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On a Theory of Sandwich Construction

By

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Summary.—The theory of sandwich construction developed in this paper proceeds from the simple assumption that the filling has only transverse direct and shear stiffnesses, corresponding to its functional requirements. This supposition permits integration of the equilibrium equations for the filling. The resulting integrals are used to study the compression buckling of a flat sandwich plate. The formulae obtained are complex, but may be simplified in practical cases. A second approach to sandwich problems is made in section 5, where a theory of ' bending ' of plates is outlined. This generalises the usual theory, making allowance for flexibility in shear. This approach is applied to overall compression buckling of a plate, and agreement with the previous calculations is found. This suggests the possibility of calculating buckling loads for curved sandwich shells. A simple example, the symmetrical buckling of a circular cylinder in compression is worked out. The theory developed would seem applicable to all cases of buckling of not too short a wave length.

1. *Assumptions.*—The construction of a plate built according to the principles of Sandwich Construction is shown in Fig. 1. Metal or plywood faces are glued to the surface of a low density filling. The faces are the principal load carrying agent. The function of the filling is to stabilise the faces against lateral buckling and to provide a shear connection between the faces without which the plate could not transmit bending actions. The filling may contribute to the load carrying capacity of the plate, but it is not essential that it should do so. The advantage of Sandwich Construction lies in the great flexural and torsional rigidity of plates constructed by this method. This rigidity arises from the stiffness of the faces in their planes combined with their relatively large separation.

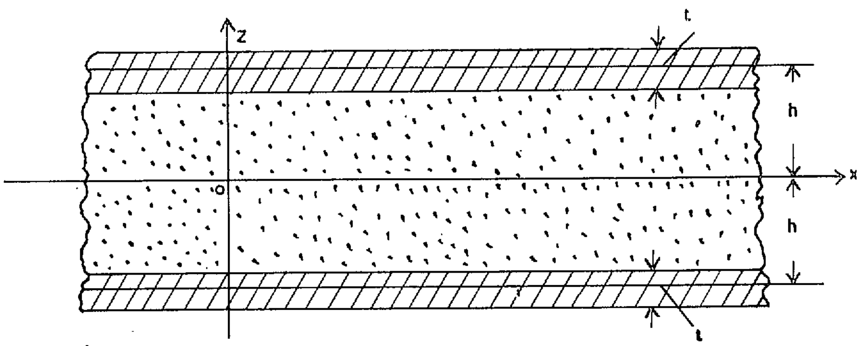

FIG. 1.

NOTE: This paper was read at the VII International Congress of Applied Mechanics (1948).
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The theory of Sandwich Construction developed in this paper proceeds from an ideal model in which the component parts fulfil their essential functions but play *no* other part at all. The faces are idealised as thin plates with a thickness t of isotropic material having Young's Modulus E and Poisson's ratio σ. The filling is assumed to extend between the middle surfaces of the faces with thickness $2h$ large compared with t. It will be assumed homogeneous, but anistropic, with direct stiffness at right-angles to the faces and shear stiffness in planes at right-angles to the faces. Other kinds of stiffness of the filling will be taken as zero. If Cartesian axes are taken with Ox and Oy in the middle surface of the filling and Oz at right-angles to the faces, the stress-strain relations for the filling can be written:—

$$\left.\begin{aligned} X_x = 0, \quad Y_y = 0, \quad Z_z = Ce_{zz}, \\ Y_z = Le_{yz}, \quad Z_x = Le_{zx}, \quad X_y = 0 \end{aligned}\right\} \quad \ldots \quad (1)$$

The notation for stress and strain components is that of Love's Treatise[1]. C is Young's Modulus in the Oz direction while L is the shear modulus in the Oyz and Ozx planes.

2. *The Displacement.*—The displacement in the filling can be calculated from equations (1) and the stress equations of equilibrium which can be written remembering (1) as:—

$$\frac{\partial Zx}{\partial z} = 0, \quad \frac{\partial Yz}{\partial z} = 0, \quad \frac{\partial Zx}{\partial x} + \frac{\partial Yz}{\partial y} + \frac{\partial Zz}{\partial z} = 0. \quad \ldots \quad (2)$$

It follows that Z_x and Y_z are functions of x and y alone, and that

$$Z_z = -z\left(\frac{\partial Z_x}{\partial x} + \frac{\partial Y_z}{\partial y}\right) + Z_{z0} \quad \ldots \quad (3)$$

where Z_{z0} is $(Z_z)_{z=0}$, a function of x and y. Using the formulae expressing the strain components in terms of the displacement (u, v, w),

$$\left.\begin{aligned} &\frac{\partial w}{\partial z} = e_{zz} \\ &\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} = e_{yz}, \quad \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = e_{zx} \end{aligned}\right\} \quad \ldots \quad (4)$$

we obtain by substitution from equation (1) and simple integration the formulae:—

$$\left.\begin{aligned} u &= \frac{z^3}{6C}\frac{\partial}{\partial x}\left(\frac{\partial Z_x}{\partial x} + \frac{\partial Y_z}{\partial y}\right) - \frac{z^2}{2C}\frac{\partial Z_{z0}}{\partial x} + z\left(\frac{Z_x}{L} - \frac{\partial w_0}{\partial x}\right) + u_0 \\ v &= \frac{z^3}{6C}\frac{\partial}{\partial y}\left(\frac{\partial Z_x}{\partial x} + \frac{\partial Y_z}{\partial y}\right) - \frac{z^2}{2C}\frac{\partial Z_{z0}}{\partial y} + z\left(\frac{Y_z}{L} - \frac{\partial w_0}{\partial y}\right) + v_0 \\ w &= -\frac{z^2}{2C}\left(\frac{\partial Z_x}{\partial x} + \frac{\partial Y_z}{\partial y}\right) + \frac{zZ_{z0}}{C} + w_0 \end{aligned}\right\} \quad \ldots \quad (5)$$

where (u_0, v_0, w_0) is the displacement of the plane $z = 0$. Equations (5) express the displacement in terms of six arbitrary functions of x and y, namely Z_x, Y_z, Z_{z0}, u_0, v_0 and w_0.

3. *Buckling in Compression.*—A sandwich plate, occupying the region $-\infty < x < +\infty$, $0 \leqslant y \leqslant b$, $-h \leqslant z \leqslant +h$, is compressed in the x-direction by a uniform load P per unit length. The edges $y = 0$, b are simply supported. The plate will become unstable at a certain critical value of P. To find this value, a small displacement (u, v, w) is imposed upon the uniform compression and the examination of the possibility of equilibrium in this buckled form is carried out in the usual way. The displacement (u, v, w) is given by equations (5). This satisfies equilibrium conditions in the filling. The six unknown functions involved are determined by the boundary conditions at the faces.

The calculations are simplified somewhat by introducing the areal dilatation Δ of the faces. This is related to the applied forces per unit area Z_x and Y_z by the equation

$$\nabla^2 \Delta = \pm \frac{(1-\sigma^2)}{Et} \left(\frac{\partial Z_x}{\partial x} + \frac{\partial Y_z}{\partial y} \right) \quad \ldots \quad (6)$$

where in this, as in subsequent equations, the upper sign refers to $z = h$ and the lower to $z = -h$. From equation (5) it follows that

$$\Delta = \nabla^2 \left\{ \pm \frac{h^3}{6C} \left(\frac{\partial Z_x}{\partial x} + \frac{\partial Y_z}{\partial y} \right) - \frac{h^2}{2C} Z_{z0} \mp h w_0 \right\}$$

$$\pm \frac{h}{L} \left(\frac{\partial Z_x}{\partial x} + \frac{\partial Y_z}{\partial y} \right) + \left(\frac{\partial u_0}{\partial x} + \frac{\partial v_0}{\partial y} \right) . \quad \ldots \quad (7)$$

Substituting from equation (7) into (6) and adding and subtracting the resulting equations:—

$$\left\{ \frac{h^3}{6C} \nabla^4 + \frac{h}{L} \nabla^2 - \frac{(1-\sigma^2)}{Et} \right\} \left\{ \frac{\partial Z_x}{\partial x} + \frac{\partial Y_z}{\partial y} \right\} = h \nabla^4 w_0 \quad \ldots \quad (8)$$

$$\nabla^2 \left(\frac{\partial u_0}{\partial x} + \frac{\partial v_0}{\partial y} \right) = \frac{h^2}{2C} \nabla^4 Z_{z0} . \quad \ldots \quad (9)$$

The remaining condition of equilibrium at the faces is that of balance of normal forces. Here the effects of the initial compression P must be introduced as well as the external force Z_z. The resulting equations are:—

$$\left(D\nabla^4 + \frac{P}{2} \frac{\partial^2}{\partial x^2} \right) (w)_{z=\pm h} \pm (Z_z)_{z=\pm h} = 0 \quad \ldots \quad (10)$$

where $$D = \frac{Et^3}{12(1-\sigma^2)} \quad \ldots \quad (11)$$

Substituting from equations (3) and (5) into equation (10) and again adding and subtracting the resulting equations:—

$$\left\{ \frac{h^2}{2C} \left(D\nabla^4 + \frac{P}{2} \frac{\partial^2}{\partial x^2} \right) + h \right\} \left(\frac{\partial Z_x}{\partial x} + \frac{\partial Y_z}{\partial y} \right) = \left(D\nabla^4 + \frac{P}{2} \frac{\partial^2}{\partial x^2} \right) w_0 \quad \ldots \quad (12)$$

$$\left(D\nabla^4 + \frac{P}{2} \frac{\partial^2}{\partial x^2} + \frac{C}{h} \right) Z_{z0} = 0 . \quad \ldots \quad (13)$$

Equations (8), (9), (12) and (13) involve only the four unknowns

$$\frac{\partial Z_x}{\partial x} + \frac{\partial Y_z}{\partial y}, \quad Z_{z0}, \quad \frac{\partial u_0}{\partial x} + \frac{\partial v_0}{\partial y} \text{ and } w_0.$$

This relative simplicity is due to the use of Δ. The calculation of critical loads is unaffected by this artifice. The equations fall into two sets. Equations (8) and (12) involve only $\partial Z_x/\partial x + \partial Y_z/\partial y$ and w_0, while equations (9) and (13) involve Z_{z0} and $\partial u_0/\partial x + \partial v_0/\partial y$. There are thus two distinct types of buckling:—

(*a*) *Symmetric.*—Here $\partial Z_x/\partial x + \partial Y_z/\partial y = w_0 = 0$ and so w is an odd function of z. The critical loads follow from equation (13).

(*b*) *Antisymmetric.*—Here $Z_{z0} = \partial u_0/\partial x + \partial v_0/\partial y = 0$ and w is an even function of z. The critical loads follow from equations (8) and (12) which yield when $\partial Z_x/\partial x + \partial Y_z/\partial y$ is eliminated:—

$$\left[\frac{h^3 D}{3C}\nabla^8 + \frac{h^3 P}{6C}\frac{\partial^2}{\partial x^2}\nabla^4 - \frac{hD}{L}\nabla^6 + \left\{h^2 + \frac{(1-\sigma^2)D}{Et}\right\}\nabla^4 - \frac{hP}{2L}\frac{\partial^2}{\partial x^2}\nabla^2 + \frac{(1-\sigma^2)P}{2Et}\frac{\partial^2}{\partial x^2}\right] w_0 = 0 . \qquad (14)$$

The critical values of P follow from equations (13) and (14) by assuming that w and hence Z_{z0} and w_0, vary as $\sin(\pi x/\lambda)\sin(\pi y/b)$, where λ is the half-wave length, as yet unknown. The formulae are:—

Type (a)
$$P = \frac{2\pi^2 D}{b^2}\left(\frac{b}{\lambda} + \frac{\lambda}{b}\right)^2 + \frac{2b^2 C}{\pi^2 h}\left(\frac{\lambda}{b}\right)^2 \qquad (15)$$

Type (b)
$$P = \frac{2\pi^2 E t h^2}{(1-\sigma^2)b^2}\left(\frac{b}{\lambda} + \frac{\lambda}{b}\right)^2 \frac{\left\{1 + \dfrac{D(1-\sigma^2)}{Eth^2} + \dfrac{\pi^2 D}{Lhb^2}\left(1 + \dfrac{b^2}{\lambda^2}\right) + \dfrac{\pi^4 hD}{3Cb^4}\left(1 + \dfrac{b^2}{\lambda^2}\right)^2\right\}}{\left\{1 + \dfrac{\pi^2 Eth}{Lb^2(1-\sigma^2)}\left(1 + \dfrac{b^2}{\lambda^2}\right) + \dfrac{\pi^4 Eth^3}{3Cb^4(1-\sigma^2)}\left(1 + \dfrac{b^2}{\lambda^2}\right)^2\right\}} \qquad (16)$$

4. *Discussion of the Buckling Formulae.*—The value of the smallest critical load follows from equations (15) and (16) by chosing λ to make P a minimum. This is easy in the case of symmetrical buckling and yields

$$b/\lambda = (1 + b^4 C/\pi^4 hD)^{1/4}. \qquad (17)$$

In practice $b^4 C/\pi^4 hd >> 1$ and so

$$\lambda = \pi\,(hD/C)^{1/4} \qquad (18)$$

which shows that symmetric buckling occurs in short wave-lengths of the order of the sandwich thickness $2h$*. The corresponding critical load is given by:—

$$P_{\text{crit}} = 4\left(\frac{CD}{h}\right)^{1/2}. \qquad (19)$$

The formula (16) for antisymmetric buckling is much more difficult to interpret. If the filling is so rigid that the effects of C and L can be disregarded, the problem reduces to that of an ordinary plate and so for minimum P, which will be written P_E, the condition is $\lambda = b$. This gives

$$P_E = \frac{8\pi^2 Eth^2}{(1-\sigma^2)b^2} \qquad (20)$$

* λ/h is proportional to $(Et^3/ch^3)^{1/4}$ which in practice is of the order of unity.

Now so long as λ is of the same order as b, inspection of equation (16) shows that of the various terms of the correcting fraction only the unities and the term involving L in the denominator need be retained. Under these conditions equation (16) can be written

$$\frac{P}{P_E} = \frac{1}{4}\left(\frac{b}{\lambda} + \frac{\lambda}{b}\right)^2 \Big/ \left\{1 + \frac{P_E}{4P_s}\left(1 + \frac{b^2}{\lambda^2}\right)\right\} \quad \ldots \quad (21)$$

where $\quad P_s = 2hL$. $\quad \ldots \quad (22)$

The minimum value of P occurs when

$$\frac{\lambda}{b} = \left(\frac{1 - P_E/4P_s}{1 + P_E/4P_s}\right)^{1/2} \quad \ldots \quad (23)$$

and this yields for P_{crit} the formula

$$\frac{1}{P_{\text{crit}}} = \frac{1}{P_E} + \frac{1}{2P_s} + \frac{P_E}{16P_s^2} \cdot \quad \ldots \quad (24)$$

The formula (24) governs the overall buckling of a sandwich panel, as opposed to the short wave wrinkling which is governed by equation (19). Its range of accuracy is revealed by equation (23), which shows that it is certainly valid for $P_E < 3P_s$. Comparison may be made with the formula for a strut with low shear stiffness which is

$$\frac{1}{P_{\text{crit}}} = \frac{1}{P_E} + \frac{1}{P_s} \cdot \quad \ldots \quad (25)$$

where P_E is now the Euler load per unit length.

The relation (16) gives a further minimum value of P when $\lambda/b << 1$. Expansion in powers of λ/b up to λ^2/b^2 gives a formula with a minimum at

$$\lambda = \pi\left(\frac{hD}{3C}\right)^{1/4}. \quad \ldots \quad (26)$$

The corresponding critical value of P is:—

$$P_{\text{crit}} = 4\sqrt{3}\cdot\left(\frac{CD}{h}\right)^{1/2} . \quad \ldots \quad (27)$$

Comparison with equation (19) shows that the critical load for anti-symmetrical wrinkling is larger than that for the symmetrical variety.

5. *Bending.*—The problem of the overall buckling of a sandwich panel may be approached *via* a theory of bending of sandwich plates. This may be developed from the displacement formulae (5) by taking that part of the displacement which is antisymmetric about $z = 0$. The displacement at the face $z = h$, written (u', v', w'), is then given by

$$\left.\begin{aligned} u' &= \frac{h^3}{6C}\frac{\partial}{\partial x}\left(\frac{\partial Z_x}{\partial x} + \frac{\partial Y_z}{\partial y}\right) + h\left(\frac{Z_x}{L} - \frac{\partial w_0}{\partial x}\right) \\ v' &= \frac{h^3}{6C}\frac{\partial}{\partial y}\left(\frac{\partial Z_x}{\partial x} + \frac{\partial Y_z}{\partial y}\right) + h\left(\frac{Y_z}{L} - \frac{\partial w_0}{\partial y}\right) \\ w' &= \frac{-h^2}{2C}\left(\frac{\partial Z_x}{\partial x} + \frac{\partial Y_z}{\partial y}\right) + w_0. \end{aligned}\right\} \quad \ldots \quad (28)$$

The stress resultants T_1', T_2' and S' in the face $z = h$ are given by

$$\left.\begin{aligned} T_1' &= \frac{Et}{(1-\sigma^2)}\left(\frac{\partial u'}{\partial x} + \sigma\frac{\partial v'}{\partial y}\right) \\ T_2' &= \frac{Et}{(1-\sigma^2)}\left(\frac{\partial v'}{\partial y} + \sigma\frac{\partial u'}{\partial x}\right) \\ S' &= \frac{Et}{2(1+\sigma)}\left(\frac{\partial v'}{\partial x} + \frac{\partial u'}{\partial y}\right). \end{aligned}\right\} \qquad (29)$$

Neglecting the contribution from the bending of the faces, the formulae for the normal stress resultants N_1 and N_2 and the stress couples G_1, G_2 and H for the sandwich plate as a whole can be written:—

$$N_1 = 2hZ_x, \quad N_2 = 2hY_z \qquad (30)$$

$$G_1 = 2hT_1', \quad G_2 = 2hT_2', \quad H = -2hS'. \qquad (31)$$

The sign convention for the quantities T_1', T_2', S', N_1, N_2, G_1, G_2 and H is given in Fig. 2.

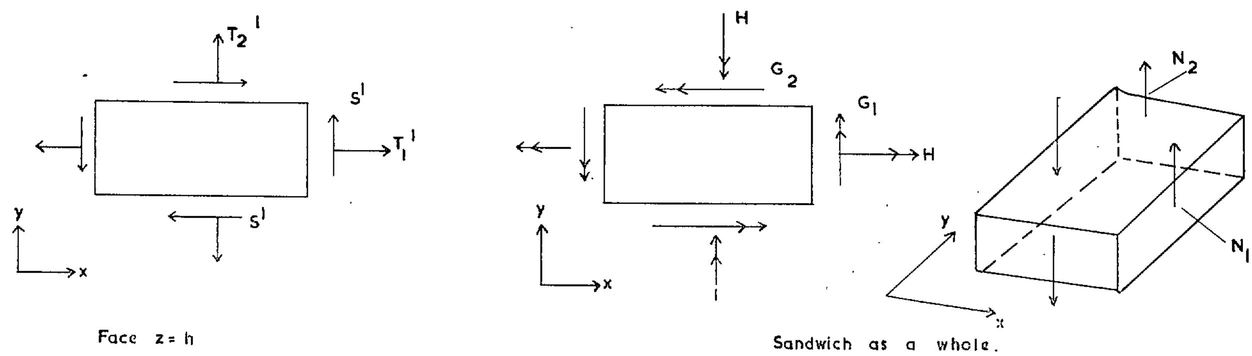

FIG. 2.

The quantities Z_x and Y_z may be eliminated using equation (30). Relations between G_1, G_2, and H and the normal displacement of the middle surface w_0 can be obtained by substituting from equation (28) into equation (29) and thence into equation (31). The result may be written:

$$\left.\begin{aligned} G_1 &= -D_1\left\{K_1 + \sigma K_2 - \frac{1}{2hL}\left(\frac{\partial N_1}{\partial x} + \sigma\frac{\partial N_2}{\partial y}\right) + \frac{h}{12C}\left(\frac{\partial^2 p}{\partial x^2} + \sigma\frac{\partial^2 p}{\partial y^2}\right)\right\} \\ G_2 &= -D_1\left\{K_2 + \sigma K_1 - \frac{1}{2hL}\left(\frac{\partial N_2}{\partial y} + \sigma\frac{\partial N_1}{\partial x}\right) + \frac{h}{12C}\left(\frac{\partial^2 p}{\partial y^2} + \sigma\frac{\partial^2 p}{\partial x^2}\right)\right\} \\ H &= D_1(1-\sigma)\left\{\tau - \frac{1}{4hL}\left(\frac{\partial N_2}{\partial x} + \frac{\partial N_1}{\partial y}\right) + \frac{h}{12C}\frac{\partial^2 p}{\partial x\,\partial y}\right\} \end{aligned}\right\} \qquad (32)$$

where $$D_1 = \frac{2Eth^2}{(1-\sigma^2)} \qquad (33)$$

(22712)

$$K_1 = \frac{\partial^2 w_0}{\partial x^2}, \quad K_2 = \frac{\partial^2 w_0}{\partial y^2}, \quad \tau = \frac{\partial^2 w_0}{\partial x \, \partial y} \qquad (34)$$

and p is the transverse load per unit area of the plate, which is given by equation (35) below. Equations (32) generalise the usual bending moment—curvature relations to allow for flexibility of the filling in shear and transverse tension and compression. In practice the terms in p are usually small and, therefore, may be omitted.

The theory of the bending of sandwich plates is completed by the usual equilibrium equations:–

$$\left.\begin{aligned} &\frac{\partial N_1}{\partial x} + \frac{\partial N_2}{\partial y} + p = 0 \\ &\frac{\partial G_1}{\partial x} - \frac{\partial H}{\partial y} - N_1 = 0 \\ &-\frac{\partial H}{\partial x} + \frac{\partial G_2}{\partial y} - N_2 = 0. \end{aligned}\right\} \qquad (35)$$

6. *Alternative Calculation of Overall Compression Buckling.*—A calculation of the buckling load for compression buckling with half-wave length λ of the order of b can be based upon the bending theory of section 5. Allowance for the initial compression P is made by writing

$$p = -P \frac{\partial^2 w_0}{\partial x^2} \qquad (36)$$

The equations (32) and (35) are solved by writing

$$\left.\begin{aligned} &w_0 = W \sin \frac{\pi x}{\lambda} \sin \frac{\pi y}{b} \\ &N_1 = n_1 \cos \frac{\pi x}{\lambda} \sin \frac{\pi y}{b} \qquad N_2 = n_2 \sin \frac{\pi x}{\lambda} \cos \frac{\pi y}{b} \\ &G_1 = g_1 \sin \frac{\pi x}{\lambda} \sin \frac{\pi y}{b} \qquad G_2 = g_2 \sin \frac{\pi x}{\lambda} \sin \frac{\pi y}{b} \\ &H = h_1 \cos \frac{\pi x}{\lambda} \cos \frac{\pi y}{b} \end{aligned}\right\} \qquad (37)$$

where W, n_1, n_2, g_1, g_2, and h_1 are constants. Substitution from equations (37) and the elimination of these constants yields the following formula for P:—

$$P = \frac{\pi^2 D_1}{b^2} \left(\frac{b}{\lambda} + \frac{\lambda}{b}\right)^2 \Big/ \left\{1 + \frac{\pi^2 D_1}{2hLb^2}\left(1 + \frac{b^2}{\lambda^2}\right)\right\} \qquad (38)$$

It is to be remarked that the terms in p in equation (32) have been omitted. Inspection of equations (33) and (20) shows that equation (38) is identical with equation (21). The approach *via* the bending theory of section 5 yields the same result for overall buckling as the more exact calculations of section 3. This suggests the possible application of the formulae (32) to more difficult problems, such as those of the buckling of curved shells.

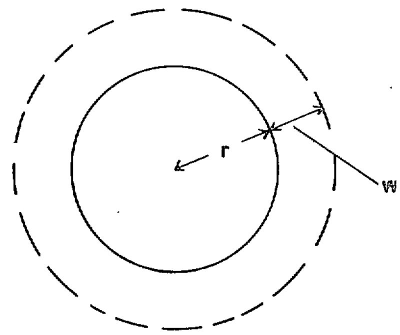

FIG. 3.

7. *Symmetrical Buckling of a Circular Cylinder in Compression.*—The application of the formulae (32) to problems of curved shells may be exemplified by the simple case of the buckling of a circular cylinder in a symmetric mode. The assumed cross-sectional deformation is shown in Fig. 3. 'w' the radial displacement is a function of x the distance along the axis of the cylinder. The hoop tensile strain ε_2 is w/r. Assuming no change in direct stress parallel to the axis, it follows that the x-wise strain ε_1 has the value $-\ \sigma w/r$.

The hoop tension T_2 is then given by

$$T_2 = \frac{2Et}{(1 - \sigma_2)} (\varepsilon_2 + \sigma\varepsilon_1) = 2Et\frac{w}{r} \quad \ldots \quad (39)$$

The equations of equilibrium are:—

$$\left.\begin{aligned} \frac{\partial N_1}{\partial x} - \frac{T_2}{r} + p = 0 \\ \frac{\partial G_1}{\partial x} - N_1 = 0 \end{aligned}\right\} \quad \ldots \quad (40)$$

where N_1 and G_1 are the shear and bending moment. The pressure p arises from the initial compression P and is given by:—

$$p = -\ P\frac{\partial^2 w}{\partial x^2} \quad \ldots \quad (41)$$

Finally the bending moment-curvature relation follows from formulae (32):—

$$G_1 = -\ D_1\left(\frac{\partial^2 w}{\partial x^2} - \frac{1}{2hL}\frac{\partial N_1}{\partial x}\right) \quad \ldots \quad (42)$$

Elimination of T_2, N_1, p and G_1 from equations (39), (40), (41) and (42) yields:—

$$\left(1 - \frac{D_1}{2hL}\frac{\partial^2}{\partial x^2}\right)\left(P\frac{\partial^2 w}{\partial x^2} + \frac{2Et}{r^2}w\right) + D_1\frac{\partial^4 w}{\partial x^4} = 0 \quad \ldots \quad (43)$$

The critical load is obtained from equation (43) by assuming w proportional to sin $\pi x/\lambda$. This yields the result:—

$$\frac{P}{P_E} = \left\{\left(\frac{\lambda_E}{\lambda}\right)^2 + \left(\frac{\lambda}{\lambda_E}\right)^2 + \frac{1}{2}\frac{P_E}{P_S}\right\}\Big/\left\{2 + \frac{P_E}{P_S}\left(\frac{\lambda_E}{\lambda}\right)^2\right\} \quad \ldots \quad (44)$$

where,

$$\left.\begin{aligned} P_E &= \frac{2}{r}(2EtD_1)^{1/2} \\ \lambda_E &= \pi\left(\frac{D_1 r^2}{2Et}\right)^{1/4} \end{aligned}\right\} \quad \ldots \quad (45)$$

P_E and λ_E are the buckling load and half wave length for the case where shear flexibility of the filling is small. P_S is given by equation (22). The minimum value of P in equation (44) occurs when

$$\left(\frac{\lambda}{\lambda_E}\right)^2 = 1 - \frac{1}{2}\frac{P_E}{P_S} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (46)$$

This gives for P_{cri} the formula:—

$$P_{\mathrm{crit}} = P_E\left(1 - \frac{P_E}{4P_S}\right) \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (47)$$

Equation (47) is valid so long as equation (46) yields a wave length sufficiently long to justify the use of the bending theory of section **5**. For practical application $P_E < \frac{3}{2}P_S$ would seem quite a reasonable limitation.

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