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A Method of Estimating the Direct Stress Concentration round Holes in Reinforced Sheet

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M. FINE, B.A., and ANNE PELLEW, B.A.

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A Method of Estimating the Direct Stress Concentration round Holes in Reinforced Sheet

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Summary.—It was shown in Aeronautical Research Council Report No. 5455[1] that the accurate stress-function solution of certain two-dimensional problems of stress distribution may be replaced, with negligible error, by the approximate stringer-sheet solution (R. & M. Nos. 2099[3], 2100[4]). This report extends the comparison of the two methods to problems of stress concentration near holes. The stringer-sheet solution is not as accurate as before but the error in the direct stress is sufficiently low for the method to be of practical use.

It is proposed to apply the method to rectangular holes and to the problem of a hole reinforced at its edge.

1. *Introduction.*—In A.R.C. Report No. 5455[1], it was shown that for certain types of loading the stress distribution in a uniform flat sheet reinforced at its longitudinal edges by heavy flanges may be obtained with sufficient accuracy by the stringer-sheet method, provided that the sheet is considered fully effective in taking direct load as well as shear. In this report, the method is used to find approximately the stress concentration near a hole. The stringer-sheet solution is compared with the accurate solution for an elliptical hole in an unreinforced sheet which is subject, at infinity, to a uniform pull in the direction of one of the axes (Fig. 2).

2. *Fundamental Stringer-Sheet Equations.*—With axes as in Fig. 2, let

u	displacement in direction Ox, *i.e.* in the direction of the applied load,
v	displacement in direction Oy, *i.e.* in the transverse direction,
t	thickness of skin,
t_s	thickness of stringer sheet, *i.e.* stringer area per unit width of sheet plus effective skin thickness,†
t_s'	equivalent thickness of material which resists transverse load,
p	stress in direction Ox,
q	shear stress,
r	stress in direction Oy,
G, E	shear modulus and Young's modulus respectively.

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* R.A.E. Report S.M.E. 3199.

† Effective skin thickness is total thickness for unreinforced sheet in tension[1].

We assume that the skin takes only shear and the stringer sheet only direct stress in the direction Ox. With the loading of Fig. 2 it follows that $\partial v/\partial x = 0$ and

$$p = E\frac{\partial u}{\partial x}, \quad q = G\frac{\partial u}{\partial y} \quad \ldots \quad (1)$$

and the condition of equilibrium in the direction Ox of a small element gives

$$t_s\frac{\partial p}{\partial x} + t\frac{\partial q}{\partial y} = 0 \quad \ldots \quad (2)$$

On substituting from equation (1) in equation (2) we obtain

$$Et_s\frac{\partial^2 u}{\partial x^2} + Gt\frac{\partial^2 u}{\partial y^2} = 0\,. \quad \ldots \quad (3)$$

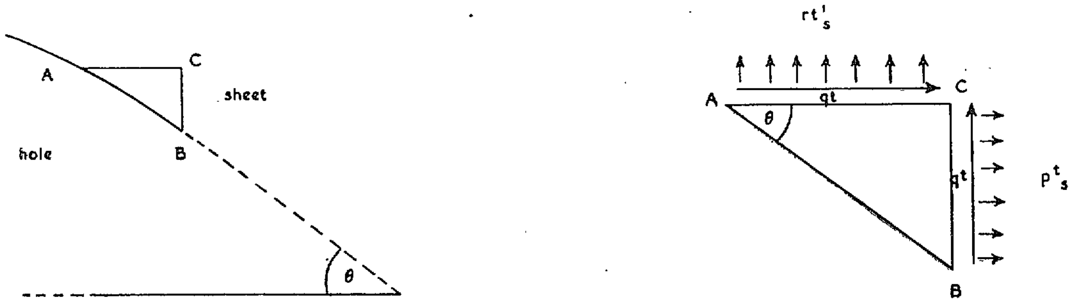
If in equation (3) we now put $x\sqrt{(Gt/Et_s)} = X$, $\quad \ldots \quad (4)$

it follows that $\quad \partial^2u/\partial X^2 + \partial^2u/\partial y^2 = 0\,. \quad \ldots \quad (5)$

Either of equations (3) or (5) may be regarded as the fundamental stringer sheet equation for a plane sheet. The equation and its solution are, of course, not exact because equilibrium in the transverse direction has been neglected. But in many cases the unbalanced transverse forces are so small that their effect is negligible and the stringer-sheet solution is sufficiently accurate for practical purposes.

The change of co-ordinates defined by equation (4) makes X less than the corresponding x and hence gives an artificial contraction in the direction Ox, the distance of any point from the y axis being divided by $\sqrt{(Et_s/Gt)}$ (*see*, for example, Fig. 2).

3. *Boundary Condition at Edge of Hole.*—



(a) (b)

FIG. 1.

Fig. 1 (a) shows a small element ABC of the sheet bounded by AB along the edge of the hole, by BC in the transverse direction and by CA in the direction of the applied pull. AB is inclined at θ to this last direction. Fig. 1 (b) is an enlarged view of the element ABC together with the forces acting on it per unit length of its boundary.

By resolving in the direction AC we obtain

$$p_h\, t_s \sin\theta + q_h t \cos\theta = 0 \quad \ldots \quad (6)$$

the suffix h indicating that the values of p, q must be taken on the boundary of the hole. The boundary condition is not completely satisfied as the element is not in equilibrium in the transverse direction. As stated above, however, it is assumed that the transverse forces rt_s' can be provided by the sheet without implying transverse movements so large as to vitiate the assumption that such displacements are negligible.

4. *Physical Analogies.*—The contraction of magnitude $\sqrt{(Gt/Et_s)}$ given by equation (4) transforms the original hole into a different hole which will be called the contracted or transformed hole. Similarly a point in the original sheet becomes a different point which will be called the transformed point.

It is readily shown that there is a complete analogy between the stringer-sheet stress distribution round the original hole and both (i) the flow of a perfect fluid past an obstacle of the same shape as the transformed hole, and (ii) the flow of electricity in a conducting plate containing a hole of the same shape as the transformed hole.

If	ϕ	velocity potential of fluid flow,
	U	velocity of flow in the direction Ox,
	V	velocity of flow in the direction Oy,
then	ϕ	corresponds to $\sqrt{(Gt/Et_s)}$,
	U	corresponds to $t_s p$
and	V	corresponds to $tq\sqrt{(Et_s/Gt)}$.

That is, direct load at a point in the sheet corresponds to longitudinal velocity at the transformed point and shear load multiplied by $\sqrt{(Et_s/Gt)}$ corresponds similarly to transverse velocity.

In the electrical analogy, ϕ is the potential and U, V the currents in the directions, Ox, Oy respectively.

5. *Elliptical Holes.*—No restriction has so far been imposed on the number, size or shape of the holes in the sheet. If the stringer-sheet solution agrees with the accurate solution, where this is known, we would expect the former to be generally reliable in cases where the latter is not known.

In order to provide a basis for comparison, an elliptical hole in a uniform unreinforced sheet is considered. The sheet is subject to a uniform pull at infinity in the direction of one of the axes. The transformed hole is also elliptical and the known fluid flow (analogy (i) of section 4) past an ellipse[5] gives the stringer-sheet stress distribution. This is compared with the known accurate solution[6].

6. *Results.—Direct Stress.*—If a and b are the semi-axes, a being in the direction of the pull, the maximum stress, which occurs at the ends of the transverse axis, is

$1 + 2b/a$ by the accurate method,

and $1 + (b/a)\sqrt{(E/G)}$, by the stringer-sheet method,

assuming in each case that there is unit stress at infinity.

The maximum stress as given by the stringer-sheet method is always less than the accurate stress since $\sqrt{(E/G)}$ is less than two. For a 2 : 1 ratio ellipse in steel or dural sheet the error is 17 per cent. too small. For ellipses of varying shape the error in this stress is plotted in Fig. 3. Although the stringer-sheet method is not accurate the error is sufficiently low to justify using the method as a good guide to maximum stress. Moreover when the sheet is stringer reinforced the error is less than for unreinforced sheet. If, therefore, the maximum stringer-sheet direct stress $1 + (b/a)\sqrt{(Et_s/Gt)}$ for an elliptic hole be evaluated and the correction indicated on Fig. 3 be added then the result will be on the safe side. Other shapes may often be replaced by roughly equivalent elliptical holes.

The distribution of direct stress at points on the transverse axis produced is shown in Fig. 4 for the circular hole. The agreement is sufficiently close to justify using the method, in conjunction with Fig. 3, in estimating the direct stress distribution along this line.

On the basis of this theory the effect of reinforcing a sheet with stringers is to increase the maximum direct stress from $1 + (b/a)\sqrt{(E/G)}$ to $1 + (b/a)\sqrt{Et_s/Gt)}$ and this gives a rough working rule for determining the maximum direct stress. This is, that in a stringer-reinforced sheet the maximum stress is the same as for an unreinforced sheet with a hole of shorter longitudinal axis, the axes being in the ratio $\sqrt{(t/t_s)}$.

Shear Stress.—The distributions of shear stress round the periphery of a circular hole are shown in Fig. 5. The agreement with the known distribution is not as good as that obtained for the direct stresses but the values obtained are useful even on a quantitative basis, as they err on the safe side.

It should be borne in mind that the stringers are assumed sufficiently numerous to be replaced by a continuous sheet. For this to be valid near the hole, it is necessary that the stringer pitch be small in comparison with the transverse axis of the hole. A pitch of less than a quarter of the transverse diameter will probably be small enough to justify the assumption of a continuous stringer sheet.

For cases where the stringer pitch is smaller than this it is probably best to treat the stringers which are cut and the immediately adjacent ones as discrete members. The very fact that there are only a few such stringers makes calculation easier.

7. *Conclusions.*—Although the stringer-sheet method does not give accurate results for the stresses near holes it gives a good guide to the distribution of direct stress and its maximum value for elliptical holes.

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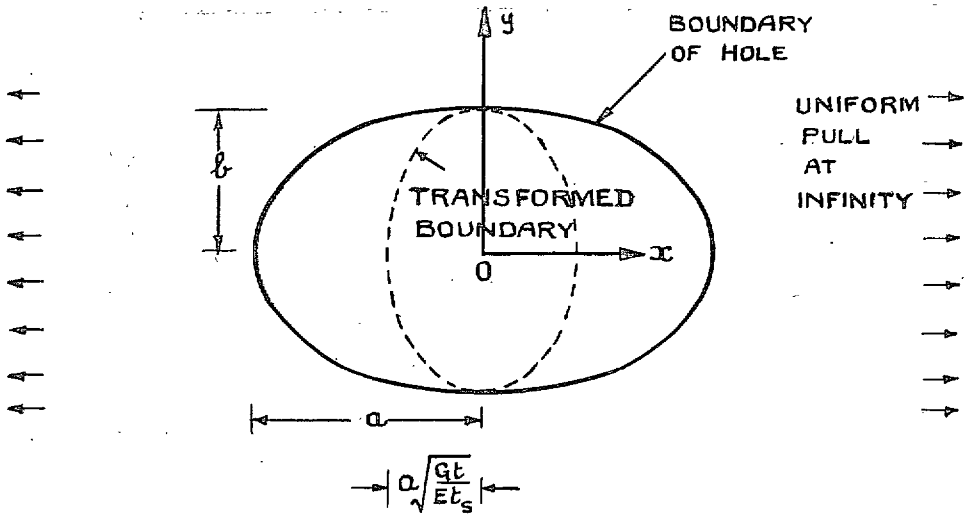

FIG. 2. Transformation of Boundary.

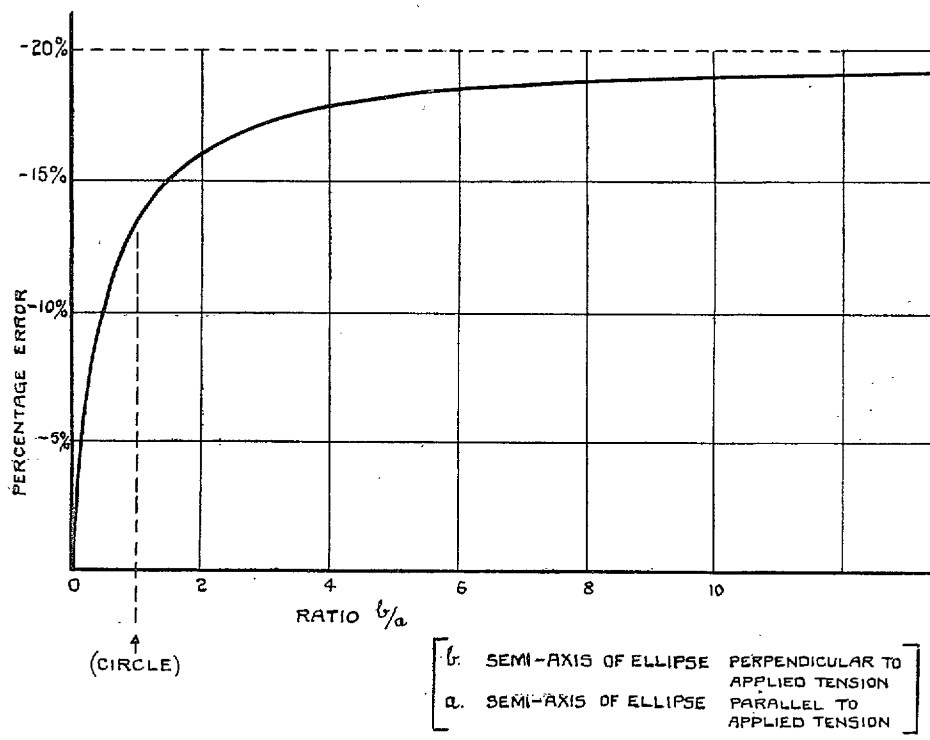

FIG. 3. Error in Maximum Direct Stress for Elliptical Hole as Calculated by Stringer-Sheet Method.

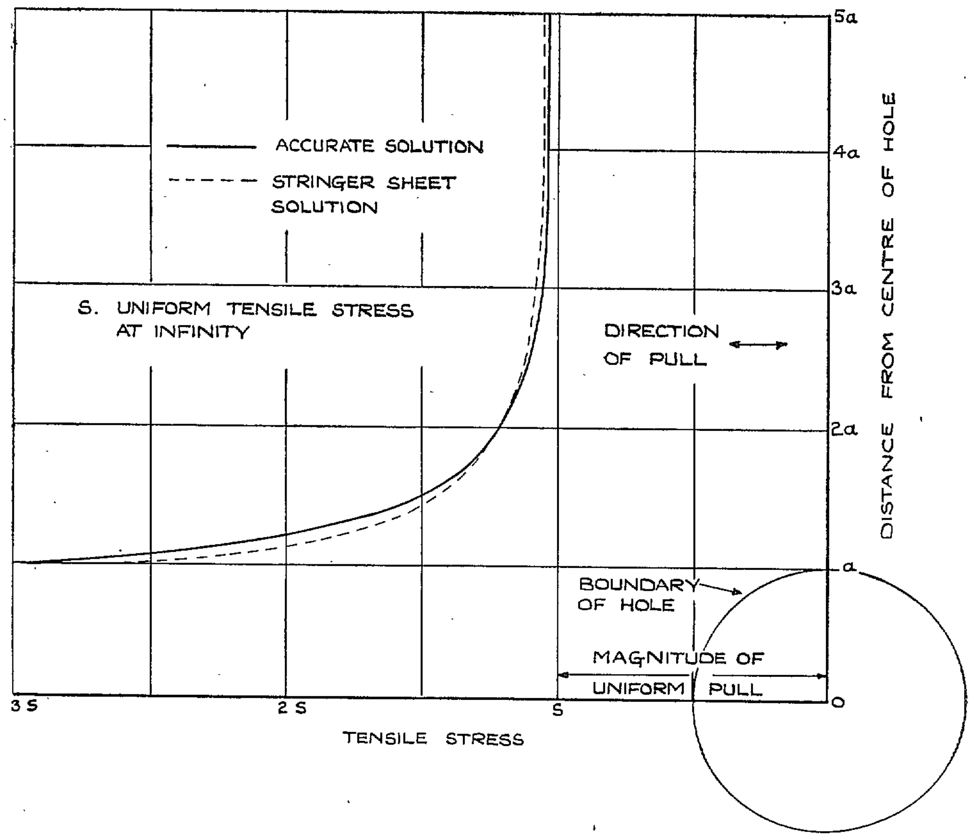

FIG. 4. Distribution of Direct Stress across Transverse Axis.

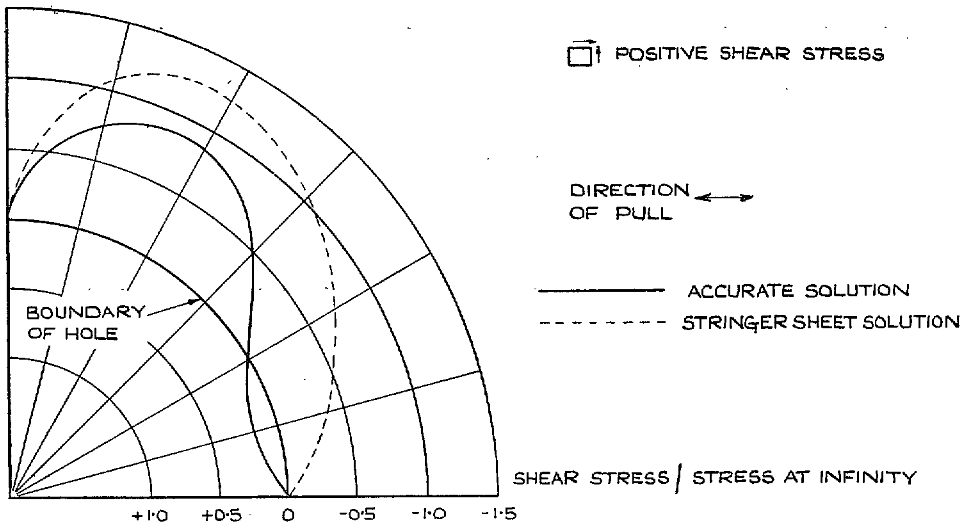

FIG. 5. Distribution of Shear Stress at Boundary of Hole.

(96204) Wt. 14/806 K 5 3/51 Hw.

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