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The Turbulent Boundary Layer in Compressible Flow

By

W. F. Cope, M.A., A.M.I.Mech.E.,
of the Engineering Division, N.P.L.

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Summary.—The flow of a compressible gas past a flat plate is investigated for a turbulent boundary layer. The local and mean skin-friction coefficients are calculated for both power and log laws of velocity distribution. The calculations show a considerable reduction of both coefficients with increasing M. In the course of the analysis assumptions have been made whose accuracy is not proven, though they are consistent with those made in incompressible gas dynamics.

The results are applied to calculate the contribution to f_R of skin friction for a typical projectile of various calibres. The calculation shows that it should be possible by a properly selected series of wind-tunnel and full-scale experiments to ascertain if the large reduction in skin friction occurs, but that it is unlikely that it will be possible to discriminate between the two hypotheses about velocity distribution.

Historical Note.—The introduction proper gives the reasons which lead to the writing of this paper and for a long time security considerations prevented its publication. Recently, however, work on similar lines both at the Royal Aircraft Establishment and in America (*e.g.*, van Driest or Wilson in *J.Ae.Sc.*) have both confirmed the general accuracy of the picture presented and to a considerable extent superseded it as a technical contribution. Nevertheless it seems still to have some value technically and to be of great interest historically as a very early contribution to the literature of the subject.

The preparation of a paper of this kind for publication raises questions of rewriting so as to bring it up to date which are very difficult to decide. In the present case it has been decided to leave it untouched except to change the details of the references if the subject matter has since been published. The main reasons for this decision are that adequate modern treatments (such as those cited above) now exist, and that therefore to rewrite it would merely add one more to them and to no useful purpose, while at the same time it would diminish almost to vanishing point the historical value of the paper.

The work was carried out as part of the National Physical Laboratory programme of work for the fighting services.

Introduction.—With the steady improvement in the aerodynamic form of projectiles the question of scale effect is becoming of increasing importance. Obviously this raises the question of the effect of compressibility on skin friction. A good deal has been done on this subject, both theoretically and experimentally, at speeds close to that of sound by Frankl and Voishel (1937), by Squire and Young (1938), and by Young and Winterbottom (1950). This work is primarily concerned with ascertaining the effect of the onset of compressibility upon transition points and so forth. In practical ballistics the viewpoint and emphasis is different as we are primarily concerned with phenomena in the region where compressible flow is fully developed and with the effect of that development on the skin-friction contribution to the total resistance. Moreover even the best projectile is, aerodynamically speaking, of bad shape in that this contribution is less than $\frac{1}{5}$ of the total resistance. Any attempt to predict the effect is handicapped by the fact that agreement has not yet been reached upon such matters as the analytical expression for the velocity distribution in the boundary layer in incompressible flow and by a complete absence of accurate experimental data in compressible flow. However, calculations have recently been made, on an incompressible-flow basis, in which the several contributions to the total resistance of a projectile are separated. It has therefore seemed worthwhile to attempt to

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evaluate the effect of compressibility on skin friction. The best that can be done, at the moment, is to calculate for both the power and log laws of velocity distribution and it is found that the skin-friction coefficient is considerably reduced by compressibility and that the reduction is about the same for both forms of velocity law.

Analysis. Preliminary.—To reduce the problem of the determination of the skin friction of a projectile to its simplest form, consider a flat plate immersed in a stream of gas at pressure p_1, temperature T_1 and density ρ_1 travelling at a speed (which may be supersonic) U_1; the Mach number of the free stream will be denoted by $M(\equiv U_1/a_1)$. Consider the boundary layer of the plate and assume it to be turbulent right to the leading edge. The flow is assumed to be two-dimensional and there is no pressure gradient in the free stream. The gas temperature rises in the layer as one approaches the plate to T_w at the plate, assumed to be the same at all points. Similarly ρ_1 rises to ρ_w. p_1 in accordance with classical boundary-layer theory is unchanged. In these circumstances

$$\frac{U^2}{2C_p} + T = \text{const}\left(= T_w = \frac{U_1^2}{2C_p} + T_1\right)$$

is an (approximate) solution of the energy equation (Cope, 1942). The problem therefore is solved if the momentum equation, which by these assumptions is reduced to its simplest form

$$\tau = \frac{d}{dx}\int_0^\delta \rho u(U_1 - u)\, dy\,,$$

can be evaluated for compressible flow and an assigned velocity distribution in the boundary layer. This proves practicable and is carried out both for power law

$$u/U_1 \propto (y/\delta)^{1/m}$$

and the log law $u/U_1 \propto \log(y/\delta)$ in subsequent sections of this report. Before doing this it is convenient to obtain some general formulae which enable us to express any quantity originally given in terms of (say) its values at the plate, in terms of its values in the free stream, and to trace its variation through the layer.

Since p is constant $\rho_w T_w = \rho T$. Therefore $\rho = \rho_w T_w/T$. From the solution to the energy equation

$$\begin{aligned}\frac{T}{T_w} &= 1 - \frac{u^2}{2C_p T_w}\\ &= 1 - \frac{\gamma - 1}{2}\,\frac{U_1^2}{\gamma(C_p - C_v)T_w}\,\frac{u^2}{U_1^2}\\ &= 1 - \frac{\gamma - 1}{2}\,B_w^2\,\frac{u^2}{U_1^2}\\ &= 1 - \alpha\frac{u^2}{U_1^2}[(\gamma - 1)/2]B_w^2\end{aligned}$$

where α has been written for $\frac{\gamma - 1}{2} B_w^2$. B_w is a Mach number formed by dividing the velocity in the free stream by the velocity of sound at the plate; it is sometimes called Bairstow's number. α and M are connected by the relation

$$(1 - \alpha)\left(1 + \frac{\gamma - 1}{2} M^2\right) = 1$$

which gives (taking $\gamma = 1{\cdot}40$)

$$\alpha = \frac{M^2}{M^2 + 5} \qquad \text{and } M^2 = \frac{5\alpha}{1 - \alpha}\,.$$

The advantage of working with α, instead of M, is that α is always less than one so that expansion in series and subsequent term by term integration become practicable.

Continuing, $T_1 = (1 - \alpha) T_w$

and $\rho_1 = (1 - \alpha)^{-1} \rho_w$

for the variation of viscosity with temperature take $\mu \propto T^{3/4}$ as a convenient working compromise between the simplest result of the kinetic theory $\mu \propto \sqrt{T}$, the complications of Sutherland's formula and the relation $\mu \propto T^{0 \cdot 768}$ recently employed by Brainherd and Emmons (1941) and (1942) in some calculations on the laminar compressible layer.

Therefore

$$\mu_1 = (1 - \alpha)^{3/4} \mu_w \qquad \text{and } \nu_1 = (1 - \alpha)^{7/4} \nu_w .$$

So that the local (c_f) and mean (C_f) skin-friction coefficients in terms of free-stream values are connected with the same quantities (c_{fw} and C_{fw}) in terms of wall values by

$$\frac{c_f}{C_f} = (1 - \alpha) \frac{c_{fw}}{C_{fw}}$$

and the two Reynolds numbers by $R = (1 - \alpha)^{-7/4} R_w$.

Analysis (2). *Power Law.*—Assume

$$v = A \eta^{1/m} \text{ where } v \equiv \frac{u}{U_\tau}, \qquad \eta \equiv \frac{\rho_w U_\tau y}{\mu_w}$$

and note that

$$V \equiv \frac{U_1}{U_\tau} = \sqrt{\left(\frac{\rho_w U_1^2}{\tau}\right)} = \sqrt{\left(\frac{2}{c_{fw}}\right)}$$

$$\frac{d\eta}{dv} = \frac{m v^{m-1}}{A^m} .$$

Substituting in the momentum equation

$$\rho_w U_\tau^2 = \frac{d}{dx} \int_0^V \frac{\rho_w U_\tau^2 v (V - v)}{1 - \alpha \frac{v^2}{V^2}} \frac{\mu_w}{\rho_w U_\tau} \frac{d\eta}{du} dv .$$

Therefore

$$\frac{\eta_w U_1}{\mu_w} = \frac{m}{A_m} V^2 \frac{d}{dx} (V^{m+1} I_\alpha)$$

where $I_\alpha \equiv \int_0^1 \frac{Z^m (1 - Z)}{1 - \alpha Z^2} dZ \left(Z \equiv \frac{v}{V}\right)$ a function of α only.

Integrating

$$R_{xw} = \frac{m(m+1)}{A^m (m+3)} V^{m-3} I_\alpha$$

or

$$c_{fw} R_{xw}^{\;2/(m+3)} = 2 \left\{\frac{m(m+1)}{A^m (m+3)}\right\}^{2/(m+3)} I_\alpha^{\;2/(m+3)} . \qquad \text{(1)}$$

Again

$$F = \int_0^\eta \tau \, dx$$

$$= \frac{m \mu_w U_1}{A^m} V^{m+1} I_\alpha .$$

So $\frac{2F}{\rho_w U_1^2 \eta} \equiv C_{fw} = \frac{2}{R_w} \frac{m}{A^m} V^{m+1} I_\alpha$ or eliminating V^{m+1} by (1)

$$C_{fw} R^{2/(m+3)} = 2 \left\{\frac{m}{A^m}\right\}^{2/(m+3)} \left\{\frac{m+3}{m+1}\right\}^{(m+1)/(m+3)} I_\alpha^{\;2/(m+3)} . \qquad \text{(2)}$$

In incompressible flow $\alpha = 0$ and I_α becomes $I_0 = 1/(m+1)(m+2)$; commonly accepted values for A and m are 8·7 and 7 respectively. In these circumstances the right-hand sides of (1) and (2) become 0·058 and 0·073 respectively.

In compressible flow it is more convenient to have the quantities in terms of free-stream values; at present they are in terms of wall values.

Converting by the formulae previously obtained and inserting the numerical values for incompressible flow we obtain

$$C_f(R)^{1/5} = 0\cdot058(1 - \alpha)^{13/20} (I_\alpha/I_0)^{1/5}$$
$$C_f(R)^{1/5} = 0\cdot073(1 - \alpha)^{13/20} (I_\alpha/I_0)^{1/5} .$$

The effect of compressibility is thus to multiply the right hand side of the equation by $(1 - \alpha)^{13/20}\, 5\sqrt{(I_\alpha/I_0)}$. Of these the first is the most important; numerically it produces about four times the effect of the second, and arises from the change in density and viscosity due to the heating of the gas. It results in a considerable decrease in the coefficient. The second arises because the change is not abrupt but takes place gradually in the boundary layer. It increases the coefficient, but by a much smaller amount than it is decreased by the first term. The net result is a decrease.

In general terms the correction is

$$(1 - \alpha)^{(2m-1)/(2m+6)} \left(\frac{I_\alpha}{I_0}\right)^{2/(m+3)} .$$

I_α is an elementary integral expressible in finite terms by the usual partial fraction methods. The integral is a polynomial plus a log term; due to the comparatively large value of m this polynomial has several terms. It is therefore in practice as quick and more convenient to expand $(1 - \alpha Z^2)^{-1}$ and integrate term by term. The result is

$$\frac{I_\alpha}{I_0} = 1 + \frac{(m+1)(m+2)}{(m+3)(m+4)}\alpha + \frac{(m+1)(m+2)}{(m+5)(m+6)}\alpha^2 + \frac{(m+1)(m+2)}{(m+7)(m+8)}\alpha^3 + \ldots$$

C_f has been calculated from the appropriate formula (for $m = 7$) for $M = 0$ (incompressible flow), 1, 2, 3 and 4. The result is plotted against R as Fig. 1. C_f for a laminar boundary layer for $M = 0$ and $3\cdot16$ is also plotted over the range $10^5 \leqslant R \leqslant 10^6$; the values are taken from the calculations of Brainherd and Emmons (1941 and 1942).

Analysis (3). *Log Law.*—In our notation this is $\eta = A \exp (v)$

$$\frac{d\eta}{dv} = A \exp (v) .$$

So the momentum integral yields

$$\frac{\rho_w U_1}{\mu_w} = A V^2 \frac{d}{dx} (V^2 I)$$

where
$$I \equiv \int_0^1 \frac{Z(1 - Z) \exp VZ}{1 - \alpha Z^2} dZ .$$

The denominator of the integrand can be expanded as in (2) and the resulting series integrated term by term. The result is a power series in α the coefficients of which are themselves power series in $1/V^2$, that is c_f. In evaluating the incompressible case ($\alpha = 0$, first term of series only) it has been customary to neglect all terms not multiplied by an exponential and to take only the first surviving term of the power series in $1/V^2$. Both these are justified on the ground that V is a large quantity. Due to the presence of the factor $Z(1 - Z)$ in the several terms the first term of the integral vanishes at both limits; the first surviving term is therefore the second.

Integrating we get (to this degree of approximation)

$$I = - \exp V\left(\frac{1-2}{V^2}\right) - \alpha \exp V\left(\frac{3-4}{V^2}\right) - \alpha^2 \exp V\left(\frac{5-6}{V^2}\right) - \ldots$$
$$= \frac{V^2}{1 - \alpha} \exp \mathrm{V} .$$

So $$R_{xw} = \frac{A}{1-\alpha} V^2 \exp V \text{ leading to}$$

$$\frac{1}{\sqrt{c_{fw}}} = \text{const} + B\{\log (R_{xw}c_{fw}) + \log (1-\alpha)\} .$$

By methods analogous to those in Analysis 2 we can show that (to the same degree of approximation)

$$\frac{1}{\sqrt{C_{fw}}} = \text{const} + B\{\log (R_{w}C_{fw}) + \log (1-\alpha)\} .$$

Converting to free-stream values

$$\frac{1}{\sqrt{C_f}} = \frac{\text{const}}{\sqrt{(1-\alpha)}} + \frac{B}{\sqrt{(1-\alpha)}}\{\log (RC_f) + \tfrac{7}{4}\log (1-\alpha)\} .$$

This formula is best regarded as semi-empirical. The analysis having indicated a probable form, the constants are determined from experimental results. This is in line with the treatment for incompressible flow, for which the accepted values are const $= 0$ and $B = 4\cdot13$ (Schoenherr). Adopting these

$$\sqrt{(C_f)} \log (RC_f) = 0\cdot242\sqrt{(1-\alpha)} - \tfrac{7}{4}\sqrt{(C_f)} \log (1-\alpha) .$$

This confirms the form conjectured by von Kármán (1936) differing from it only in the last term which is of the nature of a correction; von Kármán's value for the last term is in our notation $-\frac{1}{2}\sqrt{(C_f)} \log (1-\alpha)$. The coefficients of $\sqrt{(C_f)} \log (1-\alpha)$ differ by $\frac{5}{4}$ of which $\frac{1}{4}$ arises from the difference between the viscosity laws assumed and the remainder from the change in density in the layer. Log $(1-\alpha)$ is a negative quantity, so that once again this change operates to mitigate the reduction due to heating. Its amount is $\sqrt{(C_f)} \log (1-\alpha)$ and numerically its importance increases as R diminishes and C_f increases. Over the working range it bears much the same ratio to the heating reduction as in the case of the power law.

C_f has been calculated from this formula for the same values of M as before. The result is plotted against R as Fig. 2. The laminar boundary-layer curve is also plotted for the same range of R as Fig. 1.

Discussion.—To illustrate the effect of compressibility in general and of the two hypotheses in particular a table has been prepared. The figures relate to a hypothetical projectile $4\frac{1}{2}$ calibres long whose drag coefficient is given approximately by the Standard Law. The projectile is supposed to be tested in a tunnel at $R = 10^6$ and at Mach numbers of $1\cdot3$ and $3\cdot0$ (for which f_R is about $0\cdot7$ and $0\cdot4$), and fired at the corresponding velocities (1,460 and 3,400 ft/sec respectively) using shells of 1-in., 6-in. and 16-in. calibre. The Reynolds numbers are thus determinate, so C_f is known. The surface of the projectile is taken to be the same as the curved surface of a circular cylinder of the same length and diameter; this makes the skin-friction contribution to $f_R (f_{RS} = 9\pi C_f)$ calculable. The several f_{RS} for a given M are averaged and the result subtracted from the assumed f_R to give the form-drag contribution f_{RF}. The 'true' f_R for any velocity is then given (rows 7 and 8) by $f_{RS} + f_{RF}$. The two hypotheses give values of f_R which differ at most by one per cent. The percentage contribution to f_R made by f_{RS} is given in row 9, where again differences between the results of the two hypotheses are ignored. Finally f_{RS} for an incompressible fluid and a log law is calculated (rows 10 to 13) and expressed as a percentage of the appropriate f_R.

The most striking feature of the results is the large reduction in C_f with increase in M. This arises from two causes which produce opposite effects, one being much more potent than the other.

(1) The heating of the gas by compression. This is the important effect and confirms von Kármán's conjecture[6] that a first approximation to the compressible law would be obtained by taking the appropriate imcompressible formula and modifying it. The more detailed analysis herein presented shows that this method over-estimates the effect by (very roughly) a quarter.

(2) The temperature change is not abrupt as (1) assumes but takes place gradually in the finite thickness of the boundary layer.

In the case of the power law the contributions from the two causes are respectively

$$(1 - \alpha)^{(2m-1)/(2m+6)} \text{ which is less than 1.}$$

And $\left(\frac{I\alpha}{I_0}\right)^{2/(m+3)}$ which is greater than 1. Neither is sensitive to changes in m.

In the case of the log law the respective contributions are

$$\sqrt{(1 - \alpha)} \text{ (multiplying } 0 \cdot 242) - \tfrac{3}{4}\sqrt{(C_f)} \log (1 - \alpha)$$

in which the first term is far more important than the second. And $- \sqrt{(C_f)} \log (1 - \alpha)$ which is positive since $(1 - \alpha)$ is a proper fraction.

Over the range of R occurring in practice ($5 \times 10^5 - R - 5 \times 10^8$) the percentage change of C_f with M is much the same whichever hypothesis is adopted.

In the analysis it has been assumed (1) that the velocity distribution is independent of Mach number. This has been done for analytical convenience and because of the absence of any experimental data. An expression for the velocity distribution was given by Frankl and Voishel (1937), but it has proved impossible to verify it and, in any case, it is only given to the first power of α. So far as it goes it suggests that the change is not large. The relative insensitivity of the reduction of C_f as M increases to changes in velocity law supports the conjecture (it is no more) that such changes in distribution as may occur are not likely to vitiate the main conclusions of the analysis. (2) that the change of velocity distribution with R is adequately represented by the expressions used. In other words that m in the power law and A in the log law are absolute constants. An error here would affect the absolute value of C_f whereas an error under (1) would affect the change in C_f, at a given R, with M. On this point it appears to be generally conceded that the power law with $m = 7$ begins to underestimate C_f when R attains 20 million and that at $R = 10^9$ the error may be as much as 25 per cent. With the log law it is possible to determine the constants so that a reasonable fit to the somewhat scattered points is obtained over the whole range of values of R. This, of course, is with incompressible flow. But since the 'constants' are determined in this manner there is no guarantee that they are independent of M. Again the need for experiment is clear.

The table shows that from the point of view of practical gunnery it matters very little which hypothesis of velocity distribution is selected. The large reduction in C_f is important and its existence can only be settled by experiment. The procedure would be to determine f_R and f_{RF} in a wind tunnel, for a suitable projectile, at a high value of M. Projectiles to the same design would be fired at a velocity corresponding to the selected M and f_R determined as accurately as possible. The projectile should have a low value of f_{RF} and be as long as gunnery and tunnel conditions permit. Assuming, as is probably justified, that f_{RF} is independent of R, the programme would provide information on the variation, at constant M, of C_f with R. In the present state of experimental technique it should be possible to decide if the large decrease which theory predicts, takes place; it is unlikely unless f_{RF} can be considerably reduced that it will be possible to discriminate between the two hypotheses about velocity distribution.

Conclusions.—The values of the local and mean skin-friction coefficients of a flat plate immersed in a stream of gas, whose speed may be supersonic, have been calculated. The results show that there is a considerable reduction of both coefficients with increasing M and that this reduction is much the same whether a power law or a log law of velocity distribution is assumed.

The results thus obtained are applied to calculate the contribution to f_R of skin friction for a hypothetical but typical projectile of various calibres. The calculations suggest that it should be possible by a properly selected series of wind-tunnel and full-scale experiments to ascertain if the large reduction in skin friction occurs; but that it is unlikely that it will be possible to discriminate between the two hypotheses about velocity distribution.

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TABLE

		$M = 1\cdot3$				$M = 3\cdot0$			
		Model	Prototype			Model	Prototype		
		1 in.	1 in.	6 in.	16 in.	1 in.	1 in.	6 in.	16 in.
R (millions)		1·0	3·3	20	52	1·0	8·0	53	142
Power law	$1{,}000\ C_f$	3·94	3·10	2·16	1·74	2·67	1·75	1·20	0·99
	f_{RS}	0·111	0·089	0·061	0·051	0·076	0·050	0·034	0·028
Log law	$1{,}000\ C_f$	3·89	3·13	2·24	1·91	2·98	1·87	1·30	1·11
	f_{RS} ..	0·110	0·089	0·063	0·054	0·084	0·053	0·037	0·031
Inferred	f_{RF} ..	0·620				0·349			
'True' f_R	Power law	0·731	0·709	0·681	0·671	0·425	0·399	0·383	0·377
	Log law..	0·730	0·709	0·683	0·674	0·433	0·402	0·386	0·380
Mean f_{RS} as percentage of (mean) 'true' f_R		15·2	12·5	9·1	7·8	18·7	12·8	9·3	7·9
Incompressible Flow Log law									
$1{,}000C_f$		4·35	3·54	2·62	2·25	4·35	3·04	2·26	1·97
f_{RS}		0·123	0·100	0·074	0·064	0·123	0·086	0·064	0·056
Increase above compressible (mean) f_{RS} (rows 3 and 5)		0·013	0·011	0·012	0·012	0·043	0·034	0·028	0·026
The same as percentage of (mean) 'true' f_R		1·8	1·6	1·8	1·8	10·0	8·5	7·3	6·9

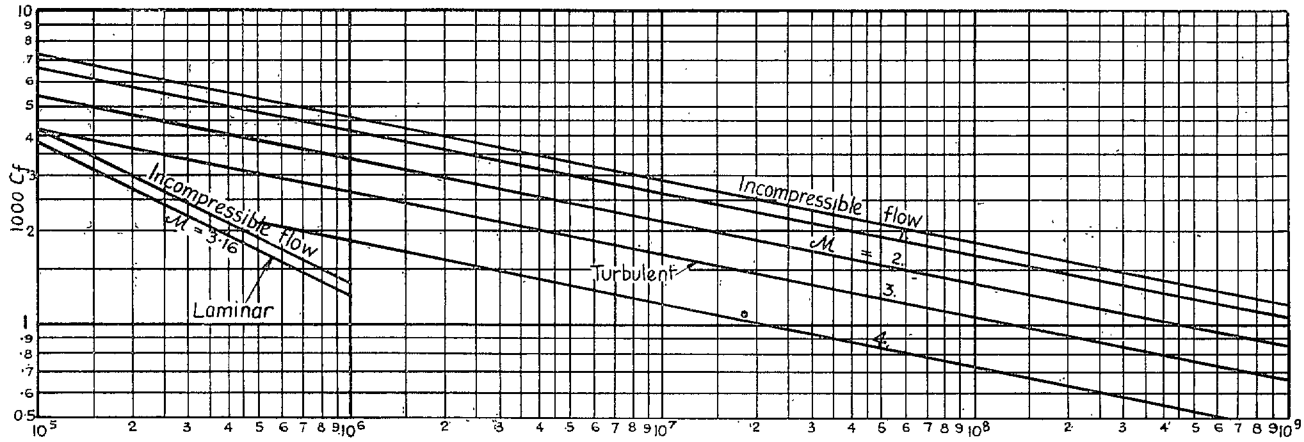

FIG. 1. The turbulent boundary layer in compressible flow. Power law of velocity distribution.

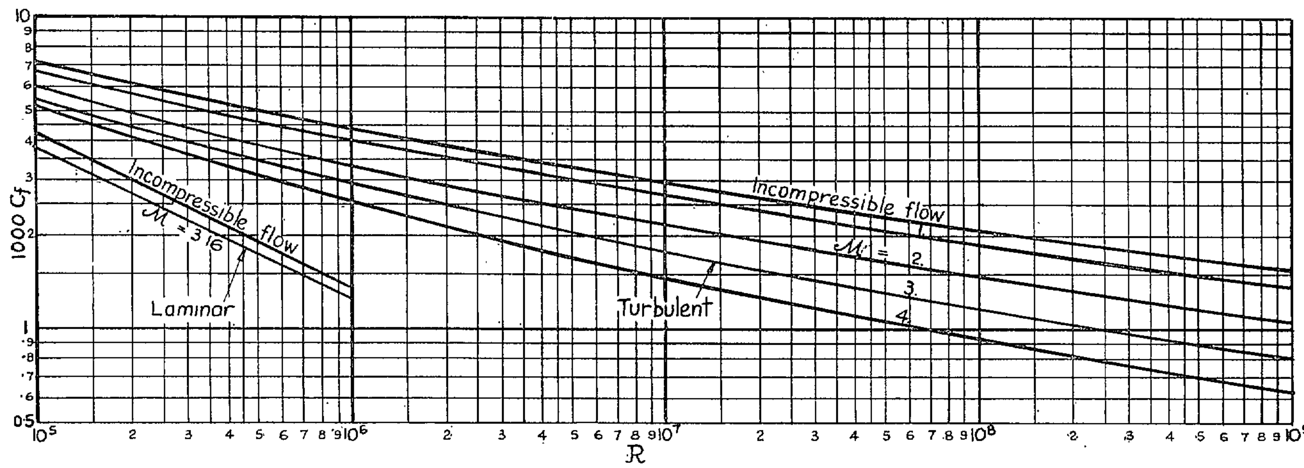

FIG. 2. The turbulent boundary layer in compressible flow. Log law of velocity distribution.

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