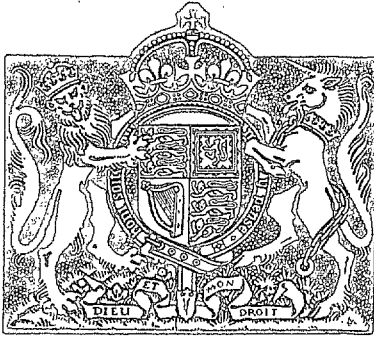


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An Approximation Simplifying Wing Flutter Calculations

By

G. A. NAYLOR, D.F.C., B.Sc., A.F.R.A.E.S.

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An Approximation Simplifying Wing Flutter Calculations

By

G. A. NAYLOR, D.F.C., B.Sc., A.F.R.A.E.S.

COMMUNICATED BY THE PRINCIPAL DIRECTOR OF SCIENTIFIC RESEARCH (AIR),
MINISTRY OF SUPPLY

*Reports and Memoranda No. 2605**

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Summary.—This report shows that the application of classical flutter theory¹ to the determination of wing flexural-torsional flutter speeds is considerably simplified by the omission of a term which is usually the very small difference between two small quantities. With this simplification it is possible to derive a formula giving the critical speed explicitly in terms of the dynamical coefficients. Numerical examples show that this approximation gives practically the same flutter speeds as the complete classical theory, even when the coefficients are given values which do not normally occur. A simpler approximate formula is obtained by a combination of the first approximation with Pugsley's simplified theory²; this second approximation gives flutter speeds for normal wings which agree with those from classical theory.

1. *Introduction.*—Several attempts have been made to simplify the classical theory of flexural-torsional wing flutter, perhaps the most notable being Pugsley's simplified theory.² In the practical application of Pugsley's theory, however, experience has indicated certain disadvantages:—

- (1) the flutter speed is normally derived graphically from the intersection of two curves; direct solution for the speed requires the use of a calculating machine;
- (2) it is troublesome to check whether or not the motion is stable at airspeeds below the flutter speed;
- (3) although the theory gives good results for 'normal' wings, it becomes inaccurate in certain cases, particularly when the separation between the flexural and inertia axes is unusually small, and when the aerodynamic torsional damping coefficient, J_s , is varied.

The present report is concerned with an approximation to classical theory¹ which enables wing-flutter speeds to be calculated more quickly than by either classical theory or Pugsley's method, and more accurately than by the latter; moreover, for this approximation a slide rule gives sufficient accuracy. The stability of the motion at any airspeed can be determined directly from terms required for the evaluation of the flutter speed. The approximation is obtained by assuming that one of the terms appearing in the classical theory—it is usually comparatively small—vanishes. In section 2 below the theoretical justification for this assumption and the resulting equation for the flutter speed are given. The corresponding formulae derived from classical theory and Pugsley's theory are also given in section 2. Numerical examples to illustrate the accuracy of the approximation are given in section 3.

* R.A.E. Report S.M.E. 3211.

The method of this report is considered to be an adequate substitute for the more elaborate classical method in all practical cases, and can be used where computational difficulties might preclude the use of the classical method.

2. *Methods for Evaluating Wing Flutter Speeds.*—2.1. *Classical Theory*¹.—For the usual co-ordinates, namely, displacement ϕl of the flexural centre at some given reference section, and wing twist θ at this section, the equations* of motion are

$$A_1\ddot{\phi} + B_1\dot{\phi} + C_1\phi + P\ddot{\theta} + J_1\dot{\theta} + K_1\theta = 0 \quad \dots \dots \dots (1)$$

$$P\ddot{\phi} + B_3\dot{\phi} + G_3\ddot{\theta} + J_3\dot{\theta} + K_3\theta = 0 \quad \dots \dots \dots (2)$$

With the notation

$$\left. \begin{aligned} B_1 &= B_1'V \\ B_3 &= B_3'V \\ J_1 &= J_1'V \\ J_3 &= J_3'V \\ K_1 &= K_1'V^2 \\ *K_3 &= m_0 + K_3'V^2 \\ C_1 &= l_\phi \end{aligned} \right\} \dots \dots \dots (3)$$

and

$$\left. \begin{aligned} a &= A_1G_3 - P^2 \\ b &= A_1J_3' + B_1'G_3 - P(J_1' + B_3') \\ c &= A_1m_0 + G_3l_\phi \\ d &= A_1K_3' + B_1'J_3' - B_3'J_1' - PK_1' \\ e &= B_1'm_0 + J_3'l_\phi \\ f &= B_1'K_3' - B_3'K_1' \\ g &= l_\phi m_0 \\ k &= l_\phi K_3' \end{aligned} \right\} \dots \dots \dots (4)$$

then from classical theory the flutter speed V_c is given by the following equation:—

$$f(bd - af) V_c^4 + \{f(bc - 2ae) - b(bk - ed)\} V_c^2 + (bce - ae^2 - b^2g) = 0 \quad \dots \dots (5)$$

The motion is stable up to the flutter speed V_c given by equation (5), if all the quantities a , b , $(c + dV^2)$, $(e + fV^2)$, $(g + kV^2)$ and $(bce - ae^2 - b^2g)$ are positive for $V < V_c$. The solution of equations (1) and (2) can also be obtained in the form of the following two simultaneous equations:—

$$V_c^2 = \frac{a\phi^4 - c\phi^2 + g}{d\phi^2 - k} = \frac{(A_1G_3 - P^2)\phi^4 - (A_1m_0 + G_3l_\phi)\phi^2 + l_\phi m_0}{(A_1K_3' + B_1'J_3' - B_3'J_1' - PK_1')\phi^2 - K_3'l_\phi} \quad \dots \dots (6a)$$

$$\phi^2 = \frac{e + fV_c^2}{b} = \frac{B_1'm_0 + J_3'l_\phi + (B_1'K_3' - B_3'K_1')V_c^2}{A_1J_3' + B_1'G_3 - P(J_1' + B_3')} \quad \dots \dots \dots (6b)$$

where the flutter frequency is equal to $\phi/2\pi$.

* The notation throughout this report is mainly that of R. & M's 1155¹, 1839² and 1782³, except that K_3 is taken as $m_0 + K_3'V^2$ whereas Pugsley² uses $m_0 - K_3'V^2$.

Unless the lengthier graphical method is adopted it is essential to use a calculating machine for the direct solution of equations (5) or (6a) and (6b) for a normal wing without wing engines, because of the small differences involved.

2.2. *Pugsley's Simplified Theory*².—This theory ignores the indirect aerodynamical damping derivatives B_3 and J_1 ; its results can be obtained by putting $B_3 = J_1 = 0$ in equations (1) to (6). Pugsley gives his results in the form of two simultaneous equations; the first is equation (6b) with $B_3 = J_1 = 0$ and the second is obtained by eliminating K_3^1 from the equations found by putting $B_3 = J_1 = 0$ in equations (6a) and (6b); the resulting equations are

$$V_c^2 = p^2 \left[\frac{P^2}{PK_1' - B_1' J_3'} \left\{ 1 + \frac{J_3'}{B_1'} \frac{A_1^2}{P^2} \left(1 - \frac{1}{p^2} \frac{l_\phi}{A_1} \right)^2 \right\} \right], \quad \dots \dots \dots (7a)$$

$$p^2 = \frac{m_0 + K_3' V_c^2 + \frac{J_3'}{B_1'} l_\phi}{G_3 + \frac{J_3'}{B_1'} A_1} \dots \dots \dots (7b)$$

Equation (7) is simpler than (6) but *direct* calculation of V_c still requires the use of a calculating machine.

2.3. *Proposed Approximation to Classical Theory*¹.—2.3.1. *Consideration of quantity f (defined by equation (4))*.—The values of the aerodynamic coefficients for a semi-rigid wing are given by equation (7) of R. & M. 1782³; with these values, if the flexural centres everywhere are at the same fraction of their respective wing chords behind the leading edge (*i.e.*, h constant) f is given by

$$f = \rho^2 l^4 c_0^3 l_w (-m_a - h l_a) \left\{ \int (c/c_0) f(\eta)^2 d\eta \times \int (c/c_0)^2 F(\eta)^2 d\eta - \int (c/c_0) f(\eta) F(\eta) d\eta \times \int (c/c_0)^2 f(\eta) F(\eta) d\eta \right\} \dots \dots \dots (8)$$

where the integrals are from root to tip.

Equation (8) shows that $f = 0$ when either

$$h = -m_a/l_a = \frac{1}{4} \quad \dots \dots \dots (9a)$$

or $f(\eta) = F(\eta) \quad \dots \dots \dots (9b)$

or $f(\eta) = c/c_0 F(\eta) \quad \dots \dots \dots (9c)$

Equation (9a) is satisfied when the flexural centre coincides with the aerodynamic centre at each section, and equation (9b) is satisfied when the flexural mode is the same as the torsional mode. In general, none of these conditions is satisfied but for most wings (without wing engines) the flexural axis is not far behind the aerodynamic centre and the flexural and torsional modes are very similar. Thus the expression in the brackets $\{ \}$ of equation (8) will usually be the difference of two nearly equal quantities and will be small. The quantity $(-m_a - h l_a)$ will also be small. Thus f will usually be very small.

For example, for the wing of R. & M. 1782³

$$f = 1.5 (0.4 - 0.48) (0.269 - 0.262) \rho^2 l^4 c_0^3$$

Experience has shown that in the case of a normal wing, f is the only one of the quantities defined by equation (4) which is the difference of two nearly equal quantities.

2.3.2. *Approximation to classical theory by taking $f = 0$.*—From equation (5), when $f = 0$ the flutter speed is given explicitly by the equation.

$$V_c^2 = \frac{bce - ae^2 - b^2g}{b(bk - ed)} \quad \dots \quad (10)$$

Since normally there are no small differences involved, this expression can be evaluated by the use of a slide rule; a calculating machine is unnecessary. Comparison of equations (5) and (10) indicates that the approximation will probably fail when $b(bk - ed)$ becomes small, since the flutter speed will then be very high, a larger error will often be permissible.

The solution in terms of flutter speed and frequency is, from (6)

$$V_c^2 = \frac{ap^4 - cp^2 + g}{dp^2 - k} = \frac{(A_1G_3 - P^2)p^4 - (A_1m_0 + G_3l_\phi)p^2 + l_\phi m_0}{(A_1K_3' + B_1'J_3' - B_3'J_1' - PK_1')p^2 - K_3'l_\phi} \quad \dots \quad (11a)$$

(same as (6a))

$$p^2 = \frac{e}{b} = \frac{J_3'l_\phi + B_1'm_0}{A_1J_3' + B_1'G_3 - P(J_1' + B_3')} \quad \dots \quad (11b)$$

The expression for V_c directly in terms of the coefficients is from equations (4) and (10)

$$V_c^2 = \frac{\{B_1'(A_1m_0 - G_3l_\phi) + Pl_\phi(J_1' + B_3')\} \{J_3'(A_1m_0 - G_3l_\phi) - Pm_0(J_1' + B_3')\} + P^2(B_1'm_0 + J_3'l_\phi)^2}{(A_1J_3' + B_1'G_3 - PJ_1' - PB_3') [-B_1'K_3'(A_1m_0 - G_3l_\phi) + P\{K_1'(B_1'm_0 + J_3'l_\phi) - K_3'(J_1' + B_3')l_\phi\} - (B_1'J_3' - B_3'J_1')(B_1'm_0 + J_3'l_\phi)]} \quad (12)$$

It is better to use equation (10) for routine calculations and equation (12) for studying the variation of the flutter speed with any one coefficient.

A further approximation is obtained by taking Pugsley's assumption of $B_3 = J_1 = 0$ with equation (12), giving

$$V_c^2 = \frac{B_1'J_3'(A_1m_0 - G_3l_\phi)^2 + P^2(B_1'm_0 + J_3'l_\phi)^2}{(A_1J_3' + B_1'G_3) \{-B_1'K_3'(A_1m_0 - G_3l_\phi) + (PK_1' - B_1'J_3')(B_1'm_0 + J_3'l_\phi)\}} \quad (13)$$

This approximation consists essentially of first taking $B_1'K_3' = B_3'K_1'$ in classical theory and then taking $B_3' = J_1' = 0$.

A more consistent approximation is obtained by taking

$$B_3' = J_1' = B_1'K_3' = 0.$$

This gives

$$V_c^2 = \frac{B_1'J_3'(A_1m_0 - G_3l_\phi)^2 + P^2(B_1'm_0 + J_3'l_\phi)^2}{(A_1J_3' + B_1'G_3)(PK_1' - B_1'J_3')(B_1'm_0 + J_3'l_\phi)} \quad (14)$$

3. *Numerical Accuracy.*—3.1. *Wing of R. & M. 1839².*—Flutter speeds have been calculated from equations (10) and (13) for the various conditions of the wing discussed in R. & M. 1839²; the results are given in Tables 1 to 4 below. In addition the effect of the variation of the value of J_3' on flutter speed has been calculated by classical theory, Pugsley's theory and the approximation of this report. The results are plotted in Fig. 2, in which it is to be noted that the differences between equation (10) and classical theory results are so small that they cannot be shown on the graph. The data used for these calculations are given in the appendix to this report and in Fig. 1. A considerable amount of laborious calculation was avoided by taking the wing of R. & M. 1839² for which flutter speeds by classical theory had already been calculated.

TABLE 1
Changes of Wing Mass Balance

Product of Inertia P	Corresponding gap between flexural and inertia axes	Flutter Speed ft/sec					
		Simplified Theory ²	Classical Theory ¹	Equation (10)		Equation (13)	Equation (14)
				Calculating Machine	Slide‡ rule	Slide‡ rule	Slide‡ rule
23·1	0·05c	1520	1530	1525	1530	1459	2664
46·2*	0·10c	1000	1010	1007	1008	1012	1208
69·3	0·15c	870	870	870	880	881	978

TABLE 2
Changes of Wing Density

Relative† Density of Wing	Flutter Speed ft/sec				
	Simplified Theory ²	Classical Theory ¹	Equation (10)		Equation (13)
			Calculating Machine	Slide‡ Rule	Slide‡ Rule
0·5	1340	1470	1453	1462	1365
1·0*	1000	1010	1007	1008	1012
∞	840	820	817	816	838

TABLE 3
Changes of Wing Flexural Stiffness

Relative† Stiffness of Wing in Flexure	Flutter Speed ft/sec				
	Simplified Theory ²	Classical Theory ¹	Equation (10)		Equation (13)
			Calculating Machine	Slide‡ Rule	Slide‡ Rule
0	1420	1300	1299	1304	1330
1·0*	1000	1010	1007	1008	1012
2·0	775	800	799	804	789
3·0	680	667	667	666	674
4·0	658	608	608	608	664
5·0	739	614	614	611	738
6·0	795	666	666	669	858
7·0	880	745	744	750	997
10·0	1558	1031	1029	1039	1430

* standard wing.

† i.e. relative to standard wing given in the Appendix.

‡ An ordinary 20-in slide rule was used.

TABLE 4

Changes of Wing Flexural Axis Position
(Inertia Axis is at 0.4c)

Flexural axis position (Distance from wing leading edge)	Flutter Speed ft/sec				
	Simplified Theory ²	Classical Theory ¹	Equation (10)		Equation (13)
			Calculating Machine	Slide Rule	Slide Rule
0.25c	1000	1004	1004	1006	999
0.30c†	1000	1010	1007	1008	1012
0.35c	1130	1100	1099	1097	1069
0.40c	No flutter	1379	1356	1363	1181
0.41c	No flutter	1481	1445	1439	1200
0.43c	No flutter	1807	1695	1692	1278
0.44c	No flutter	2055	1836	1834	1828

† standard wing.

It will be seen that the suggested approximation (equation (10)) gives practically the same flutter speed as classical theory, except when the flexural axis is behind the inertia axis (*see* Table 4); in this case the approximation gives a flutter speed less than that from classical theory. The results show that a slide rule is sufficiently accurate for calculating flutter speeds by equation (10).

Only the flutter speeds for Table 1 were calculated from equation (14), this is a poor approximation which should not be used. Equation (13) does not rest on such a sound basis as equation (10), nevertheless it gives very good results for the wing conditions which are likely to occur in practice.

3.2. *Effect of varying f.*—The value of f occurring in the calculation of the classical theory flutter speeds of section 3.1 is small. To investigate the change of flutter speed with f , flutter speeds have been calculated by classical theory but with f equal to $B_1'K_3'$ (instead of $B_1'K_3' - B_3'K_1'$), thus increasing f approximately 40 times. The results are given in Table 5 below.

TABLE 5

Flutter Speed ft/sec		Wing Condition
Classical Theory	Classical Theory with $f = B_1'K_3'$	
1010	1040	Standard.
1470	1568	Standard except wing density 0.5 of Standard (Table 2)
1300	1422	Standard except flexural stiffness zero (Table 3).
1481	No flutter	Standard except flexural axis at 0.41c (Table 4).

These results show that a large change in the value of f has little effect on the flutter speed except when the flexural axis is behind the inertia axis.

3.3. *Flutter Speeds for a Number of Aircraft.*—The wing flutter speeds of a number of aircraft, which had been estimated by classical theory, were calculated from equations (10) and (13). Classical theory had entailed the use of a calculating machine, slide rule calculations only were used for the approximations. The results (in Table 6) show little difference between the three methods:

TABLE 6
Wing Flutter Speeds for Particular Aircraft

Aircraft	Flutter Speed m.p.h.		
	Classical Theory	Equation (10) Slide Rule	Equation (13) Slide Rule
1	1390	1386	1395
2	971	972	1000
3	1249	1250	1249
4	968	968	949
5	532	533	559
6	875	874	877
7	592	592	593

4. *Conclusions.*—The approximation (equation (10)) gives practically the same flutter speeds as classical theory except when the flexural axis is appreciably behind the inertia axis. The approximation leads to a linear equation for the flutter speed V_c , which can be evaluated by the use of a slide rule, whereas classical theory leads to a quadratic equation in V_c^2 and requires the use of a calculating machine throughout if reasonable numerical accuracy is to be ensured. It should be possible to use the approximation for simple investigations into the effects of changes in various parameters upon wing flutter speeds, investigations which might be extremely laborious if the full classical theory were used.

The approximation given by equation (13) is still more simple than the approximation given by equation (10), but does not rest on such a sound basis. It is sufficiently accurate to be used for calculating the flutter speed of a present day wing without wing engines (inertia axis about $0.4c$ from leading edge, flexural axis appreciably ahead of the inertia axis.)

5. *Further Developments.*—This approximation will apply equally well to the flexure-torsion flutter of tail planes and fins; it is suggested that a similar approximation may give good results for other binary flutter cases and possibly for ternary flutter.

APPENDIX

Data for the wing used in numerical examples sections 3.1 and 3.2.—The standard wing used in sections 3.1 and 3.2 is similar to the wing of R. & Ms. 1839² and 1782³. The plan of the wing and the modes of deformation are shown in Fig. 1. The flutter coefficients {see equations (1), (2) and (3)} for the standard wing are

$$\begin{array}{ll}
 A_1 = 1323 & B_3' = -0.904 \\
 A_3 = G_1 = P = 46.2 & J_3' = 1.31 \\
 G_3 = 15.1 & K_3' = -0.0675 \\
 B_1' = 53.2 & l_\phi = 7.27 \times 10^6 \\
 J_1' = 11.46 & m_0 = 0.37 \times 10^6 \\
 K_1' = 3.88 &
 \end{array}$$

The standard wing has a density of 0.6 lb/cu. ft with the flexural axis at 0.3c and inertia axis at 0.4c from the leading edge.

The variations from the standard wing are

Table 1— P varied

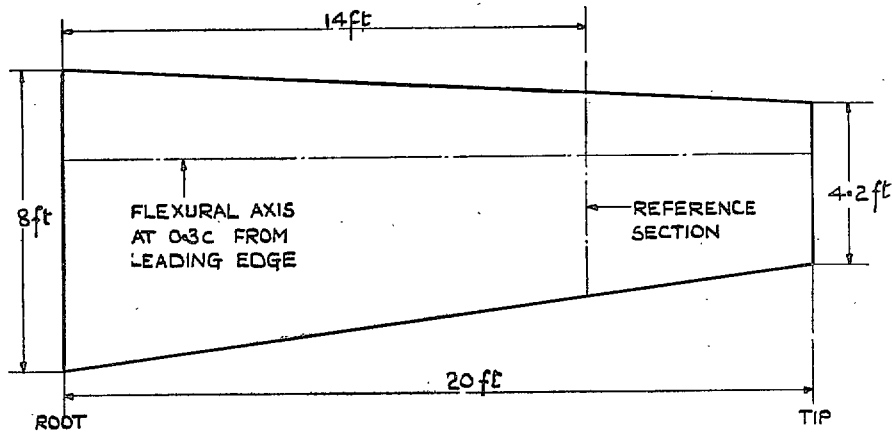
Table 2—Wing density varied; this affects A_1 , P and G_3

Table 3— l_ϕ varied

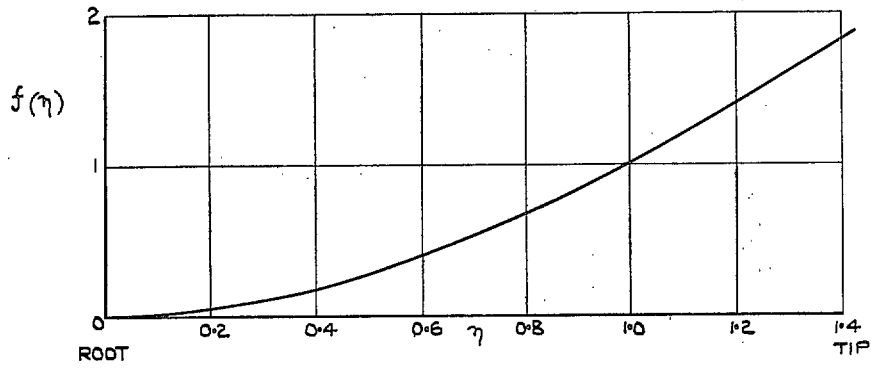
Table 4—Position of flexural axis varied; this affects P , J_1' , B_3' , J_3' and K_3' .

REFERENCES

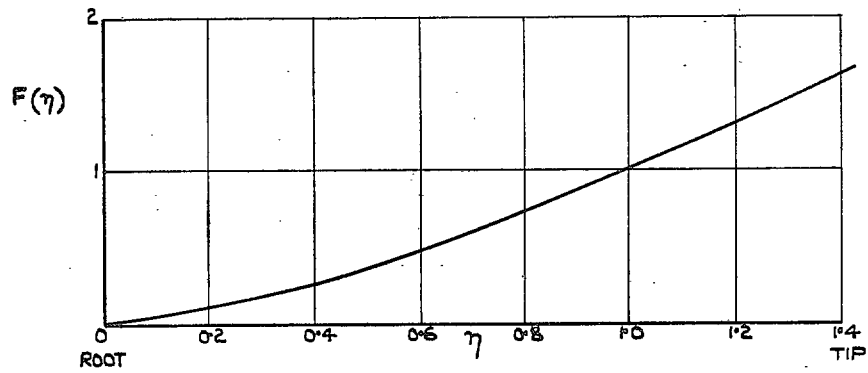
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| 2 | A. G. Pugsley | A Simplified Theory of Wing Flutter. R. & M. 1839. 1938. |
| 3 | W. J. Duncan and H. M. Lyon .. | Calculated Flexural-Torsional Flutter Characteristics of some typical cantilever wings. R. & M. 1782. 1937. |
| 4 | W. J. Duncan and C. L. T. Griffith .. | The Influence of Wing Taper on the Flutter of Cantilever Wings. R. & M. 1869. 1939. |
-



A PLAN OF WING



B FLEXURAL MODE



C TORSIONAL MODE

FIG. 1. Plan and Modes for Standard Wing used in Numerical Examples. (Sections 3.1 and 3.2).

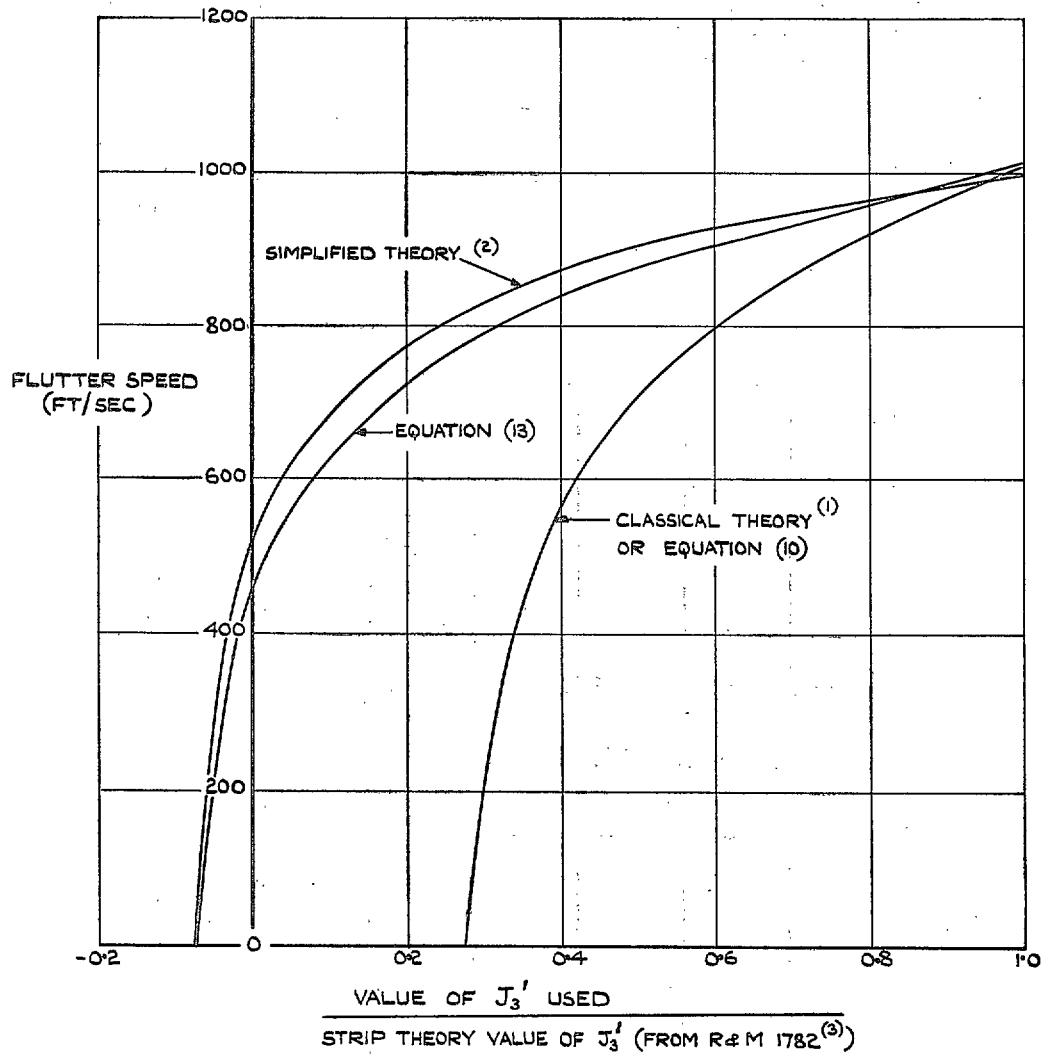


FIG. 2. Variation of Flutter Speed with Torsional Damping for Standard Wing.

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