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On the Momentum Equation in Laminar Boundary-Layer Flow.

A New Method of Uniparametric Calculation

By

B. THWAITES, M.A., PH.D., A.F.R.AE.S.

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Summary.—The general method of Pohlhausen, which is discussed in detail in Ref. 1, uses a uniparametric system of velocity distributions of the form $u/U = f(y/\delta) + \lambda g(y/\delta)$. Pohlhausen, by choosing simple forms for the functions f and g, then uses the momentum equation to find the distribution of δ with x and thence the distributions with x of the other boundary-layer characteristics. Several awkwardnesses exist in his method, especially when it is applied to problems dealing with a normal velocity at the boundary. In this paper, a new method is described of combining velocity distributions in the form $y/\theta = F(u/U) + \lambda G(u/U)$, and it is shewn that such a combination avoids several difficulties. This method of combination also allows a second parameter apart from λ, which might be found valuable in certain problems.

The method has been briefly described before[2] as part of an investigation into the effect of continuous suction on laminar boundary-layer flow under adverse pressure gradients. In that paper (R. & M. 2514[2]) a numerical example of its use was given. In this paper no example will be given because, as far as the author can see, the practical use of the method is superseded by the generalised method of Ref. 1: however it possessed considerable analytical interest.

1. *General Discussion*.—It is convenient in this general discussion to follow roughly the lines of thought that led eventually to the construction of the method of combining velocity distributions described in section 2.

Consider the problem of laminar boundary-layer flow past a flat plate in a uniform stream when there is a constant normal velocity into the plate. Several authors have attempted solutions and in Ref. 3 there is a detailed discussion of the work done upon the problem.

One method of approximate solution is the use of the momentum equation, which in this case is shewn in R. & M. 2481[4] to be:—

$$\frac{d\theta}{dx} = \frac{v_0}{U} + \frac{\nu}{U^2}\left(\frac{\partial u}{\partial y}\right)_{y=0} \qquad \ldots \qquad (1)$$

in which (x, y) are cartesian co-ordinates, (u, v) the corresponding velocity components, $\theta = \int_0^\infty \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$ is the momentum thickness and U is the stream velocity. $u \to U$ as $y \to \infty$ and the boundary is assumed to be $y = 0$, $x \geqslant 0$. ν is the kinematic coefficient of viscosity, and v_0 is the value of v at $y = 0$ and is the suction velocity (reckoned positive in the outward direction).

The boundary condition, obtained by putting $y = 0$ in the equation of motion, is

$$v_0 \left(\frac{\partial u}{\partial y}\right)_{y=0} = \nu \left(\frac{\partial^2 u}{\partial y^2}\right)_{y=0} . \qquad (2)$$

Let us attempt to apply Pohlhausen's method to equations (1) and (2). Using the distributions

$$\frac{u}{U} = 2\eta - 2\eta^3 + \eta^4 + \lambda \frac{\eta}{6} (1 - \eta)^3, \eta = \frac{y}{\delta} \qquad (3)$$

it is easy to shew that equation (2) becomes

$$\frac{v_0 \delta}{\nu} = \frac{-6\lambda}{\lambda + 12} \qquad (4)$$

and that equation (1) can be expressed in the form, after some work,

$$105\xi = 105 \frac{v_0^2 x}{U\nu} = \int_0^\lambda \frac{(12 - \lambda)(2664 + 174\lambda + 5\lambda^2)}{(\lambda + 12)(\lambda^2 - 12\lambda + 144)} d\lambda \qquad (5)$$

in which it has been arranged that $x = 0$ when $\lambda = 0$ (*i.e.*, $\delta = 0$, from equation (4)). $\xi = \frac{v_0^2 x}{U\nu}$ is a convenient variable.

Examine equation (5). We are only concerned with positive values of λ, which can be seen from equation (4), v_0 being negative. The integrand in equation (5) is positive for small values of λ, and hence ξ increases initially with λ. The integrand is always finite, since the denominator has no roots. The integrand is zero when the numerator is zero—the only root of the numerator is $\lambda = 12$. We deduce that x increases with λ for $\lambda < 12$ and reaches a maximum value at $\lambda = 12$. The actual value of this maximum is of no interest, for the problem requires a solution which allows all values of $x \geqslant 0$.

That Pohlhausen's method breaks down, in the case of solid boundaries, for $\lambda > 12$ is well-known and Dryden[5] has propounded a modification which extends the range of λ for which the general method is valid. (See also R. & M. 1632[7]). However, the fundamental difficulty still remains. The reason for this is explained in Ref. 1 which will also shew the peculiarity of the value $\lambda = 12$, and in that paper a new method is demonstrated which completely avoids the difficulty.

It is worthwhile to remark in passing on a fact that does not appear to have been mentioned before. For $\lambda > 12$, u/U has a maximum which is greater than unity, and upon this score alone, it would be unwise to continue to use the method for $\lambda > 12$ since it could no longer approximate satisfactorily to actual conditions.

In this paper, the author's first attempt to overcome the difficulty explained above is described. There is considerable general interest in the method and it simplifies very considerably the work in using the momentum equation. Before the method is described, however, it is instructive to note the characteristics of the Pohlhausen-type of solution which are undesirable.

First, a method is required which does not have the limitation described above. This leads to the requirement that the velocity distributions chosen should not have a maximum of u/U greater than unity. The use, furthermore, of the Pohlhausen-type of method in cases where, for example, the distributions at $x = 0$ and $x = \infty$ are known (as in the suction problem considered above) entails the integration of the product of the two velocity profiles apart from other awkward analytical work.* A third requirement is a general simplification of the analytical work.

* More exactly, if in the above problem, the family of distributions $u/U = B(\eta)(1 - K) + KA(\eta)$, ($B(\eta) \equiv$ Blasius' distribution, $A(\eta) \equiv$ Asymptotic suction distribution) is taken, following Preston[6], the work entails the integral, among others, $\int_0^\infty A(\eta)\, B(\eta)\, d\eta$, which it is awkward to compute.

2. *Theory*.—(1) Consider the velocity distribution given by

$$\frac{y}{\delta} = f\left(\frac{u}{U}\right)$$

or $$\frac{y}{\delta} = f(t),\ t = \frac{u}{U}, \qquad (6)$$

f being some function of t.

The conditions at the boundary and in the main stream are

$$f(0) = 0, f(1) = \infty. \qquad (7)$$

Let f be monotonic for $0 \leqslant t \leqslant 1$. Equation (6) then represents a velocity distribution which has no maximum of u/U other than unity.

The momentum and displacement thicknesses of equation (6) may be evaluated as follows.

$$\begin{aligned} \theta &= \int_0^\infty \frac{u}{U}\left(1 - \frac{u}{U}\right) dy \\ &= \delta \int_0^1 t(1-t)\, f'(t) dt \\ &= \delta \left[t(1-t)\, f(t)\right]_0^1 - \delta \int_0^1 (1-2t)\, f(t)\, dt \\ &= \delta \int_0^1 (2t-1)\, f(t)\, dt \qquad (8) \end{aligned}$$

in which it is assumed that

$$(1-t)\, f(t) \rightarrow 0 \text{ as } t \rightarrow 1^*.$$

Similarly, $$\begin{aligned} \delta^* &= \int_0^\infty \left(1 - \frac{u}{U}\right) dy \\ &= \delta \int_0^1 (1-t)\, f'(t)\, dt \\ &= \delta \int_0^1 f(t)\, dt. \qquad (9) \end{aligned}$$

So far, we have used δ as a convenient length-parameter. Its independent existence however is clearly not required, so let us put $\delta = \theta$. This, from equation (8), immediately places a condition upon $f(t)$ which is:

$$\int_0^1 (2t-1)\, f(t) = 1 \qquad (10)$$

This condition is fundamental in the simplicity of the method to follow, and henceforward, we shall assume that all functions, $f(t)$, $g(t)$ etc., representing velocity distributions satisfy a relation corresponding to equation (10)

* For Blasius' distribution $f(t) \sim A\sqrt{\log\,[1/(1-t)]}$ as $t \rightarrow 1$ and hence $(1-t)\, f(t) \rightarrow 0$ as $t \rightarrow 1$. For the asymptotic suction distribution, $f(t) \sim \log\,[1/(1-t)]$ and again the condition is satisfied. There seems little doubt that for all distribution $(1-t)\, f(t) \rightarrow 0$, but if not, we can restrict ourselves to the cases for which it is true.

We can now write

$$\frac{y}{\theta} = f(t),\ t = \frac{u}{U}. \qquad \qquad (11)$$

The significant result of this assumption is that the momentum thickness is not now a function of the ' shape ' of the velocity distribution; *i.e.*, in equation (11), $f(t)$ can be chosen in any way (provided equation (10) is satisfied) and θ remains independent. This characteristic enables velocity distributions to be ' added ' very simply, yet without imposing any condition upon θ.

Suppose now that $g(t)$ satisfies equation (10). Consider the velocity distribution given by

$$\frac{y}{\theta} = (1 - \lambda)\, f(t) + \lambda g(t). \qquad \qquad (12)$$

This is an admissible form for y/θ since

$$\int_0^1 [(1 - \lambda)\, f(t) + \lambda g(t)]\, dt = (1 - \lambda) + \lambda = 1$$

and hence the condition in equation (10) is satisfied by the form of equation (12). Also, for this form, equation (9) gives

$$\frac{\delta^x}{\theta} = H = (1 - \lambda) \int_0^1 f(t)\, dt + \lambda \int_0^1 g(t)\, dt. \qquad \qquad (13)$$

It is now evident that the Pohlhausen difficulty shewn explicitly in the footnote of the previous page does not arise in the family of distributions of equation (12) nor is there any analogous difficulty to be overcome except the satisfaction of equation (10) which is never difficult. Moreover, θ is now used as the principal length-variable, and, therefore, the work is simplified by the abolition of δ.

$(\partial u/\partial y)_{y=0}$ and $(\partial^2 u/\partial y^2)_{y=0}$ may be found for the distribution of equation (11) as follows:

Differentiation of equation (11) gives

$$\frac{dy}{\theta} = f(t)\, \frac{du}{U}$$

and so

$$\frac{\partial u}{\partial y} = \frac{U}{\theta}\, \frac{1}{f'(t)} \qquad \qquad (14)$$

the dash on f denoting differentiation with respect to t.

Equation (14) then gives

$$\frac{\partial^2 u}{\partial y^2} = \frac{U}{\theta}\, \frac{-f''(t)}{[f'(t)]^2}\, \frac{1}{U}\, \frac{\partial u}{\partial y}$$

and substitution from equation (14) gives

$$\frac{\partial^2 u}{\partial y^2} = -\frac{U}{\theta^2}\, \frac{f''(t)}{[f'(t)]^3}. \qquad \qquad (15)$$

All the necessary functions have now been obtained, and the formal expressions obtained by the use of the distributions of equation (12) in the momentum equation can be written down.

(2) For a solid boundary, the momentum equation is

$$U\frac{d\theta}{dx} = -\,\theta(H + 2)\,U' + \frac{\nu}{U}\left(\frac{\partial u}{\partial y}\right)_{y=0} \quad \ldots \quad (16)$$

and the condition at the boundary is

$$0 = UU' + \nu\left(\frac{\partial^2 u}{\partial y^2}\right)_{y=0} \quad \ldots \quad (17)$$

From equations (12) and (15), this boundary condition becomes

$$\frac{\theta^2 U'}{\nu} = \frac{f''(0) + \lambda[g''(0) - f''(0)]}{[f'(0) + \lambda(g'(0) - f'(0))]^3}. \quad \ldots \quad (18)$$

It is easy to re-arrange equation (16) and use equation (14) to obtain:

$$U\frac{d}{dx}\left(\frac{\theta^2 U'}{\nu}\right) = \frac{U'\theta^2}{\nu}\left(-\,2U'(H + 2) + \frac{UU''}{U'}\right) + \frac{2U'}{f'(0) + \lambda(g'(0) - f'(0))}. \quad (19)$$

Substitution of $\theta^2 U'/\nu$ from equation (18) will then give a differential equation for $\lambda(x)$ which can be speedily solved by a step-by-step method.

(3) The straightforward combination of equation (12) is the simplest possible. Its use is most effective when, in the problem to be solved, the distribution is known at two positions along the surface. That case suggests that $f(t)$, $g(t)$ should represent these two distributions and that distributions elsewhere should be approximated to by some such family as equation (12). This procedure was in fact adopted by the author in the constant-suction, flat-plate problem[4], and the results were quite good. In this case, the initial distribution is known to be Blasius' distribution as $x \to +0$, and as $x \to \infty$, the asymptotic suction distribution is reached.

However, in other cases, and especially when separation eventually occurs, it is probably desirable to take a more general form for the family of distributions.

Suppose $$\frac{y}{\theta} = F(t, \lambda) \quad \ldots \quad (20)$$

then we must have $$\int_0^1 (2t - 1)\,F(t, \lambda)\,dt = 1. \quad \ldots \quad (21)$$

If $F(t, \lambda)$ has been chosen originally and does not satisfy the relation in equation (21) then either of the two following forms for y/θ are clearly admissible:

$$\frac{y}{\theta} = \frac{F(t, \lambda)}{\int_0^1 (2t - 1)\,F(t, \lambda)\,dt} \quad \ldots \quad (22)$$

or $$\frac{y}{\theta} = \left(1 - \int_0^1 (2t - 1)\,F(t, \lambda)\,dt\right) A(t) + F(t, \lambda) \quad \ldots \quad (23)$$

in which $A(t)$ is an admissible function.

In the next paragraph we shall investigate the requirements of a distribution such as equation (20), and shew how a suitable function can be constructed.

3. In any flow past a solid boundary, the equation of motion gives the result that whenever $U' = 0, \left(\frac{\partial^2 u}{\partial y^2}\right)_{y=0} = 0$. The Blasius' distribution is the only distribution whose properties are well-known and for which $\left(\frac{\partial^2 u}{\partial y^2}\right)_{y=0} = 0$. It is natural, therefore, to seek a family of distributions of the form suggested by equation (23)

$$\frac{y}{\theta} = B(t)\left(1 - \int_0^1 (2t-1)\, G(t, \lambda)\, dt\right) + G(t, \lambda) \quad \ldots \quad (24)$$

in which

$$\frac{y}{\theta} = B(t), \int_0^1 (2t - 1)\, B(t)\, dt = 1,$$

represents Blasius' distribution. For this distribution:

$$\left.\begin{aligned} \left(\frac{\partial u}{\partial y}\right)_{y=0} &= 0\cdot 2204\,\frac{U}{\theta} \\ \left(\frac{\partial^2 u}{\partial y^2}\right)_{y=0} &= 0 \\ H &= 2\cdot 5911. \end{aligned}\right\}$$

Hence

$$\left.\begin{aligned} B'(0) &= 4\cdot 5345 \\ B''(0) &= 0 \\ \int_0^1 B(t)dt &= 2\cdot 5911. \end{aligned}\right\} \quad \ldots \quad (25)$$

In the choice of $G(t, \lambda)$ in equation (24), the first consideration is the obtaining of a separation distribution. A separation distribution must occur for some value of λ, and we can, therefore, take this value to be unity. The Blasius' distribution equally must correspond to some value of λ, and this value shall be zero. Thus

$$G = 0 \text{ for all } t, \text{ when } \lambda = 0. \quad \ldots \quad (26)$$

A separation distribution is obtained when $(\partial G/\partial t)_{t=0} = \infty$ for then $(\partial u/\partial y)_{y=0} = 0$. Suppose

$$\frac{\partial G}{\partial t} = \frac{1}{((1-\lambda) + t)^n}, \; n > 0. \quad \ldots \quad (27)$$

This gives a separation distribution for $\lambda = 1$.

From equation (27),

$$G(t) = \frac{1}{1-n}\,((1-\lambda) + t)^{-n+1} + g(\lambda). \quad \ldots \quad (28)$$

The condition $G(0) \equiv 0$ must be satisfied; this fixes $g(\lambda)$ so that

$$G(t) = \frac{1}{1-n}\left((1 - \lambda + t)^{-n+1} - (1 - \lambda)^{-n+1}\right). \quad \ldots \quad (29)$$

But when $\lambda = 1$, $G(0)$ must still be zero, and so $-n + 1 > 0$. This together with the condition on n in equation (27) gives

$$0 < n < 1.$$

Now from equation (15)

$$\left(\frac{\partial^2 u}{\partial y^2}\right)_{y=0} = -\frac{U}{\theta^2}\left(\frac{\partial^2 G/\partial t^2}{(\partial G/dt)^3}\right)_{t=0}$$

and so

$$\left(\frac{\partial^2 u}{\partial y^2}\right)_{y=0} = \frac{nU}{\theta^2}(1-\lambda)^{2n-1}. \quad \ldots \quad (30)$$

$(\partial^2 u/\partial y)^2_{y=0}$ is required to take both positive and negative values, which could be arranged if $2n - 1 > 0$, but that would involve the awkward factor $(1 - \lambda)^m$, $\frac{1}{2} < m < 1$ which is undesirable. Therefore, n is chosen to be $\frac{1}{2}$ which from equation (30) makes $(\partial^2 u/\partial y^2)_{y=0}$ independent of λ. This, of course, must not be so, and to avoid the difficulty, $G(t)$ may be multiplied by a function of λ. Thus equation (29) is now modified to

$$G(t) = h(\lambda)\left((1 - \lambda + t)^{1/2} - (1-\lambda)^{1/2}\right).$$

The condition in equation (26) has still to be satisfied, and so $G(t)$ is modified further to become

$$G(t) = h(\lambda)\left((1 - \lambda + \lambda t)^{1/2} - (1-\lambda)^{1/2}\right).$$

This general form can be made a little more elegant by writing

$$G(t) = C\lambda^p\left(\sqrt{\left[\left(\frac{1-\lambda^2}{2}\right)^2 + \lambda^2 t\right]} - \frac{1-\lambda^2}{2}\right)$$

so that the value at $t = 1$ takes a simpler form and the determination of the characteristic integral $\int_0^1 (2t - 1)\, G(t)\, dt$ and of H have been simplified. C and p are numbers to be fixed.

For this distribution we obtain

$$\int_0^1 (2t-1)\, G(t)\, dt = \frac{C\lambda^{p+2}}{6}\left(1 - \frac{\lambda^4}{5}\right)$$

and

$$H = \int_0^1 G(t)\, dt = C\frac{\lambda^{2+p}}{6}(\lambda^2 + 3).$$

Thus for the family of distributions, equation (24) gives

$$\frac{y}{\theta} = \left[1 - \frac{C\lambda^{p+2}}{6}\left(1 - \frac{\lambda^4}{5}\right)\right]B(t) + C\lambda^p\left[\sqrt{\left[\left(\frac{1-\lambda^2}{2}\right)^2 + \lambda^2 t\right]} - \frac{1-\lambda^2}{2}\right] \quad \ldots \quad (31)$$

in which the index p is the only remaining quantity to be fixed. It is easy to obtain from equation (31)

$$\left.\begin{aligned} \frac{\theta}{U}\left(\frac{\partial u}{\partial y}\right)_{y=0} &= \frac{1-\lambda^2}{j(\lambda)} \\ \frac{\theta^2}{U}\left(\frac{\partial^2 u}{\partial y^2}\right)_{y=0} &= \frac{2C\lambda^{p+4}}{j^3(\lambda)} \end{aligned}\right\} \quad \ldots \quad (32)$$

in which $\quad j(\lambda) = 4\cdot5345\ (1 - \lambda^2)\left[1 - \frac{C\lambda^{p+2}}{6}\left(1 - \frac{\lambda^4}{5}\right)\right] + C\lambda^{p+2}.$ .. .. .. (33)

$(\partial^2 u/\partial y^2)_{y=0}$ should take positive and negative values, and this suggests from equations (32) that either $p = -1$ or -3. $p = -3$ would involve a term in $1/\lambda$ in equation (33) which is unwanted, and so the value $p = -1$ is taken.

Therefore, we finally obtain the distribution

$$\frac{y}{\theta} = \left[1 - \frac{C\lambda}{6}\left(1 - \frac{\lambda^4}{5}\right)\right]B(t) + \frac{C}{\lambda}\left[\sqrt{\left[\left(\frac{1-\lambda^2}{2}\right)^2 + \lambda^2 t\right]} - \frac{1-\lambda^2}{2}\right] \quad .. \quad (34)$$

for which

$$\left.\begin{aligned}
\left(\frac{\partial u}{\partial y}\right)_{y=0} &= \frac{1-\lambda^2}{j(\lambda)} \\
\left(\frac{\partial^2 u}{\partial y^2}\right)_{y=0} &= \frac{2C\lambda^3}{j^3(\lambda)} \\
H &= 2\cdot5911\left[1 - \frac{C\lambda}{6}\left(1 - \frac{\lambda^4}{5}\right)\right] + \frac{C\lambda\,(\lambda^2+3)}{6} \\
j(\lambda) &= 4\cdot5345\ (1-\lambda^2)\left[1 - \frac{C\lambda}{6}\left(1 - \frac{\lambda^4}{5}\right)\right] + C\lambda.
\end{aligned}\right\} \quad .. \quad .. \quad (35)$$

4. *Comments upon the Method.*—A curious property of these distributions, and of this method of combination generally, is the presence of the constant C. Clearly, some value has to be given to C before the method can be used.

The problem to which this method was at first put is the flow past a boundary, at which there is a constant normal velocity, in a stream whose velocity is given by $U = \beta_0 - \beta_1 x$, β_0, β_1, being constants. Howarth[8], among others, has considered this problem for a solid boundary and predicted separation at $(\beta_1/\beta_0)x = 0\cdot120$. It therefore seemed desirable that, for an extension of this result for a porous boundary, the method used should give the correct result for zero normal boundary velocity. Thus in R. & M. 2514[2], the value of C was chosen so that the use of the momentum equation with the distributions in equation (34) would result in separation at $\beta_1 x/\beta_0 = 0\cdot120$. Calculations were therefore carried out for various values of C, and the value eventually chosen was $C = 5\cdot1$. For tables of the various functions employed when $C = 5\cdot1$, the reader is referred to R. & M. 2514[2].

Several authors have introduced a second parameter into a family of distributions so that a second condition, at the boundary or elsewhere, may be satisfied. For example, Sutton[9] used a second parameter in an approximation to Blasius' distribution in which he used the second integral equation (*i.e.*, that obtained by integrating across the boundary layer the equation of motion multiplied by u) in addition to the first integral or momentum equation. In a similar way, the equation of motion may be differentiated with respect to y to become $u\,\partial^2 u/\partial x\,\partial y + v\,\partial^2 u/\partial y^2 = \nu\,\partial^3 u/\partial y^3$ which will yield another boundary condition:

$$\left(\frac{\partial^3 u}{\partial y^3}\right)_{y=0} = 0 \qquad \text{for a solid boundary}$$

$$\nu\left(\frac{\partial^3 u}{\partial y^3}\right)_{y=0} = v_0\left(\frac{\partial^2 u}{\partial y^2}\right)_{y=0} = \frac{v_0^2}{\nu}\left(\frac{\partial u}{\partial y}\right)_{y=0} - \frac{UU'v_0}{\nu} \qquad \text{for a porous boundary.}$$

In these and other ways, therefore, a second parameter such as C may be used to give a greater degree of accuracy to the work. But the complexity of the work involved in the use of two independent parameters is so great in general as to outweigh completely the advantage of greater accuracy, which in any case may not amount to very much.

In the use of the method in the suction problem already referred to, a difficulty arose which is fully explained in R. & M. 2514[2] and which appeared to be an essential difficulty in continuous suction problems generally. It seemed possible that if the parameter C could be used in the right way, the difficulty might be avoided, but there was no clear indication how that was to be done. The investigation into the difficulty, however, drew the author's attention to the more fundamental aspects of the use of the momentum equation which are fully described in Ref. 1.

Conclusion.—A new method of combining velocity distributions for use in the momentum equation has been described, and the advantages of, and simplifications due to, its use are explained. Examples of its use can be found in R. & M. 2514[2] and R. & M. 2481[4], in which problems were treated with at least as much success as by other methods, and with considerably more ease.

REFERENCES

No.	*Author*	*Title, etc.*
1	B. Thwaites	Approximate Calculation of Laminar Boundary-layer Flow. *Aeronautical Quarterly*. August, 1949.
2	B. Thwaites	Investigations into the Effect of Continuous Suction in Laminar Boundary-layer Flow under Adverse Pressure Gradients. R. & M. 2514. April, 1946.
3	B. Thwaites	Notes on German Theoretical Work on Porous Suction. A.R.C. 9672. (Unpublished).
4	B. Thwaites	On the Flow past a Flat Plate with Uniform Suction. R. & M. 2481. February, 1946.
5	H. L. Dryden	Computation of the Two-dimensional Flow in a Laminar Boundary-layer. N.A.C.A. Report No. 497. 1934.
6	J. H. Preston	The Boundary-layer Flow over a Permeable Surface through which Suction is Applied. R. & M. 2244. February, 1946.
7	L. Howarth	On the Calculation of Steady Flow in the Boundary-layer near the Surface of a Cylinder in a Stream. R. & M. 1632. 1935.
8	L. Howarth	*Proc. Roy. Soc.* A, 164, 1938, 547–564.
9	W. G. L. Sutton	*Phil. Mag.* (7), 23, 1937, 1146–1152.

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