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B. THWAITES, B.A.,
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Summary.—A new method of performing boundary-layer calculations is introduced in this paper, and is applied to the problem of finding the characteristics of uniform flow past a flat plate through which there is a constant normal velocity.

An exact solution to this problem has not yet been found and it is therefore difficult to assess the accuracy of the results obtained. The results, however, are compared with those of two other methods[1, 2].

The new method will be applied to other problems and is explained in detail in Ref. 5. When the momentum equation is being used, one obvious advantage of the method is that, in " adding " velocity profiles, the momentum thickness of each may be added to give the momentum thickness of the whole. This is not so in the usual methods of boundary-layer calculations, and great simplification is thereby obtained.

1. *The Momentum Equation with Suction on the Boundary.*—The equations of motion in the boundary layer are, in the usual notation—

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{dU}{dx} + \nu\frac{\partial^2 u}{\partial y^2}, \qquad (1)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \qquad (2)$$

and the boundary conditions for the case of suction are

$$y = 0,\ u = 0,\ v = v_0(x) \qquad (3)$$

$$y = \infty, u = U. \qquad (4)$$

Writing $q = U-u$, we have at $y = 0,\ q = U$ (5)

and at $y = \infty,\ q = 0$, (6)

and equation (1) becomes

$$-q\frac{dU}{dx} - U\frac{\partial q}{\partial x} + q\frac{\partial q}{\partial x} - v\frac{\partial q}{\partial y} = +\nu\frac{\partial^2 u}{\partial y^2},$$

and after integrating with respect to y from 0 to ∞,

$$-\frac{dU}{dx}\int_0^\infty q\,dy - U\frac{\partial}{\partial x}\int_0^\infty q\,dy + \frac{1}{2}\frac{\partial}{\partial x}\int_0^\infty q^2 dy - [vq]_0^\infty + \int_0^\infty\left(\frac{\partial q}{\partial x} - \frac{\partial U}{\partial x}\right)q\,dy = \left[\nu\frac{\partial u}{\partial y}\right]_0^\infty, \qquad (7)$$

having integrated $v\dfrac{\partial q}{\partial y}$ by parts, and used (2).

Now $\delta^* = \int_0^\infty \left(1 - \frac{u}{U}\right) dy = \frac{1}{U} \int_0^\infty q dy$, hence $\int_0^\infty q dy = U\delta^*$.

Also $\theta = \int_0^\infty \frac{u}{U}\left(1 - \frac{u}{U}\right) dy = \frac{1}{U^2}\int_0^\infty q(U - q) dy = \delta^* - \frac{1}{U^2}\int_0^\infty q^2 dy$,

hence $\int_0^\infty q^2 dy = U^2\delta^* - U^2\theta$.

Hence (7) becomes

$$- 2 \frac{dU}{dx} U\delta^* - U \frac{\partial}{\partial x}(U\delta^*) + \frac{\partial}{\partial x}(U^2\delta^* - U^2\theta) + v_0(x)U = \left[\nu \frac{\partial u}{\partial y}\right]_0^\infty,$$

or

$$-2\, UU'\delta^* - UU'\delta^* - U^2 \frac{d}{dx}(\delta^*) + 2\, UU'[\delta^* - \theta] + U^2 \frac{\partial}{\partial x}(\delta^* - \theta)$$

$$+ v_0(x)\, U = \left[\nu \frac{\partial u}{\partial y}\right]_0^\infty,$$

or $$UU'(\delta^* + 2\theta) + U^2 \frac{d\theta}{dx} = v_0(x)U + \nu \left(\frac{\partial u}{\partial y}\right)_{y=0}. \qquad (8)$$

This is the momentum equation of the boundary layer on a flat plate in a stream of velocity U, and with the fluid having a velocity $v_0(x)$ normal to the surface of the plate.

For a flat plate in a uniform stream, with constant suction velocity v_0, equation (8) reduces to

$$\frac{d\theta}{dx} = \frac{v_0}{U} + \frac{\nu}{U^2}\left(\frac{\partial u}{\partial y}\right)_{y=0}. \qquad (9)$$

2. *Outline of the New Method.*—Several methods of boundary-layer calculations use the device of the 'addition' of velocity profiles. Two profiles are given: $u/U = f(y)$, $u/U = g(y)$, and a third profile is derived as $u/U = \lambda f(y) + \mu g(y)$. This type of method is awkward to use in conjunction with the momentum equation since the calculation of the momentum thickness of the derived profile presents difficulties, owing to the necessity of integrating the product $f(y)g(y)$. A second less obvious disadvantage is that when surface suction is applied such profiles in certain cases (in particular, when the Polhausen profiles are used[3]) have a maximum in u/U greater than one.

It seemed desirable to find a method which would avoid both the above mentioned difficulties Writing $t = u/U$, suppose we define a profile by the relation

$$y = f(t), \qquad (10)$$

where $f(t)$ is defined for the interval $0 \leqslant t \leqslant 1$, is monotonic in that interval and $f(0) = 0$. The second difficulty is at once disposed of. Let us now examine the displacement and momentum thickness, δ^* and θ respectively, of the profile (10).

$$\delta^* = \int_0^\infty \left(1 - \frac{u}{U}\right) dy$$

$$= \int_0^1 (1 - t)\frac{df(t)}{dt}\, dt$$

$$= \Big[(1 - t) f(t)\Big]_0^1 + \int_0^1 f(t) dt.$$

Hence $$\delta^* = \int_0^1 f(t)\, dt. \qquad (11)$$

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This is true if $f(1)$ is finite, or if $(1 - t) f(t) \rightarrow 0$ at $t \rightarrow 1$. The latter condition is true since the definition of a boundary layer implies $f(t) = o\left(\frac{1}{1-t}\right)$ as $t \rightarrow 0$.

Similarly: $$\theta = \int_0^\infty \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$
$$= \int_0^1 (t - t^2) f'(t)\, dt$$
$$= \Big[(t - t^2) f(t)\Big]_0^1 + \int_0^1 (2t - 1) f(t)\, dt$$

Hence $$\theta = \int_0^1 (2t - 1) f(t)\, dt. \qquad (12)$$

Suppose now that two profiles $y_1 = f(t)$, $y_2 = y(t)$, have momentum thicknesses θ_1, θ_2 respectively (for example, $\theta_1 = \int_0^1 (2t - 1) f(t)\, dt$) and are 'added' to form the profile $y = \lambda f(t) + \mu g(t)$, it is clear that the momentum thickness of the derived profile is $\lambda\theta_1 + \mu\theta_2$. This property is a great advantage of the method.

It is convenient to put (10) in the form

$$\frac{y}{\theta} = f(t), \qquad (13)$$

wherein the function must obviously satisfy the relation

$$\int_0^1 (2t - 1) f(t)\, dt = 1, \qquad (14)$$

having regard to equation (12).

Thus if we have two functions $f(t)$, $g(t)$, both satisfying a relation of the form of (14), and representing two profiles, we can form a new profile whose momentum thickness is θ at once

$$\frac{y}{\theta} = \lambda f(t) + (1 - \lambda)\, g(t)\,. \qquad (15)$$

Before proceeding to apply the method, which has been outlined above to the specific problem of the flat plate with constant suction in a uniform stream, we require two expressions.

Suppose
$$y/\theta = f(t),$$
then
$$\frac{\partial u}{\partial y} = \frac{U}{\theta}\frac{d(u/U)}{d(y/\theta)} = \frac{U}{\theta}\frac{1}{f'(t)}\,, \qquad (16)$$
a dash denoting differentiation.

Also $$\frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial y}\left(\frac{U}{\theta}\frac{1}{f'(t)}\right) = -\frac{U}{\theta^2}\frac{f''(t)}{[f'(t)]^3}\,. \qquad (17)$$

3. *Flat Plate in Uniform Stream with Constant Suction.*—The method to be followed is to choose two profiles, to add them together as suggested in (15), in which λ and θ are functions of x, and then satisfy the momentum equation with the derived profile. The two basic profiles will be the exact solution of the boundary-layer equations when x is large (given by Schlichting[2]) and

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the Blasius profile which is well-known. In Appendix I it is shown that when x tends to zero, towards the front of the plate, the profiles asymptotically approach the Blasius profile: Ref. 6 should be consulted for a more rigorous treatment.

For the Blasius profile we know that

$$\left.\begin{aligned} \left(\frac{\partial u}{\partial y}\right)_{y=0} &= 0\cdot 22053\,\frac{U}{\theta}, \\ \text{and}\qquad \left(\frac{\partial^2 u}{\partial y^2}\right)_{y=0} &= 0. \end{aligned}\right\} \qquad \ldots \quad (18)$$

Hence if $y/\theta = f(t)$ represents the Blasius profile, equations (16), (17) give

$$\left.\begin{aligned} f'(0) &= 4\cdot 53453 \\ f''(0) &= 0 \end{aligned}\right\} \qquad \ldots \quad (19)$$

The profile when $x \to \infty$ is the asymptotic suction profile given by

$$\left.\begin{aligned} v &= v_0, \\ u &= U\,(1 - e^{+(v_0/\nu)y}) \end{aligned}\right\}, \qquad \ldots \quad (20)$$

v_0 being the velocity through the plate. (20) represents an exact solution of the equations of motion, and was first given by Griffith and Meredith[4]. This profile is given by

$$\frac{y}{\theta} = 2\log\frac{1}{1-t} = g(t),$$

and we have

$$\left.\begin{aligned} g'(0) &= 2 \\ g''(0) &= 2 \end{aligned}\right\}. \qquad \ldots \quad (21)$$

Consider now the profile

$$y/\theta = (1-K)\,f(t) + Kg(t) \equiv F(t), \qquad \ldots \quad (22)$$

$f(t)$, $g(t)$ representing the Blasius and asymptotic suction profiles respectively and K being a function of x, such that

$$K(0) = 0,\ K(\infty) = 1 \qquad \ldots \quad (23)$$

Then

$$\left.\begin{aligned} F'(0) &= (1-K)\,4\cdot 53453 + 2K = 4\cdot 53453 - 2\cdot 53453K \\ F''(0) &= 2K. \end{aligned}\right\} \qquad \ldots \quad (24)$$

Equations (16), (17) then give

$$\left.\begin{aligned} \left(\frac{\partial u}{\partial y}\right)_{y=0} &= \frac{U}{\theta}\,\frac{1}{4\cdot 53453 - 2\cdot 534453K}, \\ \left(\frac{\partial^2 u}{\partial y^2}\right)_{y=0} &= -\frac{U}{\theta^2}\,\frac{2K}{[4\cdot 53453 - 2\cdot 53453K]^3} \end{aligned}\right\} \qquad \ldots \quad (25)$$

The equation of motion gives, at $y = 0$, the boundary condition

$$v_0\left(\frac{\partial u}{\partial y}\right)_{y=0} = \nu\left(\frac{\partial^2 u}{\partial y^2}\right)_{y=0}, \qquad \ldots \quad (26)$$

which becomes, using equations (25),

$$\theta = -\frac{\nu}{v_0}\frac{2K}{[4\cdot 53453 - 2\cdot 53453K]^2}. \qquad (27)$$

The momentum equation, (9) is

$$\frac{d\theta}{dx} = \frac{v_0}{U} + \frac{\nu}{U^2}\frac{U}{\theta}\frac{1}{[4\cdot 53453 - 2\cdot 53453K]},$$

$$= \frac{v_0}{U}\left(1 - \frac{4\cdot 53453 - 2\cdot 53453K}{2K}\right),$$

or

$$\frac{d}{dx}\left\{-\frac{\nu}{v_0}\frac{2K}{[4\cdot 53453 - 2\cdot 53453K]^2}\right\} = -\frac{v_0}{U}\frac{1-K}{K}\,2\cdot 26726, \qquad (28)$$

having used the value of θ in (27).

(28) can be written as

$$1\cdot 13363\,\frac{v_0^2}{U\nu}\,x = \int_0^K \frac{K\,(4\cdot 53453 + 2\cdot 53453K)}{(1-K)\,(4\cdot 53453 - 2\cdot 53453K)^3}\,dK.$$

It is easy to perform the integration, and we get finally

$$x\,\frac{v_0^2}{U\nu} = 0\cdot 77945\left\{\log_e\left(\frac{1\cdot 78909 - K}{1-K}\right) - \frac{0\cdot 61293}{1\cdot 78909 - K} - \frac{0\cdot 71458}{(1\cdot 78909 - K)^2} - 0\cdot 01586\right\} \qquad (29)$$

This gives the distribution of K with respect to x. The distribution of θ is then given from (27).

$$\delta^* = \theta\int_0^1\Big[(1-K)\,f(t) + Kg(t)\Big]\,dt. \qquad (30)$$

Hence

$$H = \frac{\delta^*}{\theta} = 2\cdot 5911\,(1-K) + 2K,$$

or

$$H = 2\cdot 5911 - 0\cdot 5911K \qquad (31)$$

This gives H, and δ^* in terms of x. Thus all the characteristics of the flow have been determined. Table 1 demonstrates the results.

4. *Comparison of Methods.*—No exact solution to the problem has yet been found. It is not difficult to see that no transformation of co-ordinates such as that used by Blasius will help. For suppose we take the stream function $\psi = \xi f(\eta)$, in which $\xi = Ax^a y^b$, $\eta = Bx^c y^d$, then it is shown in Appendix 2 that the equations of motion reduce to an equation in $f(\eta)$, if $\xi = (x/y)\eta^s$. We may say therefore that if $\psi = (x/y)\eta^s f(\eta)$, the function $f(\eta)$ can be found. [For example, Blasius' solution takes $\eta = \frac{1}{2}(U/\nu x)^{1/2}y$, $s = 1$.] The velocity components of this flow are

$$\left.\begin{aligned} u &= \frac{x}{y^2}\,\eta^s\{(ds-1)\,f(\eta) + \eta d f'(y)\},\\ v &= -\frac{\eta^s}{y}\,\{(cs+1)\,f(\eta) + c\eta f'(\eta)\}. \end{aligned}\right\} \qquad (32)$$

If $d > 0$, $\eta = 0$ on the plate, on which we require v to be constant. Hence η^s/y must be finite, non-zero and independent of x when $y = 0$. Hence $c = 0$, $sd = 1$, and $f(0) = -v_0$. Considering conditions at infinity, it is then clear that u cannot be independent of x, which is required in the problem. Similar remarks apply for $d < 0$. Thus an exact solution cannot be found using such substitutions.

The only other obvious method of obtaining an exact solution is to expand the stream function in powers of x, whose coefficients are functions of y, for x small, and to join this solution on to the known solution for large x. It is hoped that this task will be undertaken by means of the differential analyser.

Other solutions have been considered by Schlichting[2] and Preston[1]. Both authors have used the more orthodox method of boundary-layer calculations, in which the velocity is expressed as a function of y, and use a one-parametric family of profiles of the form

$$\frac{u}{U} = (1 - K)\, h\left(\frac{y}{\theta}\right) + Kj\left(\frac{y}{\theta}\right).$$

Both authors also take $u/U = j(y/\theta)$ as the exact solution at large distances down the plate. Preston takes for $h(y/\theta)$ the Blasius profile, and hence finds the computation of the momentum thickness awkward, while Schlichting takes an approximation to Blasius in form which enables him to do all the work analytically. It would certainly seem that Preston's method is superior to Schlichting's, but it is curious that the method of this paper agrees quite well with Schlichting's results. It is wellnigh impossible to suggest which method gives the best result, and all the results are collected and compared in Figs. 1, 2, 3.

Conclusion.—A new method of performing boundary-layer calculations has been used in considering the problem of the flat plate in a uniform stream when there is a constant normal velocity through the plate. Unfortunately no exact solution is known of the problem, and therefore comparisons are impossible. A later paper[5] will demonstrate in greater detail the new method, and its applications to other problems.

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APPENDIX I

We want to show that in the flat plate with constant suction problem the velocity profiles tend to the Blasius profiles as $x \to 0$.

Suppose that near $x = 0$, the stream function is expressible in the form

$$\psi = (\nu U x)^{1/2} f_0(\eta) + v_0 x f_1(\eta) + v_0 x^{3/2} f_2(\eta) + \dots, \qquad (33)$$

in which $\eta = \frac{1}{2}(U/\nu x)^{1/2} y$, v_0 is the velocity of suction, and $f_0(0) = f_0'(0) = 0, f_1(0) = -1$. The term $v_0 x f_1(\eta)$ is necessary for the boundary conditions along the plate. From equation (33) we obtain immediately the following relations.

$$\begin{aligned} -\frac{\partial \psi}{\partial x} = v = & -\frac{1}{2}\left(\frac{\nu U}{x}\right)^{1/2} f_0(\eta) + \frac{\eta}{2}\left(\frac{\nu U}{x}\right)^{1/2} f_0'(\eta) - v_0 f_1(\eta) \\ & + \frac{v_0}{2}\eta f_1'(\eta) - \frac{3}{2} v_0 x^{1/2} f_2(\eta) + \frac{v_0 x^{1/2}}{2}\eta f_2'(\eta) t \dots \\ = & \frac{1}{2}\left(\frac{U\nu}{x}\right)^{1/2}[\eta f_0'(\eta) - f_0(\eta)] + \frac{v_0}{2}[\eta f_1'(\eta) - 2f_1(\eta)] \\ & + \frac{v_0 x^{1/2}}{2}[\eta f_2'(\eta) - 3f_2(\eta)] + \dots \end{aligned}$$

$$\frac{\partial \psi}{\partial y} = u = \tfrac{1}{2} U f_0'(\eta) + v_0 \frac{x\eta}{y} f_1'(\eta) + \frac{v_0 x^{3/2}}{y} f_2'(\eta) + \dots$$

$$\frac{\partial u}{\partial y} = \frac{1}{2}\frac{U\eta}{y} f_0''(\eta) + v_0 \frac{x\eta^2}{y^2} f_1''(\eta) + v_0 x^{3/2}\frac{\eta^2}{y^2} f_2''(\eta) + \dots$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{2}\frac{U\eta^2}{y^2} f_0'''(\eta) + v_0 x \frac{\eta^3}{y^3} f_1'''(\eta) + v_0 x^{3/2}\frac{\eta^3}{y^3} f_2'''(\eta) + \dots$$

$$\begin{aligned} \frac{\partial u}{\partial x} = & -\frac{1}{4}\frac{U\eta}{x} f_0''(\eta) + \frac{v_0\eta}{y} f_1''(\eta) - \frac{1}{2}\frac{v_0\eta^2}{y} f_1''(\eta) + \frac{3v_0 x^{1/2}\eta}{2y} f_2'(\eta) \\ & - \frac{v_0 x^{1/2}\eta^2}{2y} f_2''(\eta) + \dots \end{aligned}$$

Hence the equation of motion (1) becomes

$$\begin{aligned} & \left[\tfrac{1}{2} U f_0'(\eta) + \tfrac{1}{2} v_0\left(\frac{Ux}{\nu}\right)^{1/2} f_1'(\eta) + 0(x)\right] \\ & \times\left[-\tfrac{1}{8}\frac{U}{x}\left(\frac{U}{\nu x}\right)^{1/2} y f_0''(\eta) + \tfrac{1}{2} v_0\left(\frac{U}{\nu x}\right)^{1/2}\{f_1'(\eta) - \tfrac{1}{4}\eta f_1''(\eta)\} + 0(1)\right] \\ & + \left[\tfrac{1}{2}\left(\frac{U\nu}{x}\right)^{1/2}\{\eta f_0'(\eta) - f_0(\eta)\} - \frac{v_0}{2}\{2f_1(\eta) - \eta f_1'(\eta)\} + 0(x^{1/2})\right] \\ & \times\left[\tfrac{1}{4} U\left(\frac{U}{\nu x}\right)^{1/2} f_0''(\eta) + \frac{v_0}{4}\frac{U}{\nu} f_1''(\eta) + 0(x^{1/2})\right] \\ & = \left[\tfrac{1}{8} U \frac{U}{\nu x} f_0'''(\eta) + \frac{v_0}{8}\left(\frac{U}{\nu}\right)^{3/2}\frac{1}{x^{1/2}} f_1'''(\eta) + 0(1)\right]\nu. \end{aligned}$$

The functions $f_0(\eta), f_1(\eta) \dots$ can now be determined by equating the coefficients of like powers of x to zero. In particular, the coefficient of x^{-1} is $f_0'''(\eta) + \frac{1}{2} f_0(\eta) f_0''(\eta)$, and this is the predominant term when x is small. Thus the predominant term in (33) is the first which in fact gives the Blasius profile. This justifies the choice in Section 3 of the Blasius profile when $K = 0$.

APPENDIX II

Let us seek a solution of the equations of motion in the form $\psi/\nu = \xi f(\eta)$, where $\xi = x^a y^b$, $\eta = x^c y^d$. We require to know what conditions on a, b, c, d exist so that the function $f(\eta)$ is determinable. With this form for ψ, we get at once :

$$\frac{u}{\nu} = \frac{b\xi}{y} f + \frac{\xi d\eta}{y} f' = \frac{\xi}{y} \bar{U}, \text{ say.}$$

$$\frac{v}{\nu} = -\frac{a\xi}{x} f - \frac{\xi c\eta}{x} f' = \frac{\xi}{x} \bar{V}, \text{ say.}$$

$$\frac{1}{\nu}\frac{\partial u}{\partial x} = \frac{b}{y}\left(\frac{a\xi}{x} f + \frac{\xi c\eta}{x} f'\right) + \frac{d}{y}\left(\frac{(a+c)\,\xi\eta}{x} f' + \frac{\xi\eta^2 c}{x} f''\right)$$

$$= \frac{\xi}{xy}\,[abf + (bc + ad + dc)\,\eta f' + dc\,\eta^2 f''] = \frac{\xi}{xy}\,\bar{U}_x\,, \text{ say}$$

$$\frac{1}{\nu}\frac{\partial u}{\partial y} = \frac{b}{y}\left(\frac{b\xi}{y} f + \frac{\xi d\eta}{y} f'\right) + \frac{d}{y}\left(\frac{(b+d)\,\xi\eta}{y} f' + \frac{\xi\eta^2 d}{y} f''\right)$$

$$-\frac{1}{y^2}\,(b\xi f + \xi d\eta f')$$

$$= \frac{\xi}{y^2}\,[(b^2 - b)\,f + (2bd + d^2 - d)\,\eta f' + d^2\eta^2 f''] = \frac{\xi}{y^2}\,\bar{U}_y,$$

$$\frac{1}{\nu}\,\frac{\partial^2 u}{\partial y^2} = [+\,(b^2 - b)\,f + (2bd + d^2 - d)\,\eta f' + d^2\eta^2 f'']\left[-\frac{2\xi}{y^3} + \frac{b\xi}{y^3}\right]$$

$$+\,\frac{\xi}{y^2}\left[(b^2 - b)\,\frac{d\eta}{y} f' + (2\,bd + d^2 - d)\,(f' + \eta f'')\,\frac{d\eta}{y}\right.$$

$$\left. +\,d^2\,(2f'' + \eta f''')\,\frac{\eta^2 d}{y}\right] = \frac{\xi}{y^3}\,\bar{U}_{yy}\,,$$

wherein $\bar{U}$, $\bar{V}$, $\bar{U}_x$, $\bar{U}_y$, $\bar{U}_{yy}$ are all functions of η and $f(\eta)$ only. The equation of motion therefore becomes

$$\frac{\xi}{y}\,\frac{\xi}{xy}\,\bar{U}\bar{U}_x + \frac{\xi}{x}\,\frac{\xi}{y^2}\,\bar{V}\bar{U}_y = \frac{\xi}{y^3}\,\bar{U}_{yy}\,,$$

or

$$\frac{\xi y}{x}\,(\bar{U}\bar{U}_x + \bar{V}\bar{U}_y) = \bar{U}_{yy}\,.$$

Since $y\xi/x$ is the only term in this equation not involving η or $f(\eta)$, the equation can be solved if $\xi y/x$ is some function of η, say $g(\eta)$, hence

$$\xi = \frac{x}{y}\,g(\eta)\,,$$

and the original form for ψ becomes

$$\frac{\psi}{\nu} = \frac{x}{y}\,g(\eta)\,f(\eta).$$

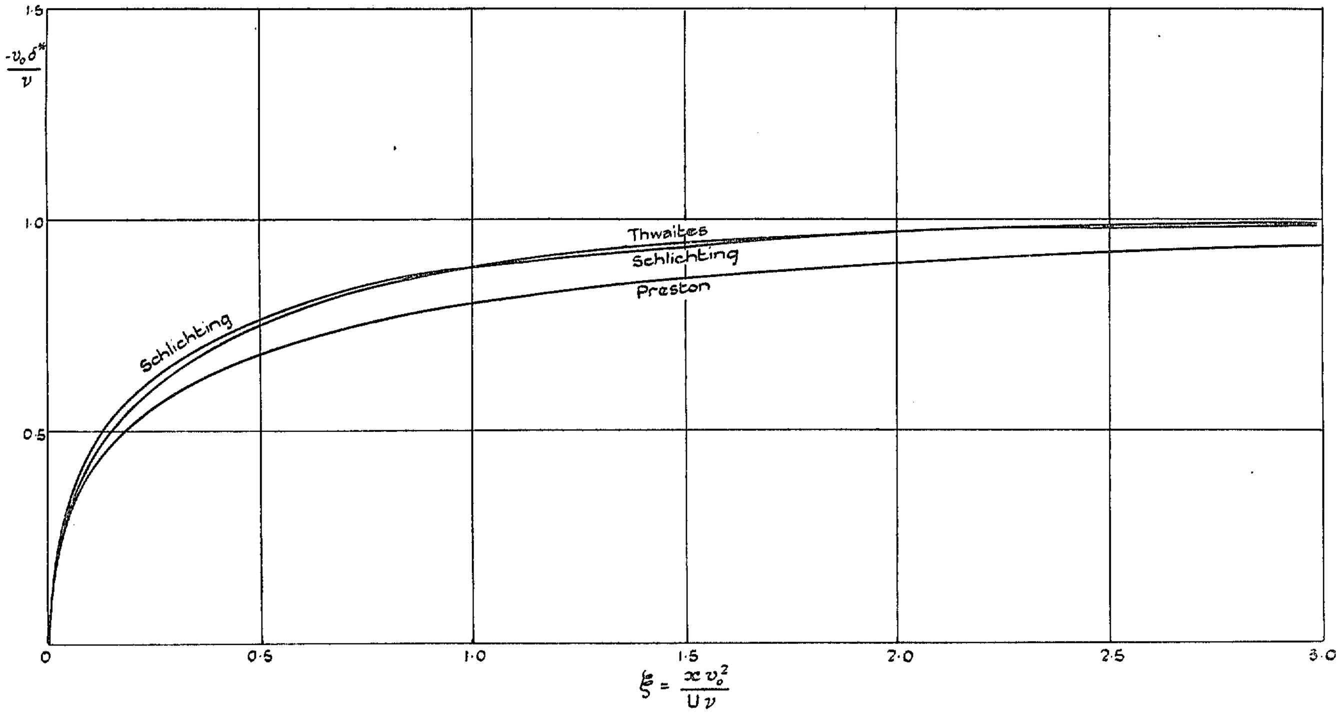
Clearly the product $g(\eta)\,f(\eta)$ could be replaced by a single function, or we may write $g(\eta) = \eta^s$: in Blasius' solution of uniform flow, $s = 1$, $c = \frac{1}{2}$, $d = 1$. Thus finally the equation of motion can be solved exactly by taking

$$\frac{\psi}{\nu} = \frac{x}{y}\,\eta^s f(\eta), \text{ wherein } \eta = x^c y^d\,.$$

TABLE

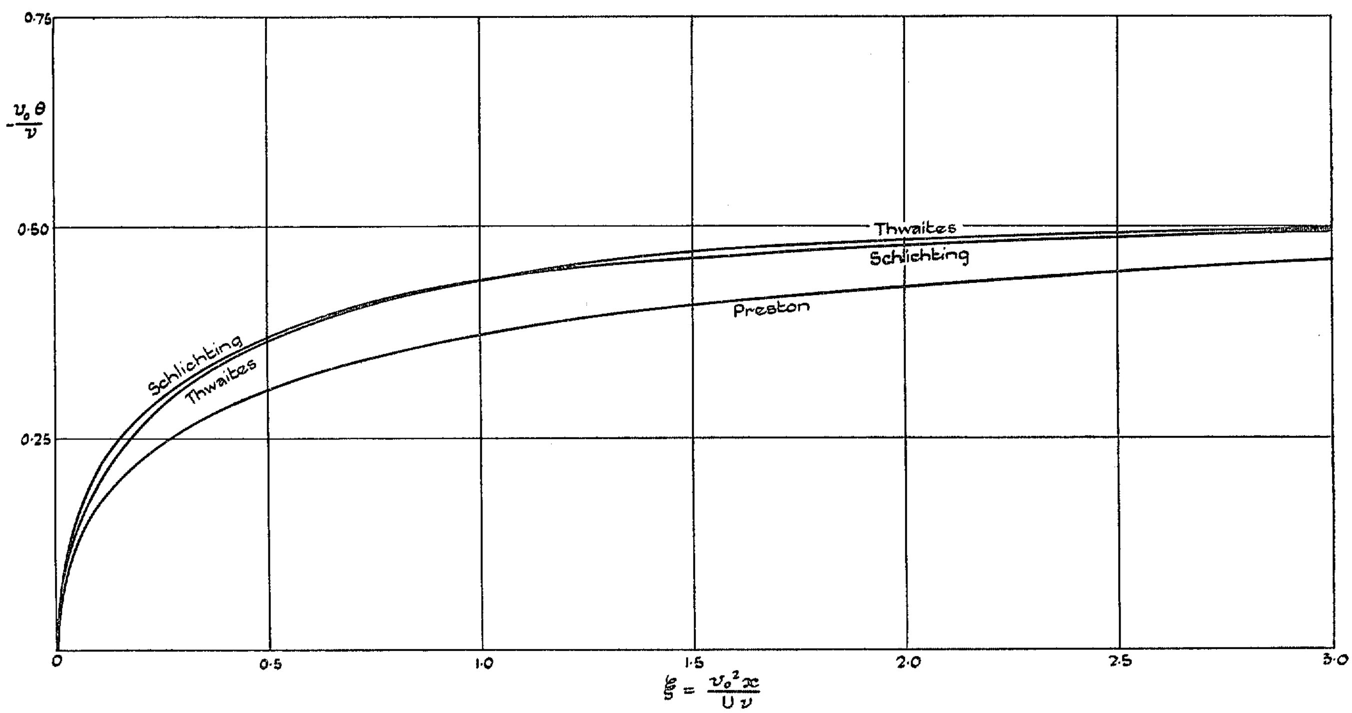
Below are tabulated the values of $\frac{-v_0\delta^*}{\nu}$, $\frac{-v_0\theta}{\nu}$, $H = \frac{\delta^*}{\theta}$, and K the " shape " parameter against values of the parameter $\frac{xv_0^2}{U\nu}$

$\frac{xv_0^2}{U\nu}$	K	$\frac{-v_0\delta^*}{\nu}$	$\frac{-v_0\theta}{\nu}$	H
0	0	0	0	2·5911
0·000287	0·1	0·02762	0·01091	2·5316
0·001382	0·2	0·06098	0·02466	2·4729
0·003953	0·3	0·1258	0·05212	2·4139
0·009381	0·4	0·1530	0·06454	2·3547
0·020067	0·5	0·2150	0·09367	2·2955
0·041134	0·6	0·2955	0·1321	2·2365
0·084344	0·7	0·4001	0·1837	2·1785
0·18122	0·8	0·5393	0·2546	2·1182
0·44880	0·9	0·7299	0·3545	2·0591
0·71783	0·94	0·8262	0·4059	2·0355
0·96401	0·96	0·8799	0·4348	2·0236
1·43044	0·98	0·9377	0·4661	2·0118
1·93224	0·99	0·9682	0·4827	2·0059
2·45283	0·995	0·9840	0·4912	2·0030
2·98316	0·9975	0·9919	0·4956	2·0015
3·69137	0·999	0·9968	0·4982	2·0006
∞	1·0	1·0	0·5	2·0



1.5
$-\frac{v_0 \delta^*}{\nu}$
1.0
0.5
0
Thwaites
Schlichting
Preston
Schlichting
0.5
1.0
1.5
2.0
2.5
3.0
$\xi = \frac{x v_0^2}{U \nu}$

FIG. 1.



0.75
$-\frac{v_0 \theta}{\nu}$
0.50
0.25
0
Thwaites
Schlichting
Preston
Schlichting
Thwaites
0.5
1.0
1.5
2.0
2.5
3.0
$\xi = \frac{v_0^2 x}{U \nu}$

FIG. 2.

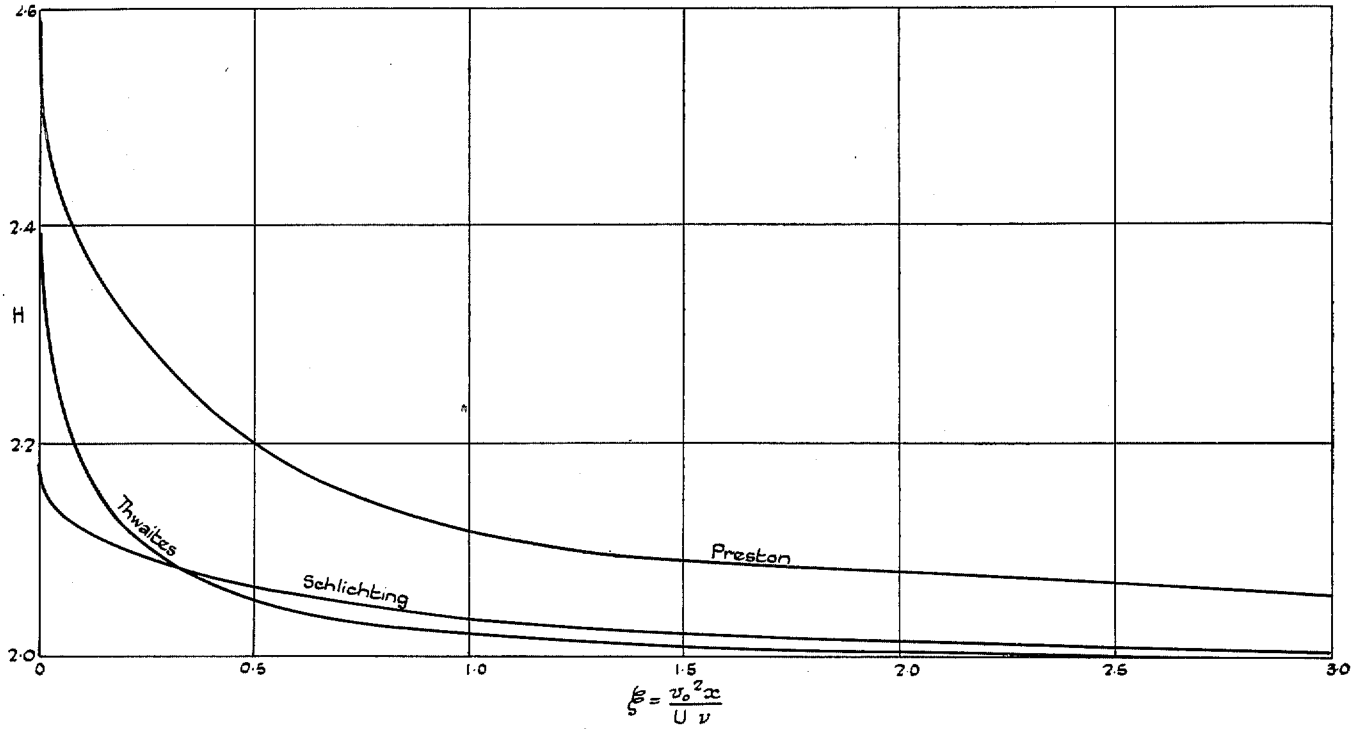


FIG. 3.

(92485) Wt. 13/806 K.5 11/52 Hw.

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