R. & M. No. 2786
(13,411)
A.R.C. Technical Report

MINISTRY OF SUPPLY

AERONAUTICAL RESEARCH COUNCIL
REPORTS AND MEMORANDA

Wind-Tunnel Interference Effects on Measurements of Aerodynamic Coefficients for Oscillating Aerofoils

By

W. P. JONES, M.A., D.Sc.
of the Aerodynamics Division, N.P.L.

LONDON: HER MAJESTY'S STATIONERY OFFICE
1958
PRICE 4*s* 6*d* NET

Wind-Tunnel Interference Effects on Measurements of Aerodynamic Coefficients for Oscillating Aerofoils

By
W. P. JONES, M.A., D.Sc.
of the Aerodynamics Division, N.P.L.

*Reports and Memoranda No. 2786**
September, 1950

Summary.—A theory is developed for estimating the effect of wind-tunnel walls on measured values of aerodynamic coefficients for two-dimensional aerofoils oscillating in an incompressible fluid. The case of an aerofoil describing translational and pitching oscillations in a wind tunnel of rectangular cross-section is considered, and it is shown in Table 1 and Figs. 3 and 4 that the damping derivatives associated with the pitching degree of freedom are very sensitive to wall effects when the frequency parameter for the motion is small, and when the axis of oscillation is not at, or near, quarter chord. When the axis is at quarter chord, the pitching-moment damping-derivative is hardly affected by the presence of the tunnel walls.

The values of the derivatives given in Table 1 refer to an axis of oscillation at mid-chord and correspond to a ratio of tunnel height to aerofoil chord of 4·75. They are used to determine the pitching-moment derivatives for an axis of oscillation at $0 \cdot 445c$ for comparison with values obtained by J. B. Bratt from measurements on a 2-in. chord aerofoil in a $9\frac{1}{2}$-in. square wind tunnel.

The theoretical values corresponding to free-stream conditions differ considerably from the experimental results, but, as shown in Figs. 4 and 5, better agreement is obtained when an allowance for tunnel-wall interference is made. The remaining difference between theory and experiment may be attributed to the influence of aerofoil thickness and to boundary-layer effects. By the use of the method developed in (R. & M. 2654[1]), these effects can also be taken into account and incorporated in the theory presented for estimating pure interference corrections for the aerodynamic derivatives. When this is done, the results given in Table 2, and plotted in Figs. 4 and 5, are obtained. A comparison of theory and experiment then shows satisfactory agreement.

1. *Introduction.*—Recent measurements at subsonic speeds of the aerodynamic damping coefficients for an aerofoil describing pitching oscillations differ widely from the results for low values of the frequency parameter predicted by theory. As wind-tunnel wall interference appeared to be the most likely cause of this difference, the method outlined in this note was developed in order to estimate interference effects on derivatives. The present theory applies only in the case of incompressible flow, but an estimate of the corresponding corrections in compressible flow up to $M = 0 \cdot 8$ is to be made. However, from comparisons of the experimental results for the pitching-moment damping-derivative at various Mach numbers with the values given by uncorrected theory, it appears that the correction factor for interference is, for the case considered, roughly independent of Mach number.

It seems likely that measurements of stability derivatives for oscillating wings of finite span would only be affected slightly by tunnel wall interference. In order to estimate the effect, the work described in (R. & M. 1912[2]) would have to be extended to include lower frequency parameter values, compressibility effects, and more general plan forms.

2. *Theory.*—An aerofoil of chord c in a tunnel of height $hc/2$ is assumed to be describing translational and pitching oscillations of small amplitude. The downward displacement z at mid-chord and the angular displacement α are defined respectively by

$$z = cz' e^{ipt}, \ \alpha = \alpha' e^{ipt}, \qquad \qquad (1)$$

*Published with the permission of the Director, National Physical Laboratory.

where $p/2\pi$ represents the frequency and t denotes time (*see* Fig. 1). The downwash $w(\equiv We^{ipt})$ at any point P on the aerofoil is then given by

$$w = \left(\frac{\partial}{\partial t} + V\frac{\partial}{\partial x}\right)(z + x\alpha) . \qquad (2)$$

Let $x = \frac{c}{2}\xi$, where $\xi = -\cos\vartheta$ on the aerofoil. From (1) and (2) it then follows that the complex amplitude W of the downwash is defined by

$$W = V[2i\omega'z' + \alpha'(1 - i\omega'\cos\vartheta)] , \qquad (3)$$

where $\omega' = pc/2V = \omega/2$. For convenience, the exponential factor e^{ipt} is omitted.

As in the theory for an oscillating aerofoil in a free stream, (R. & M. 2026[3]) the disturbed flow is assumed to be reproduced by a chordwise distribution of bound vorticity $\gamma(\equiv\Gamma e^{ipt})$. This gives rise to a free vorticity distribution $\varepsilon(\equiv Ee^{ipt})$ over the aerofoil and the wake. It is shown in R. & M. 2026[3] that

$$\begin{aligned} E &= -i\omega' e^{-i\omega'\xi}\int_{-1}^{\xi}\Gamma e^{i\omega'\xi}\,d\xi , \quad \ldots . \; -1<\xi<1 \\ &= -i\omega' e^{-i\omega'\xi}\int_{-1}^{1}\Gamma e^{i\omega'\xi}\,d\xi . \quad \xi \geqslant 1 . \end{aligned} \qquad (4)$$

Under free-stream conditions the downwash corresponding to the above vorticity distributions is given by

$$2\pi W(\xi_1) = \int_{-1}^{\infty}\frac{(\Gamma + E)\,d\xi}{\xi_1 - \xi} , \qquad (5)$$

and the general bound vorticity distribution Γ may be conveniently expressed in the form

$$\Gamma = V\sum_{n=0}^{\infty} C_n\Gamma_n , \qquad (6)$$

where

$$\begin{aligned} \Gamma_0 &\equiv 2\left[C(\omega')\cot\frac{\vartheta}{2} + i\omega'\sin\vartheta\right] , \\ \Gamma_1 &\equiv -2\sin\vartheta + \cot\frac{\vartheta}{2} + i\omega'\left[\sin\vartheta + \frac{\sin 2\vartheta}{2}\right] , \\ n \geqslant 2 \ldots . \; \Gamma_n &\equiv -2\sin n\vartheta + i\omega'\left[\frac{\sin(n+1)\vartheta}{n+1} - \frac{\sin(n-1)\vartheta}{n-1}\right] , \end{aligned} \qquad (7)$$

and the C_n's are arbitrary constants. The lift function $C(\omega')$ occurring in the definition of Γ_0 is given in terms of the Hankel functions $\overset{(2)}{H}_0(\omega')$, $\overset{(2)}{H}_1(\omega')$ by

$$C(\omega') = \overset{(2)}{H}_1(\omega')/[\overset{(2)}{H}_1(\omega') + i\overset{(2)}{H}_0(\omega')] . \qquad (8)$$

The free-vorticity distributions corresponding to $\Gamma_0, \Gamma_1, \ldots \Gamma_n$ are given by (4) and can be shown to be respectively,

$$\left.\begin{aligned} E_0 &= -2i\omega'\sin\vartheta - 2i\omega' S_0' , \\ E_1 &= -i\omega'\left(\sin\vartheta + \frac{\sin 2\vartheta}{2}\right) , \\ n \geqslant 2 \ldots . \; E_n &= -i\omega'\left(\frac{\sin(n+1)\vartheta}{n+1} - \frac{\sin(n-1)\vartheta}{n-1}\right) , \end{aligned}\right\} \qquad (9)$$

where

$$S_0' \equiv e^{-i\omega'\xi}\left[X_0\vartheta + 2\sum_{n=1}^{\infty}(-i)^n X_n \frac{\sin n\vartheta}{n}\right], \quad \ldots \quad (10)$$

$$= \pi X_0 e^{-i\omega'\xi}, \quad \ldots . \quad \xi \geqslant 1.$$

and

$$\left.\begin{aligned} X_0 &\equiv C(\omega')\,J_0(\omega') + i\{1 - C(\omega')\}J_1(\omega') \,. \\ X_n &\equiv CJ_n - i(1 - C)J_n' \,. \end{aligned}\right\} \quad \ldots \quad (11)$$

The symbol J_n represents the Bessel function of nth order, and $J_n' \equiv \frac{dJ_n}{d\omega}$. It is evident from (9) and (10) that, in the wake,

$$E_0 = -\,2i\pi\omega' X_0 e^{-i\omega'\xi}, \quad \ldots \quad (12)$$

$$E_1 = 0\,,$$

$$E_n = 0\,. \quad \ldots . \quad n \geqslant 2.$$

By the use of the above formulae, it can be shown that the downwash corresponding to the bound-vorticity distribution defined by (6) is

$$W = V\left[C_0 + C_1(\tfrac{1}{2} + \cos\vartheta) + \sum_{n=2}^{\infty} C_n \cos n\vartheta\right]. \quad \ldots \quad (13)$$

Since (3) and (13) must be identical, it follows that the arbitrary constants C_0, C_1, etc., must have the values

$$\left.\begin{aligned} C_0 &= 2i\omega' z' + \alpha'\left(1 + \frac{i\omega'}{2}\right). \\ C_1 &= -\,i\omega'\alpha'\,, \\ C_n &= 0\,. \quad \ldots . \quad n \geqslant 2\,. \end{aligned}\right\} \quad \ldots \quad (14)$$

The corresponding amplitude $L(x)$ of the lift distribution is then given by

$$L(x) = \rho V\Gamma = \rho V^2(C_0\Gamma_0 + C_1\Gamma_1)\,, \quad \ldots \quad (15)$$

where Γ_0, Γ_1 are defined by (7), and C_0, C_1 are expressed in terms of the amplitudes of the translational and pitching oscillation by (14).

The above formulae apply in the case of an aerofoil oscillating in a free stream. For oscillations in a wind tunnel, however, formula (13) for the downwash requires modification to allow for the downwash induced by the system of image vorticity distributions which arise from the presence of the tunnel walls (*see* Fig. 1). It can be deduced that the total downwash at a point P on the aerofoil due to its own vorticity distribution and that of the images is given by

$$2\pi W(\xi_1) = \int_{-1}^{\infty} \frac{\Gamma + E}{\xi_1 - \xi}\,d\xi + 2\sum_{m=1}^{\infty}(-1)^m \int_{-1}^{\infty} \frac{(\Gamma + E)(\xi_1 - \xi)}{(\xi_1 - \xi)^2 + m^2h^2}\,d\xi\,. \quad \ldots \quad (16)$$

If use is made of the relation

$$\operatorname{cosech} q = \frac{1}{q} + 2\sum_{m=1}^{\infty} \frac{(-1)^m q}{q^2 + m^2\pi^2}, \quad \ldots \quad (17)$$

equation (16) reduces to

$$2\pi W(\xi_1) = \int_{-1}^{\infty} \frac{\pi(\Gamma + E)d\xi}{h \sinh \dfrac{\pi(\xi_1 - \xi)}{h}}. \quad \ldots \quad (18)$$

To ensure tangential flow over the aerofoil, the vorticity distribution must be such that (18) gives the values of $W(\xi_1)$ prescribed by (3).

3. *Method of Solution.*—It follows from (5) and (18) that the downwash $W_I(\xi_1)$ induced by the image vorticity distributions only is given by

$$2\pi W_I(\xi_1) = \int_{-1}^{\infty} (\Gamma + E) \left[\frac{\pi}{h} \operatorname{cosech} \frac{\pi(\xi_1 - \xi)}{h} - \frac{1}{\xi_1 - \xi}\right] d\xi \, . \qquad (19)$$

By the use of (4) and differentiation with respect to ξ_1, it can also be shown that

$$2\pi \left[\frac{\partial W_I}{\partial \xi_1} + i\omega' W_I\right] = \int_{-1}^{1} \frac{\pi \Gamma}{h} \frac{\partial}{\partial \xi_1} \left[\operatorname{cosech} \frac{\pi(\xi_1 - \xi)}{h} - \frac{h}{\pi(\xi_1 - \xi)}\right] d\xi$$

$$= - \frac{\pi^2}{6h^2} \int_{-1}^{1} \Gamma \, d\xi \qquad (20)$$

when terms of higher order in $1/h$ are neglected. Equation (20) can be expressed differently in the form

$$\frac{\partial}{\partial \xi_1} \left\{\left[W_I + \frac{1}{2\pi i \omega'} \cdot \frac{\pi^2}{6h^2} \int_{-1}^{1} \Gamma \, d\xi\right] e^{i\omega' \xi}\right\} = 0 \, , \qquad (21)$$

and it is then evident that

$$W_I(\xi_1) = - \frac{1}{2\pi i \omega'} \cdot \frac{\pi^2}{6h^2} \int_{-1}^{1} \Gamma \, d\xi + e^{-i\omega' \xi_1} \left[W_I(0) + \frac{1}{2\pi i \omega'} \cdot \frac{\pi^2}{6h^2} \int_{-1}^{1} \Gamma \, d\xi\right] \qquad (22)$$

where $W_I(0)$ denotes the downwash induced at the origin by the image system of vortices. When $\xi_1 = 0$, (19) yields, in general,

$$2\pi W_I(0) = \int_{-1}^{\infty} (\Gamma + E) \left[\frac{1}{\xi} - \frac{\pi}{h} \operatorname{cosech} \frac{\pi \xi}{h}\right] d\xi \, , \qquad (23)$$

where

$$\Gamma + E \equiv V \sum_{n=0}^{\infty} C_n(\Gamma_n + E_n) \, .$$

Let I_n represent the coefficient of C_n in the expanded form of (23). Then, by the use of (12), it can be shown that

$$I_0 = \int_{-1}^{\infty} (\Gamma_0 + E_0) \left(\frac{1}{\xi} - \frac{\pi}{h} \operatorname{cosech} \frac{\pi \xi}{h}\right) d\xi$$

$$= \frac{\pi^2}{6h^2} \int_{-1}^{1} (\Gamma_0 + E_0) \xi \, d\xi \; 2\pi i \omega' X_0 (P - Q) \, , \qquad (24)$$

where

$$P \equiv \int_{\omega'}^{\infty} \frac{e^{-iy} \, dy}{y} \qquad (25)$$

and

$$Q \equiv \int_{\pi/h}^{\infty} e^{-i\gamma y} \operatorname{cosech} y \, dy \, , \qquad (26)$$

when $\gamma = \omega' h/\pi$ is substituted. The integral P is tabulated in Ref. 4, and Q may be evaluated from the series expression.

$$Q = 2e^{-i\omega'} \sum_{n=0}^{\infty} \frac{e^{-(2n+1)\pi/h}}{2n + 1 + i\gamma} \, . \qquad (27)$$

When $\omega' = 0$,

$$Q = \log_e \left(\frac{1 + \cosh(\pi/h)}{\sinh(\pi/h)}\right) \qquad (28)$$

It also follows from (7), (9) and (10) that

$$\int_{-1}^{1} (\Gamma_0 + E_0) \xi \, d\xi = 2\pi X_0 e^{-i\omega'} \left(1 - \frac{i}{\omega'}\right) - \pi + \frac{2i\pi C}{\omega'} \, .$$

Hence, finally,

$$\frac{I_0}{2\pi} = \frac{\pi^2}{6h^2}\left[X_0 e^{-i\omega'}\left(1 - \frac{i}{\omega'}\right) - \tfrac{1}{2} + \frac{iC}{\omega'}\right] - i\omega' X_0(P - Q) . \qquad (29)$$

The integrals I_1, I_2, etc., are easier to evaluate since E_1, E_2, etc., are zero in the wake. It can readily be deduced that

$$\begin{aligned} I_1 &= -\frac{\pi^3}{12h^2} , \\ I_2 &= \frac{\pi^3}{12h^2} , \\ I_n &= 0, \ldots n \geqslant 3 . \end{aligned} \qquad (30)$$

For a general vorticity distribution of the form assumed in (6), the downwash at mid-chord induced by the image system is then simply given by

$$2\pi W_I(0) = V[C_0 I_0 + C_1 I_1 + C_2 I_2] , \qquad (31)$$

where I_0, I_1, I_2 are defined above. It can also be shown that

$$\begin{aligned} \int_{-1}^{1} \Gamma_0 \, d\xi &= 2\pi\left(C + \frac{i\omega'}{2}\right), \\ \int_{-1}^{1} \Gamma_1 \, d\xi &= \frac{i\omega'\pi}{2} , \\ \int_{-1}^{1} \Gamma_2 \, d\xi &= -\frac{i\omega'\pi}{2} , \\ \int_{-1}^{1} \Gamma_n \, d\xi &= 0. \ldots n \geqslant 3 . \end{aligned} \qquad (32)$$

It then follows from (22), (23), (29), (30) and (32) that the downwash at any point P is given by

$$\frac{W_I(\xi_1)}{V} = -\frac{\pi^2}{6h^2}\left[C_0\left(\frac{C}{i\omega'} + \tfrac{1}{2}\right) + \frac{C_1 - C_2}{4}\right] + C_0 F e^{-i\omega'\xi_1} \qquad (33)$$

where

$$F \equiv \frac{\pi^2}{6h^2} X_0 e^{-i\omega'}\left(1 - \frac{i}{\omega'}\right) - i\omega' X_0(P - Q) . \qquad (34)$$

Over the aerofoil, $\xi_1 = -\cos\vartheta$, and it is known that

$$e^{i\omega'\cos\vartheta} = J_0(\omega') + 2\sum_1^n i^n J_n(\omega') \cos n\vartheta . \qquad (35)$$

By the use of (35), equation (33) may be expressed in the form

$$\frac{W_I(\xi_1)}{V} = a_0 + \sum_{n=1}^{\infty} a_n \cos n\vartheta , \qquad (36)$$

where

$$\left.\begin{aligned} a_0 &\equiv -\frac{\pi^2}{6h^2}\left[C_0\left(\tfrac{1}{2} - \frac{iC}{\omega'}\right) + \frac{C_1 - C_2}{4}\right] + C_0 F J_0 , \\ n \geqslant 1, \ldots a_n &\equiv 2 i^n J_n F C_0 . \end{aligned}\right\} \qquad (37)$$

When the downwash induced by the vorticity distribution over the actual aerofoil is added to (36), the following expression for the total downwash is obtained, namely,

$$W(\xi_1) = V\left[C_0 + \frac{C_1}{2} + a_0 + \sum_{n=1}^{\infty}(a_n + C_n)\cos n\vartheta\right] . \qquad (38)$$

The above formula for W must be identical with (3). Comparison of coefficients yields the relations

$$\left.\begin{aligned} 2i\omega'z' + \alpha' &= C_0 + \frac{C_1}{2} + a_0 \\ -i\omega'\alpha' &= C_1 + a_1 \\ 0 &= C_n + a_n \,, \ldots n \geqslant 2\,. \end{aligned}\right. \qquad \ldots \qquad (39)$$

By the use of (37) and (39), it follows that

$$\left.\begin{aligned} C_0 &= \frac{2i\omega'z' + \alpha'\left[1 + \frac{i\omega'}{2}\left(1 - \frac{\pi^2}{12h^2}\right)\right]}{D}\,, \\ C_1 &= -i\omega'\alpha' - 2iJ_1C_0F\,, \\ C_n &= -2i^nJ_nFC_0\,, \ldots n \geqslant 2 \end{aligned}\right\} \qquad \ldots \qquad (40)$$

where F is defined by (34) and

$$D \equiv 1 - \frac{\pi^2}{6h^2}\left(\frac{C}{i\omega'} + \tfrac{1}{2}\right) + F\left[J_0 - iJ_1 + \frac{\pi^2}{12h^2}(J_2 + iJ_1)\right]. \qquad \ldots \qquad (41)$$

The required bound-vorticity distribution Γ over the aerofoil is then given by

$$\Gamma = V\left[C_0\Gamma_0 - i\omega'\alpha'\Gamma_1 - 2C_0F\sum_{n=1}^{\infty} i^nJ_n\Gamma_n\right], \qquad \ldots \qquad (42)$$

and the corresponding amplitudes of the lift and the pitching moment about the mid-chord axis are respectively

$$L = \frac{\rho cV}{2}\int_0^{\pi}\Gamma \sin\vartheta \, d\vartheta\,, \qquad \ldots \qquad (43)$$

$$M = \frac{\rho c^2V}{4}\int_0^{\pi}\Gamma \cos\vartheta \sin\vartheta \, d\vartheta\,. \qquad \ldots \qquad (44)$$

4. *Aerodynamic Coefficients*.—In terms of the standard derivative coefficients l_z, $l_{\dot{z}}$, etc., referred to the mid-chord axis the force and moment can be expressed in the form

$$\frac{L}{\rho cV^2} = (l_z + i\omega l_{\dot{z}})z' + (l_\alpha + i\omega l_{\dot{\alpha}})\alpha'\,, \qquad \ldots \qquad (45)$$

$$\frac{M}{\rho c^2V^2} = (m_z + i\omega m_{\dot{z}})z' + (m_\alpha + i\omega m_{\dot{\alpha}})\alpha'\,. \qquad \ldots \qquad (46)$$

The acceleration derivatives, $l_{\ddot{z}}$, $l_{\ddot{\alpha}}$, etc., are here incorporated in the derivative coefficients in phase with the displacements. From equations (40) to (44) it follows that

$$\frac{L}{\rho cV^2} = \pi\left\{C_0\left[C + \frac{i\omega'}{2} + \frac{\omega'J_1F}{2} - \frac{i\omega'J_2F}{2}\right] + \frac{\omega'^2\alpha'}{4}\right\}, \qquad \ldots \qquad (47)$$

$$\frac{M}{\rho c^2V^2} = \frac{\pi}{4}\left\{C_0\left[C - iJ_1F\right] - \frac{i\omega'\alpha'}{2}\left(1 + \frac{i\omega'}{4}\right)\right\}, \qquad \ldots \qquad (48)$$

where C_0 is a linear function of z' and α'. Comparison of the coefficients of z' and α' in (45) and (47) then yields

$$l_z + 2i\omega'l_{\dot{z}} = \frac{2\pi i\omega'}{D}\left[C + \frac{i\omega'}{2}(1 - iJ_1F - J_2F)\right], \qquad \ldots \qquad (49)$$

$$l_\alpha + 2i\omega'l_{\dot{\alpha}} = \frac{\pi}{D}\left[1 + \frac{i\omega'}{2}\left(1 - \frac{\pi^2}{12h^2}\right)\right]\left[C + \frac{i\omega'}{2}(1 - iJ_1F - J_2F)\right] + \frac{\omega'^2\pi}{4}\,, \qquad \ldots \qquad (50)$$

where $\omega' \equiv \omega/2$. Similarly, it follows from a comparison of (46) and (48) that

$$m_z + 2i\omega' m_{\dot{z}} = \frac{\pi i \omega'}{2D}\,(C - iJ_1 F)\,, \qquad (51)$$

$$m_\alpha + 2i\omega' m_{\dot{\alpha}} = \frac{\pi}{4D}\left[1 + \frac{i\omega'}{2}\left(1 - \frac{\pi^2}{12h^2}\right)\right]\left[C - iJ_1 F\right] - \frac{i\pi\omega'}{8}\left(1 + \frac{i\omega'}{4}\right). \qquad (52)$$

When $\omega' \to 0$, it can be shown that

$$\begin{aligned} C &\to 1 - \frac{\pi\omega'}{2} + i\omega'\left(\gamma + \log_e \frac{\omega'}{2}\right), \\ P &\to -\gamma - \log_e \omega' - i\left(\frac{\pi}{2} - \omega'\right), \\ Q &\to \bar{E} - \log_e 2\,, \end{aligned} \qquad (53)$$

where

$$\bar{E} \equiv \log_e \frac{2\left(1 + \cosh \dfrac{\pi}{h}\right)}{\sinh \dfrac{\pi}{h}}\,.$$

It then follows that

$$D \to \left(1 - \frac{\pi^2}{6h^2}\right)\left[1 + i\omega'\left(\gamma + \log_e \frac{\omega'}{2}\right)\right] + i\omega'\bar{E} \qquad (54)$$

and that

$$\frac{C}{D} \to \left(1 + \frac{\pi^2}{6h^2}\right)\left[1 - i\omega'\left(1 + \frac{\pi^2}{6h^2}\right)\bar{E}\right]. \qquad (55)$$

Hence, the appropriate limiting forms of the derivatives referred to mid-chord are

$$\begin{aligned} & l_z \to 0\,; && l_{\dot{z}} \to \pi\left(1 + \frac{\pi^2}{6h^2}\right) \\ & m_z \to 0\,; && m_{\dot{z}} \to \frac{\pi}{4}\left(1 + \frac{\pi^2}{12h^2}\right) \\ & l_\alpha \to \pi\left(1 + \frac{\pi^2}{6h^2}\right); && l_{\dot{\alpha}} \to \frac{\pi}{2}\left[1 + \frac{\pi^2}{12h^2} - \bar{E}\left(1 + \frac{\pi^2}{3h^2}\right)\right] \\ & m_\alpha \to \frac{\pi}{4}\left(1 + \frac{\pi^2}{12h^2}\right); && m_{\dot{\alpha}} \to -\frac{\pi}{8}\left(1 + \frac{\pi^2}{4h^2}\right)\bar{E}\,. \end{aligned} \qquad (56)$$

It will be noticed that both $l_{\dot{\alpha}}$ and $m_{\dot{\alpha}}$ tend to finite values, whereas, for free-stream conditions, they both tend to minus infinity for oscillations about the mid-chord axis (*see* Table 1).

Applications.—The present theory is used to calculate aerodynamic coefficients for a 2-in. chord RAE 104 aerofoil oscillating in a $9\frac{1}{2}$-in. square wind-tunnel. In the first instance, the aerofoil is regarded as a thin flat plate, and the values of the aerodynamic coefficients obtained for this case are given in Table 1 for comparison with the theoretical results for free-stream conditions. It should be noticed that the effect of tunnel-wall interference on $l_{\dot{\alpha}}$ and $m_{\dot{\alpha}}$ becomes increasingly large as $\omega \to 0$. According to vortex-sheet theory for oscillations in a free stream, the damping coefficient $-m_{\dot{\alpha}}$ may be negative for positions of the axis of oscillation forward of the quarter-chord point, but it is shown in Fig. 3 that, in a wind tunnel the damping may be positive for all axis positions. It should also be noted that the amount of correction due to interference varies with frequency and axis position. At the higher values of the frequency parameter and for oscillations about the quarter-chord axis the interference effect is small.

Measurements of the aerodynamic damping and stiffness derivatives for an aerofoil of section RAE 104 oscillating about an axis at $0\cdot445c$ behind the leading edge have recently been made in the $9\frac{1}{2}$-in. square tunnel at the National Physical Laboratory for a range of Mach numbers. Values of $-m_{\dot{\alpha}}$ derived by the method here proposed for incompressible flow agree roughly with the experimental values for $M = 0\cdot5$ and are higher than the extrapolated values for $M = 0$ (*see* Fig. 4). They are, however, in much closer agreement with experiment than the values given by uncorrected theory. The remaining differences are due to thickness and boundary-layer effects and can be allowed for by making use of the steady motion characteristics of the aerofoil along the lines suggested in R. & M. 2654[1]. Some unpublished work showed that, for the particular aerofoil considered in this note, the damping was reduced by about 20 per cent while the stiffness derivative was hardly affected. These calculations were done by the method of Ref. 1 for free-stream conditions, and based on experimental data. In the next section of the present paper tunnel-wall interference, thickness and boundary-layer effects are allowed for simultaneously by using the scheme suggested in Ref. 1 in conjunction with the theory for dealing with interference presented in this note.

Thickness, Boundary Layer and Interference Effects.—The main feature of the scheme of calculation proposed in Ref. 1 is the replacement of the aerofoil at each incidence by an equivalent thin profile which gives, on the basis of linearised theory, the experimentally determined steady motion lift distribution for that incidence. For slow oscillations of small amplitude the profile is assumed to change its shape instantaneously with incidence. In the calculation of the aerodynamic forces, the linearised theory for oscillatory motion is used; variations in profile shape being taken into account.

From N.P.L. measurements of C_L and $C_M(\frac{1}{4})$ for the RAE 104 aerofoil for a range of Mach numbers, it was estimated that C_L and $C_M(\frac{1}{4})$ for incompressible flow would be given respectively by

$$C_L = 2\pi A(\alpha) = 2\pi \times 0\cdot821\alpha$$

and

$$C_M(\tfrac{1}{4}) = \frac{\pi}{4} B(\alpha) = \frac{\pi}{4} \times 0\cdot2675\alpha . \qquad (57)$$

Hence, in the notation of Ref. 1, $A' = 0\cdot821$, $B' = 0\cdot2675$, and it can be shown that the corresponding equivalent profile is defined by

$$\frac{2z}{c} = (A' + B' - 2\bar{h})\alpha + \left(A' + \frac{B'}{2}\right)\alpha\xi - \frac{B'\alpha}{2}\xi^2 , \qquad (58)$$

the axis of oscillation being at $\bar{h}c$ behind the leading edge (*see* Fig. 2).

Let $\alpha = \alpha' e^{ipt}$ as in (1). Then it follows that the downwash at any point on the chord is given by

$$w = \frac{\partial z}{\partial \alpha}\frac{d\alpha}{dt} + \frac{2V}{c}\frac{\partial z}{\partial \xi} , \qquad (59)$$

and that the amplitude

$$W(\xi_1) = V(d_0 + d_1 \cos\vartheta + d_2 \cos 2\vartheta)\alpha' ,$$

where

$$\alpha' d_0 \equiv A' + \frac{B'}{2} + i\left(A' + \frac{3B'}{4} - 2\bar{h}\right)\omega' ,$$

$$\alpha' d_1 \equiv B' - i\omega'\left(A' + \frac{B'}{2}\right), \qquad (60)$$

$$\alpha' d_2 \equiv -\frac{i\omega' B'}{4} .$$

Since (38) and (59) must be identical, the following relations are valid, namely,

$$\begin{aligned} \alpha' d_0 &= C_0 + \frac{C_1}{2} + a_0 , \\ \alpha' d_1 &= C_1 + a_1, \\ \alpha' d_2 &= C_2 + a_2, \\ n \geqslant 3 \ldots 0 &= C_n + a_n. \end{aligned} \qquad (61)$$

As in Section 4, it then follows that the pitching-moment derivatives for an axis of oscillation at $\bar{h}c$ behind the leading edge are given by

$$m_a + 2i\omega' m_{\dot{a}} = \frac{\pi}{4\alpha'} \left\{ C_0(C - iJ_1F) + \frac{\alpha' d_1}{2}\left(1 + \frac{i\omega'}{4}\right) - \frac{\alpha' d_2}{2} \right\}$$

$$- \frac{\pi(1 - 2\bar{h})}{2\alpha'} \left\{ C_0 \left[C + \frac{i\omega'}{2} + \frac{\omega' F}{2} (J_1 - iJ_2) \right] + \frac{i\omega'}{4} (d_1 - d_2)\alpha' \right\} , \qquad (62)$$

where, now,

$$C_0 \equiv \frac{\alpha'}{D} \left[d_0 - \frac{d_1}{2}\left(1 - \frac{\pi^2}{12h^2}\right) - \frac{\pi^2}{24h^2} d_2 \right] , \qquad (63)$$

and D is defined by (41). The values of m_a and $m_{\dot{a}}$ given by (62), when $\bar{h} = 0 \cdot 445$ is substituted, are compared with the corresponding experimental results in Figs. 4 and 5. When allowance is made for interference and thickness effects, theory gives good agreement with experiment.

General Remarks.—In view of the sensitivity of the pitching-moment damping-derivative to tunnel-wall interference effects, care should be taken in the interpretation of results obtained experimentally. They should not be used in stability and control calculations without appropriate correction. Interference effects may, however, be less important for wings of finite aspect ratio. An estimate of the effect of the tunnel walls on the derivatives for oscillating rectangular wings could be made by extending the method developed in R. & M. 1912[2] to deal with lower frequency parameter values, but for wings of general plan form the theory would have to be modified considerably.

In the present note the effect of compressibility is ignored, but the results shown in Fig. 4 indicate that, for a particular ω, the ratio of the free-stream theoretical to the experimental values of $m_{\dot{a}}$ is roughly independent of Mach number.

Acknowledgment.—The writer is greatly indebted to Mr. J. B. Bratt for permission to use his, as yet, unpublished results for the purpose of comparison with present theory. All the computing required in the preparation of this note was done by Miss W. M. Tafe.

REFERENCES

No.	*Author*	*Title etc.*
1	W. P. Jones	Aerofoil oscillations at high mean incidences. R. & M. 2654. April, 1948.
2	W. P. Jones	Wind tunnel interference effect on the values of experimentally determined derivative coefficients for oscillating aerofoils. R. & M. 1912. August, 1943.
3	W. P. Jones	Aerodynamic forces on wings in simple harmonic motion. R. & M. 2026. February, 1945.
4	National Bureau of Standards ..	Table of sine, cosine and exponential integrals. Vol. II.

TABLE 1

Aerodynamic Derivatives Referred to the Mid-chord Axis ($\bar{h} = 0 \cdot 5$)

(a) Wind tunnel

ω	l_z	$l_{\dot{z}}$	l_α	$l_{\dot{\alpha}}$	m_z	$m_{\dot{z}}$	m_α	$m_{\dot{\alpha}}$
0	0	3·20	3·20	−2·49	0	0·793	0·793	−1·009
0·02	0·001	3·19	3·20	−2·48	0	0·785	0·792	−1·005
0·04	0·005	3·18	3·18	−2·44	0·002	0·785	0·788	−0·998
0·08	0·019	3·13	3·14	−2·23	0·006	0·778	0·778	−0·942
0·2	0·093	2·86	2·89	−1·61	0·030	0·708	0·717	−0·784
0·4	0·166	2·41	2·48	−0·441	0·073	0·598	0·620	−0·502
0·8	−0·028	1·99	2·11	0·536	0·119	0·493	0·538	−0·260
2·0	−2·495	1·72	1·89	1·050	0·167	0·426	0·566	−0·133

(b) Free stream

ω	l_z	$l_{\dot{z}}$	l_α	$l_{\dot{\alpha}}$	m_z	$m_{\dot{z}}$	m_α	$m_{\dot{\alpha}}$
0	0	3·14	3·14	−∞	0	0·785	0·785	−∞
0·02	0·003	3·09	3·09	−5·61	0·001	0·771	0·772	−1·80
0·04	0·008	3·03	3·03	−4·36	0·002	0·757	0·758	−1·48
0·08	0·024	2·91	2·92	−3·04	0·007	0·728	0·730	−1·15
0·2	0·077	2·61	2·64	−1·27	0·027	0·653	0·661	−0·710
0·4	0·111	2·29	2·35	−0·125	0·059	0·571	0·590	−0·424
0·8	−0·088	1·96	2·07	+0·628	0·104	0·491	0·532	−0·236
2·0	−2·512	1·69	1·85	1·05	0·158	0·424	0·561	−0·130

Note: The derivatives $l_{\dot{\alpha}}$ and $m_{\dot{\alpha}}$ tend to finite values as $\omega \to 0$ when allowance is made for interference.

TABLE 2

Pitching-Moment Derivatives for the RAE 104 *Aerofoil* ($\bar{h} = 0 \cdot 445)e$

ω	m_α			$m_{\dot{\alpha}}$		
	Free-stream	Interference	Interference and thickness	Free-stream	Interference	Interference and thickness
0	0·613	0·617	0·612	−∞	−0·833	−0·658
0·02	0·602	0·616	0·611	−1·445	−0·833	−0·660
0·04	0·591	0·614	0·610	−1·210	−0·833	−0·655
0·08	0·570	0·605	0·603	−0·954	−0·785	−0·621
0·2	0·517	0·559	0·566	−0·613	−0·664	−0·523
0·4	0·464	0·486	—	−0·393	−0·452	—
0·8	0·425	0·429	—	−0·249	−0·268	—
2·0	0·475	0·479	—	−0·169	−0·172	—

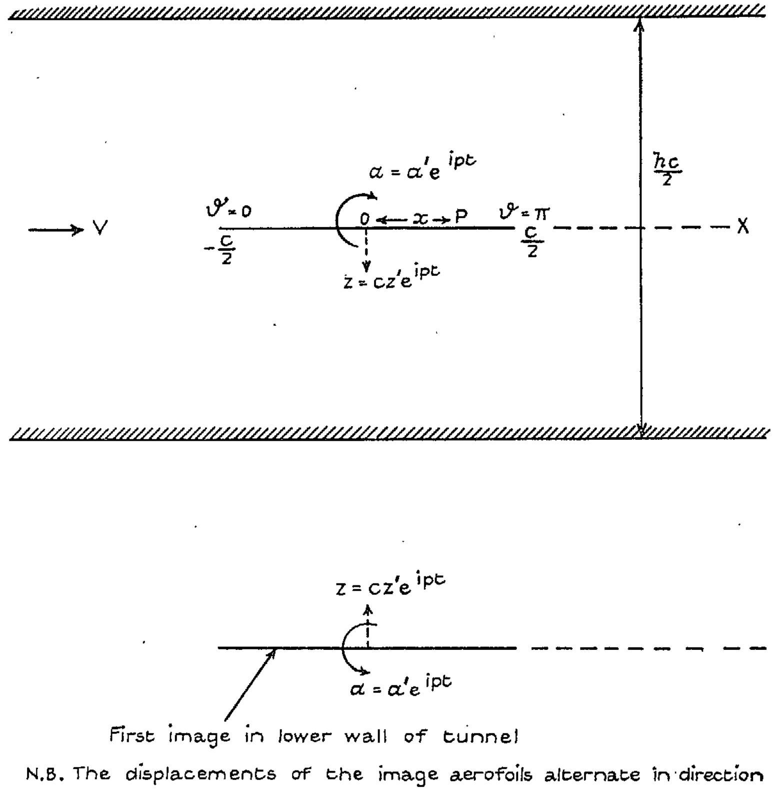

FIG. 1. Oscillating aerofoil in wind tunnel.

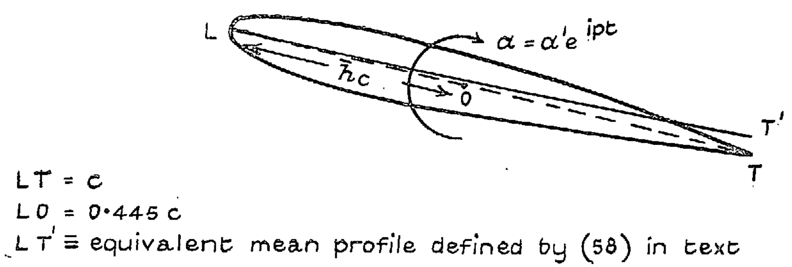

FIG. 2. Equivalent mean profile for thick aerofoil.

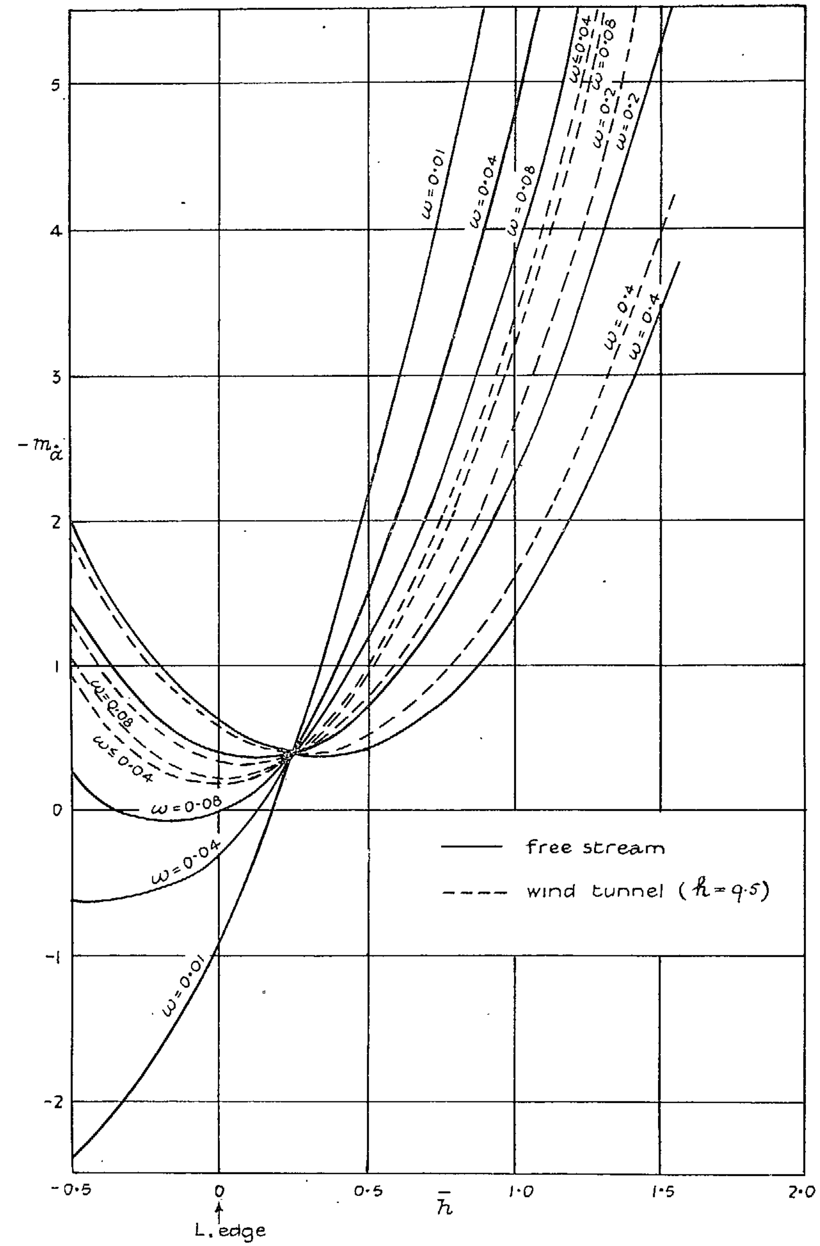

FIG. 3. Variation in damping with axis position.

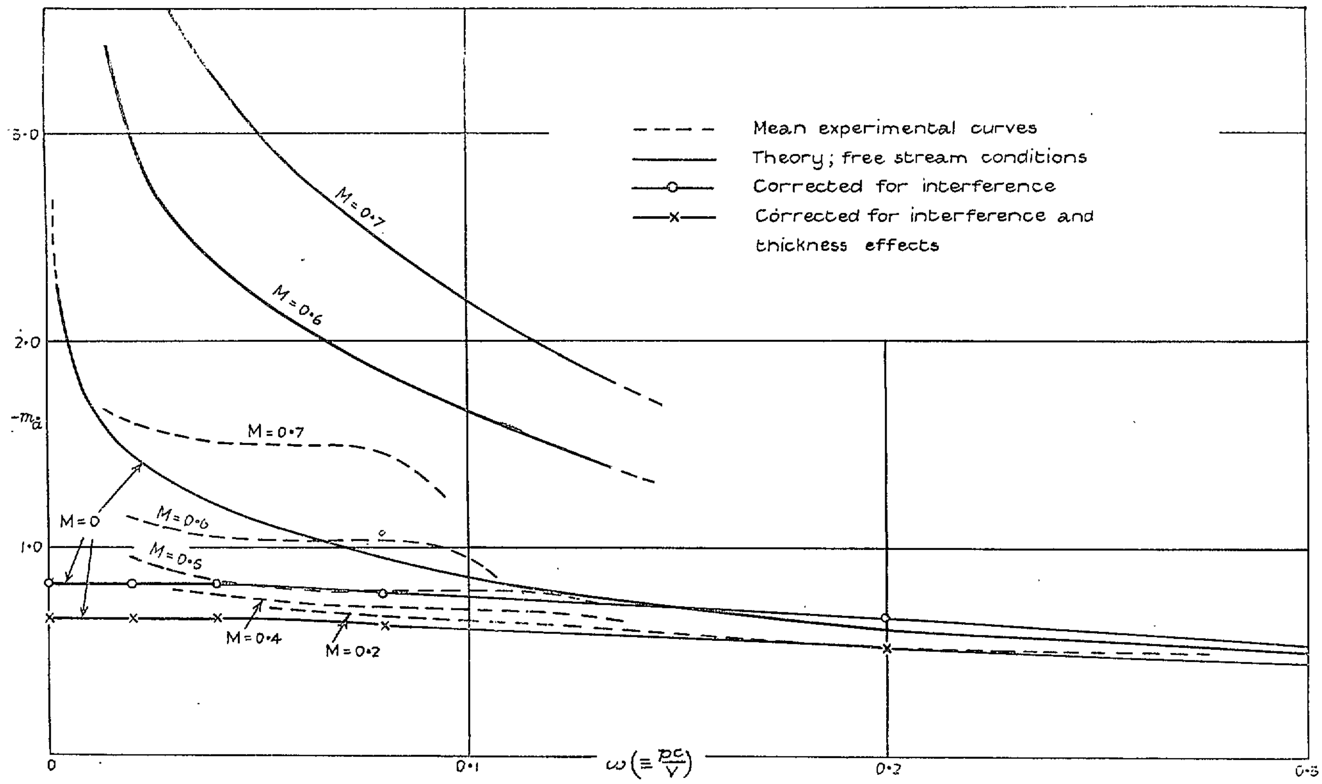

FIG. 4. Pitching-moment damping coefficient for the RAE 104 aerofoil (Axis at 0·445c).

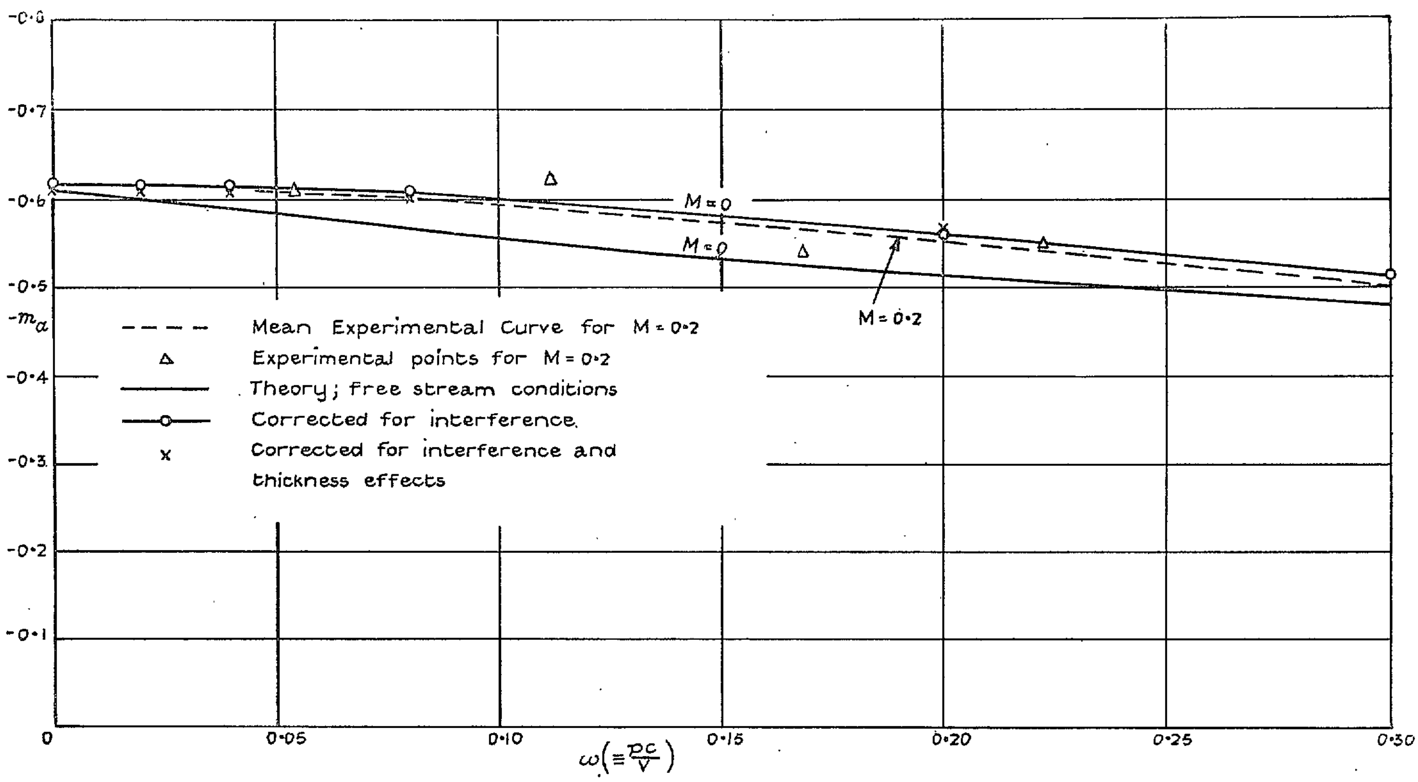

FIG. 5. Pitching-moment stiffness coefficient for the RAE 104 aerofoil (Axis at 0·445c).

Wt. 16/780 K.9 5/55 G.G. 34–262 PRINTED IN GREAT BRITAIN

Publications of the Aeronautical Research Council

ANNUAL TECHNICAL REPORTS OF THE AERONAUTICAL RESEARCH COUNCIL (BOUND VOLUMES)

1939 Vol. I. Aerodynamics General, Performance, Airscrews, Engines. 50*s.* (51*s.* 9*d.*)
Vol. II. Stability and Control, Flutter and Vibration, Instruments, Structures, Seaplanes, etc. 63*s.* (64*s.* 9*d.*)

1940 Aero and Hydrodynamics, Aerofoils, Airscrews, Engines, Flutter, Icing, Stability and Control, Structures, and a miscellaneous section. 50*s.* (51*s.* 9*d.*)

1941 Aero and Hydrodynamics, Aerofoils, Airscrews, Engines, Flutter, Stability and Control Structures. 63*s.* (64*s.* 9*d.*)

1942 Vol. I. Aero and Hydrodynamics, Aerofoils, Airscrews, Engines. 75*s.* (76*s.* 9*d.*)
Vol. II. Noise, Parachutes, Stability and Control, Structures, Vibration, Wind Tunnels. 47*s.* 6*d.* (49*s.* 3*d.*)

1943 Vol. I. Aerodynamics, Aerofoils, Airscrews. 80*s.* (81*s.* 9*d.*)
Vol. II. Engines, Flutter, Materials, Parachutes, Performance, Stability and Control, Structures. 90*s.* (92*s.* 6*d.*)

1944 Vol. I. Aero and Hydrodynamics, Aerofoils, Aircraft, Airscrews, Controls. 84*s.* (86*s* 3*d.*)
Vol. II. Flutter and Vibration, Materials, Miscellaneous, Navigation, Parachutes, Performance, Plates and Panels, Stability, Structures, Test Equipment, Wind Tunnels. 84*s.* (86*s.* 3*d.*)

1945 Vol. I. Aero and Hydrodynamics, Aerofoils. 130*s.* (132*s.* 6*d.*)
Vol. II. Aircraft, Airscrews, Controls. 130*s.* (132*s.* 6*d.*)
Vol. III. Flutter and Vibration, Instruments, Miscellaneous, Parachutes, Plates and Panels, Propulsion. 130*s.* (132*s.* 3*d.*)
Vol. IV. Stability, Structures, Wind Tunnels, Wind Tunnel Technique. 130*s.* (132*s.* 3*d.*)

Annual Reports of the Aeronautical Research Council—

1937 2*s.* (2*s.* 2*d.*) 1938 1*s.* 6*d.* (1*s.* 8*d.*) 1939–48 3*s.* (3*s.* 3*d.*)

Index to all Reports and Memoranda published in the Annual Technical Reports, and separately—

April, 1950 R. & M. 2600. 2*s.* 6*d.* (2*s.* 8*d.*)

Author Index to all Reports and Memoranda of the Aeronautical Research Council—

1909–January, 1954. R. & M. No. 2570 15*s.* (15*s.* 6*d.*)

Indexes to the Technical Reports of the Aeronautical Research Council—

December 1, 1936 — June 30, 1939.	R. & M. No. 1850.	1*s.* 3*d.* (1*s.* 5*d.*)
July 1, 1939 — June 30, 1945.	R. & M. No. 1950.	1*s.* (1*s.* 2*d.*)
July 1, 1945 — June 30, 1946	R. & M. No. 2050.	1*s.* (1*s.* 2*d.*)
July 1, 1946 — December 31, 1946.	R. & M. No. 2150.	1*s.* 3*d.* (1*s.* 5*d.*)
January 1, 1947 — June 30, 1947.	R. & M. No. 2250.	1*s.* 3*d.* (1*s.* 5*d.*)

Published Reports and Memoranda of the Aeronautical Research Council—

Between Nos. 2251–2349	R. & M. No. 2350.	1*s.* 9*d.* (1*s.* 11*d.*)
Between Nos. 2351–2449	R. & M. No. 2450.	2*s.* (2*s.* 2*d.*)
Between Nos. 2451–2549	R. & M. No. 2550.	2*s.* 6*d.* (2*s.* 8*d.*)
Between Nos. 2551–2649	R. & M. No. 2650.	2*s.* 6*d.* (2*s.* 8*d.*)

Prices in brackets include postage

HER MAJESTY'S STATIONERY OFFICE

York House, Kingsway, London W.C.2; 423 Oxford Street, London W.1
13a Castle Street, Edinburgh 2; 39 King Street, Manchester 2; 2 Edmund Street, Birmingham 3; 109 St. Mary Street, Cardiff; Tower Lane, Bristol 1; 80 Chichester Street, Belfast, or through any bookseller

S.O. Code No. 23-2786