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A Survey of Performance Reduction, with Particular Reference to Turbo-Propeller Aircraft

By

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A Survey of Performance Reduction, with Particular Reference to Turbo-Propeller Aircraft

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Summary.—Reasons for Enquiry.—Performance reduction methods will soon be required for routine tests of turbo-propeller aircraft. A survey of the types of methods available has therefore been made to find which type seemed likely to be most useful.

Scope of Enquiry.—The purpose of performance reduction is briefly examined. Methods in use are classified into experimental methods, which require no advance numerical data, and analytical methods, which require such data. The latter class is sub-divided into methods based on small corrections and methods based on performance analyses. The suitability of each class of method is discussed.

Conclusions.—Experimental methods are only practicable if any engine control linkage scheme is such as to impose dimensionally correct relations between the linked variables. They are convenient if data are required over a range of all variables or if, of the non-dimensional groups which result from dimensional analysis, all or most are susceptible to precise control. If such methods are practicable and reasonably convenient they are very attractive and probably the best to use on turbo-propeller aircraft, particularly at high altitude or Mach number, because of the lack of knowledge, as yet, of aircraft and engine characteristics under these conditions.

If experimental methods are impracticable or very inconvenient, analytical methods based on performance analyses are probably the best substitute, at least for tests at high altitude or high Mach number, until such time as numerical data on engine and airframe behaviour are available and can be easily used.

1. *Introduction.*—A beginning is now being made with performance tests on turbo-propeller aircraft, as distinct from engine development work. As performance reduction methods are required for interpretation of such tests it is necessary to select and develop suitable methods.

Performance reduction methods take, however, many very different superficial forms, as most have been developed *ad hoc* to meet particular needs. Before choosing a method it is, therefore, desirable to examine the purposes which reduction methods serve and the types of method available.

2. *Scope of Investigation.*—The purposes of performance reduction are discussed and the methods available classified. The merits and demerits of each type of method are discussed with particular reference to their application to turbo-propeller aircraft.

3. *The Problem.*—The performance of an aircraft is a function of a considerable number of independent variables. The number of these variables is sufficient to cause difficulty both in the description of the capabilities of the aircraft and their establishment by flight tests. In the description of the performance capabilities of the aircraft this difficulty is reduced by taking average values of those variables which are not in practice controllable.

Consider, for example, an aircraft with piston engine and propeller, for which the steady level speed will depend on engine speed, manifold pressure, air temperature and air pressure, and aircraft weight. In operational use the air pressure, engine speed and manifold pressure are

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freely variable within the working range. These are therefore retained as variables or parameters in the presentation of the results, the steady level speed being plotted against air pressure (height) at each of several representative combinations of engine speed and manifold pressure (*e.g.*, at the maxima for continuous weak mixture running and for combat). As regards air temperature it is not in general possible to obtain a selected value on any given occasion because the air temperature associated with a given air pressure varies from day to day. The practice is, therefore, to plot the performance against pressure (height) for a 'standard atmosphere'; in this standard atmosphere the air temperature associated with any given air pressure (or 'pressure height') is approximately the mean value for the conditions in which the aircraft is to operate. The aircraft weight as a variable is slightly different again, in that it bears some relation to the weight at take-off, which is controllable. It is therefore usual to quote steady speeds at some standard weight which is related to the take-off weight, one such standard weight is 95 per cent of the take-off weight, which is roughly the weight after climbing to operational altitude. Thus the practice is, in general, to quote performance over suitable ranges of those variables which can be controlled, and for specified 'standard' values of those variables which cannot be controlled.

If the test programme could be so arranged that the values of the uncontrolled variables were randomly distributed about the standard values their effect would appear as scatter and the required data could be obtained directly, given sufficient test results. Such a technique would, however, be very expensive in flying hours and would take many months. It is for general purposes quite impracticable. It is therefore necessary to have methods of deducing the performance under standard conditions from tests made under non-standard conditions. This process is known as performance reduction.

The information required from a particular series of tests may be anything from the steady level speed at one particular height, weight and engine rating to air speed and fuel flow data for the whole practical range of working conditions. The choice of a reduction method may depend on the type and extent of the data required and the flight test programme may depend on both; it may therefore be that no one method is superior in all circumstances.

4. *Methods of Performance Reduction.*—Methods of performance reduction may conveniently be divided into two classes, which we will refer to as the 'experimental' and the 'analytical' classes of method.

Experimental methods will be those (such as the 'non-dimensional' methods for turbo-jet aircraft) in which the performance under standard conditions is deduced directly from the flight tests on the particular aircraft, no numerical data being called up from other sources.

Analytical methods will be those in which numerical data from other sources are called up in deducing the performance under standard conditions from the tests under non-standard conditions.

5. *Experimental Methods.*—5.1. *General.*—We have observed (section 3) that the independent variables on which the performance of an aircraft depends may be *controllable* or *uncontrollable*. It is convenient to distinguish further between

(*a*) variables which can be adjusted in flight, but not exactly to any pre-determined value,

and (*b*) variables which can be adjusted in flight, with sufficient precision, to any pre-determined values in their working ranges.

For brevity type (*a*) will be referred to as *adjustable* and type (*b*) as *fully controllable*.

This distinction is of importance when results are required only at particular values of controllable variables. It is of the essence of an experimental method of reduction that the numerical data required are obtained directly from the performance tests, so if results are required at a particular value of any variable the tests must either be made at that value (as can be done with a fully controllable variable) or must cover a range about that value (as must be done with an adjustable variable). For example, in the extreme case when the performance is required at one particular value of each of n controllable variables a single test would give an answer if all the variables were fully controllable, whereas 2^n tests would be required if all were adjustable only. If results are required over a range of all variables the distinction between fully controllable and adjustable variables is not, of course, of much importance; it affects only ease of analysis and intrepretation of results.

Thus if a particular variable is adjustable but not fully controllable it may cause inconvenience, but does not make experimental methods impossible. The difficulty introduced by a completely uncontrollable variable such as air temperature is more fundamental. We have already (section 3) dismissed, as quite impracticable, so arranging the tests that the values of all uncontrolled variables were randomly distributed about their standard values. A conceivable alternative would be to make enough tests to give results over a range of each uncontrollable variable and deduce the standard performance by interpolation. This would avoid the requirement that the test values of the variables should be randomly distributed about the standard values, but a large test programme would still be required and the wait for the weather to produce a suitable range of air temperatures would be very objectionable except in fortuitously favourable circumstances. However, by recourse to dimensional analysis it is often possible to reduce the first difficulty, by reducing the number of variables involved, and to remove the second, by associating the uncontrolled variables with variables which can be controlled. This is the approach used in current experimental methods; let us consider first its application to jet aircraft, which is familiar, then its application to more complex cases.

5.2. *Application to Simple Jet Aircraft.*—It is usual to assume that the steady level speed (V) is a unique function of the engine speed (N), the aircraft weight (W) and the air temperature and pressure (θ and p).The air temperature is treated as an enthalpy, other quantities associated with temperature (such as viscosity) being neglected. Then we may write

$$f_1\,(V, N, W, \theta, p) = 0 \,. \qquad \qquad (1)$$

From this we may deduce, by dimensional analysis, that

$$f_2\left(\frac{V}{\sqrt{\theta}},\ \frac{N}{\sqrt{\theta}},\ \frac{W}{p}\right) = 0 \,. \qquad \qquad (2)$$

In equation (1) we had four independent variables, two of which (θ and W) could not be conveniently controlled. In equation (2), after the application of dimensional analysis, we have only two independent variables both of which can be controlled during any test in which N and p can be varied.

In this example dimensional analysis has reduced the number of independent variables by two and so made control of all independent variables possible. Dimensional analysis cannot reduce the number of independent variables in this case by more than two.

If, now, these independent variables ($N/\sqrt{\theta}$ and W/p) are fully controllable we can make level speed runs at the $N/\sqrt{\theta}$ and W/p which correspond to the values of N, θ, W and p at which the steady level speed is required and deduce $V/\sqrt{\theta}$ (and thence V) with very little trouble. In practice with jet aircraft it has been found that W/p can be fully controlled by adjustment of p, but full control of $N/\sqrt{\theta}$ has not usually been attempted because it is difficult to apply an accurate correction to the thermometer reading for the effects of forward speed while the tests are in progress. It is usual both for level speeds and climbs (for which a similar relation may be deduced) to make tests over a range of N sufficient* to cover the required range of $N/\sqrt{\theta}$ and deduce the answer by interpolation. Independent control of N and p is, of course, essential to this method.

5.3. *Application to More Complex Cases.*—In the above case we have the following features—

(*a*) dimensional analysis has reduced the number of independent variables by two, so making the technique practicable as this was the number of the original independent variables which could not be controlled.

(*b*) independent control of these two remaining independent variables is essential.

* An additional difficulty is that one cannot make a test on a hot day at the $N/\sqrt{\theta}$ corresponding to combat engine speed on a standard day without exceeding the engine limitations. This has caused surprisingly little trouble.

With these in mind let us now consider more complex engines. We will consider primarily the propeller turbine, but the discussion will also be applicable to any jet turbine engines which are more complicated than the simple jet engines at present in use.

First let us consider the engine without any control linkage system or automatic control (often fitted to give the pilot 'single lever' control). With most current engines there is then at least one additional independent variable relative to the simple jet engine; fuel flow or engine speed, for instance, may be variable independently of jet temperature. Dimensional analysis will still only reduce the number of independent variables by two, so all of the original independent variables except θ and W (which will always be out of control) must be variable by the operator. This condition will be met, by definition, for specifically engine variables such as N and θ_j if there is no control linkage or automatic control.

If the additional variables are fully controllable they present no additional difficulty, as tests can be made at the desired values of the associated non-dimensional quantities. If they are 'adjustable' only they may be a source of embarrassment; if so the embarrassment increases with the number of uncontrolled variables. To give a single level speed under standard conditions, for example, at least two, four or eight tests would be required if there were one, two or three uncontrolled variables respectively. If, on the other hand, full curves of performance against all variables were required no embarrassment would result from uncontrolled variables.

As an illustration of this type of case let us consider a turbo-propeller aircraft for which the steady level speed V is a function of N, W, θ and p and the jet-pipe temperature θ_j, so that we may write

$$f_3(V, N, W, \theta, p, \theta_j) = 0. \quad \ldots \quad (3)$$

After application of dimensional analysis we may write

$$f_4\left(\frac{V}{\sqrt{\theta}}, \frac{N}{\sqrt{\theta}}, \frac{W}{p}, \frac{N}{\sqrt{\theta_j}}\right) = 0. \quad \ldots \quad (4)$$

If, now, $N/\sqrt{\theta_j}$ (or any other convenient non-dimensional group containing the additional variable θ_j) is fully controllable, independently of $N/\sqrt{\theta}$, etc., the advent of the additional variable does not cause trouble. Tests must, however, be made over a range of this variable if it is adjustable only. If it cannot be varied independently of $N/\sqrt{\theta}$ and W/p experimental methods cannot be used unless its test value is exactly the required standard value fortuitously or because the controls have been so interconnected as to give this effect (*see* section 5.4). This is equivalent in our case to saying that θ_j must be varied independently of N and p; the pilot must have two independent engine controls.

5.4. *The Effect of Interconnection.*—We have seen, above, that on a turbine-propeller engine without any control linkage system, an experimental type of performance reduction method can be used, because the operator will be able to alter each of the independent variables (other than W and θ) independently of the remainder. On the other hand, most turbo-propeller engines at present proposed are to have control interconnection schemes, operated by a single pilot's lever, such that the pilot will not have independent control of these variables—and will not, for normal operational purposes, need it.

Such a linkage will impose relations between the independent engine variables which will reduce them to the one controllable by the pilot's lever. If these relations are dimensionally correct they will also reduce by the same number the independent non-dimensional groups obtained by applying dimensional analysis; it will then still be possible for the pilot to vary the remaining (two) independent non-dimensional groups separately and experimental methods of reduction will still be usable. If the relations are not dimensionally correct the pilot will be unable to vary all the independent groups separately, and as a result experimental methods will not be practicable.

Thus the practicability of experimental methods of performance reduction for engines with linked controls depends on the relation imposed by the linkage. This relation must be expressible in terms of non-dimensional groups similar to those thrown up by the dimensional analysis*. The following are examples of relations which might be used, with statements as to their suitability :—

General description	*Mathematical form*	*Suitability*
θ_j or Q (fuel flow) arbitrarily related to N	$\theta_j = f(N)$ or $Q = f(N)$	Unsuitable in general
θ_j proportional to square of N	$\dfrac{N}{\sqrt{\theta_j}} = \text{const}$	Suitable
Speeds of two turbines, not linked mechanically, in constant ratio	$\dfrac{N_1}{N_2} = \text{const}$	Suitable
Speed of power turbine held constant	$N_2 = \text{const}$	Unsuitable

5.5. *The Assumptions Required.*—Use of a purely experimental method of performance reduction is only practicable if the number of independent variables is small. As there are usually many independent variables which affect the performance to some extent it is necessary, if such a method is to be used, to be able to assume that only a few of these are of importance.

For example, it is usual to assume that the level speed and the associated mass fuel flow of a simple jet aircraft are a function of engine speed, aircraft weight, air pressure and air temperature (considered as an enthalpy) only. This implies that the following quantities, among others, can be omitted from consideration as independent variables :—

(*a*) the viscosity of the air,

(*b*) the temperature rise of the working fluid per unit mass of fuel injected.

This implication is probably correct when the test and standard conditions do not differ greatly, provided that the aircraft and its engine are not near a 'critical Reynolds number' or in a region where combustion efficiency is sensitive to working conditions, and provided that the working fluid is not changed (*e.g.*, by dilution with water or ammonia). If the test programme is so arranged that tests at a given $N/\sqrt{\theta}$ and W/p are made sometimes at a high weight (and low altitude) and sometimes at a low weight (and high altitude) any weaknesses in these assumptions can be detected and precautions taken. Large corrections should be avoided if possible.

Remarks similar to those on the above example apply to the propeller turbine. It will be convenient, indeed almost essential, to be able to assume that items (*a*) and (*b*) above can be omitted, together with the flexibility of the propeller.

6. *Analytical Methods.*—6.1. *General.*—We have classified as analytical all methods of performance reduction which require numerical data from sources other than the flight tests on the particular aircraft. Within this class there is a wide range of methods, and the choice of the type of method to be used will be influenced by the information required from the tests and by the data available about the characteristics of engine, propeller and airframe.

It is convenient to divide the methods into two sub-classes and to examine the merits and demerits of each sub-class of method with its associated flight test technique; further advance into detail is inappropriate to the present discussion. Methods of the first sub-class are essentially methods involving fairly small corrections. One makes performance tests under conditions approximating as closely to standard as is practicable and uses a reduction method to adjust individual results for the relatively small discrepancy between test and standard conditions.

* The description 'non-dimensional' is here applied to terms whose dimensions can be referred to standards from within the problem. For example W/p (equation (2), etc.) can be compared with some area, such as the wing area, associated with the aircraft.

Methods of the second sub-class involve a performance analysis, the test programme being designed to provide data suitable for analysis rather than to give test results under near-standard conditions.

6.2. *Application of the Methods.*—To estimate the corrections required for the first type of method one usually linearises the relations between the variables and estimates the slopes by a theoretical adaptation of data from other sources, preferably some fund of generalised data. In other words, one must have prior information about the response of the performance to changes in air temperature and weight, but the information need not be very precise as it is only used to estimate relatively small corrections. If such information is available and easy to use* this type of method can be very convenient. It need impose no restriction on the test programme. It does not, however, provide any internal check of the accuracy of the assumed data, nor does it provide drag data as part of the reduction process.

To use the second type of method one requires prior information about the performance of the propulsive unit and its response to changes in air temperature but not about the drag characteristics of the airframe, as drag data are obtained during the analysis. Observed or estimated jet thrusts and shaft powers combined with estimated propeller efficiencies give estimated drag or power curves†. Estimates of the thrust or thrust power which would be available under standard conditions then lead to the standard performance. A comparison of the drag curves obtained at various altitudes gives some check of the assumed performance of the propeller, etc.

The test programme should, for methods of this type, cover a fairly wide range of air speed at one altitude at least, This assists with the estimation of the effect on air speed of changes in thrust at constant altitude. The test programme should be designed primarily to provide the required drag data.

Such methods may be more convenient than the first type if the drag data required for the latter are not available, or if the drag and thrust data cannot be put into an easily used form. Also, if a performance analysis is to be made anyway, it may be more economical to merge the performance reduction with this analysis and to design the test programme accordingly. Again, if the first type of method is not much easier to use than the second it may be considered worth while to go to a little more trouble to get drag data *en passant*, even though these are not a primary requirement.

6.3. *Control Systems.*—It is necessary with either type of analytical method to be able to estimate how the thrust which the power plant under test gives varies with air temperature‡. This requires a knowledge of the behaviour not only of the basic power plant but also of its control system. If, for instance, the control system is such that the fuel flow at a given engine speed and intake pressure is independent of air temperature the jet temperature would probably increase with air temperature; the engine output might then be limited by jet temperature at high air temperatures and by engine speed at low air temperatures. On the other hand, if the control system kept constant the jet temperature associated with a given engine speed this change of regime would not occur, as the same limitation would always be the determining one. A similar change of regime can occur near the full throttle height of a piston engine. It need not be troublesome.

7. *Discussion.*—At the present stage of development of turbo-propeller aircraft it is not easy to generalise about the performance of engines and propellers, particularly at high altitude, or about the drag of the airframe at high Mach numbers. This fact will exert a strong influence on the choice of reduction methods for use in the near future.

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* In the method used for piston-engined aircraft one engine parameter (c) and two airframe parameters (C_{Dz} and eb^2) suffice to adapt generalised formulae to the particular aircraft.

† For instance, curves of (thrust power) $\times \sqrt{\sigma}/W^{1\cdot5}$ may be plotted against $V_i/\sqrt{W}$ for various values of W/p.

‡ The aim is to show what the performance of the aircraft under test would be under standard conditions. It is not, for instance, to show what the performance would be if the engine did what its designer intended it to do. One will therefore need either to deduce the engine performance under standard conditions from its observed performance under test conditions, or to estimate its performance under test conditions from power charts drawn up for standard conditions.

We have seen that experimental methods of performance reduction exist which require no prior numerical data for engine or airframe. We have also seen that such methods are

(*a*) practicable only if any automatic control or control linkage imposes only dimensionally correct relationships,

and (*b*) convenient if data are required over a range of all variables or if enough of the non-dimensional groups used in the reduction process are fully controllable.

If these two conditions are met the potential advantage of the tolerance shown by such methods towards ignorance of the aircraft is very great, especially when the test conditions or the equipment is near the border of one's knowledge and experience. This potential advantage is probably decisive for turbine engined aircraft at present, especially at high altitude or Mach number.

If, however, methods of this class are impracticable, or even very inconvenient, one is forced to consider the use of analytical methods, all of which require prior numerical knowledge about the engine and propeller including the behaviour of any control system. The type of method based on making small corrections also requires drag data, and is only convenient if this and the engine data can be conveniently used. This is unlikely to be the case with turbo-propeller aircraft in the near future, except perhaps at low altitude and low Mach numbers. Methods based on performance analyses are probably the best substitute, at present, for experimental methods. This is because if small correction methods are troublesome, one may as well make the performance analysis and obtain drag data and, possibly, an internal check on the assumed data.

It is possible to invent methods which are partly experimental and partly of the small correction analytical type. For example, in the case of equation (4) one might perhaps correct to the standard $N/\sqrt{\theta}$ analytically but deal with $N/\sqrt{\theta_j}$ experimentally, by making tests at the right value of this variable. For such methods the remarks on experimental and analytical methods apply to the respective parts of the reduction process.

8. *Conclusions.*—Experimental methods of performance reduction are, in the present state of knowledge, the most suitable for use with turbo-propeller aircraft provided that the engine control systems are suitable* and that precise control of enough of the non-dimensional groups, thrown up by dimensional analysis, is possible.

If these conditions are not met methods of reduction based on performance analysis are, at present, probably the best substitute.

* That is, that they impose only dimensionally correct relations.

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