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The Pressure Distribution, at Supersonic Speeds and Zero Lift, on some Swept-back Wings having Symmetrical Sections with Rounded Leading Edges

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Summary.—Formulae are found for the pressure distribution at supersonic speeds and at zero incidence for certain symmetrical surfaces of small finite thickness, with swept-back leading edges, the surfaces being set symmetrically to the wind direction. The solutions are only valid if the surfaces lie wholly within the Mach cone of the apex.

The results are applied to the surfaces

$$\frac{z}{2t_0} = \frac{x^2}{c^2}\left(\frac{x^2 - y^2 \cot^2 \gamma}{c^2}\right)^{1/2}, \quad \frac{z}{2t_0} = \frac{y^2 \cot^2 \gamma}{c^2}\left(\frac{x^2 - y^2 \cot^2 \gamma}{c^2}\right)^{1/2},$$

c being the chord in the vertical plane of symmetry, and t_0 a constant determining thickness.

Combining these solutions with others already available, the pressure distribution is found for a wing whose equation is of the form

$$\frac{z}{2t_0} = \left(1 + \frac{x}{a} - \frac{x^2}{b^2} + \frac{y^2}{d^2}\right)\left(\frac{x^2 - y^2 \cot^2 \gamma}{c^2}\right)^{1/2},$$

where a, b, d are constants. Some examples of the pressure distribution for wings of this type have been calculated.

1. *Introduction.*—In R. & M. 2548[1], the linearised differential equation of supersonic flow is solved in a special system of curvilinear co-ordinates, referred to as hyperboloido-conal co-ordinates; and it is shown that one of the simplest solutions corresponds to the flat delta wing at incidence. In R. & M. 2549[2], further solutions are found, corresponding to the thin elliptic cone and the elliptic hyper-cone at zero incidence, and the two solutions are combined to give the flow over a wing-like surface.

In the present report, certain general solutions are discussed, and the results are applied to the surfaces $z/2t_0 = (x^2/c^2)[(x^2 - y^2 \cot^2 \gamma)/c^2]^{1/2}$ and $z/2t_0 = (y^2 \cot^2 \gamma/c^2)[(x^2 - y^2 \cot^2 \gamma)/c^2]^{1/2}$, where x is measured downstream from the apex, y is measured to starboard and z is measured vertically upwards. The quantity c is the chord in the vertical plane of symmetry, γ is the apex semi-angle in the horizontal plane of symmetry, and t_0 is a constant determining thickness.

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The surfaces are symmetrical with respect to the xy and the zx-planes and they are set symmetrically to the wind direction, with the apex pointing against the stream. The solutions are only valid if the surfaces lie wholly within the Mach cone of the apex, and therefore, the Mach angle m ($= \operatorname{cosec}^{-1} M$) is greater than the apex semi-angle γ.

The solutions for these two surfaces are combined with those given in R. & M. 2549[2], to give the pressure distribution for wings of small finite thickness placed symmetrically to the wind direction, with straight leading edges and a hyperbolic or parabolic trailing edge. Some calculations for wing drag have also been made.

2. *Method of Solution.*—The method is essentially that used in R. & M. 2548[1] and 2549[2].

The co-ordinates used are the pseudo-orthogonal co-ordinates introduced in R. & M. 2548[1], where

$$x = \frac{\beta r \mu \nu}{hk}, \quad y = \frac{r(\mu^2 - h^2)^{1/2}(\nu^2 - h^2)^{1/2}}{h(k^2 - h^2)^{1/2}}, \quad z = \frac{r(\mu^2 - k^2)^{1/2}(k^2 - \nu^2)^{1/2}}{k(k^2 - h^2)^{1/2}} \qquad (1)$$

$$\left.\begin{aligned} \beta^2 &= M^2 - 1 = \cot^2 m = k^2 - h^2 \\ k^2 &= \cot^2 \gamma, \quad h^2 = \cot^2 \gamma - \cot^2 m \end{aligned}\right\} \qquad (2)$$

It is assumed that the surfaces all lie close to the basic plate, whose equation is $\mu = k$, and that the induced velocities on the surface are small and equal to the induced velocities on the plate. Therefore, the relation between the shape of the body and its induced velocity potential ϕ is of the form

$$\frac{\partial z}{\partial x} = \frac{1}{V}\left(\frac{\partial \phi}{\partial z}\right)_{\mu = k} \qquad (3)$$

where V is the free stream velocity.

For the linearised theory, the pressure difference Δp and the pressure coefficient C_p are given by:

$$\Delta p = -\rho V \left(\frac{\partial \phi}{\partial x}\right)_{\mu = k},$$

$$Cp = \frac{2\Delta p}{\rho V^2} = -\frac{2}{V}\left(\frac{\partial \phi}{\partial x}\right)_{\mu = k}, \qquad (4)$$

where ρ is the density of the free stream.

The linearised differential equation for the velocity potential ϕ, in terms of the co-ordinates r, μ, ν is[1]:

$$(\mu^2 - \nu^2)\frac{\partial}{\partial r}\left(r^2 \frac{\partial \phi}{\partial r}\right) - \sqrt{[(\mu^2 - h^2)(\mu^2 - k^2)]}\frac{\partial}{\partial \mu}\left(\sqrt{[(\mu^2 - h^2)(\mu^2 - k^2)]}\frac{\partial \phi}{\partial \mu}\right)$$
$$- \sqrt{[(\nu^2 - h^2)(k^2 - \nu^2)]}\frac{\partial}{\partial \nu}\left(\sqrt{[(\nu^2 - h^2)(k^2 - \nu^2)]}\frac{\partial \phi}{\partial \nu}\right) = 0, \qquad (5)$$

and it has been shown, in Appendix V of R. & M. 2548[1], that a solution of equation (5) can be found of the form $\phi = r^n f(\mu, \nu)$, where $f(\mu, \nu)$ is the product of two Lamé functions of μ, ν respectively, of degree n, n being a positive integer.

A standard Lamé function of degree n, $E_n^m(\mu)$, can be determined in $(2n + 1)$ different ways, and belongs to one of four classes K, L, M, N (Ref. 3).

Assuming that $E_n^m(\mu)$ has been determined, there is a second solution of Lamé's equation defined by[1,3]

$$F_n^m(\mu) = E_n^m(\mu)\int_\mu^\infty \frac{dt}{[E_n^m(t)]^2\,[\,|(t^2-h^2)\,(t^2-k^2)|\,]^{1/2}} .$$

For the solution of problems of the type under consideration, we require that the equation of the surface found by the integration of equation (3) shall give symmetry with respect to the xy and zx-planes, and that $(x^2 - y^2\cot^2\gamma)^{1/2}$ shall appear as a factor. It is easy to verify that the required solutions are given by combinations of solutions for the potential of the form

$$\phi_n^m = C_n r^n F_n^m(\mu)\,.\,E_n^m(\nu)\,,$$

where $E_n^m(\mu)$ is a standard Lamé function of degree n of the K class; that is, $E_n^m(\mu)$ is of the form $E_n^m(\mu) = \mu^n + a_1\mu^{n-2} + \ldots$, the last term being $a_{n/2}$ or $a_{(n-1)/2}$ according as n is even or odd. ϕ_n^m also satisfies the condition $\phi_n^m = 0$ on the Mach cone of the apex.

The solutions for odd and even values of n will be considered separately.

3. (i) *Solutions for* $n = 2N+1$.—For $n = 2N+1$, where N is a positive integer, there are $(N+1)$ K-functions of the form

$$E_{2N+1}^m(\mu) = \mu^{2N+1} + b_{1,m}\mu^{2N-1} + \ldots + b_{N,m}\mu,\ m = 1, 2, \ldots (N+1)\,. \qquad (6)$$

Equations satisfied by the coefficients $b_{1,m}, \ldots b_{N,m}$ are given in Appendix II.

The second Lamé function is given by

$$F_{2N+1}^m(\mu) = E_{2N+1}^m(\mu)\int_\mu^\infty \frac{dt}{[E_{2N+1}^m(t)]^2\,(t^2-h^2)^{1/2}\,(t^2-k^2)^{1/2}} \equiv E_{2N+1}^m(\mu)\,.\,R_{2N+1}^m(\mu)\,. \qquad (7)$$

We consider the solution

$$\phi_m = C_{2N+1} r^{2N+1} E_{2N+1}^m(\mu) E_{2N+1}^m(\nu) \mathrm{R}_{2N+1}^m(\mu)\,. \qquad (8)$$

At the plate, $\mu \to k$, and

$$r^2 = \frac{(x^2-\beta^2y^2)}{\beta^2}\,, \qquad r^2\nu^2 = \frac{h^2x^2}{\beta^2}\,, \qquad (9)$$

and

$$\frac{\partial\phi_m}{\partial z} = \frac{\partial\phi_m}{\partial\mu}\,.\,\frac{\partial\mu}{\partial z}\,.$$

Hence, it can be shown that, as $\mu \to k$,

$$\frac{\partial\phi_m}{\partial z} \to \frac{-C_{2N+1}}{E_{2N+1}^m(k)}\,\frac{r^{2N}E_{2N+1}^m(\nu)}{(k^2-\nu^2)^{1/2}} \qquad (10)$$

$$= \frac{-C_{2N+1}h}{\beta^{2N+1}E_{2N+1}^m(k)}\,\frac{x[h^{2N}x^{2N} + b_{1,m}h^{2N-2}x^{2N-2}(x^2-\beta^2y^2) + \ldots + b_{N,m}(x^2-\beta^2y^2)^N]}{(x^2-k^2y^2)^{1/2}}$$

$$\equiv \frac{-C_{2N+1}h}{\beta^{2N+1}E_{2N+1}^m(k)}\,\frac{x\sum_{r=0}^{N}(H_{r,m}x^{2r}y^{2N-2r})}{(x^2-k^2y^2)^{1/2}}\,, \qquad (11)$$

where

$$H_{r,m} = (-1)^{N-r}\beta^{2N-2r}\Big[b_{N-r,m}h^{2r} + b_{N-r+1,m}h^{2r-2}(N-r+1) + b_{N-r+2,m}h^{2r-4}\frac{(N-r+2)(N-r+1)}{2!} + \ldots + b_{N,m}\frac{N!}{r!(N-r)!}\Big], \qquad (12)$$

and $b_{0,m}$ is taken as 1.

Integrating the relation (3), it can be shown that, if the integration constant is taken as zero, z is of the form

$$z = D_{2N+1}(x^{2N} + d_{1,m}x^{2N-2}y^2 + \ldots + d_{N,m}y^{2N})(x^2 - k^2y^2)^{1/2}, \qquad (13)$$

where $D_{2N+1}, d_{1,m}, \ldots d_{N,m}$ are constants.

Therefore, ϕ_m, $m = 1, 2, \ldots (N + 1)$, gives the induced velocity potentials for the flow past the $(N + 1)$ different surfaces given by equation (13).

By constructing a potential

$$\Phi_{2N+1} = \sum_{m=1}^{N+1} (\lambda_m \phi_m),$$

where the λ's are constants to be determined, we can find the solution for any surface of the form of equation (13), where the coefficients are chosen arbitrarily. The values of λ_m are found by equating corresponding coefficients for z or for $\partial z/\partial x$, there being, in either case, $(N + 1)$ linear simultaneous equations.

In practice, it is convenient to construct solutions for surfaces whose equations are of the form

$$\frac{z}{2t_0} = \frac{x^{2s}y^{2N-2s}}{c^{2N}}\left(\frac{x^2 - k^2y^2}{c^2}\right)^{1/2}, \quad s = 0, 1, \ldots, N, \qquad (14)$$

and then to combine these solutions, since for surfaces of the form (14), $b_{1,m}, \ldots b_{N,m}$ can be more easily eliminated, and the constant coefficients expressed in terms of h and k.

To find the pressure distribution, we require the value of $\partial\phi_m/\partial x$. When $\mu \to k$,

$$\frac{\partial\phi_m}{\partial x} = \frac{C_{2N+1}h}{\beta^{2N+1}} \cdot E_{2N+1}{}^m(k) \cdot R_{2N+1}{}^m(k) \sum_{r=0}^{N} [(2r + 1)H_{r,m}x^{2r}y^{2N-2r}], \qquad (15)$$

where $H_{r,m}$ is given by equation (12).

It is shown, in Appendix II, that $R_{2N+1}{}^m(k)$ can be evaluated in terms of the complete elliptic integrals of the first and second kind of modulus h/k. Hence the pressure coefficient for a surface of the form (13) or (14) can be evaluated from the formula

$$C_p = \frac{-2}{V}\left(\frac{\partial\Phi_{2N+1}}{\partial x}\right)_{\mu=k} = \frac{-2}{V}\sum_{m=1}^{N+1}\left[\lambda_m\left(\frac{\partial\phi_m}{\partial x}\right)_{\mu=k}\right]. \qquad (16)$$

(ii) *Solutions for $n = 2N$.*—For $n = 2N$, there are $(N + 1)$ K-functions of the form

$$E_{2N}{}^m(\mu) = \mu^{2N} + a_{1,m}\mu^{2N-2} + \ldots + a_{N,m}, \quad m = 1, 2, \ldots (N + 1). \qquad (17)$$

Using the notation of equation (7), the second Lamé function is given by

$$F_{2N}{}^m(\mu) = E_{2N}{}^m(\mu) \cdot R_{2N}{}^m(\mu), \qquad (18)$$

and we consider the solution

$$\phi_m = C_{2N}r^{2N}E_{2N}{}^m(\mu)E_{2N}{}^m(\nu)R_{2N}{}^m(\mu). \qquad (19)$$

Using the relation (3), it can be shown that

$$\frac{\partial z}{\partial x} = \frac{-C_{2N}}{V\beta^{2N}E_{2N}{}^m(k)} \frac{\sum_{r=0}^{N}(H_{r,m}'x^{2r}y^{2N-2r})}{(x^2 - k^2y^2)^{1/2}}, \qquad (20)$$

where $H'_{r,m}$ is given by equation (12) if a is written for b . Hence

$$z = \frac{-C_{2N}}{V\beta^{2N}E_{2N}{}^{m}(k)} \left[\{d_{1,m}' x^{2N-1} + d_{2,m}' x^{2N-3}y^2 + \ldots . + d_{N,m}' xy^{2N-2}\} (x^2 - k^2y^2)^{1/2} + D_m y^{2N} \int \frac{dx}{(x^2 - k^2y^2)^{1/2}} \right] , \text{ where } d_{1,m}', d_{2,m}', \ldots . d_{N,m}' \quad \ldots \quad (21)$$

are constants, and

$$D_m = \sum_{r=0}^{N} \left[\frac{(2r)!}{2^{2r}(r!)^2} k^{2r} H_{r,m}' \right] . \quad \ldots \quad (22)$$

If we construct a potential

$$\Phi_{2N} = \sum_{m=1}^{N+1} (\lambda_m \phi_m) ,$$

where the λ's satisfy the condition

$$\sum_{m=1}^{N+1} \left(\frac{\lambda_m D_m}{E_{2N}{}^{m}(k)} \right) = 0 , \quad \ldots \quad (23)$$

we obtain the solution for a surface whose equation is

$$z = (c_1x^{2N-1} + c_2x^{2N-3}y^2 + \ldots . + c_N xy^{2N-2})(x^2 - k^2y^2)^{1/2} , \quad \ldots \quad (24)$$

where c_1, c_2, ... c_N are constants.

In particular, we can construct the solutions for the N surfaces whose equations are of the form

$$\frac{z}{2t_0} = \frac{x^{2s+1}y^{2N-2s-2}}{c^{2N-1}} \left(\frac{x^2 - k^2y^2}{c^2} \right)^{1/2} , \quad \ldots \quad (25)$$

for which the coefficients are more easily evaluated. For example, for $n = 2$, the condition (23) gives

$$\frac{\lambda_1}{\lambda_2} = - \frac{(k^2 - a_1)[k^2h^2 + a_2(k^2 - 2h^2)]}{(k^2 - a_2)[k^2h^2 + a_1(k^2 - 2h^2)]} = - \frac{a_2}{a_1} ,$$

since $3a_1a_2 = h^2k^2$. (*See* equation (1) in Appendix II.) Therefore, the potential $\Phi_2 = \phi_1/a_1 - \phi_2/a_2$, with the appropriate value of C_2, gives the solution for the one surface $z/2t_0 = (x/c)[(x^2 - k^2y^2)/c^2]^{1/2}$. This solution is given in R. & M. 2549[2].

Returning to the general case, for $n = 2N$, when $\mu \rightarrow k$,

$$\frac{\partial \phi_m}{\partial x} \rightarrow \frac{-C_{2N}}{\beta^{2N}} E_{2N}{}^{m}(k) R_{2N}{}^{m}(k) \sum_{r=1}^{N} [2rH_{r,m}' x^{2r-1}y^{2N-2r}] ,$$

the formula for $R_{2N}{}^{m}(k)$ being given in Appendix II. Hence the pressure coefficient C_p for any surface of the form (24) or (25) is given by

$$C_p = \frac{-2}{V} \sum_{m=1}^{N+1} \left[\lambda_m \left(\frac{\partial \phi_m}{\partial x} \right)_{\mu=k} \right] , \quad \ldots \quad (26)$$

the λ's being found as before, by equating the coefficients of z or of $\partial z/\partial x$.

By combining the solutions found for the surfaces given by equations (14), (25), it is possible to find formulae for the velocity distribution and the pressure coefficient for any surface whose equation is of the form

$$z = f(x, y^2)(x^2 - k^2y^2)^{1/2} ,$$

where $f(x, y^2)$ is a rational algebraic function of x and y^2 .

4. *Examples*

(i) The surface $\dfrac{z}{2t_0} = \dfrac{x^2}{c^2}\left(\dfrac{x^2 - k^2y^2}{c^2}\right)^{1/2}$

(ii) The surface $\dfrac{z}{2t_0} = \dfrac{k^2y^2}{c^2}\left(\dfrac{x^2 - k^2y^2}{c^2}\right)^{1/2}$

} at zero incidence.

For the two surfaces (i), (ii), the solution for $n = 3$ is taken. We assume

$$E_3{}^m(\mu) = \mu^3 - a_m\mu , \quad m = 1, 2 . \qquad \ldots \qquad (27)$$

Relation (2) of Appendix II gives the equation

$$5a_m{}^2 - 4(h^2 + k^2)a_m + 3h^2k^2 = 0 ,$$

and therefore,

$$a_1 + a_2 = \tfrac{4}{5}(h^2 + k^2), \quad a_1a_2 = \tfrac{3}{5}h^2k^2 . \qquad \ldots \qquad (28)$$

We consider the solution

$$\phi_m = C_3r^3E_3{}^m(\mu)E_3{}^m(\nu)R_3{}^m(\mu) , \quad m = 1, 2 . \qquad \ldots \qquad (29)$$

and it can be shown that, as $\mu \to k$,

$$\frac{\partial\phi_m}{\partial z} \to \frac{- C_3hx\,[h^2x^2 - a_m\,(x^2 - \beta^2y^2)]}{k\beta^3(k^2 - a_m)(x^2 - k^2y^2)^{1/2}} , \qquad \ldots \qquad (30)$$

and

$$\frac{\partial\phi_m}{\partial x} \to \frac{C_3hk}{\beta_3}\,(k^2 - a_m)[3(h^2 - a_m)x^2 + a_m\beta^2y^2]R_3{}^m(k) , \qquad \ldots \qquad (31)$$

where (*see* Appendix II)

$$R_3{}^m(k) = \frac{1}{2kh^2a_m(k^2 - a_m)(h^2 - a_m)}\left[(3a_m - 2k^2 - 2h^2)E\left(\frac{h}{k}\right)\right.$$
$$\left. - (3a_m - h^2 - 2k^2)K\left(\frac{h}{k}\right)\right] , \qquad \ldots \qquad (32)$$

$K\left(\frac{h}{k}\right)$, $E\left(\frac{h}{k}\right)$ being the complete elliptic integrals of the first and second kind respectively.

We construct a potential

$$\Phi_3 = \lambda_1\phi_1 + \lambda_2\phi_2 , \qquad \ldots \qquad (33)$$

where λ_1, λ_2 are constants to be determined after using relation (3).

(i) <u>The surface $\dfrac{z}{2t_0} = \dfrac{x^2}{c^2}\left(\dfrac{x^2 - k^2y^2}{c^2}\right)^{1/2}$ at zero incidence.</u>

If Φ_3 is the induced velocity potential for flow past surface (i), it can be shown, after some simplification that

$$\frac{\lambda_1}{\lambda_2} = -\frac{(5a_2 - h^2)}{5a_1 - h^2} .$$

Therefore, the induced velocity potential can be taken as

$$\Phi_3 = \frac{5}{h^2}(a_2\phi_1 - a_1\phi_2) - (\phi_1 - \phi_2) . \quad \ldots \quad (34)$$

By comparing coefficients, it is found that this requires

$$C_3 = \frac{2t_0 V k^3 \beta^3}{5c^3 h(a_1 - a_2)} . \quad \ldots \quad (35)$$

Hence, by using relation (4), and eliminating a_1, a_2 by means of relations (28), it can be shown that the pressure coefficient C_p is given by

$$C_p\sqrt{(M^2 - 1)} = \frac{4t_0}{c^3}\left[x^2 G_1\left(\frac{h}{k}\right) + k^2y^2 G_2\left(\frac{h}{k}\right)\right]$$

$$= \frac{4t_0}{c^3}\left[x^2 F_1\left(\frac{\tan\gamma}{\tan m}\right) + y^2\cot^2\gamma F_2\left(\frac{\tan\gamma}{\tan m}\right)\right], \quad \ldots \quad (36)$$

since $\frac{h^2}{k^2} = 1 - \frac{\tan^2\gamma}{\tan^2 m}$.

Writing $\frac{h^2}{k^2} = \varkappa^2$,

$$\left.\begin{aligned} F_1\left(\frac{\tan\gamma}{\tan m}\right) &\equiv G_1(\varkappa) = \frac{\sqrt{(1-\varkappa^2)}}{2\varkappa^6}[(3\varkappa^4 + \varkappa^2 + 8)K(\varkappa) - (6\varkappa^4 + 5\varkappa^2 + 8)E(\varkappa)] \\ F_2\left(\frac{\tan\gamma}{\tan m}\right) &\equiv G_2(\varkappa) = \frac{\sqrt{(1-\varkappa^2)}}{2\varkappa^6}[(\varkappa^4 - 9\varkappa^2 + 8)K(\varkappa) + (\varkappa^4 + 5\varkappa^2 - 8)E(\varkappa)] , \end{aligned}\right\} \quad \ldots \quad (37)$$

where $K(\varkappa)$, $E(\varkappa)$ are the complete elliptic integrals of the first and second kind respectively, with modulus $\varkappa$.

The functions F_1, F_2 are finite and continuous for $0 \leqslant \left|\frac{\tan\gamma}{\tan m}\right| \leqslant 1$, that is for $1 \geqslant |\varkappa| \geqslant 0$.

It can be verified that when $\varkappa \to 0$, $F_1 \to 2{\cdot}798$ and $F_2 \to 0{\cdot}4416$.

(ii) <u>The surface $\frac{z}{2t_0} = \frac{k^2y^2}{c^2}\left(\frac{x^2 - k^2y^2}{c^2}\right)^{1/2}$ at zero incidence.</u>

If Φ_3 is the induced velocity potential for surface (ii), it can be shown that

$$\frac{\lambda_1}{\lambda_2} = -\frac{(k^2 - a_1)(h^2 - a_2)}{(k^2 - a_2)(h^2 - a_1)} = -\frac{(8h^2k^2 - 5a_1h^2 - 5a_2k^2)}{8h^2k^2 - 5a_2h^2 - 5a_1k^2} .$$

Therefore, we construct the potential

$$\Phi_3 = 8h^2(\phi_1 - \phi_2) - 5\frac{h^2}{k^2}(a_1\phi_1 - a_2\phi_2) - 5(a_2\phi_1 - a_1\phi_2) . \quad \ldots \quad (38)$$

It is found that in this case

$$C_3 = \frac{-2t_0 V k^5}{5c^3 \beta h^3 (a_1 - a_2)} \quad \ldots \quad (39)$$

Hence it can be shown that the pressure coefficient C_p is given by

$$C_p \sqrt{(M^2 - 1)} = \frac{4t_0}{c^3}\left[x^2 G_3\left(\frac{h}{k}\right) + k^2 y^2 G_4\left(\frac{h}{k}\right)\right]$$

$$= \frac{4t_0}{c^3}\left[x^2 F_3\left(\frac{\tan\gamma}{\tan m}\right) + y^2 \cot^2\gamma F_4\left(\frac{\tan\gamma}{\tan m}\right)\right]. \quad \ldots \quad (40)$$

Writing $\frac{h^2}{k^2} = \varkappa^2$,

$$\left.\begin{aligned} F_3\left(\frac{\tan\gamma}{\tan m}\right) &\equiv G_3(\varkappa) = \frac{\sqrt{(1-\varkappa^2)}}{2\varkappa^6}[(8 - 5\varkappa^2)K(\varkappa) - (8 - \varkappa^2)E(\varkappa)] \\ F_4\left(\frac{\tan\gamma}{\tan m}\right) &\equiv G_4(\varkappa) = \frac{\sqrt{(1-\varkappa^2)}}{2\varkappa^6}[(8 - 15\varkappa^2 + 7\varkappa^4)K(\varkappa) - (8 - 11\varkappa^2 + 2\varkappa^4)E(\varkappa)]. \end{aligned}\right\} \quad (41)$$

The functions F_3, F_4 are finite and continuous for $0 \leqslant \left|\frac{\tan\gamma}{\tan m}\right| \leqslant 1$, that is, for $1 \geqslant |\varkappa| \geqslant 0$.

It can be verified that when $\varkappa \to 0$, $F_3 \to 0{\cdot}1472$ and $F_4 \to 0{\cdot}4416$.

The values of F_1, F_2, F_3, F_4 are tabulated in Appendix I, and are shown graphically in Fig. 1.

5. *The surface* $\frac{z}{2t_0} = \left(1 + \frac{x}{a} - \frac{x^2}{b^2} + \frac{y^2}{d^2}\right)\left(\frac{x^2 - y^2\cot^2\gamma}{c^2}\right)^{1/2}$, *at zero incidence.*—By combining the solutions found in section 4 and the solutions given in R. & M. 2549[2], a formula can be found for the pressure coefficient for a wing of small finite thickness, with straight leading edges and a hyperbolic (or parabolic) trailing edge. If the constants are chosen so that $d/b > \tan m$, the whole of the surface will lie outside the Mach cones of points on the trailing edge. If this condition is not satisfied, there will be small corrections to be made for the portions of the wing within these Mach cones.

The pressure coefficients for the surfaces

$$\frac{z}{2t_0} = \left(\frac{x^2 - k^2y^2}{c^2}\right)^{1/2} \quad \text{and} \quad \frac{z}{2t_0} = \frac{x}{c}\left(\frac{x^2 - k^2y^2}{c^2}\right)^{1/2}$$

are given by

$$C_p \sqrt{(M^2 - 1)} = \frac{4t_0}{c} f_1\left(\frac{\tan\gamma}{\tan m}\right) \quad \ldots \quad (42)$$

$$C_p \sqrt{(M^2 - 1)} = \frac{4t_0}{c^2} x f_2\left(\frac{\tan\gamma}{\tan m}\right), \quad \ldots \quad (43)$$

respectively, where

$$f_1\left(\frac{\tan\gamma}{\tan m}\right) = \frac{\sqrt{(1-\varkappa^2)}}{\varkappa^2}[K(\varkappa) - E(\varkappa)] \quad \ldots \quad (44)$$

$$f_2\left(\frac{\tan\gamma}{\tan m}\right) = \frac{\sqrt{(1-\varkappa^2)}}{\varkappa^4}[(\varkappa^2 + 2)K(\varkappa) - 2(\varkappa^2 + 1)E(\varkappa)] \quad \ldots \quad (45)$$

f_1, f_2 are tabulated in R. & M. 2549[2] and are repeated in Appendix I to this report.

Hence, combining the formulae (42), (43), (36), (40), the pressure coefficient for the surface

$$\frac{z}{2t_0} = \left(1 + \frac{x}{a} - \frac{x^2}{b^2} + \frac{y^2}{d^2}\right)\left(\frac{x^2 - y^2 \cot^2 \gamma}{c^2}\right)^{1/2} \text{ is} \quad \ldots \ldots \ldots \quad (46)$$

$$C_p \sqrt{(M^2 - 1)} = \frac{4t_0}{c}\left[f_1 + \frac{1}{a} f_2 x - \frac{1}{b^2}(x^2 F_1 + k^2 y^2 F_2) + \frac{1}{k^2 d^2}(x^2 F_3 + k^2 y^2 F_4)\right] \ldots \quad (47)$$

The induced drag coefficient is $D = D_p + D_n$, where D_p is the pressure drag and D_n is the drag due to the high pressure at the rounded leading edges of the wing[4].

As an example, the drag has been calculated for the surface

$$\frac{z}{T_0} = 0 \cdot 94\left(1 + \frac{x}{c} - \frac{2x^2}{c^2} + \frac{y^2}{4c^2}\right)\left(\frac{x^2 - y^2 \cot^2 \gamma}{c^2}\right)^{1/2}$$

for different values of γ. c is the maximum chord in the vertical plane of symmetry, T_0 is the maximum thickness and the thickness ratio T_0/c is taken as $0 \cdot 10$. It can be shown that the pressure coefficient C_p is given by

$$C_p \sqrt{(M^2 - 1)} = 0 \cdot 189\left[0 \cdot 6303 + 1 \cdot 5038\frac{x}{c} - 4 \cdot 911\frac{x^2}{c^2} + 0 \cdot 723\frac{y^2}{c^2}\right].$$

The pressure drag is found by integrating the component pressure along the wind direction over the planform. Therefore, the pressure drag coefficient $C_{D,p}$ is given by

$$C_{D,p} \times (\text{area of planform}) = +2\int\int C_p \frac{\partial z}{\partial x}\, dx\, dy, \text{ integrated over the planform.}$$

z is zero on the leading and trailing edges, therefore, integrating by parts,

$$C_{D,p} \times (\text{area of planform}) = -2\int\int z\frac{\partial C_p}{\partial x}\, dx\, dy\,. \quad \ldots \ldots \ldots \ldots \ldots \quad (48)$$

Hence

$$C_{D,p} \sqrt{(M^2 - 1)} = 0 \cdot 03585\pi\left[0 \cdot 0490 f_2 + 0 \cdot 1479\, F_1 - 0 \cdot 0185\frac{F_3}{k^2}\right].$$

R. T. Jones' formula for the force per unit length normal to the leading edge at any point is[4]

$$F_n = \pi r \frac{\rho V^2}{2} \frac{\sin^2 \gamma}{(1 - M^2 \sin^2 \gamma)^{1/2}} \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad (49)$$

where r is the radius of curvature of the leading edge and the other symbols are as defined in the report. This leads to the additional drag which is given by

$$C_{D,n}\sqrt{(M^2 - 1)} = 0 \cdot 017672\pi\frac{\tan \gamma}{\tan m}\left(1 - \frac{\tan^2 \gamma}{\tan^2 m}\right)^{1/2}\left\{\frac{17}{20} + \frac{9}{40}(\tan^2 \gamma - 8) + \frac{1}{96}(\tan^2 \gamma - 8)^2\right\}.$$

The total induced drag coefficient is $C_D = C_{D,p} + C_{D,n}$.

The drag coefficient C_D, based on the area of the wing, is plotted against M in Fig. 3. The strip-theory values for the centre section are also shown.

As examples of the pressure distribution, some calculations have been made for

(a) the surface $z = 0\cdot 94\,\frac{T_0}{c}\left(1 + \frac{x}{c} - \frac{2x^2}{c^2} + \frac{1}{4}\frac{y^2}{c^2}\right)(x^2 - y^2)^{1/2}$,

for $M = 1\cdot 118$;

(b) the surface $z = 2\,\frac{T_0}{c}\left(1 - \frac{x}{c} + \frac{y^2}{c^2}\right)(x^2 - 3y^2)^{1/2}$,

for $M = 1\cdot 442$;

(c) the surface $z = 0\cdot 6095\,\frac{T_0}{c}\left(1 + \frac{3x}{c} - \frac{4x^2}{c^2} + \frac{4y^2}{c^2}\right)(x^2 - y^2)^{1/2}$;

for $M = 1\cdot 118$;

(d) the surface $z = \frac{2T_0}{c}\left(1 - \frac{x}{c} + \frac{1}{2}\frac{y^2}{c^2}\right)(x^2 - y^2)^{1/2}$,

for $M = 1\cdot 118$;

(e) the surface $z = 3\cdot 375\,\frac{T_0}{c}\left(1 - \frac{2x}{c} + \frac{x^2}{c^2} - \frac{3y^2}{c^2}\right)(x^2 - 3y^2)^{1/2}$,

for $M = 1\cdot 442$;

(f) the surface $z = 2\cdot 598\,\frac{T_0}{c}\left(1 - \frac{3}{2}\frac{x}{c} + \frac{1}{2}\frac{x^2}{c^2} - \frac{1}{4}\frac{y^2}{c^2}\right)(x^2 - 3y^2)^{1/2}$,

for $M = 1\cdot 709$.

In each case, c is the chord in the vertical plane of symmetry, and T_0 is the maximum thickness in this plane. The thickness ratio T_0/c is taken as $0\cdot 10$. The pressure distributions and the shapes of surfaces (a), (b), (c), (d), (e), (f) are shown in Figs. 2, 4, 5, 6, 7, 8 respectively.

It is easy to show that for any surface of the form given by equation (46), if $b^2 \geqslant 0$, the maximum thickness in the vertical plane of symmetry is at $x = x_t$, where $c/2 \leqslant x_t \leqslant 2c/3$, c being the chord in this plane, and also that the leading edges are slightly rounded, except at the apex. If $b^2 < 0$, as in surfaces (e), (f), $c/3 \leqslant x_t < c/2$.

For surface (a), it can be shown that, for $M = 1\cdot 118$,

$$C_p = 0\cdot 378\left[0\cdot 6303 + 1\cdot 5038\,\frac{x}{c} - 4\cdot 911\,\frac{x^2}{c^2} + 0\cdot 723\,\frac{y^2}{c^2}\right],$$

for all points on the surface.

For surface (b), for $M = 1\cdot 442$,

$$C_p = 0\cdot 3849\left[0\cdot 6740 - 1\cdot 5838\,\frac{x}{c} + 0\cdot 0702\,\frac{x^2}{c^2} + 0\cdot 2949\,\frac{y^2}{c^2}\right],$$

for all points on the surface ahead of the Mach cones of points M, N on the trailing edges (Fig. 4).

For surface (c), for $M = 1\cdot 118$,

$$C_p = 0\cdot 2438\left[0\cdot 6303 + 4\cdot 514\,\frac{x}{c} - 9\cdot 0244\,\frac{x^2}{c^2} + 2\cdot 3232\,\frac{y^2}{c^2}\right],$$

for all points on the surface ahead of the Mach cones of points M, N on the trailing edge (Fig. 5).

For surface (d), for $M = 1 \cdot 118$,

$$C_p = 0 \cdot 8 \left[0 \cdot 6303 - 1 \cdot 5038 \frac{x}{c} + 0 \cdot 1139 \frac{x^2}{c^2} + 0 \cdot 1253 \frac{y^2}{c^2} \right],$$

for all points on the surface ahead of the Mach cones of the points M, C on the trailing edge (Fig. 6).

For surface (e), for $M = 1 \cdot 442$,

$$C_p = 0 \cdot 6495 \left[0 \cdot 6740 - 3 \cdot 1676 \frac{x}{c} + 2 \cdot 3785 \frac{x^2}{c^2} - 1 \cdot 9725 \frac{y^2}{c^2} \right],$$

for all points ahead of the line MPN (Fig. 7).

For surface (f), for $M = 1 \cdot 709$,

$$C_p = 0 \cdot 3749 \left[0 \cdot 7393 - 2 \cdot 5390 \frac{x}{c} + 1 \cdot 3453 \frac{x^2}{c^2} - 0 \cdot 7068 \frac{y^2}{c^2} \right],$$

for all points on the surface.

6. *Conclusion.*—Similar calculations could be made for surfaces formed by including solutions for higher values of n. But for n greater than 3, the work involved in obtaining the formulae, in a form suitable for computation, would be considerably longer.*

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* Calculations have since been made for $n = 4, 5, 6$. The results will be published in R. & M. 2865.

LIST OF SYMBOLS

c	Chord in the vertical plane of symmetry
t_0	Constant determining thickness
γ	Apex semi-angle
T_0	Maximum thickness of wing in the vertical plane of symmetry
x	Chordwise co-ordinate (measured downstream from the apex)
y	Spanwise co-ordinate (positive to starboard)
z	Normal co-ordinate (positive upwards)
r, μ, ν	*cf.* equations (1), (2)
m	Mach angle
M	Mach number
β	$(M^2 - 1)^{1/2}$
k	$\cot \gamma$
h	$(\cot^2 \gamma - \cot^2 m)^{1/2}$
ϕ, Φ	Induced velocity potential
V	Free-stream velocity
ρ	Free-stream density
Δp	Pressure difference
$E_n(\mu)$	Standard Lamé function of degree n
$F_n(\mu)$	Lamé function of the second kind
$R_n(\mu)$	$F_n(\mu)/E_n(\mu)$
$\varkappa$	h/k
$K(\varkappa)$	Complete elliptic integral of the first kind, with modulus $\varkappa$
$E(\varkappa)$	Complete elliptic integral of the second kind, with modulus $\varkappa$
C_D	Drag coefficient, based on the area of the wing

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APPENDIX I

Values of the functions f_1, f_2, F_1, F_2, F_3, F_4

$\frac{\tan \gamma}{\tan m}$	0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0
$h^2/k^2 \equiv \varkappa^2$	1	0·99	0·96	0·91	0·84	0·75	0·64	0·51	0·36	0·19	0
f_1	0	0·2707	0·4095	0·5048	0·5755	0·6303	0·6740	0·7097	0·7393	0·7642	0·7854
f_2	0	0·7148	1·0438	1·2528	1·3979	1·5038	1·5838	1·6458	1·6927	1·7347	1·7672
F_1	0	1·286	1·821	2·132	2·342	2·484	2·587	2·659	2·720	2·760	2·798
$-F_2$	0	0·0950	0·1745	0·2385	0·2895	0·3302	0·3626	0·3893	0·4088	0·4260	0·4416
F_3	0	0·2139	0·2550	0·2572	0·2452	0·2279	0·2105	0·1916	0·1770	0·1610	0·1472
F_4	0	0·0507	0·1026	0·1539	0·2036	0·2506	0·2949	0·3321	0·3746	0·4100	0·4416

APPENDIX II

Evaluation of the Second Lamé Function $F_n(k)$

The second Lamé function $F_n(\mu)$ is given by :

$$F_n(\mu) = E_n(\mu) \int_{\mu}^{\infty} \frac{dt}{[E_n(t)]^2 (t^2 - h^2)^{1/2} (t^2 - k^2)^{1/2}} \equiv E_n(\mu) R_n(\mu) ,$$

where $E_n(\mu)$ is a standard Lamé function of degree n. The class of Lamé function considered here is the K class of functions[3]; that is $E_n(\mu)$ is of the form $\mu^n + a_1\mu^{n-2} + a_2\mu^{n-4} + \ldots$, where $a_1, a_2, \ldots$ are constants, and the last term is of the form $a_{n/2}$ or $a_{(n-1)/2}\mu$ according as n is even or odd.

It is shown in Ref. 3 that the roots of the equation $E_n(\mu) = 0$ are all real and unequal, and not equal to $\pm h$ or $\pm k$. Therefore, if n is even and equal to $2N$, we can express $E_n(\mu)$ in the form $E_{2N}(\mu) = (\mu^2 - c_1)(\mu^2 - c_2) \ldots (\mu^2 - c_N)$, where $c_1, c_2, \ldots c_N$ are real, positive and unequal.

Substituting for E in Lamé's equation

$$(\mu^2 - h^2)(\mu^2 - k^2) \frac{d^2E}{d\mu^2} + \mu(2\mu^2 - h^2 - k^2) \frac{dE}{d\mu} + \{(h^2 + k^2)p - n(n+1)\mu^2\}E = 0 ,$$

and substituting the value $\mu^2 = c_r$, we obtain, after some simplification, the relation

$$\frac{1}{2c_r}\left(3 + \frac{h^2}{c_r - h^2} + \frac{k^2}{c_r - k^2}\right) + 2\left(\frac{1}{c_r - c_1} + \frac{1}{c_r - c_2} + \ldots + \frac{1}{c_r - c_s} + \ldots + \frac{1}{c_r - c_N}\right) = 0, \quad \ldots \quad (1)$$

$r = 1, 2, \ldots N, \quad s \neq r .$

Similarly, if n is odd and equal to $2N + 1$, $E_n(\mu)$ can be expressed in the form

$$E_{2N+1}(\mu) = \mu(\mu^2 - d_1)(\mu^2 - d_2) \ldots (\mu^2 - d_N) ,$$

where $d_1, d_2, \ldots d_N$ are real, positive and unequal, and it can be shown that

$$\frac{1}{2d_r}\left(5 + \frac{h^2}{d_r - h^2} + \frac{k^2}{d_r - k^2}\right) + 2\left(\frac{1}{d_r - d_1} + \frac{1}{d_r - d_2} + \ldots + \frac{1}{d_r - d_s} + \ldots + \frac{1}{d_r - d_N}\right) = 0, \quad \ldots \quad (2)$$

$r = 1, 2, \ldots N, \quad s \neq r .$

To evaluate

$$R_{2N}(k) = \int_k^{\infty} \frac{dt}{(t^2 - c_1)^2 (t^2 - c_2)^2 \ldots (t^2 - c_N)^2 (t^2 - h^2)^{1/2} (t^2 - k^2)^{1/2}} ,$$

put $t = k \operatorname{sn} u$, and write $h^2/k^2 = \varkappa^2$, $c_r/k^2 = \alpha_r^2$, $r = 1, 2, \ldots N$, $\operatorname{sn} u$ being a Jacobian elliptic function of modulus $\varkappa$.

Hence

$$R_{2N}(k) = \frac{1}{k^{4N+1}} \int_0^{K(\varkappa)} \frac{\operatorname{sn}^{4N} u \, du}{(1 - \alpha_1^2 \operatorname{sn}^2 u)^2 (1 - \alpha_2^2 \operatorname{sn}^2 u)^2 \ldots (1 - \alpha_N^2 \operatorname{sn}^2 u)^2},$$

$K(\varkappa)$ being the complete elliptic integral of the first kind, with modulus $\varkappa$.

Similarly, it can be shown that

$$R_{2N+1}(k) = \frac{1}{k^{4N+3}} \int_0^{K(\varkappa)} \frac{\mathrm{sn}^{4N+2} u \, du}{(1-\beta_1^2 \mathrm{sn}^2 u)^2 (1-\beta_2^2 \mathrm{sn}^2 u)^2 \ldots (1-\beta_N^2 \mathrm{sn}^2 u)^2},$$

where $d_r/k^2 = \beta_r^2$, $r = 1, 2, \ldots N$.

By expressing $\dfrac{\mathrm{sn}^{2N-2} u}{(1-\alpha_1^2 \mathrm{sn}^2 u)(1-\alpha_2^2 \mathrm{sn}^2 u) \ldots (1-\alpha_N^2 \mathrm{sn}^2 u)}$ in partial fractions, we obtain, after some further simplification,

$R_{2N}(k) =$

$$\frac{1}{k^{4N+1}} \sum_{r=1}^{N} \Bigg[\frac{1}{(\alpha_r^2-\alpha_1^2)^2 (\alpha_r^2-\alpha_2^2)^2 \ldots (\alpha_r^2-\alpha_s^2)^2 \ldots (\alpha_r^2-\alpha_N^2)^2} \Bigg\{ \int_0^{K(\varkappa)} \frac{\mathrm{sn}^4 u}{(1-\alpha_r^2 \mathrm{sn}^2 u)^2} du$$

$$- 2\left(\frac{1}{\alpha_r^2-\alpha_1^2} + \frac{1}{\alpha_r^2-\alpha_2^2} + \ldots + \frac{1}{\alpha_r^2-\alpha_s^2} + \ldots + \frac{1}{\alpha_r^2-\alpha_N^2}\right) \int_0^{K(\varkappa)} \frac{\mathrm{sn}^2 u}{1-\alpha_r^2 \mathrm{sn}^2 u} du \Bigg\} \Bigg]$$

$$s \neq r,$$

and

$R_{2N+1}(k) =$

$$\frac{1}{k^{4N+3}} \sum_{r=1}^{N} \Bigg[\frac{1}{(\beta_r^2-\beta_1^2)^2 (\beta_r^2-\beta_2^2)^2 \ldots (\beta_r^2-\beta_s^2)^2 \ldots (\beta_r^2-\beta_N^2)^2} \Bigg\{ \int_0^{K(\varkappa)} \frac{\mathrm{sn}^6 u}{(1-\beta_r^2 \mathrm{sn}^2 u)^2} du$$

$$- 2\left(\frac{1}{\beta_r^2-\beta_1^2} + \frac{1}{\beta_r^2-\beta_2^2} + \ldots + \frac{1}{\beta_r^2-\beta_s^2} + \ldots + \frac{1}{\beta_r^2-\beta_N^2}\right) \int_0^{K(\varkappa)} \frac{\mathrm{sn}^4 u}{1-\beta_r^2 \mathrm{sn}^2 u} du \Bigg\} \Bigg]$$

$$s \neq r.$$

It has been shown in the Appendix of R. & M. 2549[2] that

$$\int_0^{K(\varkappa)} \frac{\mathrm{sn}^4 u}{(1-\alpha^2 \mathrm{sn}^2 u)^2} du = \frac{(1-\alpha^2)K(\varkappa) - E(\varkappa)}{2\alpha^2(\varkappa^2-\alpha^2)(1-\alpha^2)} - \frac{1}{2\alpha^2}\left(3 + \frac{\varkappa^2}{\alpha^2-\varkappa^2} + \frac{1}{\alpha^2-1}\right) \int_0^{K(\varkappa)} \frac{\mathrm{sn}^2 u}{1-\alpha^2 \mathrm{sn}^2 u} du,$$

where $K(\varkappa)$, $E(\varkappa)$ are the complete elliptic integrals of the first and second kind, with modulus $\varkappa$.

It can also be shown that

$$\int_0^{K(\varkappa)} \frac{\mathrm{sn}^6 u}{(1-\beta^2 \mathrm{sn}^2 u)^2} du = \frac{(3\beta^2 - 2 - 2\varkappa^2)E(\varkappa) - (3\beta^2 - 2 - \varkappa^2)K(\varkappa)}{2\beta^2\varkappa(\varkappa^2-\beta^2)(1-\beta^2)}$$

$$- \frac{1}{2\beta^2}\left(5 + \frac{\varkappa^2}{\beta^2-\varkappa^2} + \frac{1}{\beta^2-1}\right) \int_0^{K(\varkappa)} \frac{\mathrm{sn}^4 u}{1-\beta^2 \mathrm{sn}^2 u} du.$$

Therefore, using relations (1) and (2), it is seen that the coefficients of

$$\int_0^{K(\varkappa)} \frac{\mathrm{sn}^2 u}{1-\alpha_r^2 \mathrm{sn}^2 u} du, \quad \int_0^{K(\varkappa)} \frac{\mathrm{sn}^4 u}{1-\beta_r^2 \mathrm{sn}^2 u} du$$

in the expressions for $R_{2N}(k)$, $R_{2N+1}(k)$ respectively, vanish.

Hence, $(s \neq r)$,

$$R_{2N}(k) =$$

$$\frac{1}{k^{4N+1}} \sum_{r=1}^{N} \left[\frac{1}{(\alpha_r^2 - \alpha_1^2)^2 (\alpha_r^2 - \alpha_2^2)^2 \ldots (\alpha_r^2 - \alpha_s^2)^2 \ldots (\alpha_r^2 - \alpha_N^2)^2} \cdot \frac{(1 - \alpha_r^2)K(\varkappa) - E(\varkappa)}{2\alpha_r^2(\varkappa^2 - \alpha_r^2)(1 - \alpha_r^2)} \right]$$

$$R_{2N+1}(k) =$$

$$\frac{1}{k^{4N+3}} \sum_{r=1}^{N} \left[\frac{1}{(\beta_r^2 - \beta_1^2)^2 (\beta_r^2 - \beta_2^2)^2 \ldots (\beta_r^2 - \beta_s^2)^2 \ldots (\beta_r^2 - \beta_N^2)^2} \cdot \right.$$

$$\left. \frac{(3\beta_r^2 - 2 - 2\varkappa^2)E(\varkappa) - (3\beta_r^2 - \varkappa^2 - 2)K(\varkappa)}{2\beta_r^2\varkappa^2(\varkappa^2 - \beta_r^2)(1 - \beta_r^2)} \right].$$

Therefore, substituting for $\varkappa$, a_r, β_r,

$$F_{2N}(k) =$$

$$\frac{1}{k} E_{2N}(k) \sum_{r=1}^{N} \left[\frac{1}{(c_r - c_1)^2 (c_r - c_2)^2 \ldots (c_r - c_s)^2 \ldots (c_r - c_N)^2} \cdot \frac{(k^2 - c_r)K\left(\frac{h}{k}\right) - k^2E\left(\frac{h}{k}\right)}{2c_r(h^2 - c_r)(k^2 - c_r)} \right]$$

$$F_{2N+1}(k) =$$

$$\frac{1}{k} E_{2N+1}(k) \sum_{r=1}^{N} \left[\frac{1}{(d_r - d_1)^2 (d_r - d_2)^2 \ldots (d_r - d_s)^2 \ldots (d_r - d_N)^2} \cdot \right.$$

$$\left. \frac{(3d_r - 2k^2 - 2h^2)E\left(\frac{h}{k}\right) - (3d_r - h^2 - 2k^2)K\left(\frac{h}{k}\right)}{2d_r h^2(h^2 - d_r)(k^2 - d_r)} \right].$$

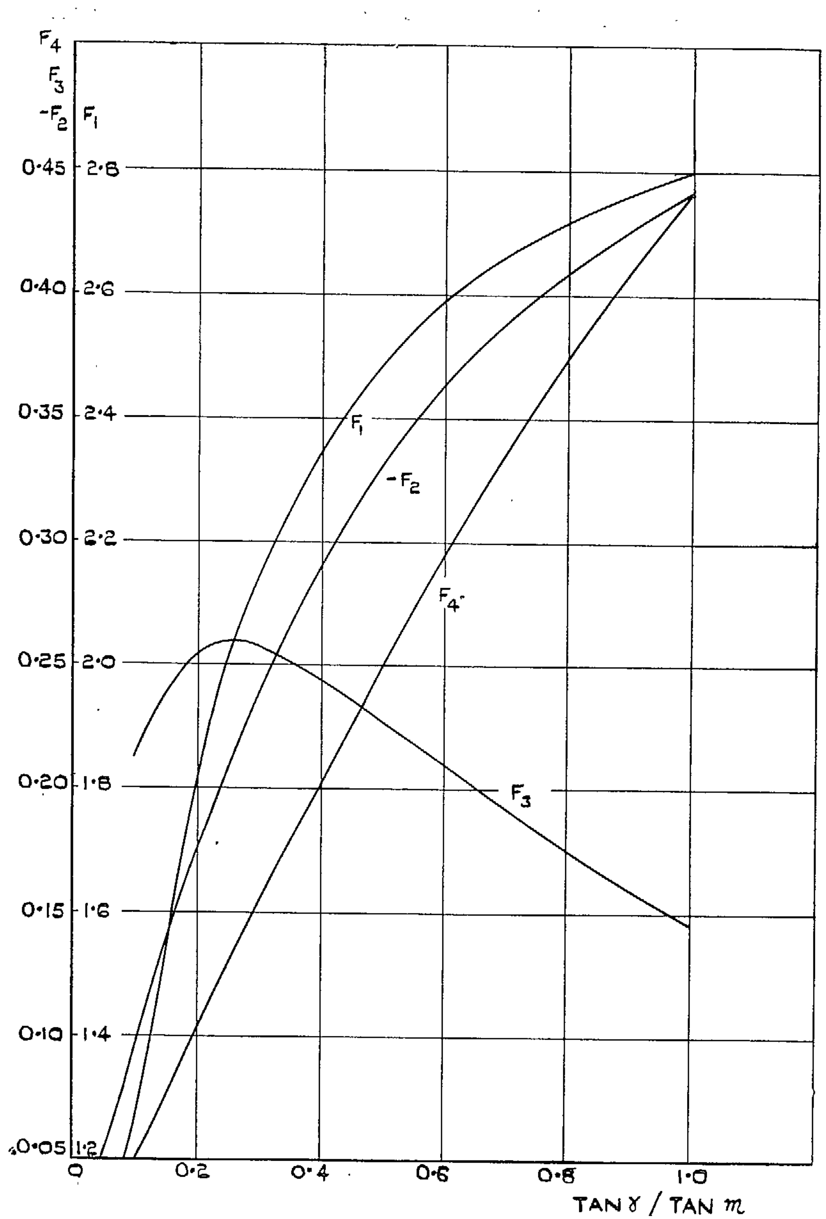

FIG. 1. The functions F_1 (tan γ/tan m), $-F_2$ (tan γ/tan m), F_3 (tan γ/tan m), F_4 (tan γ/tan m).

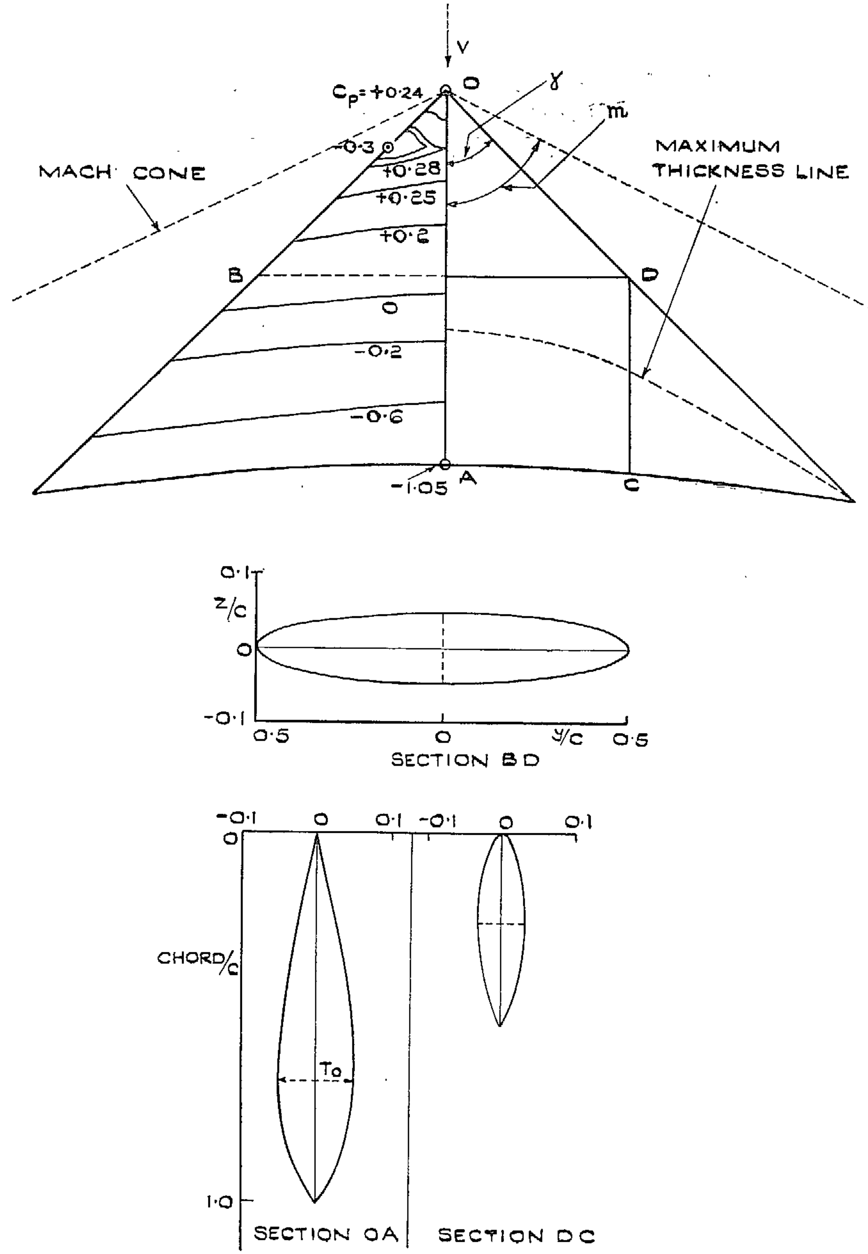

FIG. 2. Pressure distribution and shape of surface (a). $T_0/c = 0 \cdot 10$, $M = 1 \cdot 118$.

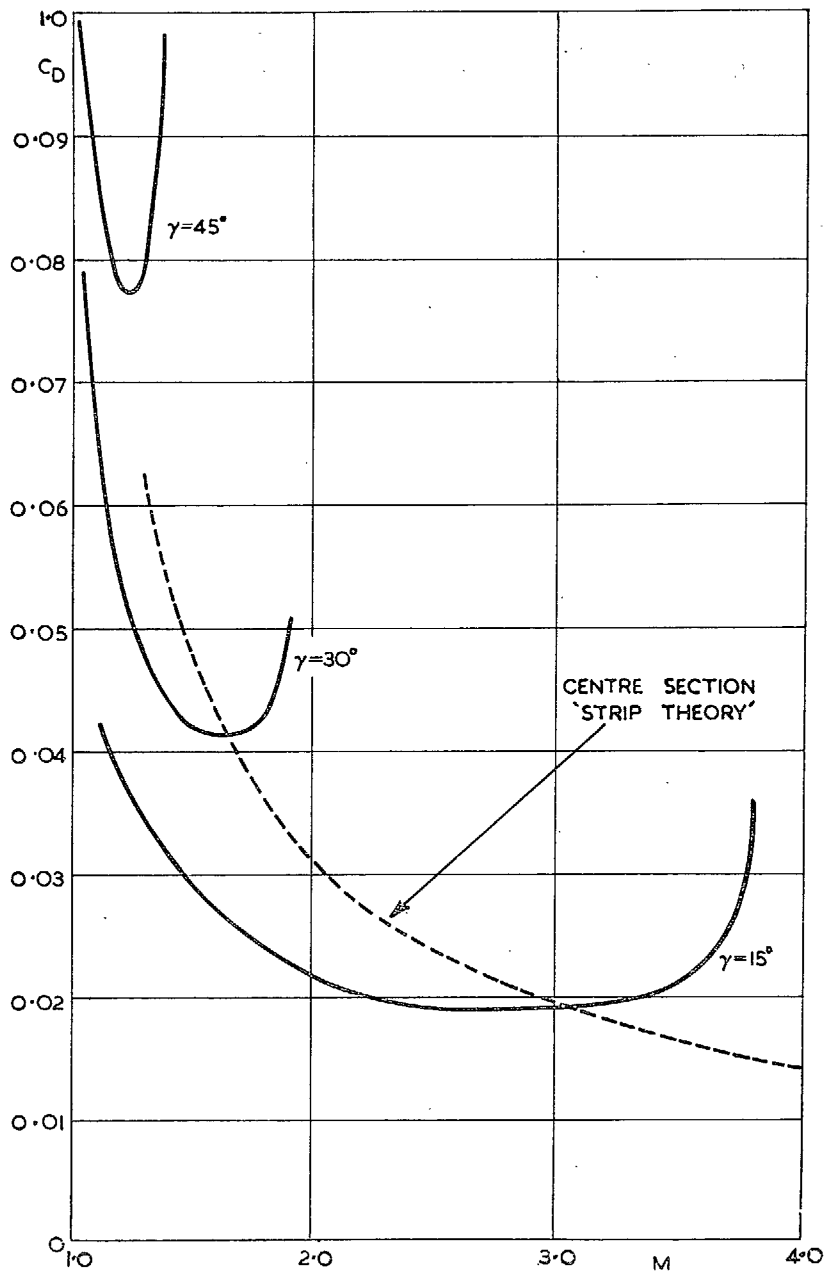

Fig. 3. Calculated drag of wing in Fig. 2 for different values of γ. C_D based on wing area. $T_0/c = 0 \cdot 10$.

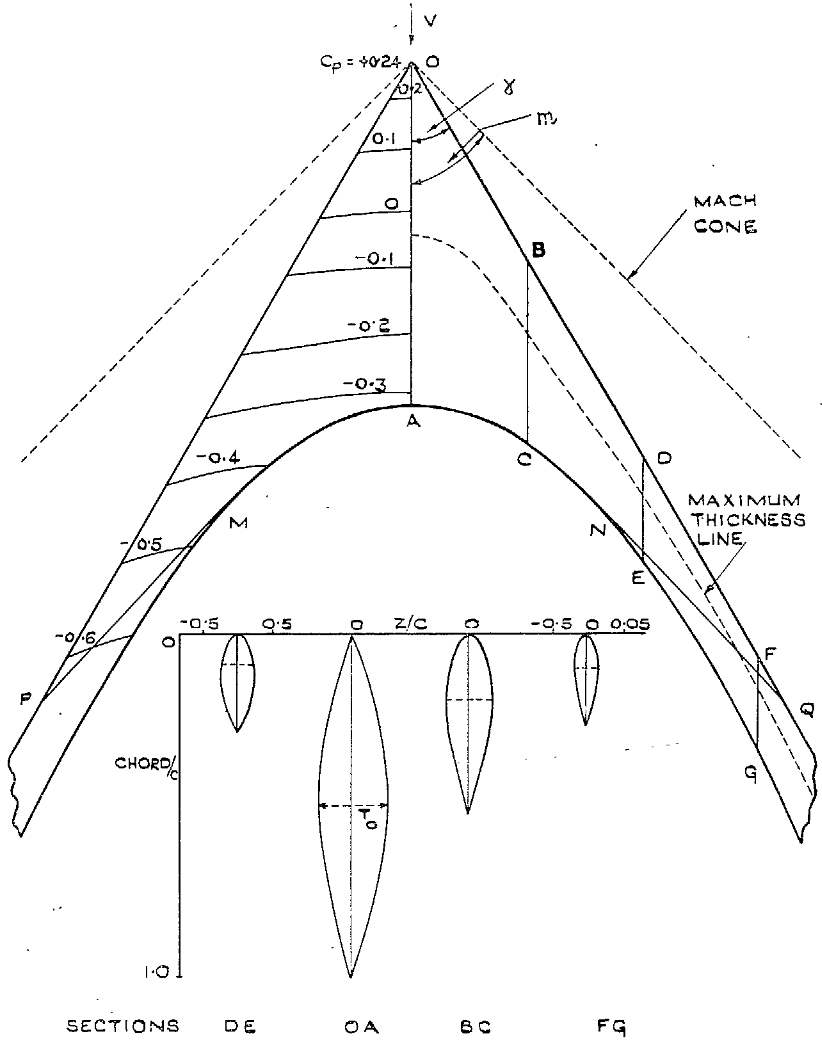

Fig. 4. Pressure distribution and shape of surface (b). $M = 1 \cdot 44$, $T_0/c = 0 \cdot 10$. Solution not valid behind the lines MP, NQ.

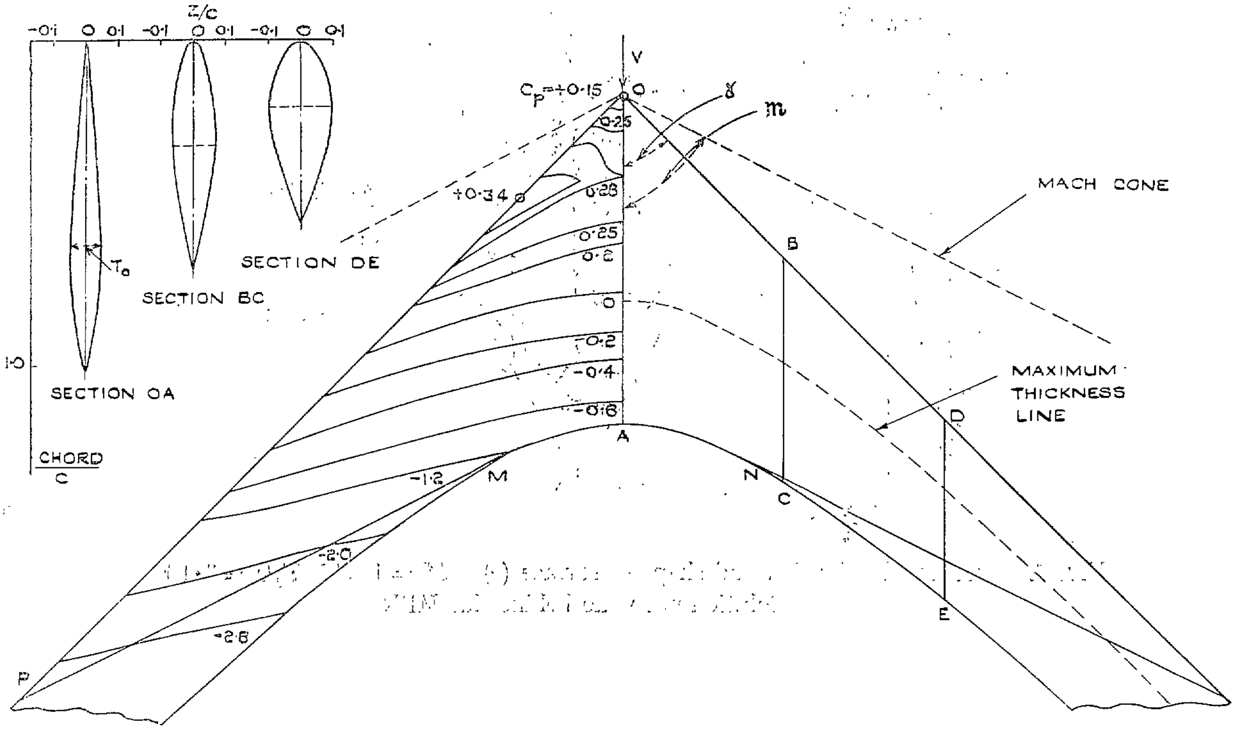

FIG. 5. Pressure distribution and shape of surface (c). $M = 1\cdot 118$, $T_0/c = 0\cdot 10$. Solution not valid behind the lines MP, NQ.

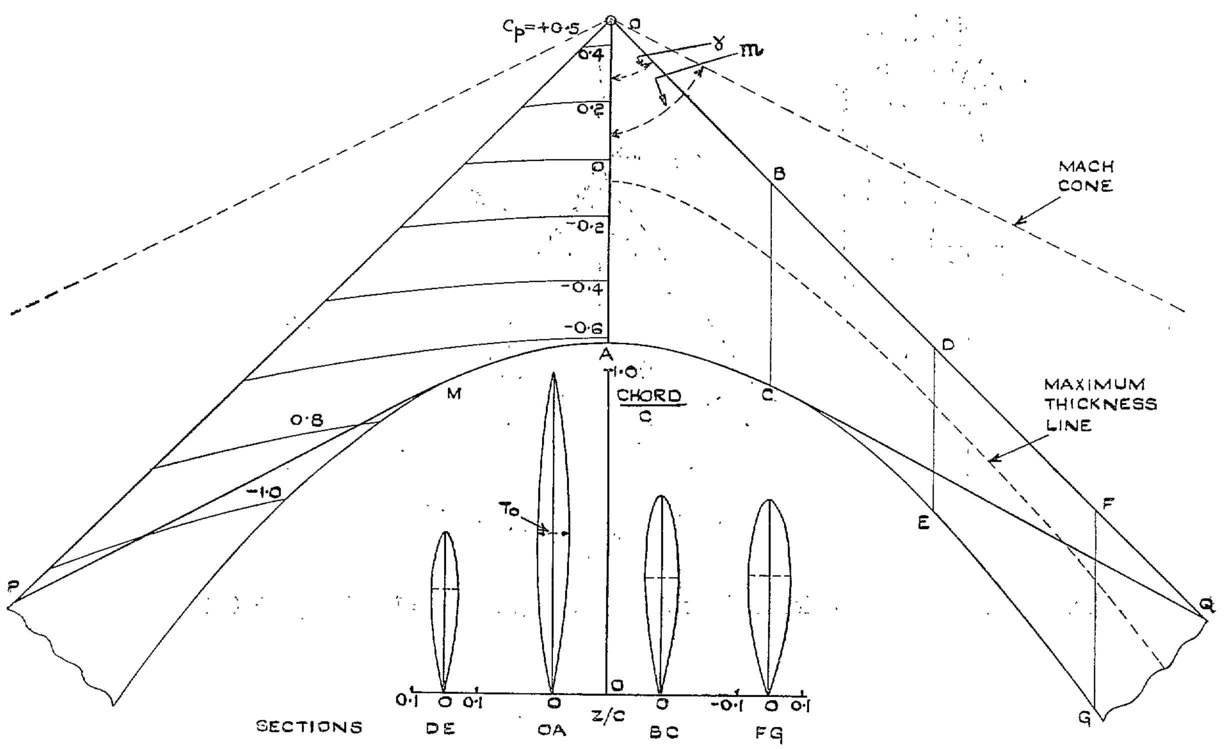

FIG. 6. Pressure distribution and shape of surface (d). $M = 1\cdot 118$, $T_0/c = 0\cdot 10$. Solution not valid behind the lines MP, CQ.

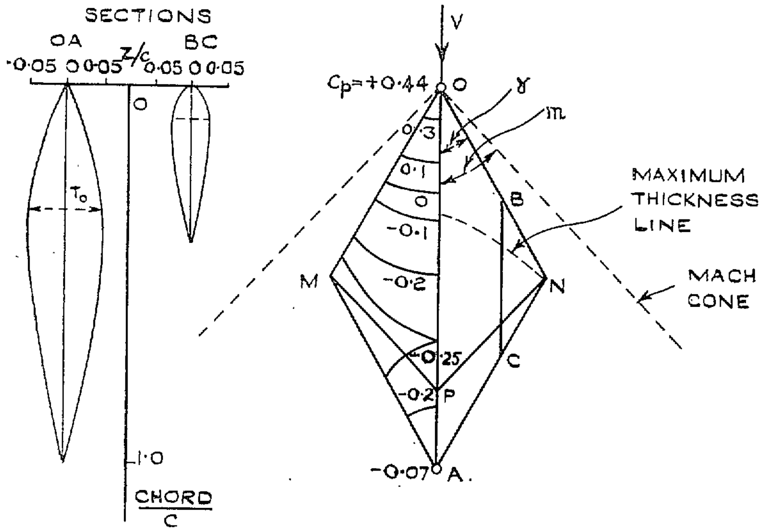

FIG. 7. Pressure distribution and shape of surface (e). $M = 1 \cdot 442$, $T_0/c = 0 \cdot 10$. Solution not valid behind line MPN.

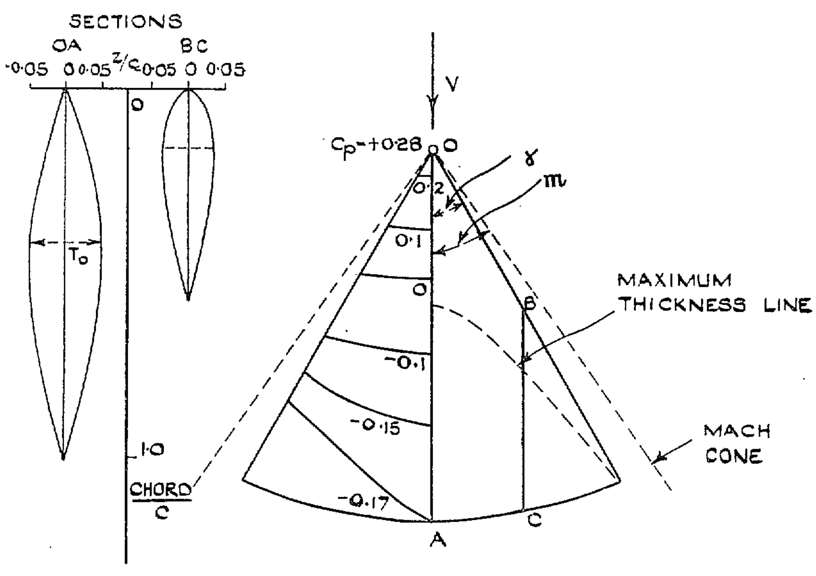

FIG. 8. Pressure distribution and shape of surface (f). $M = 1 \cdot 709$, $T_0/c = 0 \cdot 10$.

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