R. & M. No. 2921
(15,642)
A.R.C. Technical Report

MINISTRY OF SUPPLY

AERONAUTICAL RESEARCH COUNCIL
REPORTS AND MEMORANDA

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LONDON: HER MAJESTY'S STATIONERY OFFICE
1956
PRICE 4s 6d NET

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Reports and Memoranda No. 2921*

February, 1953

Summary.—A relatively simple method for calculating the aerodynamic forces on an oscillating aerofoil is developed and used to derive the aerodynamic coefficients for $M = 0 \cdot 7$, $0 \cdot 8$ and $0 \cdot 9$ for a range of frequency parameter values.

The two-dimensional aerofoil is represented by a flat plate and the usual assumptions of linearized theory for unsteady flow are made. The problem is reduced to one of finding the solution of an integral equation for the velocity potential of the disturbed flow. This is solved by the use of the known solution of a related problem in incompressible flow in which the aerofoil oscillates at a frequency increased by the factor $(1 - M^2)^{-1}$ and for which the condition for tangential flow is suitably modified. By successive approximation to this modified boundary condition, it is possible to obtain solutions to any desired accuracy. Formulae for the aerodynamic coefficients may also be derived for each approximation. Those given by the first approximation are of sufficient accuracy for use in stability calculations when the frequency parameters involved are low. For higher values, more complicated formulae corresponding to higher-order approximations could be derived if required.

The results obtained confirm that values given in Ref. 6 which were derived by Dietze's method for $M = 0 \cdot 7$ and by Schade for $M = 0 \cdot 8$ are substantially correct.

Introduction.—The problem of the oscillating aerofoil in two-dimensional subsonic flow has been considered by many writers. Possio[1], in 1938, reduced it to one of finding the solution of an integral equation. He obtained some numerical values for the aerodynamic coefficients corresponding to plunging and pitching oscillations of the aerofoil for particular Mach numbers. Frazer[2], in 1941, repeated Possio's calculations and improved the accuracy of the numerical solution given by the latter. Then followed the work of Eichler[3] (1942), Schade[4] (1944) and Dietze[5] (1944). Both Eichler and Schade reduce the problem to the solution of a set of linear algebraic equations, while Dietze solved Possio's integral equation by an iterative method. Tables of aerodynamic coefficients for different Mach numbers and a range of frequency parameter values have been given by Minhinnick[6]. They are based mainly on results obtained by Dietze's iterative method and Schade's values.

An alternative method of approach suggested by Reissner and Sherman[7] (1944), Timman[8] (1946) and Billington[9] (1949) is to solve the wave equation directly in terms of series of Mathieu functions. Timman, Van de Vooren and Greidanus[10] (1951) have given tables of the aerodynamic coefficients derived on the basis of Timman's earlier analytical treatment of the problem. As shown in Table 1 their values differ appreciably in some cases from those given in Ref. 6.

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In view of the discrepancies between the various theories it seemed desirable that the exact solution should be found. With this object in view, the method of solution suggested in Ref. 11, with slight modifications, is used to calculate the aerodynamic coefficients for $M = 0 \cdot 7$ and $0 \cdot 8$ for a suitable range of values of the frequency parameter. The results obtained agree closely with those given in Ref. 6 and it appears that the coefficients tabulated in Ref. 10 are in error.

As a matter of mathematical interest, calculations were also done for $M = 0 \cdot 9$ for a smaller range of frequency parameter values. The results corresponding to Approx. III (3) given in Table 1 are believed to be reasonably accurate except possibly those for the largest frequency parameter value considered. However, it did not seem worthwhile to proceed to Approx. IV as the values obtained would in any case require modification to allow for thickness and boundary-layer effects in practice.

LIST OF SYMBOLS AND DEFINITIONS

$c(= 2l)$	Chord
$x(= lX = -l \cos \vartheta$	Distance along OX axis of point P
$z(= lz' \, e^{ipt})$	Downward displacement at mid-chord
$\alpha(= \alpha' \, e^{ipt})$	Angular displacement
ζ	Downward displacement at P (*see* Fig. 1)
U_0	Wind speed
$t(= lT/U_0)$	Time
$w(= w' \, e^{ipt})$	Downwash distribution
$\phi(= l\Phi \, e^{i(\lambda X + \omega T)})$	Velocity potential
M	Mach number
$\bar{\omega} = 2\omega = pc/U_0$	Frequency parameter
$\nu = \dfrac{\omega}{1 - M^2}$;	$\varkappa = M\nu$; $\lambda = M^2\nu$; $\beta = \sqrt{(1 - M^2)}$
$k(= lK \, e^{i(\lambda X + \omega T)})$	Discontinuity in velocity potential at surface of aerofoil and wake
$K(= \Phi_a - \Phi_b)$	Discontinuity in Φ
$W\left(= \dfrac{w'}{\beta} \, e^{-i\lambda X}\right)$	Downwash distribution corresponding to Φ

K_n distributions

$$K_0 = 2\left\{\sin \vartheta + e^{i\nu} \cos \vartheta \left[X_0(\nu)\vartheta + 2 \sum_{n=1}^{\infty} (-i)^n . X_n(\nu) \frac{\sin n\vartheta}{n}\right\}\right]$$

$$= 2\pi X_0(\nu) \, e^{-i\nu X} \, ... \, X \geqslant 1$$

$$K_1 = \sin \vartheta + \frac{\sin 2\vartheta}{2}$$

$$K_n = \frac{\sin (n + 1)\vartheta}{n + 1} - \frac{\sin (n - 1)\vartheta}{n - 1} \, , ... \, n \geqslant 2$$

Γ_n distributions

$$\Gamma_n = i\nu K_n + \frac{\partial K_n}{\partial X}$$

$$\Gamma_0 = 2\left[C(\nu)\cot\frac{\vartheta}{2} + i\nu\sin\vartheta\right]$$

$$\Gamma_1 = -2\sin\vartheta + \cot\frac{\vartheta}{2} + i\nu\left(\sin\vartheta + \frac{\sin 2\vartheta}{2}\right)$$

$$\Gamma_n = -2\sin n\vartheta + i\nu\left[\frac{\sin(n+1)\vartheta}{n+1} - \frac{\sin(n-1)\vartheta}{n-1}\right]; \ldots . n \geqslant 2$$

$$C(\nu) = \frac{H_1^{(2)}(\nu)}{H_1^{(2)}(\nu) + iH_0^{(2)}(\nu)}$$

$$X_0(\nu) = C(\nu)J_0(\nu) + i[1 - C(\nu)]J_1(\nu)$$

$$X_n(\nu) = C(\nu)J_n(\nu) - i[1 - C(\nu)]J_n'(\nu)$$

Lift and Moment Integrals

$$R_0 = 2\pi\left\{C(\nu)[J_0(\lambda) - iJ_1(\lambda)] + \frac{i\nu}{2}(J_0(\lambda) + J_2(\lambda))\right\}$$

$$R_1 = -\pi\left(1 - \frac{\nu}{\lambda}\right)[J_2(\lambda) + iJ_1(\lambda)]$$

$$n \geqslant 2, \ldots R_n = (-i)^{n+1}\pi\left(1 - \frac{\nu}{\lambda}\right)[J_{n+1}(\lambda) + J_{n-1}(\lambda)]$$

$$R_0' = 2\pi\left\{C(\nu)[J_0'(\lambda) - iJ_1'(\lambda)] + \frac{i\nu}{2}[J_0'(\lambda) + J_2'(\lambda)]\right\}$$

$$R_1' = -\pi\left(1 - \frac{\nu}{\lambda}\right)[J_2'(\lambda) + iJ_1'(\lambda)] - \frac{\pi\nu}{\lambda^2}[J_2(\lambda) + iJ_1(\lambda)]$$

$$n \geqslant 2, \ldots R_n' = (-i)^{n+1}\pi\left\{\left(1 - \frac{\nu}{\lambda}\right)[J_{n+1}'(\lambda) + J_{n-1}'(\lambda)] + \frac{\nu}{\lambda^2}[J_{n+1}(\lambda) + J_{n-1}(\lambda)]\right\}$$

Equations of Motion.—Let U_0, p_0, ρ_0 be the uniform velocity, pressure, and density respectively of the air stream in the undisturbed state, and let V_s denote the velocity of sound. Then if $U_0 + u$, w denote the velocity components of the disturbed flow at a point x, z at time t due to the presence of the oscillating aerofoil, the linearized equations for the motion will be

$$\frac{du}{dt} = -\frac{1}{\rho_0}\frac{\partial p}{\partial x}; \quad \frac{dw}{dt} = -\frac{1}{\rho_0}\frac{\partial p}{\partial z} \quad \ldots \quad (1)$$

where p is the pressure and $d/dt \equiv \partial/\partial t + U_0\,\partial/\partial x$. The corresponding equation of continuity is

$$\frac{dp}{dt} + \rho_0 V_s^2\left(\frac{\partial u}{dx} + \frac{\partial w}{\partial z}\right) = 0\,. \quad \ldots \quad (2)$$

Let ϕ denote the velocity potential of the disturbance superimposed on the steady flow. Then $u = \partial\phi/\partial x$, and $w = \partial\phi/\partial z$, and substitution in (1) and integration yields

$$\frac{d\phi}{dt} = \frac{\partial\phi}{\partial t} + U_0\frac{\partial\phi}{\partial x} = -\frac{p - p_0}{\rho_0} + f(t) \, . \quad \ldots \quad (3)$$

Since ϕ and $p - p_0$ are zero at an infinite distance away from the aerofoil and its wake, $f(t)$ must also vanish everywhere.

Let ϕ_a and ϕ_b represent the value of ϕ immediately above and just below the sheet of discontinuity representing the aerofoil and its wake. Then, it follows from (3) that the lift distribution $\bar{l}(x)$ is given by

$$\bar{l}(x) = p_b - p_a = \rho_0\left(\frac{\partial k}{\partial t} + U_0\frac{\partial k}{\partial x}\right) \quad \ldots \quad (4)$$

where $k \equiv \phi_a - \phi_b$. In the wake, since there is no discontinuity in pressure, the condition

$$\frac{\partial k}{\partial t} + U_0\frac{\partial k}{\partial x} = 0 \quad \ldots \quad (5)$$

must be satisfied. From (2) and (3) it may also be deduced that ϕ satisfies the equation

$$\frac{d^2\phi}{dt^2} = V_s^{\,2}\left(\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial z^2}\right) \quad \ldots \quad (6)$$

over the whole field of flow. Furthermore, it must be such that the condition of tangential flow at the aerofoil's surface is satisfied. If $\zeta(x, t)$ is the downward displacement at any point at time t, the corresponding downwash at that point is

$$w = \frac{\partial\phi}{\partial z} = \frac{\partial\zeta}{\partial t} + U_0\frac{\partial\zeta}{\partial x} \, . \quad \ldots \quad (7)$$

The problem is then reduced to one of finding a solution of (6) which satisfies (5) in the wake and condition (7) on the aerofoil.

Method of Solution.—Firstly, the variables x, z, t are replaced by X, Z, T, where

$$x = lX \; ; \quad z = \beta^{-1}lZ \; ; \quad t = \frac{lT}{U_0} \quad \ldots \quad (8)$$

and where $\beta = \sqrt{(1 - M^2)}$. The symbol l denotes half-chord, and $M(\equiv U_0/V_s)$ is the Mach number.

If f is the frequency of the oscillation, the velocity potential ϕ of the disturbance may be expressed conveniently in the form

$$\phi = l\Phi\, e^{i(\lambda X + \omega T)}, \quad \ldots \quad (9)$$

where $\omega = 2\pi fl/U_0$ and $\lambda = M^2\omega/\beta^2$. It then follows from (6) that Φ must satisfy the wave equation

$$\frac{\partial^2\Phi}{\partial X^2} + \frac{\partial^2\Phi}{\partial Z^2} + \varkappa^2\Phi = 0 \quad \ldots \quad (10)$$

where $\varkappa = M\omega/\beta^2$. The corresponding boundary condition is

$$W = \frac{\partial\Phi}{\partial Z} = \frac{w}{\beta}\, e^{-i(\lambda X + \omega T)} \, , \quad \ldots \quad (11)$$

where w is defined by (7). Furthermore, by the use of (4) and (9), it may be shown that

$$\bar{l}(X) = \rho_0 U_0 \left(\frac{\partial}{\partial T} + \frac{\partial}{\partial X}\right) K\, e^{i(\lambda X + \omega T)} \quad \ldots \quad (12)$$

where $K \equiv \Phi_a - \Phi_b$.

Write

$$\nu = \lambda + \omega = \omega/\beta^2 \quad \ldots \quad (13)$$

and

$$\Gamma = i\nu K + \frac{\partial K}{\partial X} \quad \ldots \quad (14)$$

Equation (12) then yields

$$\bar{l}(X) = \rho_0 U_0 \Gamma\, e^{i(\lambda X + \omega T)} \quad \ldots \quad (15)$$

Since $l(X)$ is zero in the wake, $\Gamma = 0$, and $K(X)$ is defined in terms of its value at the trailing edge by

$$K(X) = K(1)\, e^{-i\nu(X-1)} \ldots . \; X \geqslant 1 \quad \ldots \quad (16)$$

If $K(X)$ represented the circulation in incompressible flow, (16) would be the wake condition corresponding to an oscillation of frequency f/β^2, since $\nu = \omega/\beta^2$.

It is shown in Ref. 11 that the solution of (10) may be derived from the integral equation

$$2\pi W(X_1) = -\int_{-1}^{\infty} K(X) \frac{\partial^2}{\partial Z_1^2}\left[\frac{\pi}{2} i H_0^{(2)}\{\varkappa\sqrt{(x - x_1{}^2 + z_1{}^2)}\}\right] dX\,, \quad \ldots \quad (17)$$

where $H_0^{(2)}$ ($\equiv J_0 - iY_0$) is Hankel's function of zero order. When $z_1 \to 0$, (17) yields, after integration by parts, the relation

$$2\pi W(X_1) = \int_{-1}^{\infty} \frac{1}{x - x_1} \frac{\partial}{\partial x}\left[K(X) \,.\, \frac{\pi\sigma}{2}\,(Y_1(\sigma) + iJ_1(\sigma))\right] dX\,, \quad \ldots \quad (18)$$

where $\sigma = \varkappa|x - x_1|$ and J_1, Y_1 are Bessel functions. This may be expressed as

$$2\pi(W + I) = \int_{-1}^{\infty} \frac{1}{x_1 - x} \frac{\partial K}{\partial X}\, dX\,, \quad \ldots \quad (19)$$

where

$$2\pi I = \int_{-1}^{\infty} \frac{1}{x_1 - x} \frac{\partial}{\partial X}\,[K(X)\,\psi(\sigma)]\, dX \quad \ldots \quad (20)$$

and

$$\left.\begin{aligned} \psi(\sigma) &= 1 + \frac{\pi}{2}\sigma(Y_1 + iJ_1) \\ &= \frac{\sigma^2}{2}\left(\gamma - \tfrac{1}{2} + \log_e\frac{\sigma}{2} + \frac{i\pi}{2}\right) - \frac{\sigma^4}{16}\left(\gamma - \frac{5}{4} + \log_e\frac{\sigma}{2} + \frac{i\pi}{2}\right) \\ &\qquad + \text{etc.} \end{aligned}\right\} \quad \ldots \quad (21)$$

The function ψ is represented fairly accurately by the first two terms of the above series over the range $0 < \sigma < 2$. For values of $\sigma > 2$ more terms would have to be taken into account. It should be noted that equation (19) is precisely the equation that arises in incompressible flow if $W + I$ is regarded as a known downwash distribution, and if the frequency of oscillation is changed to $f\beta^{-2}$ to satisfy the wake condition $\Gamma = 0$ and equation (16). However, I is unknown since it is dependent on the distribution $K(X)$ which is to be determined.

The solution of (19) has been discussed in Ref. 11, and it is suggested in that report that by iteration and successive approximation to I the problem could be solved to any required accuracy. Only the first approximation to I, in which terms of higher order than the first in frequency were neglected, was considered, and formulae for the aerodynamic coefficients were obtained. These gave good agreement with the values of Ref. 6 for $M = 0 \cdot 7$ at low values of ω and fair agreement for higher values in the flutter range. In the present paper a slightly different method is used.

If $W + I$ were known, equation (19) could be solved exactly by the use of known results from incompressible-flow theory. However, I is unknown and the solution can only be obtained by iteration, as in Ref. 11, or by transforming (19) to an equivalent set of linear algebraic simultaneous equations. The latter more direct approach is used in this paper.

Let the distribution $K(X)$ ($\equiv \Phi_a - \Phi_b$) be represented by the linear combination

$$K = U_0 \sum_{n=0}^{\infty} C_n K_n \qquad \ldots \quad (22)$$

where K_0, K_1, etc., are well-known functions which occur in incompressible-flow theory and $C_0, C_1, \ldots C_n$ are arbitrary constants. The K_n distributions are defined in the list of symbols, and it should be noted that only K_0 does not vanish at the trailing edge, $\vartheta = \pi$, and in the wake. By substituting (22) in (19) and (20), the following equations are obtained:

$$W + I = U_0[C_0 + C_1(\tfrac{1}{2} + \cos\vartheta_1) + \sum_{n=2}^{\infty} C_n \cos n\vartheta_1] \qquad \ldots \quad (23)$$

and

$$I = U_0 \sum_{n=0}^{\infty} C_n I_n\,, \qquad \ldots \quad (24)$$

where I_n, the function corresponding to K_n, is expressible in the form

$$I_n = \sum_{r=0} I_{nr} \cos r\vartheta_1\,. \qquad \ldots \quad (25)$$

For an aerofoil describing plunging and pitching oscillations about mid-chord as indicated in the following diagram:

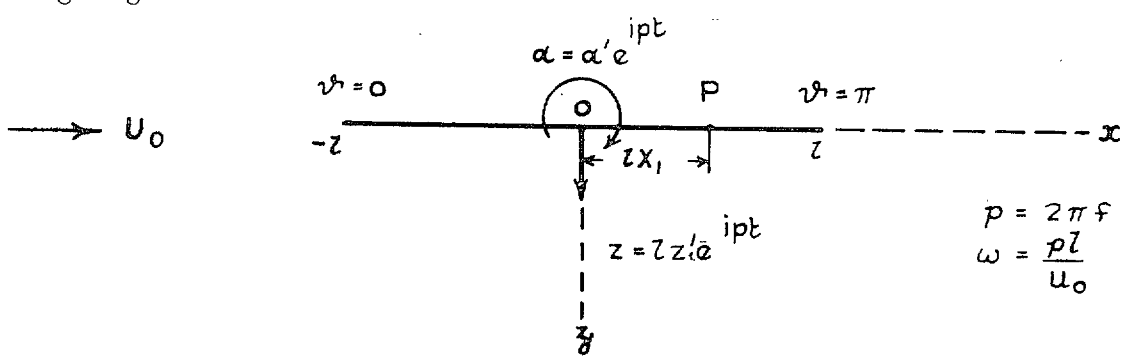

Fig. 1.

the downward displacement at the point P is

$$\zeta = l(z' + X_1\alpha')\,e^{ipt}. \qquad \ldots \quad (26)$$

By the use of (7) and (11) it may then be shown that the amplitude w' of w is given by

$$w' = U_0[i\omega z' + \alpha' + i\omega X_1 \alpha'] \qquad \ldots \quad (27)$$

and that

$$W = \beta^{-1} w'\, e^{-i\lambda X_1}\,. \qquad \ldots \quad (28)$$

Since $X_1 = -\cos\vartheta_1$ the exponential term may be expressed in terms of Bessel functions of parameter λ. Thus

$$e^{i\lambda\cos\vartheta_1} = J_0(\lambda) + 2\sum_{r=1}^{\infty} i^r J_r(\lambda)\cos r\vartheta_1 \qquad \ldots \quad (29)$$

and, hence,

$$\left.\begin{aligned} W &= \frac{U_0}{\beta}\left[\bar{a} - \omega\alpha' \frac{\partial}{\partial\lambda}\right] e^{i\lambda\cos\vartheta_1} \\ &= \frac{U_0}{\beta}\left[\bar{a} - \omega\alpha' \frac{\partial}{\partial\lambda}\right]\left[J_0(\lambda) + 2\sum_{r=1}^{\infty} i^r J_r(\lambda)\cos r\vartheta_1\right] \end{aligned}\right\} \quad \ldots \quad \ldots \quad (30)$$

where $\bar{a} \equiv \alpha' + i\omega Z'$. Then substitution in (20) and comparison of coefficients of $\cos r\vartheta_1$ yield the following infinite set of equations :

$$\left.\begin{aligned} C_0 + \frac{C_1}{2} - \sum_{n=0}^{\infty} C_n I_{n0} &= \frac{1}{\beta}\left(\bar{a} - \omega\alpha' \frac{\partial}{\partial\lambda}\right) J_0(\lambda) \\ C_r - \sum_{n=0}^{\infty} C_n I_{nr} &= \frac{2i^r}{\beta}\left(\bar{a} - \omega\alpha' \frac{\partial}{\partial\lambda}\right) J_r(\lambda) \end{aligned}\right\} \quad \ldots \quad \ldots \quad (31)$$

where $r = 1, 2, 3, \ldots \infty$. The coefficients I_{nr} are given in Appendix I in the form of series in ascending powers of $\varkappa (\equiv M\nu)$ up to the fourth—terms of order $\varkappa^6 \log_e \varkappa$ and higher being neglected. For particular values of frequency and Mach number, equation (31) may then be solved to give C_0, C_1, C_2, etc. It was found that solutions of sufficient accuracy could be obtained by solving the first four equations with $C_n = 0$, $n \geqslant 4$ assumed. For the frequencies and Mach numbers considered, the differences in the values of the derivatives obtained from solutions based on the first three and the first four equations are small. It also appears that there is little loss of accuracy due to the neglect of terms of order $\varkappa^6 \log_e \varkappa$ and higher in the I_{nr} coefficients when $\varkappa$ is not much greater than unity. This is shown by a comparison of the results given in Table 1 for Approximations I, II and III.

Approximation I was obtained by neglecting terms of order higher than the first in frequency. In this case, as shown in Ref. 11, $I_{00} \sim i\nu\delta$, where

$$\delta \equiv \log_e \frac{M}{2} + \sqrt{(1 - M^2)} \log_e \frac{1 + \sqrt{(1 - M^2)}}{M}, \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (32)$$

and the infinite set of equations reduces to two equations, namely

$$\left.\begin{aligned} \bar{a} &= \beta C_0 (1 - i\nu\delta) + \frac{\beta C_1}{2} \\ i(\bar{a}\lambda - \omega\alpha') &= \beta C_1 \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (33)$$

These yield

$$\left.\begin{aligned} \beta C_0 &= i\omega Z'.a + \alpha'.b \\ \beta C_1 &= i\lambda . i\omega Z' + i(\lambda - \omega)\alpha' \end{aligned}\right\}, \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (34)$$

$$\left.\begin{aligned} \text{where} \quad a &= \left(1 - \frac{i\lambda}{2}\right) \Big/ (1 - i\nu\delta) \\ \text{and} \quad b &= \left[1 + \frac{i(\omega - \lambda)}{2}\right] \Big/ (1 - i\nu\delta) \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (35)$$

Approximation II includes terms of second order in frequency in the I_{nr} coefficients. Solutions obtained by solving two, three and four equations of the infinite set defined by (31) correspond to Approx. II(1), II(2) and II(3) respectively in Table 1. It was found that Approx. II(2), obtained when only C_0, C_1 and C_2 were assumed to have non-zero values, gave results in close agreement with those derived for Approx. II(3) when the C_3 term was included. Hence, without loss of accuracy, the infinite set of equations could be reduced to four equations at most. These could be solved algebraically but, since the I_{0r} coefficients are rather complicated and would not lead to simple formulae for the aerodynamic coefficients, they were solved numerically.

Approximation III takes terms of fourth order in $\varkappa$ into account but neglects those of order $\varkappa^6 \log_e \varkappa$. Since the results corresponding to Approx. III(3) differ little from those given by Approx. II(3), which neglects terms of order $\varkappa^4 \log_e \varkappa$ and higher, it is assumed that terms of order $\varkappa^6 \log_e \varkappa$ neglected in Approx. III(3) have negligible effect on derivative values for the Mach numbers and frequency parameter values considered. Furthermore, since Approx. III(2) and Approx. III(3) give almost identical results, it is not necessary to solve more than four equations of the infinite set defined by (31). For values of $\varkappa < 1$, the method appears to be rapidly convergent.

Aerodynamic Coefficients.—When the arbitrary constants C_0, C_1, C_n appropriate to the prescribed motion have been determined, the K distribution is given by (8). The corresponding Γ distribution is then known and the lift distribution is expressible in the form

$$\bar{l}(X) = \rho_0 U_0 \left(\frac{\partial}{\partial T} + \frac{\partial}{\partial X}\right) K \, e^{i(\lambda X + \omega T)}$$

$$= \rho_0 U_0^2 \sum_{n=0}^{\infty} C_n \Gamma_n \, e^{i(\lambda X + \omega T)} \qquad (36)$$

where $\Gamma_n = i\nu K_n + \dfrac{\partial K_n}{\partial X}$ and $\nu = \omega\beta^{-2}$. Let

$$\left.\begin{aligned} R_n &= \int_{-1}^{1} \Gamma_n \, e^{i\lambda X} \, dX \\ \text{and} \quad iR_n' &= -\int_{-1}^{1} \Gamma_n \, . \, X \, . \, e^{i\lambda X} \, dX \end{aligned}\right\} \qquad (37)$$

where $R_n' = \partial R_n / \partial \lambda$ and Γ_n is here assumed to be independent of λ. The lift L and pitching moment M about half-chord are then given by

$$\left.\begin{aligned} L &= \rho_0 l U_0^2 \sum_{n=0} C_n R_n \, e^{i\omega T} \\ \text{and} \quad M &= \rho_0 l^2 U_0^2 \sum_{n=0} i C_n R_n' \, e^{i\omega T} \end{aligned}\right\} \qquad (38)$$

where the integrals R_n, R_n' are the known functions of M and ν given in the list of symbols.

The aerodynamic coefficients are defined in terms of the chord $c (= 2l)$ as standard length by the formulae

$$\left.\begin{aligned} \frac{L}{\rho_0 c U_0^2} &= (l_z + i\bar{\omega} l_{\dot{z}}) \frac{z}{c} + (l_\alpha + i\bar{\omega} l_{\dot{\alpha}})\alpha \\ \frac{M}{\rho_0 c^2 U_0^2} &= (m_z + i\bar{\omega} m_{\dot{z}}) \frac{z}{c} + (m_\alpha + i\bar{\omega} m_{\dot{\alpha}})\alpha \end{aligned}\right\} \qquad (39)$$

where $\bar{\omega} = 2\omega = pc/U_0$.

The coefficients C_0, C_1, etc., in (31) are linearly dependent on z and α. By comparison of (38) and (39), it is then possible to derive formulae or numerical values for the derivative coefficients. In the simplest case (Approx. I), it may be deduced from (34), (35) and (38) that

$$\left.\begin{aligned} l_z + i\bar{\omega} l_{\dot{z}} &= \frac{i\omega}{\beta}\,[aR_0 + i\lambda R_1] \\ l_\alpha + i\bar{\omega} l_{\dot{\alpha}} &= \frac{1}{2\beta}\,[bR_0 + i(\lambda - \omega)R_1] \\ m_z + i\bar{\omega} m_{\dot{z}} &= -\frac{\omega}{2\beta}\,[aR_0' + i\lambda R_1'] \\ m_\alpha + i\bar{\omega} m_{\dot{\alpha}} &= \frac{i}{4\beta}\,[bR_0' + i(\lambda - \omega)R_1'] \end{aligned}\right\} \quad \ldots \quad (40)$$

Similar formulae are given in Ref. 11, but in that report $a = 1 + i\nu\delta - \frac{1}{2}i\lambda$ and $b = 1 + i\nu\delta + \frac{1}{2}i(\omega - \lambda)$, whereas, for (40), a and b are defined by (35).

Concluding Remarks.—The method of calculation used in this report could be extended to higher frequency parameter values by taking more terms in the series expansion for the function ψ in (20). However, since the results given for the frequency and Mach numbers considered agree closely with those of Ref. 6, it did not seem worthwhile to embark on further confirmatory calculations for $M = 0 \cdot 7$ and $M = 0 \cdot 8$. Fettis[12] has also done some calculations by a different method which add further support to the view that values of the derivatives given by Dietze's method are correct.

In practice, however, aerodynamic coefficients calculated on the basis of linearized theory require some modification to allow for thickness and boundary-layer effects. Some allowance for such effects can be made by the equivalent thin-profile method of Ref. 13 in which the measured steady-motion characteristics of the aerofoil are introduced into the unsteady linearized theory. Recently this process has been used with some success to estimate the pitching-moment damping on an oscillating aerofoil in subsonic compressible flow[14].

Acknowledgment.—The numerical work required for this report was done by Miss Sylvia W. Skan of the Aerodynamics Division, N.P.L.

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APPENDIX I

Evaluation of Integrals

(i) *Integral I_0*

Since $K_0(X) = K_0(1)\, e^{-i\nu(X-1)}$ in the wake, equation (6) may be expressed in the form

$$2\pi I_0 = \int_{-1}^{1} \frac{1}{X_1 - X} \frac{\partial}{\partial X} [K_0(X)\, \psi(\varkappa |X - X_1|)]\, dX$$

$$+ \int_{1}^{\infty} \frac{K_0(1)}{X_1 - X} \frac{\partial}{\partial X} [\psi\, e^{-i\nu(X-1)}]\, dX$$

$$= \int_{-1}^{1} \frac{1}{X_1 - X} \frac{\partial}{\partial X} [K_0 \psi]\, dX - \int_{X_1}^{1} \frac{K_0(1)\, e^{i\nu}}{X_1 - X} \frac{\partial}{\partial X} (\psi\, e^{-i\nu X})\, dX$$

$$+ i\nu\delta K_0(1)\, e^{-i\nu(X_1-1)} \quad \ldots \quad (41)$$

where, as shown in Ref. 11, Appendix I,

$$\delta = \log_e \frac{M}{2} + \sqrt{(1 - M^2)} \log_e \frac{1 + \sqrt{(1 - M^2)}}{M} . \quad \ldots \quad \ldots \quad \ldots \quad (42)$$

Furthermore, the function $\psi(\varkappa|X - X_1|)$ defined by (7) is expressible in the form

$$\psi = \frac{\varkappa^2(X - X_1)^2}{2} [G + L] - \frac{\varkappa^4(X - X_1)^4}{16} [G + L - \tfrac{3}{4}] + O(\varkappa^6 \log_e \varkappa) , \quad (43)$$

where

$$G = \gamma - \tfrac{1}{2} + \log_e \frac{\varkappa}{4} + \frac{i\pi}{2}$$

$$L = \log_e 2|X - X_1| = -2 \sum_{n=1}^{\infty} \frac{\cos n\vartheta \cos n\vartheta_1}{n} .$$

By substituting (43) in (41) and integrating by parts, it may be deduced with the aid of (14) that

$$I_0 = \sum_{r=0}^{\infty} I_{0r} \cos r\vartheta_1 , \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (44)$$

where $I_{00} = i\nu X_0(\nu) J_0(\nu) P + \frac{M^2}{2}\left[1 - C(\nu) + i\nu G\left(C(\nu) + \frac{i\nu}{2}\right)\right]$

$$+ \frac{M^4}{32}\left\{G\left[\frac{3\nu^4}{4} - 2i\nu^3 C + 2\nu^2(C - 1) + 4i\nu C\right]\right.$$

$$\left. + \frac{\nu^4}{16} - \frac{i\nu^3 C}{2} + \frac{3\nu^2(C - 1)}{2} + 4(1 - C) + 3i\nu C\right\}$$

$$I_{01} = -2\nu X_0(\nu) J_1(\nu) P + \frac{i\nu M^2(1 - C)}{2}$$

$$- \frac{M^4}{16}\left\{G[2\nu^2 C + i\nu^3(1 - C)] + \frac{i\nu^3(1 - C)}{2} + \frac{3C\nu^2}{2} + 2i\nu(C - 1)\right\}$$

$$I_{02} = -2i\nu X_0(\nu) J_2(\nu) P - \frac{M^2\nu^2}{8} - \frac{i\nu^3 M^4 G}{32}\left(C + \frac{i\nu}{2}\right)$$

$$+ \frac{M^4}{16}\left[\frac{5\nu^4}{48} - \frac{3i\nu^3 C}{8} + \frac{\nu^2}{2}(C - 1)\right]$$

$$I_{03} = 2\nu X_0(\nu) J_3(\nu) P - \frac{i\nu^3(1 - C)M^4}{192}$$

$$I_{04} = 2i\nu X_0(\nu) J_4(\nu) P + \frac{M^4\nu^4}{1536}$$

$$n > 4, \ldots I_{0n} = 2i^{n+1}\nu X_0(\nu) J_n(\nu) P . \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (45)$$

In the above formulae $M\nu = \varkappa$ and

$$C(\nu) = H_1^{(2)}(\nu)/[H_1^{(2)}(\nu) + IH_0^{(2)}(\nu)]$$

$$\sim 1 + i\nu f - \nu^2 f^2 - i\nu^3\left(f^3 - \frac{f^2}{2} + \frac{f}{2} - \frac{1}{4}\right) + \nu^4\left(f^4 - f^3 + f^2 - \frac{f}{2}\right)$$

$$X_0(\nu) = C(\nu)J_0(\nu) + i[1 - C(\nu)]J_1(\nu)$$

$$P = \delta + \frac{M^2}{2}\left(\tfrac{1}{2} - \log_e \frac{M}{2}\right) - \frac{M^4}{8}\left(\log_e \frac{M}{2} + \tfrac{1}{4}\right)$$

and $$f = \gamma + \frac{i\pi}{2} + \log_e \frac{\nu}{2} = G + \tfrac{1}{2} - \log_e \frac{M}{2}\,. \quad \ldots \quad (46)$$

(ii) *Integral* I_n

When $n = 1, 2, 3$, etc., $K_n(1) = 0$ and equation (6) yields

$$2\pi I_n = \int_{-1}^{1} \frac{1}{X_1 - X} \frac{\partial}{\partial X} [K_n(X) \,.\, \psi(\varkappa|X - X|)]\, dX$$

$$\sim -\frac{\varkappa^2}{2}\int_{-1}^{1} K_n\left[G + L - \frac{\varkappa^2}{8}(X - X_1)^2 (G + L - \tfrac{3}{4})\right] dX \quad \ldots \quad (47)$$

which may be expressed in the form

$$2\pi I_n = \sum_{r=0} 2\pi I_{nr} \cos r\vartheta_1\,. \quad \ldots \quad (48)$$

From (47) is readily follows that

$$I_{10} = -\frac{\varkappa^2 G}{8} + \frac{3\varkappa^4}{256}\left(G + \frac{1}{12}\right)$$

$$I_{11} = \frac{\varkappa^2}{16} - \frac{\varkappa^4}{128}\left(G + \frac{5}{12}\right)$$

$$I_{12} = -\frac{\varkappa^2}{16} + \frac{\varkappa^4}{128}\left(G + \frac{5}{12}\right)$$

$$I_{13} = -\frac{\varkappa^2}{48} - \frac{\varkappa^4}{1024}$$

$$I_{14} = \frac{\varkappa^4}{64 \times 48}$$

$$I_{15} = -\frac{\varkappa^4}{64 \times 160} \quad \ldots \quad (49)$$

when terms of order $\varkappa^6 \log_e \varkappa$ are neglected. Similarly,

$$I_{20} = \frac{\varkappa^2 G}{8} - \frac{\varkappa^4 G}{96} \qquad ; I_{21} = 0$$

$$I_{22} = \frac{\varkappa^2}{12} - \frac{\varkappa^4}{128}\left(G + \frac{7}{24}\right) \qquad ; I_{23} = 0$$

$$I_{24} = -\frac{\varkappa^2}{96} - \frac{\varkappa^4}{1920} \qquad ; I_{25} = 0$$

$$I_{26} = \frac{\varkappa^4}{32 \times 1440}$$

$$I_{31} = -\frac{\varkappa^2}{16} + \frac{\varkappa^4}{128}\left(G + \frac{3}{8}\right) \qquad ; I_{30} = 0$$

$$I_{33} = \frac{\varkappa^2}{32} + \frac{3\varkappa^4}{2560} \qquad ; I_{32} = 0$$

$$I_{35} = -\frac{\varkappa^2}{160} - \frac{\varkappa^4}{32 \times 240} \qquad ; I_{34} = 0$$

$$I_{37} = \frac{\varkappa^4}{128 \times 840} \qquad ; I_{36} = 0$$

$$I_{40} = -\frac{\varkappa^4}{32 \times 24}\left(G - \frac{3}{8}\right) \qquad ; I_{41} = 0$$

$$I_{42} = -\frac{\varkappa^4}{48} - \frac{\varkappa^4}{960} \qquad ; I_{43} = 0$$

$$I_{44} = \frac{\varkappa^2}{60} + \frac{\varkappa^4}{32 \times 120} \qquad ; I_{45} = 0$$

$$I_{46} = -\frac{\varkappa^2}{240} - \frac{\varkappa^4}{32 \times 630} \qquad ; I_{47} = {}_2$$

$$I_{48} = \frac{\varkappa^4}{32 \times 120 \times 56}. \qquad \text{.. .. ..} \qquad (50)$$

TABLE 1

Aerodynamic Coefficients for Mid-chord Axis

$M = 0{\cdot}7$

Derivative	$\bar{\omega}$	Approx. I	Approx. II			Approx. III			Ref. 6	Ref. 10*
			1	2	3	1	2	3		
l_z	0·2	0·1900	0·1849	0·1848		0·1848	0·1848		0·1849	0·193
	0·4	0·3160	0·2984	0·2972	0·2972	0·2979	0·2967	0·2967	0·2975	0·313
	0·6	0·3505	0·3189	0·3134	0·3134	0·3164	0·3109	0·3108	0·3120	0·360
	0·8	0·3299	0·2835	0·2678	0·2677	0·2751	0·2594	0·2593	0·2613	0·317
	1·0		0·2211	0·1883	0·1879	0·2001	0·1672	0·1668	0·1678	0·241
$l_{\dot z}$	0·2	3·054	3·054	3·054		3·054	3·054		3·054	3·050
	0·4	2·483	2·504	2·505	2·505	2·504	2·505	2·505	2·504	2·505
	0·6	2·223	2·265	2·270	2·270	2·264	2·269	2·269	2·269	2·250
	0·8	2·094	2·157	2·172	2·172	2·155	2·170	2·170	2·172	2·120
	1·0		2·111	2·146	2·146	2·108	2·143	2·143	2·148	2·077
$-m_z$	0·2	−0·0657	−0·0629	−0·0629		−0·0629	−0·0629		−0·0629	−0·063
	0·4	−0·1441	−0·1325	−0·1329	−0·1329	−0·1324	−0·1329	−0·1329	−0·1330	−0·147
	0·6	−0·2206	−0·1989	−0·2014	−0·2014	−0·1988	−0·2014	−0·2014	−0·2016	−0·212
	0·8	−0·2953	−0·2672	−0·2756	−0·2755	−0·2669	−0·2759	−0·2758	−0·2768	−0·288
	1·0		−0·3371	−0·3588	−0·3587	−0·3370	−0·3603	−0·3602	−0·3626	−0·375
$-m_{\dot z}$	0·2	−0·7417	−0·7398	−0·7424		−0·7398	−0·7424		−0·743	−0·745
	0·4	−0·5611	−0·5705	−0·5807	−0·5807	−0·5706	−0·5809	−0·5809	−0·5808	−0·582
	0·6	−0·4462	−0·4726	−0·4956	−0·4956	−0·4734	−0·4964	−0·4964	−0·4960	−0·487
	0·8	−0·3542	−0·3983	−0·4383	−0·4383	−0·4008	−0·4407	−0·4407	−0·4406	−0·427
	1·0		−0·3298	−0·3892	−0·3891	−0·3358	−0·3946	−0·3946	−0·3948	−0·380
l_α	0·2	3·105	3·117	3·117		3·117	3·117		3·117	3·11
	0·4	2·582	2·638	2·638	2·638	2·637	2·638	2·638	2·637	2·63
	0·6	2·359	2·470	2·471	2·471	2·469	2·471	2·471	2·471	2·435
	0·8	2·265	2·444	2·447	2·447	2·442	2·446	2·446	2·448	2·38
	1·0		2·496	2·505	2·505	2·493	2·503	2·503	2·508	2·40
$l_{\dot\alpha}$	0·2	−3·977	−3·878	−3·878		−3·877	−3·877		−3·881	−3·85
	0·4	−1·336	−1·276	−1·277	−1·273	−1·273	−1·274	−1·274	−1·277	−1·45
	0·6	−0·3970	−0·3736	−0·3749	−0·3749	−0·3656	−0·3670	−0·3670	−0·3705	−0·46
	0·8	+0·0284	+0·0231	0·0202	0·0200	+0·0391	0·0359	0·0355	+0·032	−0·07
	1·0		0·2086	0·2020	0·2016	0·2367	0·2285	0·2283	0·225	+0·15
$-m_\alpha$	0·2	−0·7590	−0·7595	−0·7594		−0·7595	−0·7594		−0·7595	−0·755
	0·4	−0·6022	−0·6167	−0·6164	−0·6164	−0·6169	−0·6166	−0·6166	−0·6166	−0·617
	0·6	−0·5146	−0·5474	−0·5467	−0·5467	−0·5484	−0·5476	−0·5476	−0·5474	−0·532
	0·8	−0·4542	−0·5027	−0·5013	−0·5013	−0·5058	−0·5042	−0·5042	−0·5043	−0·488
	1·0		−0·4623	−0·4595	−0·4595	−0·4702	−0·4663	−0·4664	−0·4656	−0·445
$-m_{\dot\alpha}$	0·2	1·728	1·668	1·668		1·668	1·668		1·669	1·670
	0·4	1·027	0·9737	0·9758	0·9758	0·9734	0·9756	0·9756	0·9761	1·010
	0·6	0·7627	0·7296	0·7343	0·7343	0·7289	0·7342	0·7342	0·7350	0·770
	0·8	0·6291	0·6192	0·6277	0·6278	0·6182	0·6282	0·6282	0·6301	0·648
	1·0		0·5603	0·5740	0·5741	0·5594	0·5758	0·5759	0·5779	0·592

* Values were derived by interpolation of results given in Ref. 10.

TABLE 1—*continued*

$M = 0 \cdot 8$

Derivative	$\bar{\omega}$	Approx. I	Approx. II			Approx. III			Ref. 6	Ref. 10
			1	2	3	1	2	3		
l_z	0·2	0·2569	0·2475	0·2473						0·264
	0·4	0·4189	0·3934	0·3910		0·3908	0·3884		0·3886	0·421
	0·6	0·5004	0·4601	0·4529	0·4527	0·4480	0·4403	0·4401		0·502
	0·8	0·5532	0·4932	0·4868	0·4856	0·4599	0·4500	0·4489	0·4514	0·520
	1·0	0·5998	0·4970	0·5201	0·5153	0·4342	0·4425	0·4398		0·511
$l_{\dot{z}}$	0·2	3·180	3·190	3·191						3·17
	0·4	2·488	2·532	2·539		2·531	2·539		2·530	2·50
	0·6	2·186	2·252	2·281	2·281	2·251	2·280	2·280		2·20
	0·8	2·008	2·088	2·161	2·161	2·088	2·161	2·160	2·149	2·06
	1·0	1·859	1·952	2·092	2·092	1·954	2·093	2·093		1·985
$-m_z$	0·2	−0·0907	−0·0845	−0·0846						−0·089
	0·4	−0·1882	−0·1676	−0·1703		−0·1675	−0·1703		−0·1680	−0·178
	0·6	−0·2675	−0·2384	−0·2529	−0·2528	−0·2387	−0·2542	−0·2541		−0·266
	0·8	−0·3133	−0·2869	−0·3325	−0·3321	−0·2913	−0·3395	−0·3390	−0·3343	−0·348
	1·0	−0·3108	−0·2964	−0·3961	−0·3946	−0·3132	−0·4189	−0·4170		−0·425
$-m_{\dot{z}}$	0·2	−0·7371	−0·7396	−0·7430						−0·746
	0·4	−0·4640	−0·5000	−0·5357		−0·5013	−0·5371		−0·5403	−0·521
	0·6	−0·2662	−0·3395	−0·4141	−0·4145	−0·3452	−0·4195	−0·4199		−0·400
	0·8	−0·0967	−0·2007	−0·3144	−0·3157	−0·2152	−0·3278	−0·3293	−0·3322	−0·307
	1·0	+0·0504	−0·0822	−0·2203	−0·2238	−0·1077	−0·2443	−0·2484		−0·225
l_α	0·2	3·243	3·275	3·275						3·25
	0·4	2·598	2·706	2·710		2·706	2·709		2·696	2·665
	0·6	2·331	2·521	2·534	2·534	2·521	2·534	2·534		2·418
	0·8	2·190	2·462	2·493	2·493	2·467	2·499	2·499	2·481	2·340
	1·0	2·084	2·433	2·486	2·487	2·455	2·510	2·510		2·335
$l_{\dot{\alpha}}$	0·2	−5·603	−5·436	−5·434						−5·95
	0·4	−1·959	−1·912	−1·908		−1·895	−1·890		−1·902	−2·16
	0·6	−0·8045	−0·8452	−0·8429	−0·8429	−0·8045	−0·8026	−0·8026		−0·96
	0·8	−0·3263	−0·4318	−0·4429	−0·4430	−0·3600	−0·3721	−0·3720	−0·3817	−0·44
	1·0	−0·1031	−0·2517	−0·2911	−0·2912	−0·1483	−0·1894	−0·1892		−0·21
$-m_\alpha$	0·2	−0·7608	−0·7672	−0·7706						−0·762
	0·4	−0·5192	−0·5603	−0·5733		−0·5618	−0·5748		−0·5771	−0·561
	0·6	−0·3567	−0·4273	−0·4539	−0·4539	−0·4345	−0·4603	−0·4603		−0·434
	0·8	−0·2241	−0·3020	−0·3391	−0·3391	−0·3222	−0·3559	−0·3560	−0·3602	−0·318
	1·0	−0·1125	−0·1765	−0·2128	−0·2129	−0·2166	−0·2438	−0·2442		−0·210
$-m_{\dot{\alpha}}$	0·2	2·401	2·268	2·271						2·27
	0·4	1·364	1·275	1·288		1·274	1·289		1·269	1·37
	0·6	0·9648	0·9353	0·9658	0·9658	0·9358	0·9698	0·9699		1·00
	0·8	0·7381	0·7508	0·8034	0·8036	0·7584	0·8174	0·8176	0·8042	0·82
	1·0	0·5810	0·6093	0·6802	0·6807	0·6325	0·7126	0·7131		0·71

TABLE 1—*continued*

$M = 0{\cdot}9$

Derivative	$\bar{\omega}$	Approx. I	Approx. II			Approx. III		
			1	2	3	1	1	3
l_z	0·2 0·4 0·6	0·3762 0·5692 0·6422	0·3569 0·5184 0·4794	0·3577 0·5504 0·6688	0·3578 0·5501 0·6631	0·3488 0·4983 0·4486	0·3496 0·5257 0·5909	0·3496 0·5255 0·5880
$l_{\dot{z}}$	0·2 0·4 0·6	3·179 2·713 1·854	3·228 2·356 1·949	3·239 2·433 2·126	3·239 2·433 2·137	3·261 2·356 1·886	3·272 2·430 2·076	3·272 2·430 2·080
$-m_z$	0·2 0·4 0·6	−0·1361 −0·1837 −0·0592	−0·1193 −0·1630 −0·1050	−0·1217 −0·1990 −0·2148	−0·1217 −0·1987 −0·2125	−0·1187 −0·1677 −0·1257	−0·1213 −0·2053 −0·2466	−0·1213 −0·2049 −0·2438
$-m_{\dot{z}}$	0·2 0·4 0·6	−0·5362 +0·0373 0·3636	−0·5930 −0·1469 +0·0463	−0·6554 −0·3274 −0·1481	−0·6557 −0·3334 −0·1781	−0·6085 −0·1644 +0·0226	−0·6713 −0·3451 −0·1818	−0·6717 −0·3516 −0·2168
l_α	0·2 0·4 0·6	3·254 2·401 1·955	3·353 2·581 2·243	3·360 2·631 2·343	3·360 2·631 2·350	3·385 2·586 2·215	3·393 2·635 2·321	3·393 2·635 2·324
l_α	0·2 0·4 0·6	−8·541 −2·913 −1·288	−8·270 −2·947 −1·254	−8·288 −3·106 −1·670	−8·288 −3·105 −1·662	−8·056 −2·794 −1·094	−8·073 −2·933 −1·420	−8·073 −2·933 −1·416
$-m_\alpha$	0·2 0·4 0·6	−0·5744 −0·0405 0·2633	−0·6342 −0·1984 +0·0362	−0·6784 −0·3232 −0·0894	−0·6786 −0·3265 −0·1069	−0·6500 −0·2206 −0·0048	−0·6944 −0·3434 −0·1267	−0·6946 −0·3471 −0·1476
$-m_{\dot{\alpha}}$	0·2 0·4 0·6	3·673 1·503 0·5505	3·319 1·440 0·6580	3·372 1·636 0·9138	3·372 1·635 0·9135	3·302 1·474 0·7506	3·359 1·682 1·036	3·359 1·681 1·036

J4347 Wt.19/8411 K7 2/56 D&Co. 34/263

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