N.A.E.

R. & M. No. 2729
(11,885)
A.R.C. Technical Report

MINISTRY OF SUPPLY

AERONAUTICAL RESEARCH COUNCIL
REPORTS AND MEMORANDA

The Diffusion of Load into a Panel Bounded by Constant Stress Booms and a Transverse Beam

By

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LONDON: HER MAJESTY'S STATIONERY OFFICE
1953
PRICE 3s. 6*d*. NET

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E. H. Mansfield, M.A.

Communicated by the Principal Director of Scientific Research (Air),
Ministry of Supply

*Reports and Memoranda No. 2729**
August, 1948

Summary.—A theoretical investigation is made into the diffusion of symmetrical, concentrated loads into a long stiffened panel having constant stress edge members and a transverse loading beam.

Both pin-jointed and clamped end conditions for the beam are considered. Curves are given for determining the peak shear stress near the boom, the variation of this shear stress along the length of the panel, the proportion of load transferred by the beam, and the bending moment at the ends of the beam.

1. *Introduction.*—It has been shown (R. & M. 2663)[1] that, in the diffusion of end load into a panel lying between parallel booms, the shear stresses in the panel are considerably reduced in the presence of a transverse beam attached to the free edge of the sheet and with its ends fastened to the main booms.

Booms of constant cross-section were considered in R. & M. 2663[1], but in practice booms are usually designed for constant stress, or nearly so. The purpose of this report is to consider the effect of a transverse beam when the booms are tapered for a constant stress, and in particular to determine the taper required and the stiffness of the cross-beam for efficient diffusion.

By combining these results with those of R. & M. 2663[1], it is also possible to obtain a reasonable estimate of the stresses in the panel for a structure in which the booms have some taper between zero and that giving constant stress.

2. *Statement of Problem and Assumptions.*—The problem is to determine the stress distribution in a long, flat, rectangular, stiffened panel which is bounded on its longer edges by constant stress booms to which are applied equal end loads. A uniform beam is attached to the free edge of the skin and the ends of the beam are either pin-jointed or clamped to the booms.

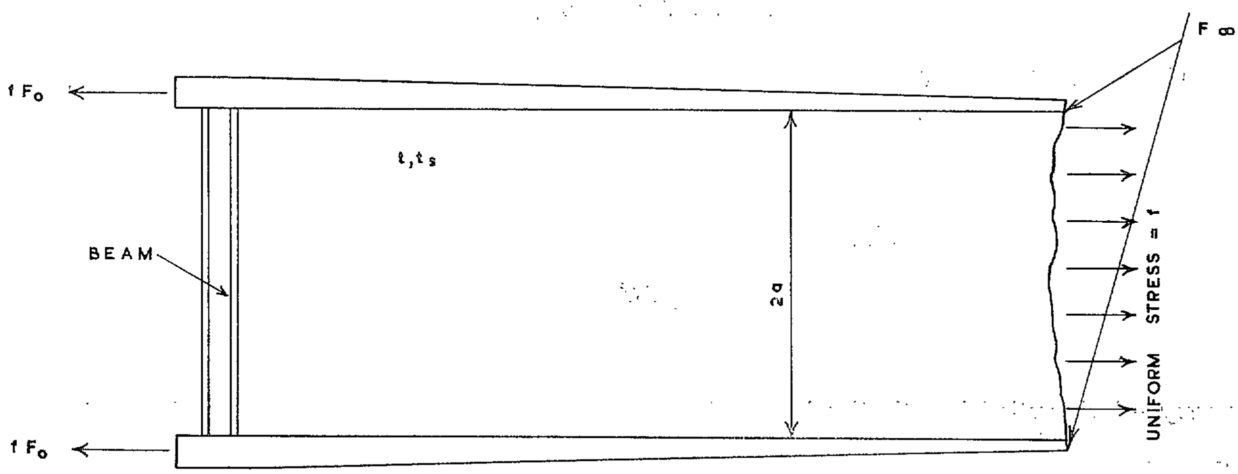
Simple engineering theory is assumed for the beam, and its deflection due to shear is neglected. The stringer-sheet method (R. & M. 2663,[1] 2618[2] and 2670[3]) is used to determine the panel stresses.

3. *Description of Results.*—The complete stress distribution is shown to depend primarily on one non-dimensional parameter $\beta = kI/a^3t_s$, and it is therefore possible in presenting the results to include all practical values of structure dimensions.

* R.A.E. Report Structures 31, received 3rd November, 1948.

Another parameter would be introduced if the shear flexibility of the beam were included, but as this would increase the computation considerably and as, moreover, it has been shown[1] (R. & M. 2663) that this effect is not of great importance, it has been decided to ignore the shear flexibility.

3.1. *Boom Taper Consistent with Constant Stress Booms*



If the boom area at some considerable distance from the root, where the stresses in the sheet have become uniform, is F_∞, the total direct load at that section will be $2f(F_\infty + at_s)$. This must also be the total load at any other section, and in particular at the root we have:—

$$2f(F_\infty + at_s) = 2fF_0$$

or

$$F_0 - F_\infty = at_s.$$

The variation of flange area between the two end values of F_0 and F_∞ is shown in Figs. 1 and 2 for various values of the parameter β. It is evident that this variation is independent of F_∞: for if we increase the boom area everywhere by an amount δF, say, and increase the total load by $2f\delta F$ the stress in the modified boom remains unaltered. This means that the boundary conditions for the panel are unchanged although F_∞ is different.

The beam itself transfers part of the load to the panel and in order to keep the boom stress constant there must therefore be a sudden decrease in boom area from one side of the beam to the other.

3.2. *Shear Stress Adjacent to the Booms.*—A family of curves showing the distribution of shear stress adjacent to the booms for various values of β is shown in Fig. 3. The beam is assumed to be clamped at its ends, which implies that there will be no rotation between beam and boom, so that the shear stress adjacent to the boom will be zero initially.

It will be noticed that the shear stress rises rapidly to a maximum value and then dies away in much the same way as if there had been no beam.*

The shear stress in the less practical case in which the beam is pin-jointed at its ends is shown in Fig. 4.

* The following is a simple closed expression for the shear stress when there is no beam:—

$$q = \frac{4}{\pi} f\bar{k} \coth^{-1} \{e^{x/2ak}\}$$

3.3. *Peak Value of Shear Stress Adjacent to Booms.*—The value of the peak shear stress adjacent to the booms is of practical importance and may be obtained from Fig. 5 where both clamped and pin-jointed end conditions for the beam are considered. If the beam is pin-jointed its stiffness must be very high for the shear stress to be kept within reasonable limits, but if β is not less than $0 \cdot 002$ clamping the beam reduces the peak shear stress to $0 \cdot 8\ \bar{k}f$ (probably a safe value, depending on the magnitude of $\bar{k}$).

3.4. *End Load Transferred by the Beam.*—Owing to the stiffness of the beam a proportion of the load which the sheet and stringers will eventually take is transferred immediately by the beam. This proportion is plotted in Fig. 6 where it is seen that if, for example, β is equal to $0 \cdot 002$ and the beam is clamped, 32 per cent of the panel load (*i.e.*, $2fat_s$) is transferred to it by way of the beam. As the total load is equal to $2f(F_\infty + at_s)$ the load transferred in this case by the beam will be $32/(1 + F_\infty/at_s)$ per cent of the total load. If the beam is pin-jointed, however, only $9 \cdot 7$ per cent of the panel load is transferred by way of the beam for the same value of β.

3.5. *Bending Moment at Ends of Beam.*—It has been shown that the clamped beam reduces the peak shear stress in the sheet to a much greater extent than the pin-jointed beam. This clamping has the disadvantage however of causing a relatively high bending moment at the ends of the beam. This bending moment, expressed as a fraction of fa^2t_s, is shown in Fig. 7, where it will be seen that over the practical range of β this fraction varies approximately as $\beta^{5/9}$.

This means that the bending moment varies as $a^{1/3}$ (with other dimensions unchanged), and for beams of similar cross-section the bending stress in the beam varies inversely as (linear dimension of beam section)$^{7/9}$.

4.1. *Example* 1—*A Design Problem.*—A panel has the following dimensions:

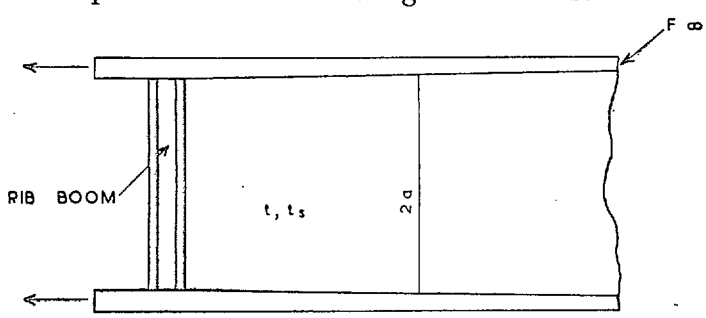
$$2a = 30 \text{ in.}$$

$$t = 0 \cdot 05 \text{ in.}$$

$$t_s = 0 \cdot 08 \text{ in.}$$

$$F_\infty = 1 \cdot 0 \text{ in.}^2$$

$$G/E = 0 \cdot 4$$



There is a rib-boom of approximate area $0 \cdot 7$ in.^{2} which is built-in to the booms and may be regarded as a beam.

Required to find moment of inertia of beam so that peak shear stress in the panel will not exceed 13 ton/in.^{2} and the boom area and taper so that there will be a uniform stress in the booms of 20 ton/in.^{2}. The total end load is 88 tons.

k and $\bar{k}$ are equal to $2 \cdot 0$ and $0 \cdot 8$ respectively, and hence $\dfrac{q_{\max}}{\bar{k}f} = \dfrac{13}{0 \cdot 8 \times 20} = 0 \cdot 81.$

This implies, from Fig. 5, that $\beta (= kI/a^3t_s) \nless 0 \cdot 002$, giving

$$I \nless 0 \cdot 002 a^3 t_s / k$$

$$\nless \underline{0 \cdot 027 \text{ in.}^4}$$

This value of β determines the bending moment at the ends of the beam, and from Fig. 7 we have

$$\text{bending moment} = 0 \cdot 0375 fa^2 t_s$$

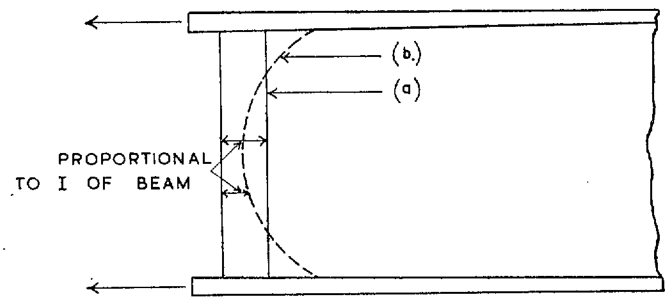
$$= 13 \cdot 5 \text{ ton in.}$$

If we limit the maximum stress in the beam to 20 ton/in.^{2} the permissible depth of the beam may be found from ordinary engineering theory:

$$\text{total beam depth} = \frac{2 \times 20 \times 0 \cdot 27}{13 \cdot 5}$$

$$= 0 \cdot 8 \text{ in.}$$

If we take $I = 0 \cdot 27$ in.^{4} this restriction on the beam depth necessitates a beam width of at least 6 in., which is clearly out of the question. But suppose that we have a varying rib-boom moment of inertia, such as that depicted by the broken line below:

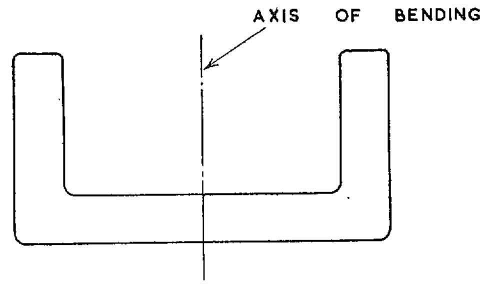


(a) = constant beam section scheme

(b) = variable beam section scheme

From physical considerations the bending moment at the ends for case (b) will not be appreciably different from case (a). (In fact there are many forms for case (b) which leave the end moments unaltered.)

There is now greater freedom in the design of the rib-boom. A suitable cross-section for the beam at its ends is that drawn full scale below:—



which has a section area of 1 in.^{2}, a depth of 2 in. and a moment of inertia of $0 \cdot 67$ in.^{4} (*cf.* $0 \cdot 27$ in.^{4}); the greatest direct stress due to the $13 \cdot 5$ ton in. bending moment is again 20 ton/in.^{2}.

It is suggested that the beam should be tapered down to the value of $0 \cdot 27$ in.^{4} at a distance of about $\frac{1}{6}$ panel width, *i.e.*, 5 in. from each boom, and then tapered down to some smaller value, say $0 \cdot 17$ in.^{4} at the centre of the panel. This tapering of the beam may be conveniently done by machining off the requisite amount of beam flange. In the present example this would imply that at the mid-point of the panel the beam would consist of the original beam web alone, and it would therefore have an area of $0 \cdot 5$ in.^{2}.

The semi-load transferred by the beam to the panel is, from Fig. 6, equal to

$$0 \cdot 325 f a t_s = 7 \cdot 8 \text{ tons.}$$

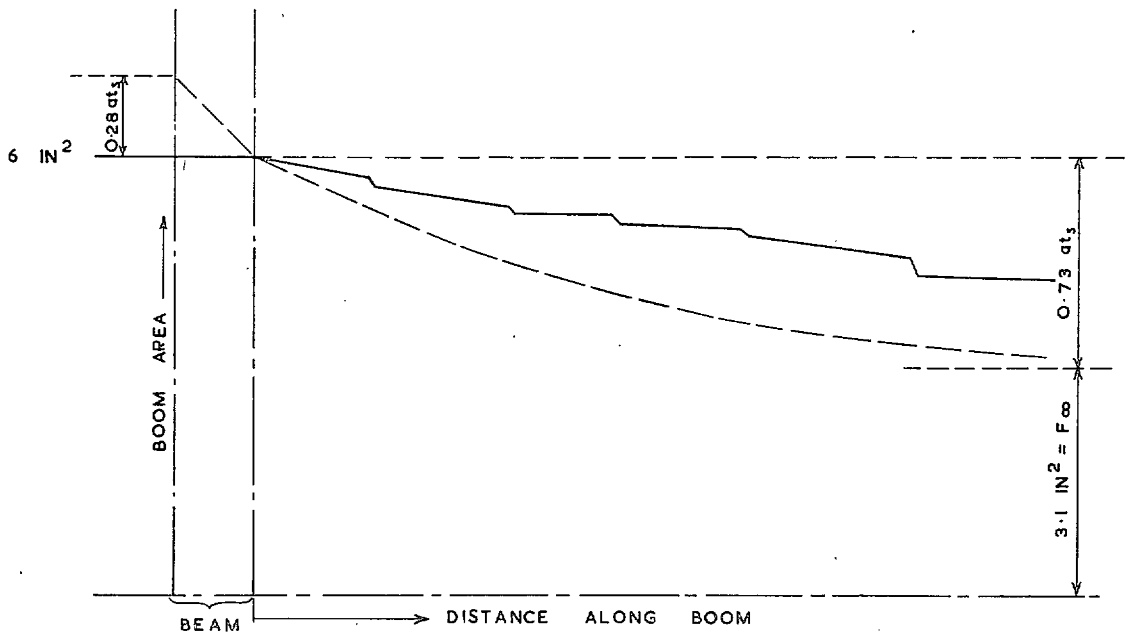
The boom area at the root is $(F_\infty + a t_s) = 2 \cdot 2$ in.^{2} and has a value just inboard of the beam (*see* Fig. 1) of

$$F_\infty + 0 \cdot 66 a t_s = 1 \cdot 8 \text{ in.}^2$$

The variation of boom area is completely determined from Fig. 1; for example, at a distance from the beam of $0 \cdot 4ak$, *i.e.*, 12 in., the boom area is

$$F_{\infty} + 0 \cdot 41 at_s = 1 \cdot 49 \text{ in.}^2$$

4.2. *Example 2—Stress Determination.*—The full line in the diagram below represents the variation of boom cross-section in a particular design. The boom area at the root is 6 in.^{2}, $at_s = 4$ in.^{2} and $\beta = 10^{-3}$. It is required to find the peak shear stress in the panel in terms of the direct stress at the root f_0.



The top broken curve corresponds to the case of constant boom area treated in R. & M. 2663[1]. The peak shear stress is $0 \cdot 6 f_0 \bar{k}$.

The bottom broken curve represents a constant stress boom corresponding to the same value of β and with F_{∞} chosen to make the boom area 6 in.^{2} inboard of the beam. For this case the peak shear stress is $0 \cdot 9\ \bar{k} f$, which may be written as

$$0 \cdot 9 \bar{k} f_0 \left(\frac{6}{6 + 1 \cdot 12}\right) = 0 \cdot 76 f_0 \bar{k}.$$

It is obvious that the actual peak stress will be about half-way between these two values, say

$$q_{\text{max.}} = 0 \cdot 68 f_0 \bar{k}.$$

5. *Conclusions.*—The diffusion of symmetrical, concentrated loads into a long stiffened panel bounded by constant stress booms and a transverse beam has been examined theoretically.

Information relating to the loads acting on the beam and the shear stresses adjacent to the booms is given in graphical form. Using these graphs it is possible to design such a diffusion structure with reasonable efficiency; and by combining the results presented here with those of R. & M. 2663[1] a fair estimate may be obtained of the stresses in a panel bounded by booms with any degree of taper.

If the ends of the beam are clamped to the booms the peak shear stress adjacent to the booms is considerably smaller than when the beam has its ends pin-jointed. For the clamped case, however, there is a large bending moment at the ends of the beam which necessitates additional strength in this region.

LIST OF SYMBOLS

$2a$	Width of panel
t	Thickness of sheet
t_s	Thickness of stringer-sheet
F	Cross-sectional area of each boom, $= F(x)$
F_0	Cross-sectional area of each boom at root
F_∞	Cross-sectional area of each boom at a considerable distance from the root
I	Moment of inertia of beam (bending about a line perpendicular to plane of sheet)
E, G	Elastic moduli
f	Direct stress in booms
k	$= (Et_s/Gt)^{1/2}$ — non-dimensional parameters
$\bar{k}$	$= (Gt_s/Et)^{1/2}$ — non-dimensional parameters
β	$= kI/a^3t_s$ — non-dimensional parameters

Additional symbols used only in Appendix

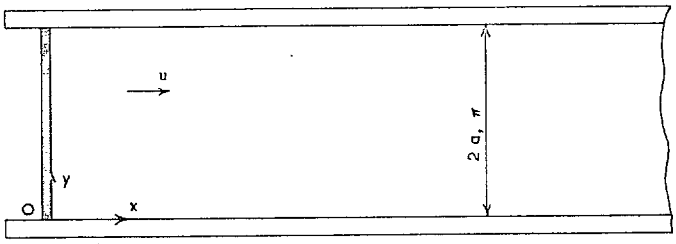
Ox, Oy	Orthogonal axes, x measured along the panel
u	Displacement in x direction
u_b	Displacement of beam in x direction
σ, τ	Direct and shear stresses in panel
n	Positive integer
Σ	Summation for n. (It is shown that *odd* values only of n are needed).

Other symbols introduced where necessary.

REFERENCES

No.	*Author*	*Title, etc.*
1	E. H. Mansfield	Effect of Spanwise Rib-boom Stiffness on Stress Distribution near a Wing Cut-out. R. & M. 2663. December, 1947.
2	M. Fine and H. G. Hopkins ..	Stress Diffusion Adjacent to Gaps in the Interspar Skin of a Stressed-skin Wing. R. & M. 2618. May, 1942.
3	E. H. Mansfield	Diffusion of Load into a Semi-infinite Sheet. Part II. R. & M. 2670. June, 1948.

APPENDIX I



Let f be the stress at a section where the stress distribution has become steady; then a suitable form for the displacement u will be given by

$$\frac{E}{f}\frac{\partial u}{\partial x} = \frac{\sigma}{f} = 1 - \Sigma D_n \sin (n\pi y/2a) \exp (- n\pi x/2ak) \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

Equation (1) satisfies the differential equation for u:

$$\frac{\partial^2 u}{\partial x^2} + \frac{1}{k^2}\frac{\partial^2 u}{\partial y^2} = 0$$

where $k^2 = Et_s/Gt$,

and the conditions at $y = 0, 2a$ (where $\sigma_x = f$) and the conditions a long way from the root (where the stresses are steady).

Integrating (1) gives

$$\frac{E}{f} u = x + \frac{2ak}{\pi} \Sigma \frac{D_n}{n} \sin (n\pi y/2a) \exp (- n\pi x/2ak) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

+ a function of y alone, which will be zero by virtue of the end conditions.

Considering the beam in bending we may write:

$$\frac{\partial^4 u_b}{\partial y^4} = \frac{\sigma t_s}{EI} \; at \; x = 0$$

$$= \frac{t_s}{I}\frac{\partial u}{\partial x} \text{ at } x = 0$$

$$= \frac{ft_s}{EI}\left\{1 - \Sigma D_n \sin (n\pi y/2a)\right\} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

Integrating equation (3) and using the condition of similarity of the displacements about the centre-line of the panel and the fact that u is zero at the two ends gives:

$$u_b = \frac{ft_s}{EI}\left(\frac{(y-a)^4 - a^4}{24} + Ca^2y(2a - y) - \frac{16a^4}{\pi^4} \Sigma \frac{D_n}{n^4} \sin (n\pi y/2a)\right) \quad \ldots \quad \ldots \quad (4)$$

where C is a constant dependent on the degree of end fixity of the beam, and n has all odd positive values.

Also from eqn. (2)

$$u_b = \frac{2akf}{E\pi} \Sigma \frac{D_n}{n} \sin (n\pi y/2a), \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

since the deflection of the beam is the same as the deflection of the end of the panel.

Equating equations (4) and (5) gives an equation for determining the D_n.

Now,

$$a^2y(2a - y) = \frac{32}{\pi^3} a^4 \sum_{n \text{ odd}} \frac{\sin (n\pi y/2a)}{n^3}$$

and

$$(y - a)^4 - a^4 = \frac{192a^4}{\pi^3} \sum_{n \text{ odd}} \left(\frac{8}{(n\pi)^2} - 1\right) \frac{\sin (n\pi y/2a)}{n^3} \quad \ldots \quad (6)$$

Equating the coefficients of sin $(n\pi y/2a)$ in equations (4) and (5) therefore gives:

$$\frac{ft_s}{EI}\left\{\frac{8a^4}{\pi^3 n^3}\left(\frac{8}{n^2\pi^2} - 1\right) + \frac{32a^4C}{\pi^3n^3} - \frac{16a^4D_n}{\pi^4n^4}\right\} = \frac{2akfD_n}{E\pi n} \quad \ldots \quad (7)$$

Re-arranging, and writing $\beta = \dfrac{kI}{a^3t_s}$,:

$$D_n = \frac{4\pi n}{(8 + \beta\pi^3n^3)}\left(4C - 1 + \frac{8}{n^2\pi^2}\right) \quad \ldots \quad (8)$$

Determination of C

For a clamped beam

$$\frac{\partial u_b}{\partial y} = 0 \text{ at } y = 0,\ 2a$$

and hence

$$C = \frac{1}{12} + \frac{4}{\pi^3} \sum \frac{D_n}{n^3} \quad \ldots \quad (9)$$

And for a pin-jointed beam

$$\frac{\partial^2 u_b}{\partial y^2} = 0 \text{ at } y = 0,\ 2a$$

and hence $C = 1/4$ .. .. (10)

Confining our attention to the clamped case, and substituting in (8) and simplifying gives

$$D_n = \left(\frac{4n/\pi}{1 + \beta n^3\pi^3/8}\right)\left(\frac{1}{n^2} - \frac{S}{T}\right) \quad \ldots \quad (11)$$

where

$$S = \sum \frac{1}{n(1 + \beta n^3\pi^3/8)}\ ,$$

$$T = \sum \frac{n}{(1 + \beta n^3\pi^3/8)}\ , \quad \ldots \quad (12)$$

For the simply supported case:

$$D_n = \frac{4}{n\pi(1 + \beta n^3 \pi^3/8)} \quad \ldots \quad (13)$$

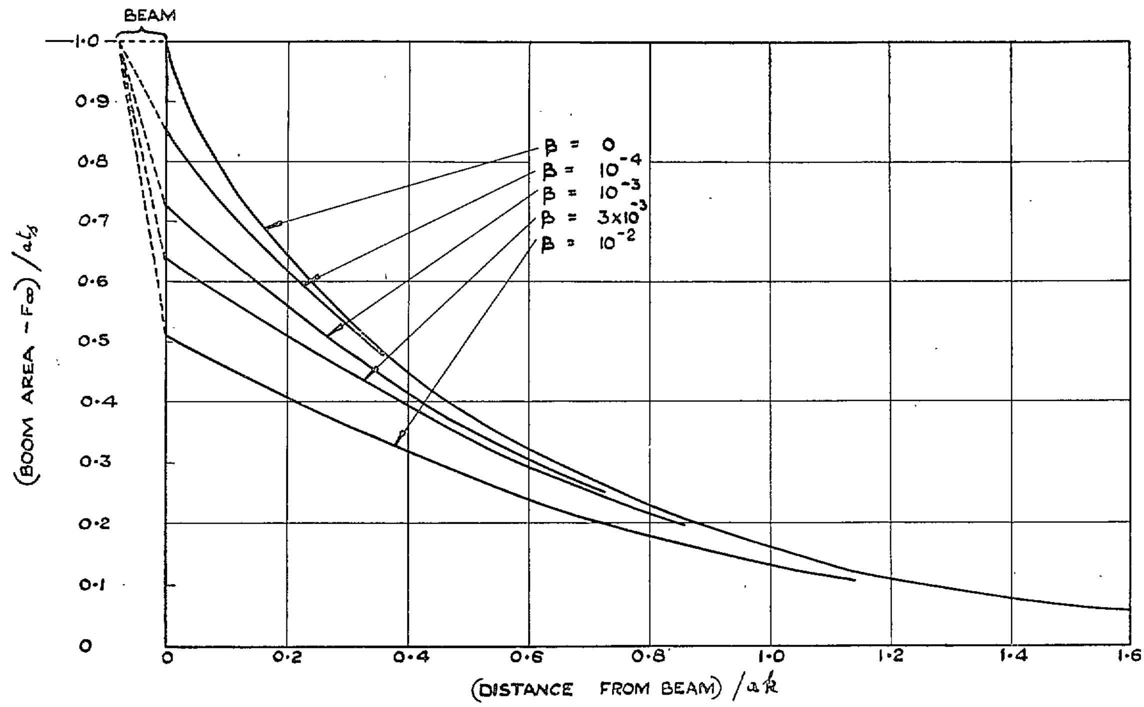


FIG. 1. Boom taper consistent with constant stress booms (beam clamped).

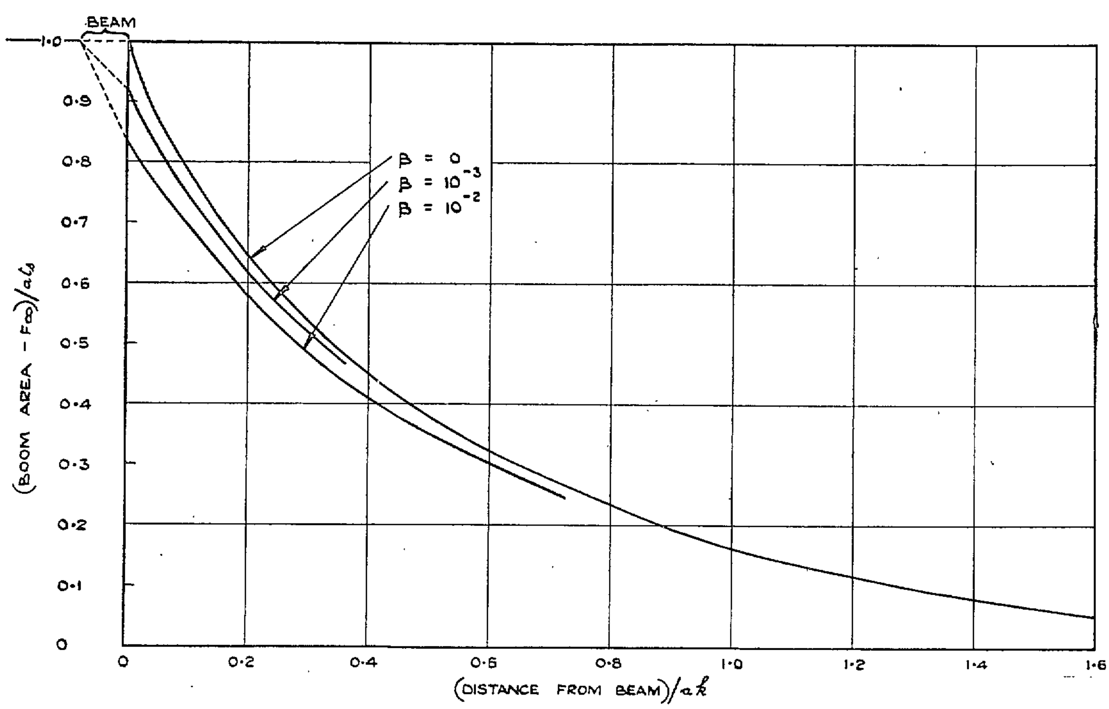


FIG. 2. Boom taper consistent with constant stress booms (beam pin-jointed).

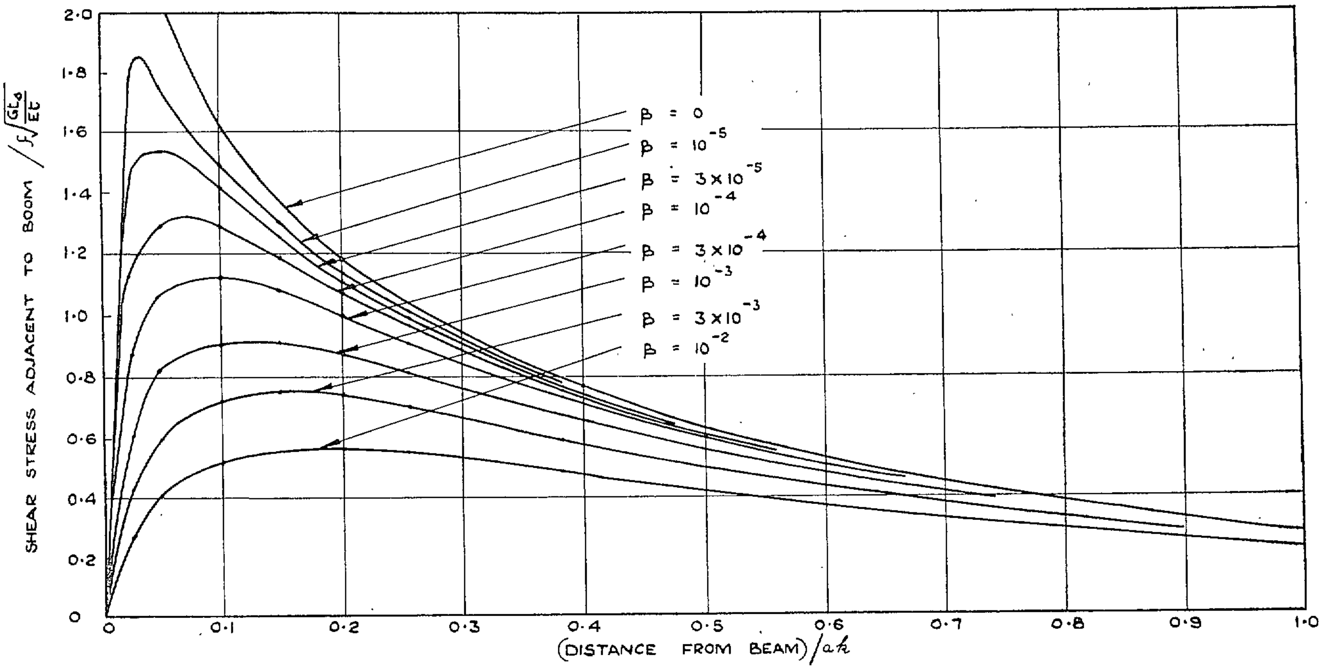


FIG. 3. Shear stress adjacent to boom (beam clamped).

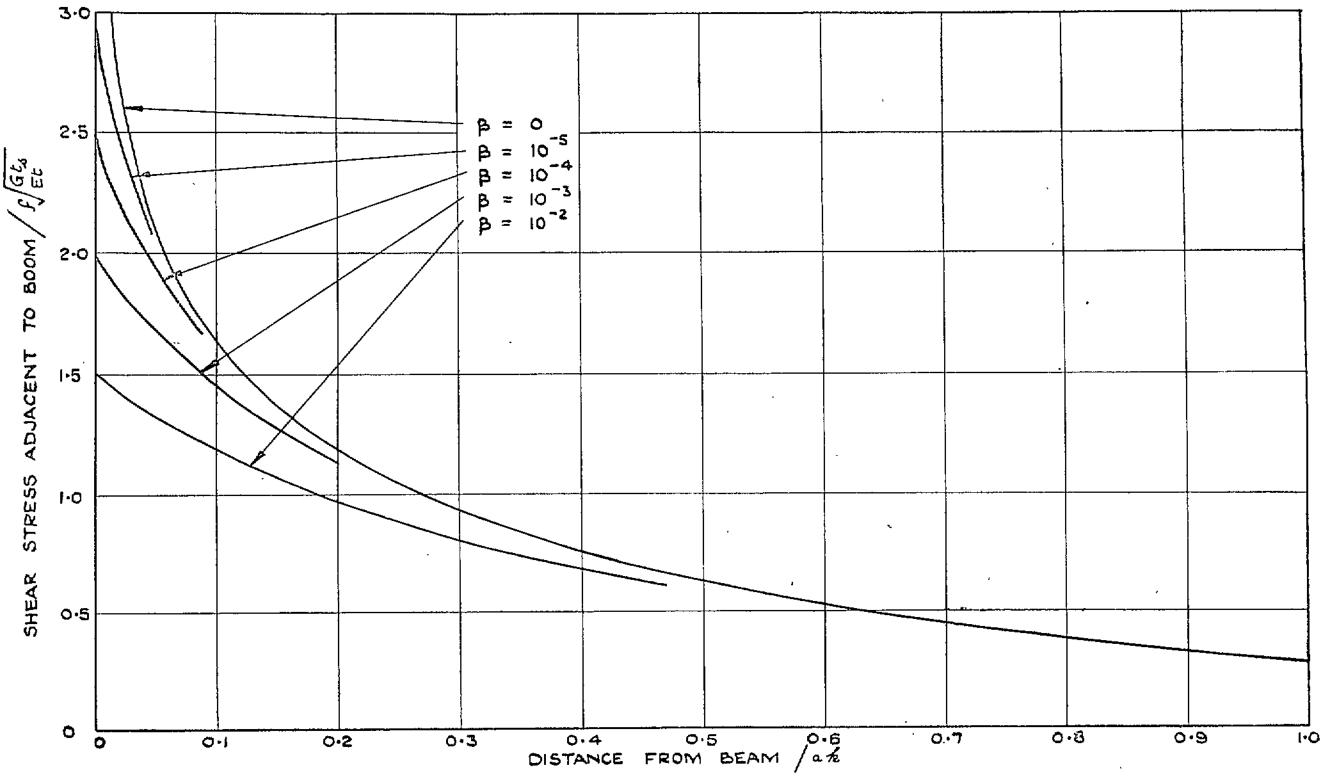


FIG. 4. Shear stress adjacent to boom (beam pin-jointed).

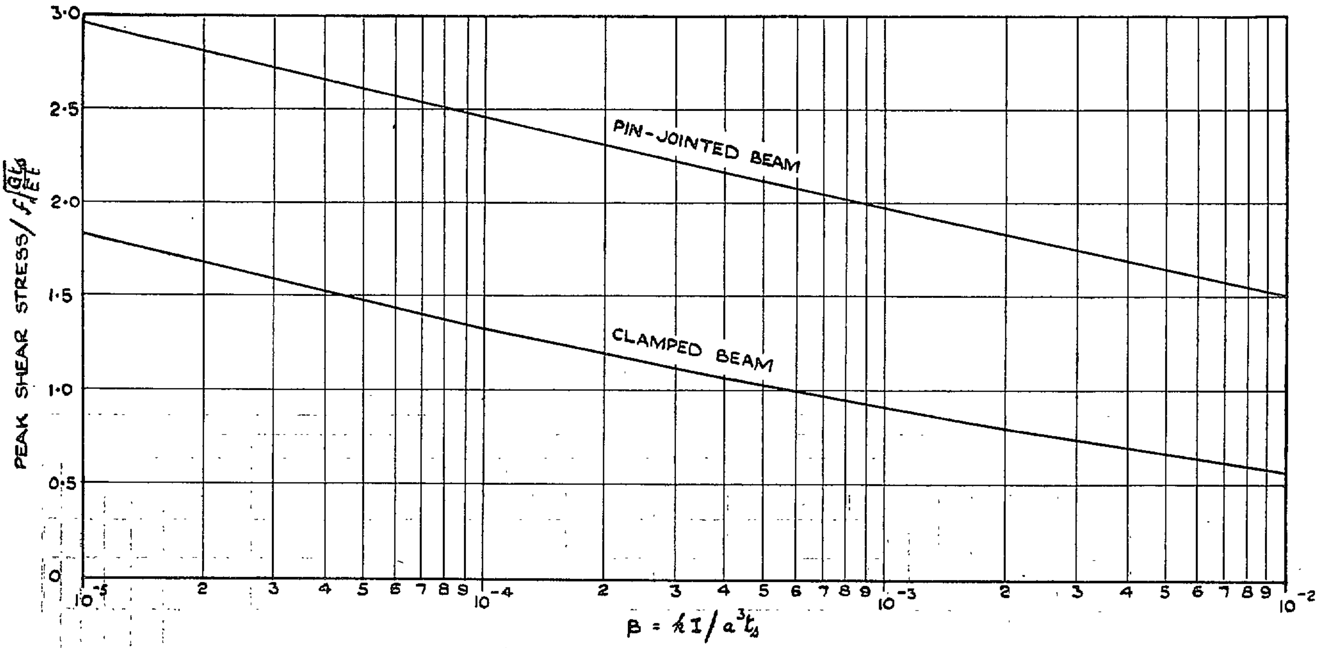


Fig. 5. Peak shear stress adjacent to a boom.

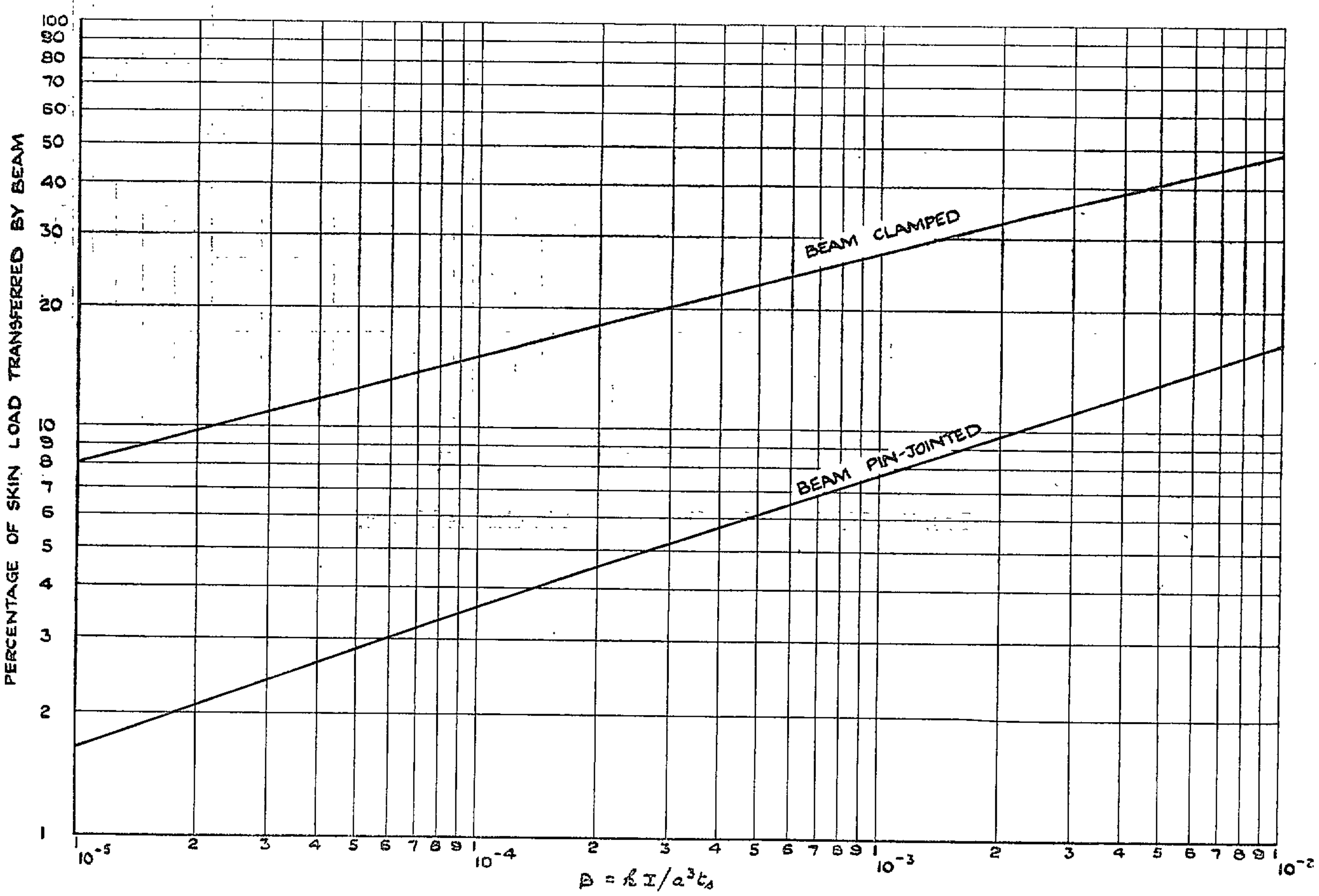


Fig. 6. Percentage of skin load transferred by beam.

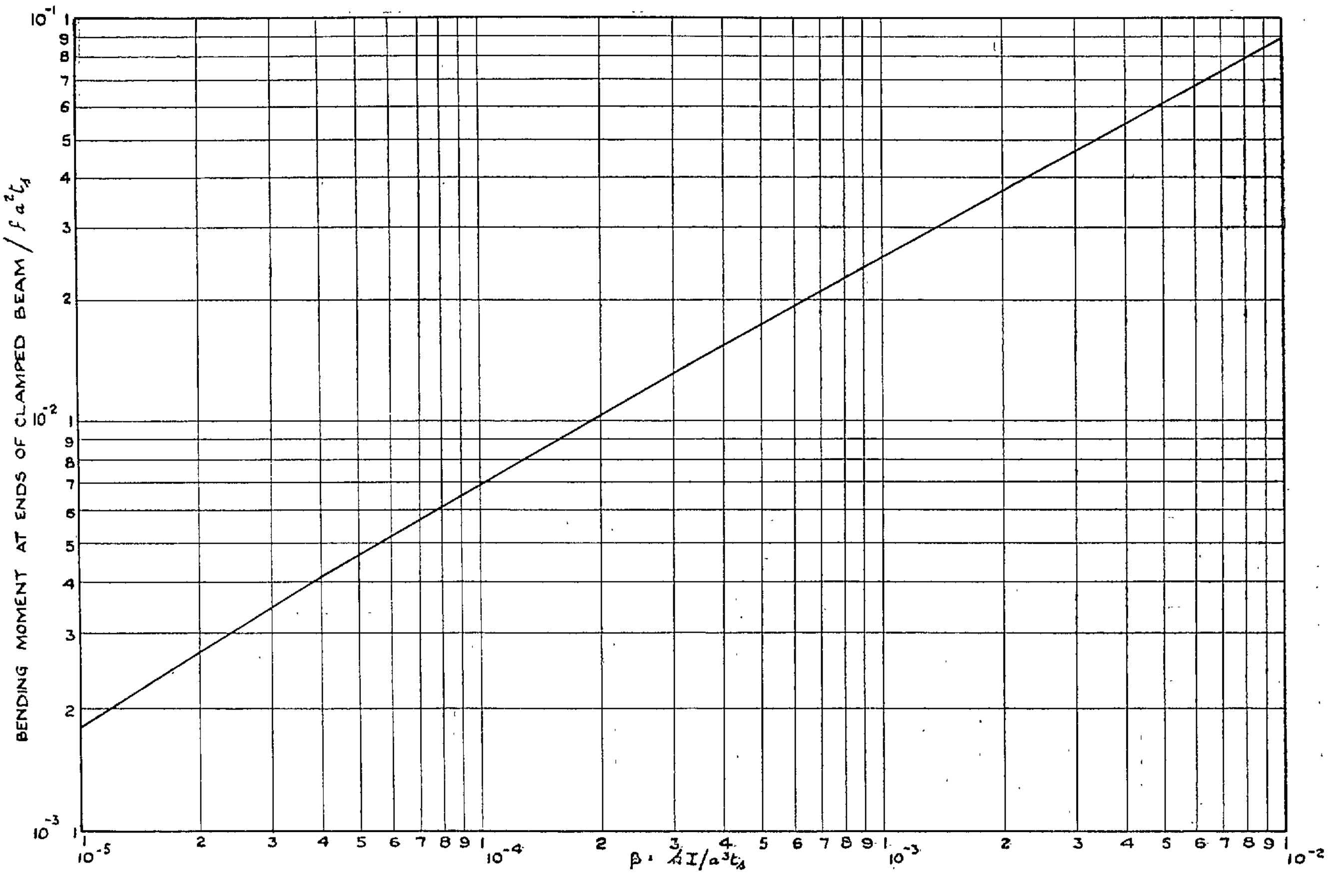


FIG. 7. Maximum bending moment in clamped beam.

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