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Summary of the Theoretical Work done on the Stability of Thin Plates 1939 to 1946

By
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Summary of the Theoretical Work done on the Stability of Thin Plates* 1939 to 1946

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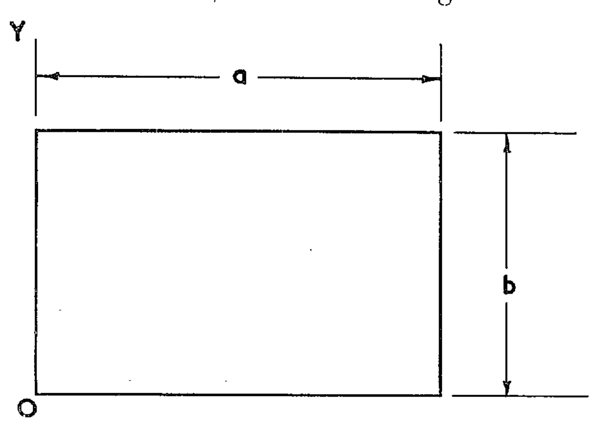
1. *Introduction.*—Most of the work done during the war on the stability of thin plates has been written up and published in reports (Refs. 1 to 8). These reports do not however form a connected series, and the object of this summary is to draw attention to the more important stability problems which were requiring solution at the beginning of the war, and to indicate the progress made towards their solution during the subsequent seven years.

In 1939 there were three main problems, or types of problem, for which existing solutions were inadequate: (A) the critical buckling load of a flat rectangular plate when the edges are not all simply supported, with special reference to a plate under shear, (B) the post-buckling behaviour of a long† flat plate under shear, (C) the initial and post-buckling behaviour of a curved plate under various combinations of shear, compression, and normal pressure. Of these three classes of problem (A) and (B) are primarily important in the design of plate web spars, while (C) is clearly of much wider application. For in any aeroplane of predominantly stressed-skin construction, the surface of the wings and fuselage consists very largely of thin and slightly curved plates.

Moreover, in addition to the general and ever-present problem of making aircraft structures lighter, aerodynamic developments during the war presented the aircraft designer with the further task of constructing wings that would remain sufficiently smooth to provide laminar flow over a considerable portion of the wing surface (R. & M. 2193[32]). As the degree of smoothness required is incompatible with buckling, this gave added importance to knowledge of the loads at which flat and slightly curved plates begin buckling.

2. *The Critical Buckling Loads of Flat Rectangular Plates when the Edges are not all Simply Supported.*—2.1. To indicate the difficulties which arise when the edges of a plate are not all simply supported, and the way by which such difficulties can be overcome, we shall outline the method described by Timoshenko[9] for cases in which the edges *are* all simply supported.



* No reference is made in this summary to work done on sandwich construction.

† For the purpose of this summary a plate is termed long when its length is more than three times its width.

Consider a flat rectangular plate loaded along its edges by forces acting in its plane, and suppose that the plate undergoes a small transverse displacement consistent with the boundary conditions. Then, if the distortion is such that there is no stretching of the middle surface, the increase in strain energy of the plate is solely due to bending. At the same time work will be done by the forces acting in the middle surface, and if the plate is on the point of buckling in the assumed form of distortion, this will equal the strain energy due to bending. Moreover, if the plate undergoes any small displacement from this particular configuration, the work done by the forces acting in the middle surface will be equal to the increase in the plate's strain energy.

Denoting by w* the transverse displacement of any point in the middle surface, and by N_x, N_y, N_{xy}, the stress resultants in the plane of the plate, the fundamental equation from which to deduce the critical buckling load is

$$\frac{D}{2}\int_0^a\int_0^b \{(w_{xx}+w_{yy})^2 - 2(1-\sigma)(w_{xx}w_{yy} - w_{xy}^2)\}\,dx\,dy$$
$$= -\tfrac{1}{2}\int_0^a\int_0^b \{N_x w_x^2 + 2N_{xy}w_x w_y + N_y w_y^2\}\,dx\,dy\,, \qquad (1)$$

or, if the distribution of stress in the plate is uniform, so that

$$N_x = -P_x,\ N_{xy} = -S,\ N_y = -P_y\,,$$

where P_x, S and P_y are constants; (1) is

$$\frac{D}{2}\int_0^a\int_0^b \{(w_{xx}+w_{yy})^2 - 2(1-\sigma)(w_{xx}w_{yy} - w_{xy}^2)\}\,dx\,dy$$
$$= \tfrac{1}{2}P_x\int_0^a\int_0^b w_x^2\,dx\,dy + S\int_0^a\int_0^b w_x w_y\,dx\,dy + \tfrac{1}{2}P_y\int_0^a\int_0^b w_y^2\,dx\,dy \qquad (2)$$

The next step is to obtain an analytical expression for the transverse displacement w which is general, which satisfies the boundary conditions, and which it is possible to substitute in equation (2). If w is expressed as an infinite series, it must be capable of term-by-term differentiation. In order to be general the expression for w must contain an infinite number of parameters, A_{mn} (say), and these are found by expressing the fact that equation (2) remains valid for all arbitrarily small changes in the A_{mn}'s. This in general leads to an infinite system of equations, linear and homogeneous in the A_{mn}'s, and the condition that these equations should possess a non-zero solution gives an infinite determinantal equation from which to deduce the critical loads.

The method as outlined is standard, and has been applied by Timoshenko[9] and others to a wide range of thin plate problems in which the edges of the plate are all simply supported. In such cases the transverse displacement w is expressed as the double sine series

$$w = \sum_1^\infty \sum_1^\infty A_{mn} \sin n\pi\frac{x}{a} \sin n\pi\frac{y}{b}\,, \qquad (3)$$

which is quite general, satisfies the conditions for simple support at the edges, and is differentiable term by term.

If however the edges are *not* all simply supported, the expression for w given by (3) is inadequate, as it does not satisfy the boundary conditions.

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* $w_x \equiv \dfrac{\partial w}{\partial x}$; $w_{xx} \equiv \dfrac{\partial^2 w}{\partial x^2}$, *etc.*

2.2. On physical grounds any partial derivative of the transverse displacement w is a continuous function of x and y in the range $0 < x < a$, $0 < y < b$, and can therefore be expanded as a double Fourier series valid throughout this range. In particular, $\partial w^8/\partial x^4\, \partial y^4$ may be so expanded, whence

$$\frac{\partial w^8}{\partial x^4\, \partial y^4} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} m^4 n^4 A_{mn} \sin m\pi \frac{x}{a} \sin n\pi \frac{y}{b} . \qquad (4)$$

By the properties of double Fourier series

$$\left| A_{mn} \right| < \frac{K}{m^5 n^5} ,$$

where K is a positive constant, and it is therefore legitimate to integrate the infinite series on the right of (4) term by term[10]. Carrying out this process eight times, a possible form for w is

$$\sum_1^{\infty} \sum_1^{\infty} A_{mn} \left\{ \sin m\pi \frac{x}{a} + t_m + p_m x + q_m x^2 + s_m x^3 \right\} \left\{ \sin n\pi \frac{y}{b} + h_n + e_n y + f_n y^2 + g_n y^3 \right\} \qquad (5)$$

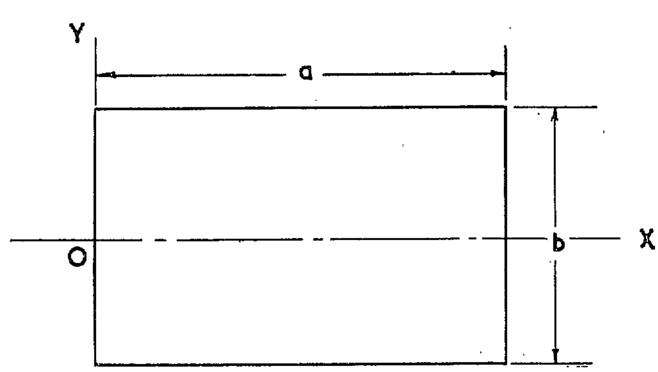
where $t_m \ldots g_n$ are arbitrary constants. Since each term in the series (5) is of the form $A_{mn} f_{mn}(x) \phi_{mn}(y)$ and f and ϕ each contain four arbitrary constants, these are uniquely determined by the edge conditions—two for each edge—and an expression has thus been found for w which is general, satisfies the appropriate boundary conditions, and is differentiable term by term.

The analysis when w is expressed in the form (5) and the constants are not all zero, is much more involved than when the expression (3) is applicable, but apart from greater complexity the procedure is unaltered.

Using this method, the critical buckling load for a square plate under shear, when one pair of opposite edges is clamped and the other pair is simply supported, has been worked out by Leggett (R. & M. 1991[2]), and the more general problem, including as a special case a square plate with all edges clamped, by Hopkins (R. & M. 2234[7]).

Other methods, suitable for dealing with particular cases when the edges are not all simply supported, have been developed by Taylor[11], Iguchi[12], Weinstein[13], Levy[14] and Leggett[15].

3. *The Post-Buckling Behaviour of a Long Flat Plate under Shear.**—3.1. Before considering the particular problem of a long flat plate under shear, an outline will be given of the method, developed by Prescott[16], for investigating the post-buckling behaviour of a flat rectangular plate of thickness $2h$ and flexural rigidity D due to loads acting in the plane of the plate and applied at its edges.



* A very considerable literature exists for the problem of the post-buckling behaviour of a long flat plate under compression.

Suppose that at any point in the middle surface of the plate u, v, w, are the co-ordinate displacements, ε_x, ε_y, γ_{xy} are the components of strain, and N_x, N_y, N_{xy} are the stress resultants, then the equations of equilibrium are

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 , \qquad (1)$$

$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0 , \qquad (2)$$

$$D\nabla^4 w = N_x \frac{\partial^2 w}{\partial x^2} + 2 N_{xy} \frac{\partial^2 w}{\partial x\, \partial y} + N_y \frac{\partial^2 w}{\partial y^2} , \qquad (3)$$

and the following relations hold between the displacements and the components of stress and strain:—

$$\varepsilon x = \frac{\partial u}{\partial x} + \tfrac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 , \qquad 2Eh\varepsilon_x = N_x - \sigma N_y , \qquad (4)$$

$$\varepsilon y = \frac{\partial v}{\partial y} + \tfrac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 , \qquad 2Eh\varepsilon_y = N_y - \sigma N_x , \qquad (5)$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} , \qquad 2Eh\gamma_{xy} = 2(1 + \sigma)N_{xy} . \qquad (6)$$

Introducing a stress function ϕ defined by

$$N_x = 2h \frac{\partial^2 \phi}{\partial y^2} , \qquad N_y = 2h \frac{\partial^2 \phi}{\partial x^2} , \qquad N_{xy} = -\,2h \frac{\partial^2 \phi}{\partial x\, \partial y} \qquad (7)$$

equations (1) and (2) are satisfied identically, and the problem reduces to that of solving the two von Kármán equations[17],

$$D\nabla^4 w = 2h\left\{ \frac{\partial^2 \phi}{\partial y^2}\frac{\partial^2 w}{\partial x^2} - 2\,\frac{\partial^2 \phi}{\partial x\, \partial y}\frac{\partial^2 w}{\partial x\, \partial y} + \frac{\partial^2 \phi}{\partial x^2}\frac{\partial^2 w}{\partial y^2} \right\} \qquad (8)$$

$$\nabla^4 \phi = E\left\{ \left(\frac{\partial^2 w}{\partial x\, \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} \right\} . \qquad (9)$$

Unfortunately, however, the only exact solutions of these two equations which exist at present are trivial, and for all cases of practical interest recourse is made to an approximate method which uses a strain energy condition in place of equation (8). The procedure now falls into three clearly defined stages. First a form is chosen for w which includes various parameters and is a good approximation to the type of distortion which is observed in practice. Next, the stress function ϕ is obtained from equation (9), after the form assumed for w has been substituted in the right-hand side of this equation, and the arbitrary functions of integration have been determined from the appropriate boundary conditions. Finally, w and ϕ are substituted in the expression for the strain energy of the plate, which can be expressed in the form

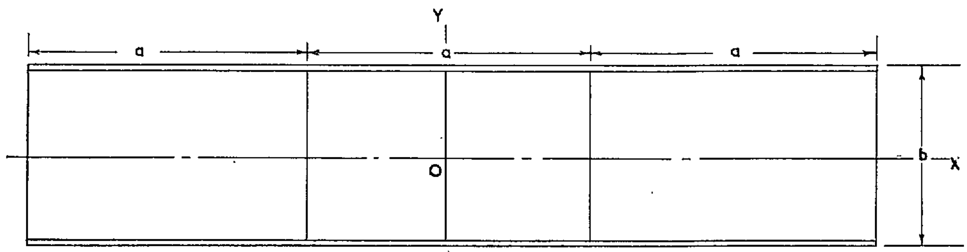
$$\frac{h}{E}\int_0^a\!\int_0^b \left\{ (\nabla^2\phi)^2 + 2(1+\sigma)\left[\left(\frac{\partial^2 \phi}{\partial x\, \partial y}\right)^2 - \frac{\partial^2 \phi}{\partial x^2}\frac{\partial^2 \phi}{\partial y^2}\right] \right\} dx\, dy$$

$$+ \frac{D}{2}\int_0^a\!\int_0^b \left\{ (\nabla^2 w)^2 + 2(1-\sigma)\left[\left(\frac{\partial^2 w}{\partial x\, \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right] \right\} dx\, dy , \qquad (10)$$

and the as yet undetermined parameters, *i.e.*, those contained in the expression assumed for w, are found by applying the static analogue of Kelvin's minimum energy theorem[18].

Thus described the method is standard; but its application to the case of a long flat plate (spar web), attached at its edges to heavy members (spars), corrected at regular intervals by light members (verticals), raises several points which require further consideration.

3.2.



This particular problem, that of the post-buckling behaviour of a spar web under shear, was first studied by Wagner[19], and later by Kromm and Marguerre[20]. The former propounded his now celebrated theory of tension fields. The latter, using the method outlined above, obtained a solution which was a considerable advance on Wagner's, expecially for applied shear loads which are only a few times the buckling load. At the same time this solution assumed that the edges are simply supported, whereas in actual fact the edges are more nearly clamped, no proper account is taken of the finite stiffness of the verticals, and the form assumed for w fails to satisfy the edge conditions even of simple support.

3.3. By representing the normal displacement of the plate w by the expression

$$w = A\left(1 + \cos\frac{2\pi y}{b}\right)\cos\lambda(y - mx)\,, \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (11)$$

where A, λ and m are constants, and assuming that the edges of the plate are attached to heavy members which are kept apart by verticals of *finite* stiffness, Leggett (R. & M. 2430[1]) has met most of the criticisms to which the solution of Kromm and Marguerre[20] is open. The condition of clamped edges is satisfied exactly, and the only assumption of importance which is not in accord with practice is that no account is taken of the stabilising effect of the verticals on the web. The finite stiffness of the verticals is allowed for by adding to the expression (10) a term representing the strain energy of the verticals, and by including among the boundary conditions an analytical expression of the fact that the pull of the web on the spars must equal the compressive load in the verticals.

Although not exact, Leggett's solution is in fair agreement with experimental work by Lahde and Wagner[21], Kuhn[22], and Crowther[23], so long as the applied shear load does not exceed five or six times the buckling load.

4. *The Initial and Post-Buckling Behaviour of a Curved Plate under Various Combinations of Shear, Compression, and Normal Pressure.*—Stated generally, this problem is of great complexity and only special cases have been solved at all accurately.

4.1. *Initial Buckling of a Complete Cylinder.*—For the special case of the initial buckling of a perfectly formed complete cylinder, accurate solutions have been obtained by Southwell[24] and Dean[25] if the cylinder is loaded in compression, and an approximate solution by Donnell[26] if the cylinder is loaded in torsion. In the latter case Donnell's solution has been extended by Hopkins and Brown (R. & M. 2423[8]) to include the stabilising effects of normal pressure.

4.2. *The Initial Buckling of Curved Plates.*—For the initial buckling of perfectly formed, curved plates in compression, approximate theoretical solutions have been developed by Timoshenko[27] and Redshaw[28]. But these solutions do not in general satisfy both the equations of equilibrium and the boundary conditions, and for more accurate solutions of cases in which the plate is long and the curvature small, recourse must be made to a method originally developed by Leggett[29] for finding the buckling stress of a long and slightly curved plate under shear.

The main points of this method are the expression of the normal displacement of any point in the middle surface of the plate in a form which is general, which satisfies the boundary conditions, and which can be differentiated term by term (*cf.* section 2.2); the derivation, from two of the three equations of equilibrium, of expressions for the tangential displacement of any point; and then, finally, the substitution of the expressions obtained for the normal and tangential displacement in the third equation of equilibrium. This last step gives rise to an infinite determinantal equation from which the critical buckling stress can be obtained by a process of successive approximation.

4.3. *Post-Buckling Behaviour of Curved Plates and Complete Cylinders.*—No accurate solutions yet exist for cases of post-buckling, although approximate solutions when the loading is compression have been obtained by von Kármán and Tsien[30] and by Leggett and Jones (R. & M. 2190[3]) for a complete cylinder, and by Levy[31] for a curved plate. In each case the methods employed are extensions of that outlined in section 3.1, although the presence of terms due to curvature render the analysis much more complicated. Von Kármán and Leggett both assume a form for the normal displacement which involves four parameters; but whereas von Kármán only uses the criterion of minimum strain energy for determining two of these parameters, and adopts an arbitrary and not very accurate procedure for determining the other two, Leggett adopts the laborious but more exact method of determining all four parameters from strain energy considerations. Levy's approach is, to begin with, more accurate, in that he expresses the displacements quite generally as Fourier series. But as the method proceeds, it becomes impossible to deal with more than a few terms of the series, with the result that in the end his solution also is only approximate.

5. *Conclusion.**—The methods which now exist for finding the stress at which a flat or perfectly formed curved plate will begin to buckle are sufficient for practical purposes. The same is not true, however, of the buckled state, and the great need now is the development of methods of analysis which can be applied with reasonable accuracy and without involving too much arithmetic to investigate the behaviour of curved plates and cylinders after they have buckled.

The effect of only very slight initial irregularities on reducing the stress at which a curved plate or cylinder first buckles is most striking, and it now seems clear that a satisfactory explanation of this phenomenon will only follow from a thorough understanding, physical as well as analytical of what happens after buckling.

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* Although much work has been done since the end of the war on the various types of problem referred to in this summary—*see* list of references to papers published since 1946—the remarks made in section 5 are still largely applicable.

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