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# Fracture of Glass Rods in Bending and under Radial Pressure

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# Fracture of Glass Rods in Bending and under Radial Pressure

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*Summary.*—A simple theoretical argument leads to the conclusion that if Griffith's crack hypothesis is true, rods of brittle materials when subjected to radial pressure should fracture at a mean pressure equal to the average tensile stress at failure in a tensile test. In practice, it is not convenient to do tensile tests on account of the difficulty of gripping the test pieces and of procuring axial loading. In the present series of tests, the mean radial pressure at fracture of three types of glass rod has been compared with the tensile stress at fracture computed from bending tests. The mean fracture stresses developed in the two types of test differ significantly though not greatly. When departures of the experimental test conditions from ideal conditions are considered, they appear adequate to account for the difference. The results, therefore, are not in disagreement with the deduction made from Griffith's crack hypothesis that fracture in radial pressure occurs at a pressure numerically equal to the tensile breaking stress.

1. *Introduction.*—A previous report[1] describes experiments in which plastic rods were fractured in tension and under radial pressure. Included in that report was a general discussion of the cause of failure under radial pressure, and it was concluded that brittle materials such as mineral glass would be expected to fracture in radial pressure, at a pressure equal to the tensile strength of the material. For convenience the argument will be recapitulated. Following Griffith[2,3], mineral glass if free from flaws would have a strength of some millions of lb./sq. in. whereas the strength of commercial glass rod *in vacuo* is of the order of 20,000 lb./sq. in. Griffith suggested that the weakness of ordinary glass was due to its containing sharp ended cracks, which results in a concentration of stress when pulled in tension of the order of 100 : 1. As pointed out by Bridgman[4] (1938), radial pressure on a cylindrical rod may be regarded as a longitudinal tension, superimposed on a hydrostatic pressure. If the fluid used to exert the hydrostatic pressure is allowed to penetrate surface cracks, no concentration of stress at the ends of these cracks results, whereas the longitudinal tension results in a concentration of stress of the order of 100 : 1. The stress at the ends of surface cracks in a rod subjected to radial pressure acting alone, is thus tensile, and very nearly equal to that caused by a longitudinal tension acting alone, numerically equal in magnitude to the radial pressure. Fracture under radial pressure, may therefore be expected to occur at a pressure numerically equal to the tensile strength of the material.

In practice, it is not convenient to make tension tests on brittle materials. Apart from the difficulty of gripping the test piece, the difficulty of obtaining axial loading is insuperable with parallel rods of the diameter tested ($\frac{3}{16}$ in.). To limit adventitious bending stresses to 1 per cent. of the axial stress the line of action of the load would need to be located to 0·00025 in. which is experimentally impracticable. In the present experiments, tests in bending were made instead, the tensile stress at fracture being computed from simple bending theory. Radial pressure tests were made in an apparatus previously described[1]. Three different types of glass were tested.

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2. *Material.*—Experiments were made on glasses of three different compositions.

(*a*) A heat resisting glass with the following percentage analysis :—

| | | | |
|---|---|---|---|
| Silica .. | 81·98 | Potash .. | 0·98 |
| Lime .. | 0·12 | Boric oxide.. | 8·97 |
| Soda .. | 5·18 | Nickel oxide | slight trace. |

Alumina, titantia, and ferric oxide 2·52.

(*b*) Silica glass of 99·9 per cent. purity.

(*c*) Soda glass with the following analysis for which the authors are indebted to Professor W. E. S. Turner :—

| | | | |
|---|---|---|---|
| Silica .. | 68·78 | Magnesia .. | 3·17 |
| Alumina .. | 2·76 | Boric oxide | 0·43 |
| Iron as Ferric Oxide .. | 0·061 | Soda .. | 17·95 |
| Lime.. .. | 5·67 | Potash .. | 0·48 |
| Titantia .. | 0·080 | | |

The glass was received in 4–5 ft. lengths and was broken up and tested without any further heat treatment. No special care was taken to avoid surface damage to the glass caused by contact with other glass.

The soda glass was annealed by the manufacturers, but the other two glasses were not. However, owing to their low coefficients of expansion, it would be expected that internal stresses would be small. The rods were somewhat variable in diameter, having a mean diameter of about $\frac{3}{16}$ in. Rods for the radial pressure tests were selected to have diameters within a few thousandths of an inch of that of the bore of the plungers, and the remainder of the glass was used for the bending tests. The statistical allocation of test pieces to the two sorts of test was therefore not ideal, and because the tests for significant differences between the bending and the radial pressure results are based on a random allocation of test pieces, the significance of differences is somewhat exaggerated.

Additional tests were made on $\frac{1}{4}$-in. dia. soda glass from the same batch as that used for fatigue experiments previously reported[5]. All tests were made at 20–22 deg. C. and 40–60 per cent. R.H.

3. *Experiments.*—(*a*) *Bending Tests.*—The bending tests were made in a simple four point bend test rig housed in a constant temperature box. Glass for test was stored in the box prior to testing.

To avoid the need to attach the loading wires to each test piece in turn and to adjust their relative poisitons, the loading wires were permanently attached to a flexible plastic tube, into which the test pieces could be inserted. The length of test piece between the inner loading wires was $1\frac{1}{2}$ in. The two central wires were attached by cup hooks to a beam and load was applied to the centre of this by a wire whose lower end was attached to a lever carrying a movable weight The weight engaged with a screw, driven by a small electric motor mounted on one end of the lever. The rate of loading was controlled by controlling the speed of the motor. A microswitch was mounted under the lever which fell when the test piece broke, and by operating the switch stopped the motor. The distance of the weight from the fulcrum of the lever could be read off on a scale and the load to fracture the test piece could then be calculated. The maximum load which could be applied to the test piece was $9\frac{1}{2}$ lb. and at this load an accuracy (range of 10 tests) of about $\pm \frac{1}{32}$ lb. was obtained. The test pieces were smeared with oil before testing, so that surface conditions should be as near as possible the same as during the radial pressure tests.

The observations made during each test were as follows :—

(1) Duration of loading.

(2) Deflection of the end of the beam at failure.

(3) Position of weight at failure.

(4) Diameter of test piece at the point of fracture.

From (1) and (3) the rate of loading was calculated.

From (2), the time interval between fracture and interruption of the motor circuit was estimated, and a correction of the order of 1 or 2 per cent. made to the indicated load to fracture. The results of the bending tests are summarised in the right-hand part of Table 1. The large variation in rate of stressing is due in part to variation in diameter of the test pieces ; the rate of loading was constant apart from variation in friction. In all about one hundred and fifty tests were made in bending.

The appearance of the fractures in bending has been described in a previous report[5]. The fractured surfaces contained a small mirror-like surface at which fracture appeared to have started. This merged into a finely fissured surface having a hazy appearance, which in turn merged into a surface containing large radiating fissures. This merged into a comparatively smooth rippled surface, which completed the fracture. The mirror-like surface was approximately at right angles to the axis of the test piece. The remainder of the fractured surface curved first toward the axis and later away from it, making in side view a rough letter S.

(*b*) *Tests under Radial Pressure.*—The apparatus used to produce radial pressure around the glass rods was that shown in a previous report[1]. Briefly, the test piece, $1\frac{1}{2}$ in. long $\times$ $\frac{3}{16}$ in. or $\frac{1}{4}$ in. diameter was surrounded by a length of rubber tubing about $1\frac{1}{4}$ in. long which slides inside a large steel cylinder of $\frac{1}{2}$ in. internal diameter. Tubular plungers at each end fit the hole in the cylinder and slide over the test piece so that they bear on the end of the rubber tubing. By applying load to the plungers, pressure is transmitted to the curved surface of the rods, the ends of which are free from external forces.

The load was applied using a universal testing machine in the $1\frac{1}{2}$ and 3 ton capacity ranges. A control of the rate of strain was provided, and with this the approximate value of the rate of increase of stress required was obtained in one or two trials.

The test pieces were kept smeared in oil ready for test in the constant temperature box. The observations made during test were as follows :

(1) Diameter of test piece.

(2) Duration of loading from 0·2 ton upwards.

(3) Failing load.

(4) Distance of fracture from end of test piece.

The failing stress and rate of stress increase were calculated from (2) and (3). As the diameter of the glass varied some 5-thousandths of an inch, the diameter of each test piece was measured in order to determine whether the closeness of fit of the test piece in the ends of the plungers had any effect on the failing load. No variation, of failing stress and diameter of any significance, was detected. In the same way, from the measurements made in (4), less than 10 per cent. of the test pieces could possibly have failed where the rubber and plungers meet.

A total of 300 tests were made under radial pressure. The results are summarised in the left-hand part of Table 1.

The appearance of the fracture under radial pressure was of interest. The fractured surface was a smooth mirror-like plane at right angles to the axis of the test piece.

4. *Discussion of Results.*—From Table 1 it is seen that in three of the four series of tests, the difference in mean breaking stress in the two types of test is significant ($t>2$, indicating that the chance of obtaining equal or greater difference in the means of samples from the same population is less than 0·05). For only one material do the standard deviations differ significantly.

There are a number of factors which may contribute to the differences between the nominal stresses developed in the two types of test. The surface area subjected to the highest stress, is much greater in the radial pressure tests than in the bending tests where the maximum stress occurs along a line. It is probable, therefore, that a larger crack will be subjected to the highest stress in the radial pressure tests than in bending tests. For this reason, fracture in radial pressure might be expected to occur at a lower load than fracture in bending. Other factors leading to a difference in breaking stress of the same sign are non-uniformity of stress in the radial pressure tests as discussed previously[1], although concentration of stress at the section of entry into the plungers was not so great as to cause an undue proportional of fractures to occur at the ends; and the greater free energy of the surrounding fluid in the radial pressure tests. In the latter test the fluid is subjected to hydrostatic pressure and will therefore be more chemically active and thus more likely to attack the glass than in the bending tests in which the fluid was at atmospheric pressure. Any internal stress present in the test pieces would not be expected to affect the comparison of strengths in the two types of test. The internal stress system would be simply additive to that caused by the applied forces and would have the effect of altering the mean strengths by the same amount in both types of test. Friction in the apparatus used for the radial pressure tests would tend to make the nominal stress developed too high, while differences in rates of loading and lack of statistical homogeneity of the test pieces, may favour either test. Referring to Table 1, it is seen that in three out of four cases, the stress developed in bending was higher than that in radial pressure, a result not in disagreement with expectations.

The difference in the appearance of the fractures in the two types of test needs comment. The appearance of the bending fracture has already been discussed[5]. When the crack is small, the greatest principal stress is axial, and fracture accordingly starts along a transverse surface. As the crack proceeds, transverse tensile stresses, and longitudinal shear stresses occur at the root of the crack, and these increase relative to the longitudinal tensile stress, causing the direction of greatest principal stress to become inclined to the axis of the rod. The general S shape of the broken surface is due to the fracture tending to take place at right angles to the direction of greatest principal tensile stress. The fissures in the fractured surface are due to inertia stresses.

The transverse stresses occurring at the root of a crack in rods partly fractured in bending are a consequence of the curvature of the rod. When subjected to radial pressure acting alone, the curvature of the rod is zero and transverse stresses are consequently zero. The direction of greatest principal stress at the end of a crack in a partly fractured rod therefore remains axial, and the fractured surface is everywhere transverse. The absence of fissures in the surface of fracture is due to the relative smallness of the tensile inertia stresses during fracture under radial pressure, compared with those during bending fracture. In the radial pressure tests, the rate of change of the pressure exerted on the rod will depend on the rate of flow of the rubber into the space between the two surfaces of fracture; when those are still close together the rate of flow of the rubber will be relatively slow owing to its high viscosity. The rate of change of applied load and consequently the inertia stresses during fracture in radial pressure, are therefore much less than those during bending fracture. The predominantly compressive nature of the applied forces will also tend to reduce fissuring.

It is concluded that the difference in appearance of the fractured surfaces in the two types of test can be readily accounted for, and is not inconsistent with the cause of fracture—the tensile stresses at the roots of cracks initially present in the material—being the same in both types of test.

5. *Conclusions.*—The mean values of the radial pressure at fracture are significantly, though not greatly, different from the mean values of tension stress of fracture computed from bending tests. When departures of the experimental test conditions from ideal conditions are considered, they appear adequate to account for the difference. The results therefore are not in disagreement with the deduction made from Griffith's crack hypothesis that fracture in radial pressure occurs at a pressure numerically equal to the tensile breaking stress.

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## TABLE 1

*Fracture Stresses*

| Material | Radial Pressure | | | | | Bending | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Mean stress lb./in.$^2$ | $\sigma$ stress / mean stress | Rate of stress increase. lb./in.$^2$/sec. | $\sigma$ rate / mean stress | No. of tests | Mean stress lb./in.$^2$ | $\sigma$ stress / mean stress | Rate of stress increase. lb./in.$^2$/sec. | $\sigma$ rate / mean rate | No. of tests | Value of $t$ in Students' test for comparison of mean strengths | Value of $t$ in Students' test for comparison of standard deviations |
| Heat resisting glass | 10,000 | 0·21 | 700 | 0·15 | 40 | 11,600 | 0·17 | 950 | 0·35 | 40 | 3·2 | 0·4 |
| Silica glass .. | 14,700 | 0·19 | 1,400 | 0·08 | 40 | 13,200 | 0·17 | 1,450 | 0·29 | 40 | 2·6 | 1·3 |
| Soda glass $\frac{3}{8}$-in. dia. | 14,000 | 0·14 | 2,350 | 0·06 | 40 | 15,200 | 0·21 | 2,350 | 0·21 | 40 | 2·0 | 2·9 |
| Soda glass $\frac{1}{4}$-in. dia. | 13,500 | 0·25 | 1,500 | 0·16 | 180 | 15,000 | 0·21 | 1,500 | 0·17 | 20 | 1·9 | 0·4 |

$\sigma$ = standard deviation

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