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The Theory of Parachutes with Cords over the Canopy

By

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Summary.—The basis for designing parachutes of R. & M. 862³ did not appear to be correct for parachutes with cords over the canopy. Moreover, the presence of these cords was essential in a practical design for heavy duty purposes and it was obvious from appearance that their presence produced a stress distribution considerably different from that of the earlier theory.

This report investigates the distribution of stress in a parachute with cords over the canopy, particularly when the cords are kept shorter than the length of the fabric gore which is permitted to bulge out between the cords under the excess pressure of the air inside the parachute.

An approximate theory of shape and of the distribution of stress is developed by making certain assumptions, particularly that the tension in the fabric in any axial section can be reduced to a negligible amount, and that the pressure difference all over the parachute can be regarded as uniform. On the basis of this theory a method of calculating the shape of a gore is developed and an example given. A brief statement is made on the degree to which a parachute so designed departs from the shape and maximum stress calculated.

1. *Introduction.*—A basis for designing parachutes is given in R. & M. 862³ but it is assumed that the canopy is a surface of revolution. The direct consequence of this assumption is that the cords over the canopy must be slack and cannot serve any useful purpose; in fact, the rigging lines might just as well terminate at the canopy hem.

If, on the other hand, the cords over the canopy are tighter than the canopy itself, then the canopy will no longer be a surface of revolution; instead, the fabric between adjacent cords will bow out and the fabric tension will produce a force component necessary and sufficient to support tension in the cords. It is then possible to arrange that the hoop stress in the fabric is nearly constant over the whole parachute. This is clearly preferable to the stress distribution in a parachute designed on the basis of R. & M. 862³, where the stress increases continuously as the apex is approached, and, in the absence of elasticity of the fabric, becomes infinite at the apex.

The present report discusses the manner in which an even stress distribution of the type mentioned above may be achieved, but it has been found impossible at present to produce an exact theory for the design of a parachute with taut cords over the canopy. Certain simplifying assumptions can however be made which in practice lead to a greatly improved design for heavy duty purposes.

2. *Theory of a Parachute with Cords over the Canopy.*—2.1. *Influence of General Considerations of Design.*—It is convenient to make a parachute canopy from a number of equal sections, or gores. The cords which pass over the canopy are threaded into the seams between the gores, and are continuous over the canopy; that is, a cord is threaded up one seam and down the opposite seam. In order that a large number of seams shall not meet in a point at the crown,

* R.A.E. Report No. Exe 115 received 31st August, 1942.

it is usual for the points of the gores to be cut away, leaving a vent. In the particular application for which these parachutes were designed, it is not necessary to have a hole at the crown, so the vent is covered by a patch of material.

The rigging cords are stitched to the canopy at the periphery and may or may not be stitched to it at the vent hem; between the periphery and the vent, however, the cords are free in the seams.

In any structure which is wholly of fabric, the stressed surface must be bounded by a curved member in tension, such as a cord or reinforced hem. For this reason it is usual to terminate the gores both at the vent and periphery in hems which are reinforced, usually by strips of tape.

2.2. *The Distribution of Fabric Stresses in the Canopy.*—The theory of the stress distribution in a parachute and the design for optimum distribution may be considerably simplified if the tension in the fabric in the generator* direction is made negligible. Moreover, it will be made a condition of the design that the fabric is cut so that the tension across the panel, at right angles to the generator direction, is nearly constant over the length of a gore.

It is clearly impossible to remove all the generator tension from the fabric because there will be skin friction. However, the drag from this source must be only a small fraction of the total drag so that the above conditions should be practicable to achieve.

If the hoop stress has a value T_1 and the generator stress has a different value T_2 at an element of the parachute canopy then the pressure difference p between the two sides of the canopy can be related to these stresses through the radii of curvature, ρ_1 and ρ_2 , respectively in the hoop and generator directions by

$$p = \frac{T_1}{\rho_1} + \frac{T_2}{\rho_2} \quad \dots \quad (1)$$

This supposes, of course, that the hoop and generator directions are principal directions of curvature which, although a close approximation, will be seen during the development of the theory not to be strictly true.

Now it is possible to vary ρ_1 appreciably, particularly when there is a fairly large number of cords, without much change in the width of the gores; ρ_1 may be made small compared with ρ_2 . Moreover, by gathering the fabric along the cords much of the generator tension is removed from the fabric. The net result is that the second term of equation(1) can be made negligible in comparison with the first.

Thus one may write

$$p = \frac{T_1}{\rho_1} \quad \dots \quad (2)$$

The maximum pressure difference which the canopy can withstand will be the value of p when T_1 approaches the tensile strength of the fabric.

Since a spherical parachute, although not the optimum design for other reasons³, stresses the fabric most efficiently because T_1 , T_2 , ρ_1 , and ρ_2 are all equal, it should be apparent already that the design under investigation cannot be far from the optimum because ρ_1 is going to be very much smaller than the radius of the parachute, certainly much less than half. This more than compensates for the fact that only in one direction are the fibres of the cloth really being stressed.

It will be convenient to assume that p is constant over the whole canopy surface which, for an impervious parachute, is a reasonable assumption³. Yet one usually must design with porous materials and little is known on the effect of porosity on the pressure distribution. However, if pressure distributions were discovered for which the above assumption was definitely unreasonable then it should still be possible to correct the calculations by at least imperical methods.

Since a cord can be considered as one of the generator lines, the tension in the fabric where it meets a cord will everywhere be at right-angles to the cord, and so the tension in a cord will be

* The term "generator" has been used throughout as meaning a line in the fabric which lies in an axial plane. In actual fact such a line is not a generator, since the canopy is not a surface of revolution.

3. *Theory of a Particular Type of Parachute.*—3.1. *The Geometry of the Type of Parachute Considered.*—The shroud lines are evenly spaced meridian lines or generators on a surface of revolution. The surface of a gore of the canopy according to the design adopted can be supposed generated as follows:—Let a circle of constant radius always lie in the plane containing the normals to a pair of adjacent generators at corresponding points situated at equal distances from the vertex and pass through these points. Then except for the presence of crinkles the surface of the gore is swept out by the circle as it passes down the shroud lines from the vertex to the hem.

3.2. *Calculation of the Shape of the Gores.*—Suppose that, in Fig. 1, OA, OB are adjacent cords of a parachute, their shape agreeing with the Taylor curve, and O is the apex of the parachute. Let P, Q be corresponding points on these cords, at distance r from the axis OZ of the parachute. Suppose M is a point on OZ such that MP, MQ are normal to the cords OA, OB at P and Q respectively. Then

$$MP = MQ = r \operatorname{cosec} \phi . \quad \dots \quad (10)$$

Suppose a gore is cut to some shape SOT, Fig. 2, and that it is placed between the two cords so that the fabric along OS is ranged along the cord OA, and the fabric along OT is ranged along OB. Let the points P, Q, of the fabric be made to correspond to P, Q on the cords and also suppose that the line P, HQ, in its bowed-out position lies in the plane MPQ, as shown in Figs. 1 and 3. In this case

$$OH > OP,$$

$$\text{so that } OP_1 > OH > OP, \quad \dots \quad (11)$$

and the fabric near a cord will be slack compared with the cord itself.

Without considering any pressure difference on the two sides of the fabric, suppose that PHQ is an arc of a circle of radius h , centre R, as in Fig. 3, h being independent of r . Let arc HQ = w , QRH = α . Let the circle centre M, radius MP, cut MH at G.

Then

$$\text{arc GQ} = \frac{\pi r}{n}, \text{ approximately, } \dots \quad (12)$$

and

$$GMQ = \frac{\pi}{n} \sin \phi . \quad \dots \quad (13)$$

From $\triangle QRM$,

$$\frac{h}{\sin \left(\frac{\pi}{n} \sin \phi \right)} = \frac{r \operatorname{cosec} \phi}{\sin \alpha},$$

or

$$\sin \alpha = \frac{\pi r}{nh} \text{ approximately, } \dots \quad (14)$$

if $\frac{\pi \sin \phi}{n}$ is small, which is usually true.

Then

$$w = h \cdot \alpha,$$

i.e.

$$\frac{w}{a} = \frac{h}{a} \sin^{-1} \frac{\pi r}{nh} . \quad \dots \quad (15)$$

If H' corresponds to P', Q' on the cords, and HH' = dS , PP' = QQ' = ds , then

$$\frac{dS}{ds} = 1 + GH \cdot \frac{d\phi}{ds}, \quad \dots \quad (16)$$

approximately.

But
$$GH = MR + RM - GM$$

$$= h + \frac{r \operatorname{cosec} \phi \cdot \sin \left(\alpha - \frac{\pi}{n} \sin \phi \right)}{\sin \alpha} - r \operatorname{cosec} \phi ;$$

or
$$GH = h + r \operatorname{cosec} \phi \left[\cos \left(\frac{\pi}{n} \sin \phi \right) - 1 - \cot \alpha \cdot \sin \frac{\pi}{n} \left(\sin \phi \right) \right] \dots \dots (17)$$

From equation (14)

$$\cos \alpha = 1 - \frac{1}{2} \cdot \left(\frac{\pi r}{nh} \right)^2 ,$$

if $\frac{\pi r}{nh}$ is small, so that

$$\cot \alpha = \frac{nh}{\pi r} - \frac{1}{2} \cdot \frac{\pi r}{nh} .$$

Equation (17) may therefore be written

$$GH = h + r \operatorname{cosec} \phi \left[-\frac{1}{2} \cdot \frac{\pi^2}{n^2} \cdot \sin^2 \phi - \left(\frac{nh}{\pi r} - \frac{1}{2} \frac{\pi r}{nh} \right) \cdot \frac{\pi}{n} \sin \phi \right]$$

$$= h + r \operatorname{cosec} \phi \left[-\frac{h}{r} \sin \phi - \frac{1}{2} \cdot \frac{\pi^2}{n^2} \sin^2 \phi + \frac{1}{2} \cdot \frac{\pi^2}{n^2} \cdot \frac{r}{h} \sin \phi \right] ,$$

or
$$GH = \frac{1}{2} \cdot \frac{\pi^2}{n^2} \cdot \frac{r^2}{h} - \frac{r}{2} \cdot \frac{\pi^2}{n^2} \sin \phi . \dots \dots \dots (17a)$$

From equations 16 and (17a),

$$\frac{dS}{ds} = 1 + \frac{\pi^2}{2n^2} \left[\frac{r^2}{h} - r \sin \phi \right] \frac{d\phi}{ds} . \dots \dots \dots (18)$$

Substituting from equations (6) and (7) for $\frac{d\phi}{ds}$ and $\sin \phi$, this becomes

$$\frac{dS}{ds} = 1 + \frac{\pi^2}{2n^2} \left[\frac{r^2}{h} - \frac{r^3}{a^2} \right] \cdot \frac{2r}{a^2} ,$$

i.e.
$$\frac{dS - ds}{ds} = \frac{\pi^2}{n^2} \left[\frac{a}{h} \cdot \frac{r^3}{a^3} - \frac{r^4}{a^4} \right] \dots \dots \dots (19)$$

Taking h as constant (*see* para. 3.1), this may be integrated to give

$$S - s = \frac{\pi^2}{n^2} \int_0^s \left[\frac{a}{h} \cdot \frac{r^3}{a^3} - \frac{r^4}{a^4} \right] ds , \dots \dots \dots (20)$$

if both S and s are measured from the apex O .

But from equations (4) and (7),

$$\begin{aligned} ds &= dr/\cos \phi \\ &= dr/\sqrt{(1 - r^4/a^4)} \end{aligned}$$

so that (20) becomes

$$\begin{aligned} \frac{S - s}{a} &= \frac{\pi^2}{n^2} \cdot \int_0^{r/a} \left[\frac{a}{h} \cdot \frac{r^3}{a^3} - \frac{r^4}{a^4} \right] \cdot \frac{d(r/a)}{(1 - r^4/a^4)} \\ \text{i.e. } \frac{S}{a} &= \int_0^{r/a} \frac{d(r/a)^4}{\sqrt{(1 - r^4/a^4)}} \\ &= \frac{\pi^2}{2n^2} \cdot \frac{a}{h} \left[1 - \left(1 - \frac{r^4}{a^4} \right)^{1/2} \right] - \frac{\pi^2}{n^2} \int_0^{r/a} \frac{r^4}{a^4} \cdot \frac{d(r/a)}{\sqrt{(1 - r^4/a^4)}}, \end{aligned}$$

which may be written

$$\frac{S}{a} = \int_0^{r/a} \frac{d(r/a)}{1 - r^4/a^4} + \frac{\pi^2}{2n^2} \cdot \frac{a}{h} \left[1 - \left(1 - \frac{r^4}{a^4} \right)^{1/2} \right] - \frac{\pi^2}{n^2} \int_0^{r/a} \frac{r^4}{a^4} \cdot \frac{d(r/a)}{\sqrt{(1 - r^4/a^4)}} \quad (21)$$

Equation (21) may be solved using tables of elliptic functions to give the value of S/a for any value of r/a . The corresponding value of w/a may be found from equation (15). Therefore by taking a series of values of r/a , corresponding values of w/a and S/a may be found. Thus, for any value of a , corresponding values of w and S may be obtained. Since w corresponds to the semi-width of the gore SOT at distance S from the apex, it is therefore possible to plot the gore shape SOT.

3.3. Stresses in the Parachute.—It is possible to estimate the stresses in the fabric when a parachute made with gores of this type is towed through the air, but an accurate solution has not been attempted. Since there is negligible generator tension in the fabric (the cords are tighter than the corresponding fabric) the cords will still approximate to the Taylor shape. In order to calculate the stress in the canopy equation (2) can be used.

Now ρ will be some function of h and $\frac{ds}{d\phi}$, and the value of this function will not be very different from h , if the stretch of the fabric under load is neglected. Therefore as an approximation it may be written

$$p = \frac{T}{h}, \dots \dots \dots (22)$$

which, with (5), gives

$$T = \frac{L \cdot h}{a^2} \dots \dots \dots (23)$$

From (23) it is seen that the maximum load which such a parachute will withstand is given by

$$L_{\max} = \frac{a^2}{h} \cdot T_0,$$

where T_0 should be considered to be the tensile strength of the fabric across a seam. Thus

$$L_{\max} = C \cdot D \cdot T_0, \dots \dots \dots (24)$$

$$\text{where } C = \frac{\pi}{2} \cdot \frac{a}{h}, \dots \dots \dots (25)$$

and $D = 2a$ is the diameter of the parachute.

From the last two equations it is seen that L_{\max} is directly proportional to a/h , so that the greater the value of a/h , the greater the break-up load of the parachute.

4. *Practical Considerations in the Design of a Parachute of this Type.*—4.1. *Choice of the Ratio a/h .*—It is evident that before the gore shape can be worked out, some specific value of the ratio a/h has to be chosen. The value of a/h determines the “fullness” of the gores, and the larger the value of this ratio, the smaller the tension in the fabric for a given load on the parachute, and so the smaller the tensile strength of fabric which is needed.

The bulk of the parachute will be directly proportional to the area of one gore, and also to the thickness of the fabric. The area of a gore increases as the value of a/h increases, but simultaneously the stresses in the fabric are reduced, so that a weaker (and therefore thinner) fabric may be used; consequently there is a possibility of a minimum value for the product of these two quantities. It is found in practice that a minimum does occur when a/h is approximately equal to two although the value depends upon the quality of the fabric from which the parachute is made.

For this reason practically all the parachutes which have been used for experimental purposes have been designed on the basis of $a/h = 2$. However, there are other considerations, such as opening characteristics of the parachute, which enter into the final choice of this ratio.

When the values of a/h and of n have been chosen, the gore shape may be worked out from equations (15) and (21)*, namely

$$\frac{w}{a} = \frac{h}{a} \sin^{-1} \frac{\pi r}{nh} \quad \dots \dots \dots \quad (15)$$

$$\frac{S}{a} = \int_0^{r/a} \frac{d(r/a)}{\sqrt{1 - r^4/a^4}} + \frac{\pi^2}{2n^2} \cdot \frac{a}{h} \left[1 - \left(1 - \frac{r^4}{a^4} \right)^{1/2} \right] - \frac{\pi^2}{n^2} \int_0^{r/a} \frac{r^4}{a^4} \cdot \frac{d(r/a)}{\sqrt{1 - r^4/a^4}} \quad (21)$$

Table 1 gives the values of w/a and S/a in the case where $a/h = 2$. These correspond to the gore co-ordinates of a parachute of unit radius.

4.2. *Departure of Parachutes from the Theoretical Shape.*—A large number of questionable approximations have been made in working out the above theory, and, as might be expected, in practice parachutes designed on this approximate theory do not take up quite the shape which the theory predicts. The shape assumed by the cords is found to be not so flat as the Taylor shape, and the actual diameter of the parachute in flight is less than the nominal diameter D . Photographs of parachutes of this type are shown in Figs. 4 and 5.

Because of this departure from the theoretical shape it might be expected that equations (24) and (25) would not be accurately obeyed in practice. In actual fact, from the experimental work which has been done, there is no reason to doubt the practical correctness of equation (24); the value of C obtained is, however, 12 per cent. smaller than the value calculated from equation (25). This may be partly due to the fact that the method of manufacture of the parachute causes local overstraining or local weakening of the fabric beyond that allowed for in the seaming.

* Alternatively a graphical treatment may be adopted for working out the gore shape.

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| 3 | R. Jones | Aerodynamic Characteristics of Parachutes. A Comprehensive Account of Researches Incorporated in Various Papers submitted to the Advisory Committee for Aeronautics. R. & M. 862. June, 1923. |

TABLE 1

Gore Co-ordinates for a Parachute of Unit Radius

(see page 7)

$$\frac{a}{h} = 2.0$$

$\frac{r}{a}$	$\frac{S}{a}$	$\frac{w}{a}$
0	0	0
0.10	0.100	0.0262
0.20	0.200	0.0524
0.30	0.301	0.0789
0.40	0.402	0.1055
0.50	0.505	0.1324
0.60	0.612	0.1597
0.70	0.725	0.1875
0.80	0.852	0.2160
0.85	0.924	0.2305
0.90	1.005	0.2453
0.94	1.085	0.2572
0.97	1.164	0.2662
1.00	1.349	0.2754

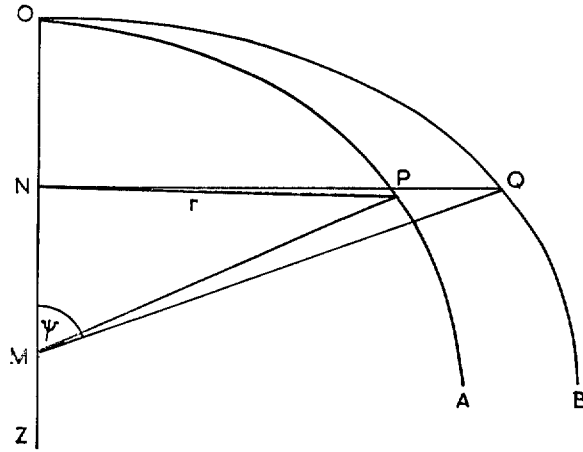


FIG. 1.

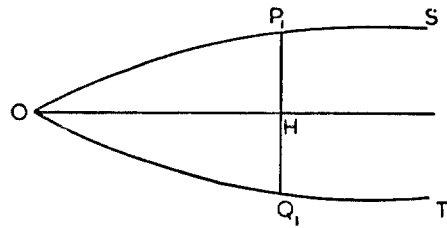


FIG. 2.

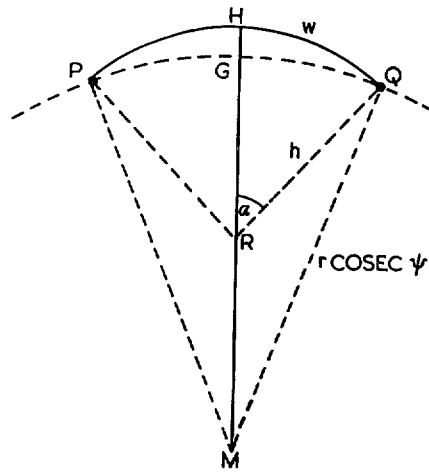


FIG. 3.

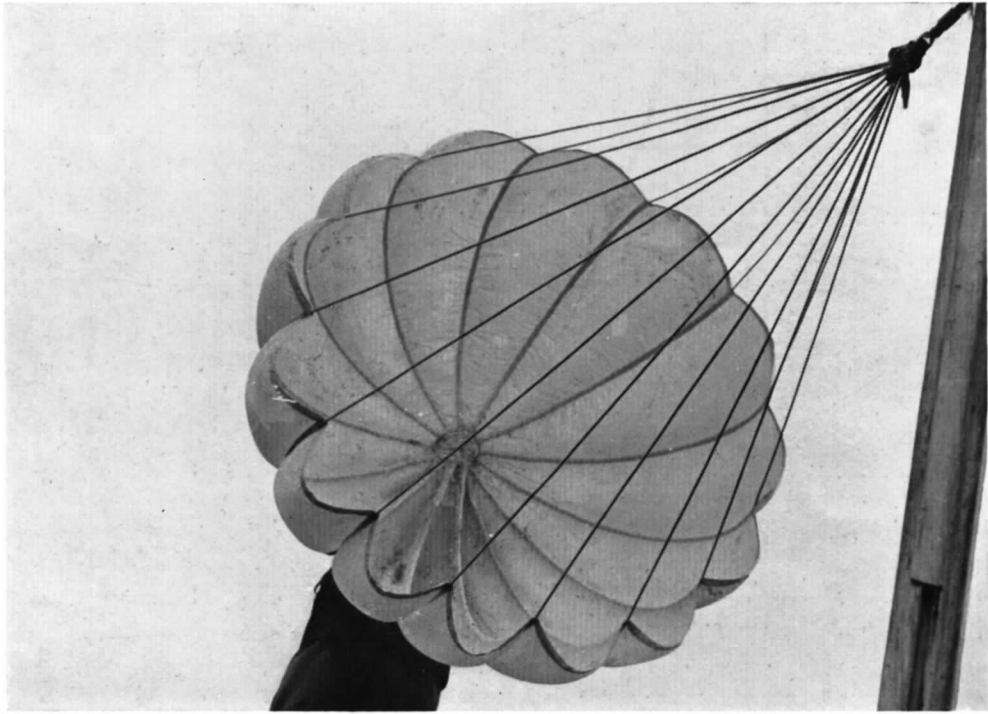


FIG. 4.

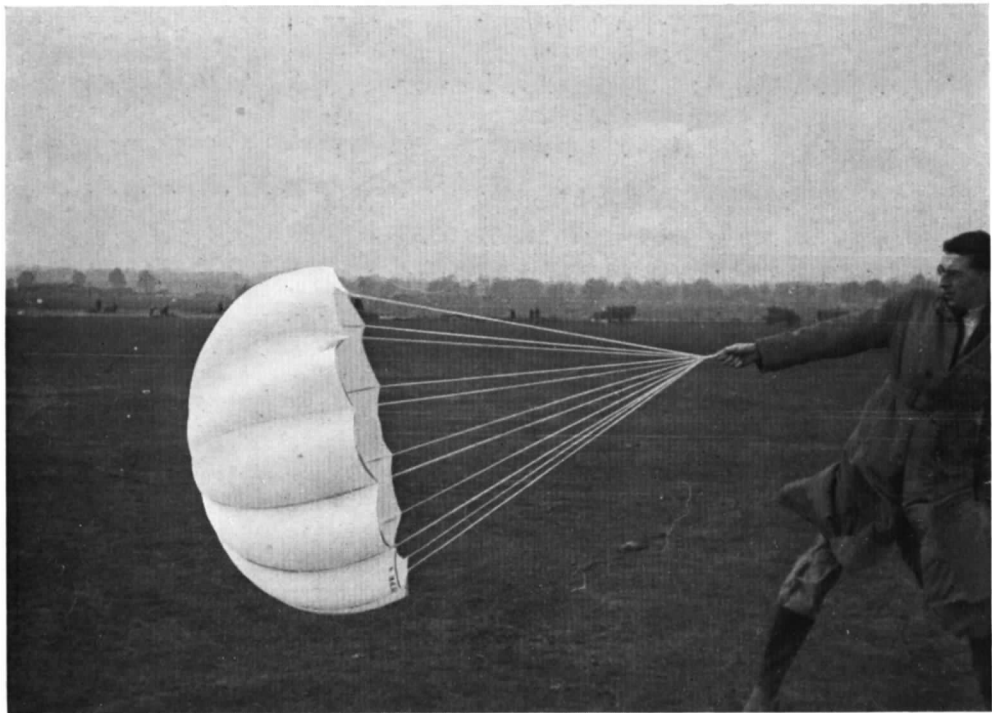


FIG. 5.

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