aircraft would be equal to the sum of the potential and kinetic energies at the actual height  $h$ . This definition is the same as that used by Lush, but it is now necessary to allow for the effect of the wind in calculating the kinetic energy  $E_K$ .

Thus, if  $V$  is constant and  $W$  is small compared with  $V \cos \theta$ , the rate of increase of energy height (defined as above) is independent of wind gradient. If  $W$  is not small compared with  $V \cos \theta$  this is no longer true; there is then a transfer of energy from the wind to the aircraft (with the aircraft flying down-wind), represented by the term  $d(\frac{1}{2}mW^2)/dt$  in equation (13).

In the extreme case where  $\theta = 90$  deg, equation (9) shows that the wind gradient has no effect on the rate of climb. The motion of the aircraft is then similar to that of a balloon with a constant rate of climb.







9. *Conclusions and Discussion.*—It has been shown that a wind gradient may have an important effect on the rate of climb of an aircraft, and that the effect is *not* confined to low altitudes. Nevertheless, the wind gradient will usually only be large enough to have an appreciable effect over a limited range of altitude, so that the effect on the total time to climb to a given height will usually be fairly small.

The result obtained in Ref. 1 is only correct if the angle of climb is small and if the true air speed is constant during the climb. In fact, the true air speed is usually *not* constant during the climb, and for typical subsonic aircraft this may change the effect of a wind gradient by about 10 per cent. For an aircraft climbing at supersonic speed, accelerating during the climb, the acceleration may reduce considerably the effect of a wind gradient. In a typical case of this kind the effect of a wind gradient is only about one third of the value calculated by neglecting the acceleration.

It should be noted that a wind gradient can have no effect on the ceiling of an aircraft, because the *change* of rate of climb becomes zero when the rate of climb itself is zero.

The effect of a wind gradient on the initial rate of climb, immediately after take-off, may sometimes be large, but in this case the acceleration term on the right-hand side of equation (8) is likely to be important and the assumption made in deriving equations (8) and (9), that at a given height  $V$  is independent of wind gradient, will not be correct. Fortunately, however, the effect of a wind gradient in this case is favourable, so that no difficulty should arise if take-off tests for acceptance purposes are always made in conditions of very small wind gradient. If an aircraft turns down wind immediately after take-off, however, the rate of climb may be seriously reduced by a wind gradient.

The present practice at the Aircraft and Armament Experimental Establishment is to apply a correction based on Ref. 1 (equation (10) of this paper), where climbing tests are made in the presence of a known wind gradient. It is usual to make climbing tests on reciprocal courses, and whenever possible these are made with the plane of the flight path normal to the wind velocity. If this latter condition is satisfied the wind gradient should have no effect on the rate of climb.

10. *Acknowledgements.*—I am grateful to Mr. J. K. Bannon of the Meteorological Office for information about wind gradients, and to Mr. T. V. Somerville and Mr. A. S. Taylor of the Royal Aircraft Establishment for their helpful criticisms and suggestions.

## REFERENCES

<i>No.</i>	<i>Author</i>	<i>Title, etc.</i>
1	Presented by the Superintendent, Royal Aircraft Factory	The effect of a wind gradient on the rate of climb of an aeroplane. R. & M. 379. 1917.
2	E. L. Deacon .. .. .	Vertical diffusion in the lowest layers of the atmosphere. <i>Q. J. Roy. Met. Soc.</i> Vol. 75, pp. 89 to 103. 1949.
3	F. B. Bradfield and J. Cohen .. ..	Wind tunnel test of lift and drag measured in a velocity gradient. R. & M. 1489. 1932.
4	S. Petterssen and W. C. Swinbank ..	On the application of the Richardson criterion to large-scale turbulence in the free atmosphere. <i>Q. J. Roy. Met. Soc.</i> Vol. 73, pp. 335 to 345. 1947.
5	J. K. Bannon .. .. .	Shear frequencies in the upper troposphere and lower stratosphere over England. <i>Meteorological Magazine</i> , Vol. 79, pp. 161 to 165. 1950.
6	K. J. Lush .. .. .	A review of the problem of choosing a climb technique, with proposals for a new climb technique for high performance aircraft. R. & M. 2557. 1948.
7	Wing Cdr. L. Kelly .. .. .	Optimum climb technique for a jet propelled aircraft. College of Aeronautics Report No. 57. A.R.C. 15,057. 1952.



# Publications of the Aeronautical Research Council

## ANNUAL TECHNICAL REPORTS OF THE AERONAUTICAL RESEARCH COUNCIL (BOUND VOLUMES)

- 1938 Vol. I. Aerodynamics General, Performance, Airscrews. 50s. (51s. 8d.)  
Vol. II. Stability and Control, Flutter, Structures, Seaplanes, Wind Tunnels, Materials. 30s. (31s. 8d.)
- 1939 Vol. I. Aerodynamics General, Performance, Airscrews, Engines. 50s. (51s. 8d.)  
Vol. II. Stability and Control, Flutter and Vibration, Instruments, Structures, Seaplanes, etc. 63s. (64s. 8d.)
- 1940 Aero and Hydrodynamics, Aerofoils, Airscrews, Engines, Flutter, Icing, Stability and Control, Structures, and a miscellaneous section. 50s. (51s. 8d.)
- 1941 Aero and Hydrodynamics, Aerofoils, Airscrews, Engines, Flutter, Stability and Control, Structures. 63s. (64s. 8d.)
- 1942 Vol. I. Aero and Hydrodynamics, Aerofoils, Airscrews, Engines. 75s. (76s. 8d.)  
Vol. II. Noise, Parachutes, Stability and Control, Structures, Vibration, Wind Tunnels. 47s. 6d. (49s. 2d.)
- 1943 Vol. I. Aerodynamics, Aerofoils, Airscrews. 80s. (81s. 8d.)  
Vol. II. Engines, Flutter, Materials, Parachutes, Performance, Stability and Control, Structures. 90s. (91s. 11d.)
- 1944 Vol. I. Aero and Hydrodynamics, Aerofoils, Aircraft, Airscrews, Controls. 84s. (86s. 9d.)  
Vol. II. Flutter and Vibration, Materials, Miscellaneous, Navigation, Parachutes, Performance, Plates and Panels, Stability, Structures, Test Equipment, Wind Tunnels. 84s. (86s. 9d.)

## ANNUAL REPORTS OF THE AERONAUTICAL RESEARCH COUNCIL—

1933-34	1s. 6d. (1s. 8½d.)	1937	2s. (2s. 2½d.)
1934-35	1s. 6d. (1s. 8½d.)	1938	1s. 6d. (1s. 8½d.)
April 1, 1935 to Dec. 31, 1936	4s. (4s. 5½d.)	1939-48	3s. (3s. 3½d.)

## INDEX TO ALL REPORTS AND MEMORANDA PUBLISHED IN THE ANNUAL TECHNICAL REPORTS, AND SEPARATELY—

April, 1950 - - - - - R. & M. No. 2600. 2s. 6d. (2s. 7½d.)

## AUTHOR INDEX TO ALL REPORTS AND MEMORANDA OF THE AERONAUTICAL RESEARCH COUNCIL—

1909-January, 1954 - - - - - R. & M. No. 2570. 15s. (15s. 5½d.)

## INDEXES TO THE TECHNICAL REPORTS OF THE AERONAUTICAL RESEARCH COUNCIL—

December 1, 1936 — June 30, 1939.	R. & M. No. 1850.	1s. 3d. (1s. 4½d.)
July 1, 1939 — June 30, 1945.	R. & M. No. 1950.	1s. (1s. 1½d.)
July 1, 1945 — June 30, 1946.	R. & M. No. 2050.	1s. (1s. 1½d.)
July 1, 1946 — December 31, 1946.	R. & M. No. 2150.	1s. 3d. (1s. 4½d.)
January 1, 1947 — June 30, 1947.	R. & M. No. 2250.	1s. 3d. (1s. 4½d.)

## PUBLISHED REPORTS AND MEMORANDA OF THE AERONAUTICAL RESEARCH COUNCIL—

Between Nos. 2251-2349.	R. & M. No. 2350.	1s. 9d. (1s. 10½d.)
Between Nos. 2351-2449.	R. & M. No. 2450.	2s. (2s. 1½d.)
Between Nos. 2451-2549.	R. & M. No. 2550.	2s. 6d. (2s. 7½d.)
Between Nos. 2551-2649.	R. & M. No. 2650.	2s. 6d. (2s. 7½d.)

*Prices in brackets include postage*

## HER MAJESTY'S STATIONERY OFFICE

York House, Kingsway, London W.C.2; 423 Oxford Street, London W.1 (Post Orders: P.O. Box 569, London S.E.1);  
13a Castle Street, Edinburgh 2; 39 King Street, Manchester 2; 2 Edmund Street, Birmingham 3; 109 St. Mary Street,  
Cardiff; Tower Lane, Bristol 1; 80 Chichester Street, Belfast, or through any bookseller

S.O. Code No. 23-2953

R. & M. No. 2953