Notes on the Automatic Control of a Blowdown Wind Tunnel

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OF A BLOWDOWN WIND TUNNEL

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SUMMARY

This report briefly describes salient results obtained during commissioning of the stagnation pressure control system of the N.P.L. 15 in x 10 in (38.1 cm x 25.4 cm) blowdown wind tunnel. The very different problems of low and high Mach number operation are examined, and the representation of the various flow processes in a manner suitable for an analogue computer is discussed.

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Symbols

\( C_b \) Pneumatic capacitance cf air storage tanks \( (= \frac{\partial M_b}{\partial P_b}) \)

\( C_s \) Pneumatic capacitance of settling chamber \( (= \frac{\partial M_s}{\partial P_o}) \)

\( h \) Heat lost by air to settling chamber walls per unit temperature difference

\( k_c \) D.C. gain cf controller

\( M \) Mach no.

\( M_b \) Mass of air stored in storage tanks

\( M_s \) Mass of air stored in settling chamber

\( m_v \) Mass flow through control valve

\( m_t \) Mass flow through nozzle

\( p_b \) Storage pressure

\( P_o \) Stagnation pressure in settling chamber

\( P_v \) Air pressure immediately upstream of control valves

\( R \) Gas constant

\( T_o \) Bulk stagnation temperature (mass-averaged temperature of air in settling chamber)

\( T_v \) Air temperature immediately upstream of control valves

\( T_w \) Temperature of settling chamber wall

\( V_s \) Volume of settling chamber

\( Y \) Ratio of specific heats for air

\( \rho_o \) Bulk stagnation density

\( \theta \) Control-valve opening

Suffices

\( a \) Set point or demanded level

\( f \) "Final", level i.e., after initial transient

N.B. \( \left( \frac{\partial P_o}{\partial t} \right) \approx 0 \).
1. Introduction

The control system for a supersonic blowdown wind tunnel has to perform two distinct (in some ways conflicting) functions. It must:

a) speedily attain the desired test conditions so as to minimise loss of run time (and reduce the intensity of starting loads imposed on models in the test section).

b) maintain these conditions as accurately as possible for the duration of the test, avoiding both excessive errors and excessive rates of change of error.

Since the conditions in the working-section are uniquely related to the conditions in the settling-chamber and respond to changes in the latter with negligible lag (transit times being normally of the order of a few milliseconds), the control function reduces to control of settling-chamber conditions. This note is concerned primarily with control of stagnation pressure. The inlet air to the tunnel is heated in a pebble-bed storage heater. The relationship between inlet air temperature and test-section stagnation temperature is complex, but it will be shown that good stagnation-pressure control implies a small rate of change of the bulk temperature of the air in the settling-chamber once the initial pressure transients have ceased.

2. The control system of the N.P.L. 15 in x 10 in tunnel

2.1 Background

The control system of the N.P.L. 15 in x 10 in (38.1 cm x 25.4 cm) tunnel is shown in schematic form as Fig 1. This tunnel draws air from 38,000 ft³ (10761 m³) of compressed-air storage at a maximum pressure of 365 psia (2.52 MN/m²) and exhausts either to a 36,000 ft³ (1020 m³) vacuum sphere or to atmosphere*. For the purposes of this note it may be taken that during operation at test section Mach numbers (M) of 2 or 3 the tunnel is discharged to atmosphere so that the run time is limited by the compressed-air storage capacity. Conversely at M = 5 or M = 7 the tunnel is discharged to the vacuum sphere, whose capacity is then the factor limiting the run time.

The maximum stagnation pressure in the settling chamber is 225 psia (1.55 MN/m²).

The tunnel is of fairly conventional design but two features are worth note in this context. Firstly, the maximum steady mass-flow rate through the nozzle is 250 times the minimum; and, secondly, the settling chamber is large in comparison to normal practice for M = 5 and M = 7 operation. Both features are consequences of the wide Mach number and Reynolds number ranges of this tunnel. The mass flow range arises directly from the combination of wide Mach number and Reynolds number ranges. The settling-chamber size resulted indirectly from the wide Mach number range because conservative design criteria were adopted to

* These capacities will shortly be augmented to 48,000 ft³ (1,365 m³) and 72,000 ft³ (2,040 m³) of compressed air and vacuum storage respectively.
ensure low turbulence levels in \( M = 2 \) operation. On balance, this advantage has been outweighed by the disadvantages of the large settling-chamber volume during operation at \( M = 7 \). These disadvantages arise in stagnation pressure control and from free convection if the settling-chamber is not pre-heated (as is currently the case with the 15 in \( \times \) 10 in tunnel). Following other recent experience, it should be possible to eliminate free convection by straightforward modifications to the system. However, in the present context, the settling-chamber size has merely served to throw into sharper focus the normal problems of blowdown tunnel control. Two distinct set of problems may be distinguished. These are:

(i) At low Mach numbers \(( M = 2 \) or \( M = 3 \) say) the stagnation pressure responds very rapidly to changes in control-valve setting. On the other hand, the storage air pressure falls rapidly during the run. The control system must continuously increase the opening of the control valve to maintain a constant mass flow rate, and hence a constant stagnation pressure.

(ii) At higher Mach numbers \(( M = 5 \) or \( M = 7 \) say) the mass flow rates through the nozzle are much smaller so that any volume between control valve and nozzle throat results in sluggish response of stagnation pressure to changes in control valve opening. However, the low mass flows normally imply that the storage pressure remains effectively constant throughout the run.

2.2 The control loop

The control system currently used (Fig (1)) consists of an assemblage of commercially available units. This applies to the control valves in the air lines to the tunnel as well as to the systems that control the settings of these valves. One butterfly valve is located in each of three pipe-lines in parallel through which air can be admitted to the settling-chamber. These lines are of 12 in (30.5 cm), 5 in (12.7 cm), and 2\( \frac{1}{2} \) in (6.35 cm) diameter. An appropriate size may thus be selected for the flow rates required for any test. The leak rate (mass flow at zero valve opening) through these valves is approximately 5\% to 10\% of the maximum (fully open) flow rate for the same upstream and downstream pressures. A good overlap between the mass flow ranges available with individual valves is obtained by the valve sizes adopted, and the required sensitivity in control is provided over the entire tunnel mass-flow range. Each butterfly valve is preceded by a parallel slide gate valve. The gate valves fulfil the rapid on-off and the sealing functions, and permit of pre-setting control valves before the run, if so desired. Important additional advantages accrue from this divorce of the on-off and the control functions, and their allocation to separate valves. Firstly, commercially available valves are readily obtainable complete with actuators and other ancillaries; notably the difficulties of obtaining absolutely reliable edge or face seals for butterfly valves suitable for use at 365 psia (2.52 MN/m\(^2\)) and 350°C are avoided. Secondly, the safety interlocks are separated from the stagnation-pressure control loop. The control loop includes
only the butterfly valves, while safety interlocks are provided in the electrical feeds to the gate valve actuators. The latter can thus remain inviolate and are unaffected by any modifications to the control loop. Safety is materially improved in this way.

Stagnation pressure is sensed by a differential transducer (backed off by a trapped volume reference pressure) with a range of 0 to 25 psi (0 to 0.17 MN/m²). The electrical signal from this transducer is compared with a set-point signal in a conventional process controller whose output is related to the input by the usual integral, proportional, and differential actions. To avoid transmission lags this electrical command signal is carried to the immediate vicinity of the control valves where it is converted to a pneumatic signal which operates pilot valves which in turn control the position of the actuator of one of the three butterfly control valves. The signal levels are noted on the diagram together with the approximate -3dB frequencies of the transfer functions of each unit. The controller is normally used in the proportional mode only. Rate action has been found to be of little effect, while integral action results in a slow drift of valve opening prior to a run (which is initiated by opening a quick-acting gate valve) giving a large unpredictable difference between set point and the stagnation pressure at the end of the initial transient.

3. Outline of operating experience at low Mach numbers

3.1 Open loop gain and the effect of falling storage pressure

The problems of low-Mach number operation are essentially ones of obtaining enough open-loop gain to reduce the stagnation pressure error to acceptable levels while avoiding instability. The significance of the control system open-loop gain is, of course, bound up with the "steady state" error.

The proportional action of the controller means that under steady state (d.c.) or low frequency conditions the valve opening is linearly related to the difference between demanded (set point) and actual stagnation pressures. This difference can be regarded as the "steady state" error and it must be present in order to produce the valve opening required for any given stagnation pressure. However, the higher the open-loop gain the smaller is the error needed to produce a given change in valve opening. The error can, of course, be set to zero for one particular valve opening during each run - by the "manual reset facilities" in the controller which adjust the controller output for zero error input. Nevertheless, since the valve opening has to increase continuously during a run at low Mach numbers (to allow for changes in storage pressure) the steady-state error has to be allowed to increase continuously and will only be zero at one instant. But the rate of change will still depend on the open-loop gain; the higher the gain the smaller will be the rate of change of stagnation pressure.

The fall in storage pressure also reacts on the open-loop gain itself. In considering this it may first be noted that the rate of fall of pressure is normally sufficiently small that the high frequency response of elements in the control loop can be
neglected. Hence, apart from the starting transient, the run can be considered as a succession of virtually steady state conditions. The steady state error, whose dependence upon the open-loop gain was outlined above, thus constitutes the total error. A useful analysis of the uncontrolled blowdown tunnel\(^3\) has brought out this point with regard to lag time due to finite settling-chamber volume. Storage pressure affects the open-loop gain through the static gain of the control valve (mass-flow change per unit change in valve opening) which, as pointed out by others\(^2\) falls as the storage pressure falls. Thus, the open-loop gain tends to fall throughout the run giving rise to an increasing rate of change of stagnation pressure.

Various installations (e.g. those at B.A.C., Warton, England and N.L.R., Amsterdam, Holland) have incorporated means for modulating the open-loop gain by a subsidiary control system which is used so as to keep the open loop gain invariant during the run. These methods often necessitate an expensive special valve. An alternative approach has been to use an adaptive control system\(^2\) in which the increasing error is detected and excessive error increases controller gain (and hence open-loop gain) in a stepwise fashion. Both methods demand non-standard elements in the control loop.

However, in the 15 in x 10 in tunnel it has not been found necessary to incorporate such sophisticated devices. Sufficiently high controller gains can be used to avoid excessive errors without giving rise to instabilities in the initial (high storage pressure - high loop gain) part of the run. This is attributed to two factors which will be discussed in turn in the next two sections.

### 3.2 Control valve characteristics

The mass flow per unit open area through the butterfly valves has been found to be sensibly constant for given upstream conditions. It has also been found to be proportional to the pressure upstream of the valve for pressure ratios of upstream pressure to downstream (stagnation) pressures of as low as 1.5 to 1. These points are well brought out in Figs 2 and 3. In Fig 2 the stagnation-pressure/storge-pressure relation traced out during a number of uncontrolled (constant valve opening) runs is shown. The proportionality of these quantities can be seen; and their ratio is plotted as a function of valve opening in Fig 3. Comparison between this curve and the \((1 - \cos \theta)\) dependence of valve opening on valve opening confirms that the valve is, in fact, acting like a variable area choked orifice plate\(^3\), except at low values of pressure ratio.

Excepting low pressure ratios it will be seen that \(\Delta (P_0/P_1)/\partial \theta\) (which is proportional to the valve static gain under these conditions) increases with decreasing pressure ratio. Lower pressure ratios correspond to conditions towards the end of a run so that the valve non-linearities tend to compensate for the effect of lower storage pressure on the open loop gain.

\(*\) with constant discharge coefficient
The results presented in Figs 2 and 3 are the most convenient and accurate form of this type of data that can be obtained during normal use of the tunnel. However, in using only one valve a unique relationship is implied; for a given Mach number, between valve opening and pressure ratio across the valve. The dependence of flow rate per unit open area per unit upstream pressure on pressure ratio thus tends to obscure the effects of flow rate with valve opening at constant pressure ratio. However, the analogy with an orifice plate suggests that for \( P_o/p_v < 0.528 \) flow rate is independent of pressure ratio. It is reasonable, therefore, to infer that the \((1 - \cos \theta)\) dependence of flow rate (and hence \( P_o \)) on valve opening \((\theta)\) closely exhibited for \( 0 < P_o/p_v < 0.528 \) reflects the true dependence of flow rate on valve opening and that the levelling out of the curves of Fig 3 at high values of \( P_o/p_v \) represents the effect of pressure ratio. Confirmation of this view can be obtained from observation of the effects of using the 5 in valve as a "trim" control. With the 12 in valve set fully open \((\theta = 60^\circ)\), opening the 5 in valve to \(30^\circ\) results in a 1.5% increment of stagnation pressure. Further opening of the 5 in valve to \(60^\circ\) gives an additional 3.5% increase in stagnation pressure \((M = 3, \text{ constant } P_v)\).

### 3.3 Automatic control in a trim mode

Further, the control system is in fact normally used to control the setting of the 5 in butterfly valve while the 12 in valve opening remains fixed at some preset value chosen so as to pass the required flow rate at the start of the run. During a low Mach number run, flow is first established by opening the gate valve in series with the 12 in butterfly valve. When the desired stagnation pressure has been attained in this way, the gate valve in series with the 5 in butterfly valve is opened to bring the automatic control into play. The control system then gradually opens the 5 in butterfly valve so as to maintain a constant stagnation pressure in the face of a falling storage pressure.

The advantage of this system lies in the fact that it prevents any large error signal being sensed by the control system at any time when movement of the valve under automatic control has an effect on the mass flow rate into the settling-chamber. If the main (12 in) valve were controlled directly, the control system would see the full desired stagnation-pressure as an error signal at the start of the run. With the open-loop gain necessarily high, the system would be marginally stable and hence lightly damped. The initial overshoot and subsequent oscillation of stagnation pressure would take long to die out. The amplitude of the initial overshoot is of course determined by the damping of the system and by the error signal seen by the controller at the time the control loop is closed (i.e. by opening the gate valve to make the butterfly valve effective, thus establishing a relationship between flow rate and butterfly valve opening). Both these conditions are unfavourable in the case of a single valve passing the total mass-flow. Thus well before actual instability would be reached with increasing controller gain, the practical limit to the use of a single valve is reached when the initial transients occupy the whole tunnel run-time. This condition was demonstrated on the tunnel and hence
confirmed the advantages of using the control system in the trim mode described above. In this, open-loop gains right up to onset of instability can be used. Although transients are only lightly damped the system is operated in such a way that no large perturbations are imposed upon it. The control loop is closed at a time when the error in stagnation pressure is small - ideally zero. Thus only true dynamic instability imposes a limit on useable open-loop gain. Indeed the system can become unstable at the end of a run due to the non-linear valve characteristics noted earlier. Ideally, instability should occur just when the valve reaches the full range of its normal travel (60° open).

The penalty for such simplicity is, of course, that the 5 in valve cannot produce as much total area change as the 12 in valve. While its deficiency of area change per unit valve opening is easily compensated for by increasing controller gain to keep the open-loop gain high, such artifice cannot increase the maximum mass-flow that can be passed by the valve for a given storage pressure. Thus, during a run, the 5 in valve can reach the fully open position. control is then lost, and the experiment has to be terminated. In practice progress of the experiment is merely halted; the 5 in line gate valve is closed, the 12 in butterfly valve reset, the 5 in line reopened and the experiment recommenced. The whole process takes about 10 secs. A 40 to 50 sec. tunnel run is accomplished without resetting the 12 in valve and two resetttings only are needed for the maximum (air storage capacity limited) run of approx. 150 secs.

A typical stagnation pressure history at \(M = 3\) is given as Fig 4 together with a histogram showing the run-to-run variations in rate of change of stagnation pressure.

4. Operating experience at high Mach numbers

At higher Mach numbers (\(M = 5\) and \(M = 7\)) it is found that the primary problem is the sluggish response of the stagnation pressure to changes in valve opening. Changes in stagnation pressure (\(P_0\)) correspond to changes in stored mass (\(M_0\)) and even if the change in \(P_0\) is small this represents the mass flow \(m_t\) integrated over a considerable time. In fact tests such as those shown in Figs 6 and 7, where constant valve openings were held throughout, show that under these conditions the stagnation pressure rises in a quasi-exponential fashion with a rise time (to 1/e of final valve) which may exceed the whole run time (typically 30 to 90 secs). The whole run is taken up with the starting transient.

Clearly the problem is again one of getting a reasonably good open-loop gain, this time passing air through the 5 in valve (occasionally the 2 in valve) only. The opening of the valve must initially be a maximum to fill the setting-chamber quickly with air so as to improve the rise time. At the right moment, the valve must almost close so as to maintain \(m_v = m_t\) and thus keep the stagnation pressure constant at the desired value. The success that has been attained may be judged from a comparison of Fig 7 with Fig 8 which shows a typical stagnation-pressure history for \(M = 7\) operation under automatic control.
The valve non-linearities discussed earlier seem to have little bearing on this problem which is instead dominated by two other non-linearities. These are:

(i) there is a maximum mass-flow that can be passed by the valve for any given storage pressure. Control cannot be achieved unless the leak rate through the valve is less than the flow rate through the nozzle at the desired test conditions. This requirement imposes a limit on the maximum size of valve that can be used. Thus, in turn, implies a certain maximum mass flow through the valve \( m_0 \) (when the valve is in its fully open position).

(ii) negative (reverse) flows are not possible, as \( p_v > p_o \) the flow must be towards the tunnel (indeed it can only be reduced to the level of the leak rate).

The first condition imposes a limit on the improvement in rise time to be obtained from an automatic control system. The second means that severe penalties are paid for any overshoot of stagnation pressure. Thus if stagnation pressure overshoots the desired value the valve terminates the flow into the settling-chamber; the net outflow is then equal to the flow rate down the nozzle. As has been noted earlier, this is so small as to take a long tune to accomplish any significant change in stagnation pressure. Whilst rise times can be improved by increasing the in-flow there is no way in the conventional tunnel of increasing the outflow beyond its "natural" value.

The long time required for the decay of any overshoot of stagnation pressure means that at high Mach numbers the optimum controller gain is no longer the maximum that can be obtained without inducing dynamic stability. Very high controller gains result in fast average rates of rise of stagnation pressure at the start of a run but the narrow proportional band consequent upon high controller gains results in the valve staying fully open until the stagnation pressure has risen almost exactly to the desired level. The command from the controller, to close the valve in order to maintain a constant stagnation pressure, is thus left till too late. The inevitable lags (due to response times of controller and valve actuator, valve inertia etc.) result in late closure of the valve and hence large stagnation pressure overshoot.

On the other hand, too low a controller gain results in premature closure of the valve before the stagnation pressure has risen sufficiently. The final approach to the desired stagnation pressure is thus made too slowly.

An optimum controller gain found by experience for \( M = 7 \) operation is that which gives full scale valve opening for a 22.5 psi \( (0.155 \text{ MN/m}^2) \) change in stagnation pressure (d.c. conditions).

Not a great deal of attention need be given to the problem of maintaining a steady stagnation pressure once it is achieved under these conditions. The very sluggishness of its response means that small differences between flow rates into and out of the settling chamber do not give rise to troublesome rates of change of stagnation pressure.
5. **Use of an analogue computer - (and a simple representation of settling-chamber processes)**

The "feel" for the basic elements of the problem was developed using the tunnel in conjunction with a small analogue computer. This made it possible to repeat simulated runs quickly and at repeatable values of $p_v$. Neither is possible on the tunnel itself because it shares its compressed air storage with other intermittent tunnels. All the elements of the control loop shown in Fig 1 are easily represented by conventional linear analogue-computing elements with the exception of the control valves and the settling-chamber. The control valve mass-flow/valve-opening characteristics were approximated by $m_v = k_y p_v^{62}$ which could be easily generated by two analogue multipliers. The good correspondence between this formula and the true relationship is shown in Fig 9 where $p_{of}$, which is proportional to $m_v (m_v = m_t$ when $\partial p_0/\partial t = 0)$, is shown as a function of $\theta$ for two values of $p_v$ at $M = 5$. Non-linear, biased-diode elements were used to limit $\theta$, as noted above, between $0 \leq \theta \leq 60^\circ$.

The settling-chamber poses more difficult problems. The capacity of the settling-chamber to store mass, and the outflow of this mass through the nozzle, is broadly analogous to an electrical resistance/capacitance system in a simple RC low-pass network, and leads naturally to the expressions for characteristic time of $C_s/k_t = \frac{V_R}{k_t R_0}$ (where $k_t = \frac{d m_t}{d p_0}$).

Other workers\(^{(2,4)}\) have suggested that the correct value for the pneumatic capacitance $C_s$ is $\frac{V_R}{k_t R_0}$ and not $\frac{V_s}{R_0}$; the latter corresponding to isothermal conditions. They have argued that small changes in $p_v$ will be accomplished in an adiabatic fashion and have proceeded to use the isentrope relationship $(p_v/p_0)^{\gamma - 1} = constant$. However, even in the adiabatic case, the process will not be isentropic since the incoming air will mix with the air already present in the settling-chamber which may well be at a different temperature. This mixing creates entropy. The present authors believe that the assumption of isothermal conditions is the correct procedure on two grounds:-

(i) Any heat transfer from fluid to walls will cause the situation to move towards the isothermal rather than the adiabatic case.

(ii) In the case of small pressure changes, the thermal capacity of the air initially present in the settling-chamber will enable it to absorb any excess energy due to the incoming air being at a higher temperature. The fact that changes in $p_v$ are small results in isentropy only when the changes are vanishingly small, and then the process will be isothermal as well as isentropic.

The present authors also maintain that this simple representation of the settling-chamber as a simple linear "low-pass filter" can be used for pressure changes of many times the initial level (including the whole stagnation pressure history of a typical run). To do this the temperature at which $C_s$ is evaluated must be that corresponding to the higher pressure level.

The most severe test of such a proposition is provided by a step change in flow rate through the valve. This effectively occurs at the start of a run at constant valve opening or at the end of any run.
Fig 10 shows the decay of stagnation pressure after flow into the settling-chamber was terminated to end a $M = 5$ run. The near exponential decay can be clearly seen and the good agreement between results from tunnel and analogue computer is apparent.

This is, as just mentioned, the most severe test that can be devised for the proposition. The stagnation temperature falls during the decay of stagnation pressure and this is reflected in the increasing rate of decay at lower pressure (long elapse time). However, the apparently large disparity at $(P_0/P_{of}) \approx 0.8$ corresponds, in fact, to a difference between analogue computer and wind tunnel of approx 1 atm in a fall of approx 10 atm in pressure.

Better results are obtained in analysing the start of a run (Fig 11). Here, under more favourable circumstances, the agreement is excellent. This latter case is the one of most practical interest.

The long time constants, of course, preclude the use of an actual RC filter but an active system with the same transfer function is easily constructed in the usual way.

To examine the reasons for this result, and to add further confirmation, a simple digital computer programme was written. This performs numerically the integrations necessary to solve the complete set of exact equations describing nozzle mass-flow rate, and the conservation of mass and energy. This calculation can include heat transfer to the settling-chamber walls. It is, however, non-exact in that it does not represent the true conditions of extensive free convection effects under such conditions but rather uses a bulk-average stagnation temperature.

Fig 11 compares analogue computer, digital computer, and measured stagnation pressure histories for a run at $M = 7$ with a constant valve opening*. Figs 12a and 12b show digital computer results for typical values of settling chamber volume and Mach number. The response to step changes in flow rate for different initial pressures and different heat-transfer rates are shown. As postulated earlier, the variations in temperature are much less than the wide excursions implied by the isentropic relationships and virtually steady temperatures seem to be reached much more quickly than steady pressures.

A good example of the application of the analogue computer is illustrated in Fig 8b. This shows the time between first maximum and first minimum of stagnation pressure during initial transients of $M = 5$ operations. The marked dependence of this time, an important contribution to the total time needed to attain steady conditions, on the extent of the initial stagnation pressure overshoot is clearly shown. Also evident is the good correlation between analogue computer and wind tunnel results. The analogue computer runs shown here were, in fact, performed before attempting use of automatic control at high Mach numbers. The understanding thus gained reduced considerably the number of tunnel runs needed to find good controller settings.

* The digital computer result includes estimated heat transfer effects.
6. Conclusions

(i) Control problems have been found to fall sharply into two categories.

(a) At low Mach numbers the stagnation pressure responds quickly to changes in valve opening but the storage air pressure falls rapidly and the control system must continuously adjust valve opening to maintain a constant stagnation pressure. The highest stable open loop gain is the best and use of the automatically controlled valve in a "trim mode" has been found to be a useful expedient.

(b) At high Mach numbers the problem is one of getting the stagnation pressure onto the desired valve quickly despite its very sluggish response to valve opening. An intermediate open loop gain was found to be optimum.

(ii) The mass flow/stagnation pressure transfer function of the settling-chamber could be represented as a simple time lag (low pass RC filter) over the whole range of stagnation pressure provided that the pneumatic capacitance was evaluated for isothermal conditions at an appropriate bulk air temperature (equal to the inlet air temperature for adiabatic conditions).

(iii) The simple classical approach to calculation of response, stability, etc. through the use of linear differential equations to model the system was inadequate. Non-linearities due to the finite limits of valve travel and the non-linear valve angle/open area relationship are important.

(iv) A simple analogue computer proved to be an invaluable aid to understanding the essential features of the problems.
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APPENDIX

This append-ix sets out a number of relations pertinent to the **discussion** of the **main body** of the paper. These are intended to supplement the main descriptive **approach** and to illustrate the approximations used in setting up the simple **analogue** computer.

(a) **Basic Equations**

1. **Flow through the control valve**
   
   \[ m_v = k_v \cdot p_v \cdot f'(\theta) \cdot f''(T_v) \cdot f'''(p_v/p_o) \]
   
   if gate valve is open

   \[ m_v = 0 \]
   
   if gate valve is shut

   \( k_v \) includes effect of valve size and discharge coefficient.

   For a butterfly valve \( k_v \cdot f(\theta) \) is proportional to \( k_v \cdot (1 - \cos \theta) \), or to a close approximation through the working range \( 0 \leq \theta \leq 60^\circ \)

   \[ k_v \cdot f'(\theta) = k_v \cdot \theta^2 \]

   For all but the lowest pressure **ratios**

   \[ f'''(p_v/p_o) = 1 \]

   As upstream temperature is normally constant throughout each run

   \( f(T_v) \) can be combined with \( k_v \)

   **Taken together these approximations give**

   \[ m_v = k_v \cdot p_v \cdot \theta^2 \]

2. **Flow through nozzle throat**

   As sonic conditions exist at the throat for the vast majority of a run

   \[ m_t = k_t \cdot p_0 \cdot f''(T_0) \]

   If it is assumed that conditions in the settling chamber may be taken as **isothermal** (assuming a proper choice of \( T_0 \)) then this temperature may be combined with the effects of the **throat** area in the constant \( k_t \) giving

   \[ m_t = k_t \cdot p_0 \]

3. **Mass of air in the settling chamber**

   \[ M = p_0 \cdot V_s \]

   \[ = p_0 \cdot V_s / R \cdot T_0 \]

   or for isothermal conditions

   \[ M = C_s \cdot p_0 \]

   Where \( C_s \) the pneumatic **capacitance** \( = \Delta M/\Delta p_0 = V_s / R \cdot T_0 \)
Note that if it is erroneously assumed that the isentropic relation
\[ p_o = C_p \rho_o \] held true, then the pneumatic capacitance \( CM \) be evaluated by
considering a small change \( dp_o \) in stagnation pressure resulting in a small change
\( dM \) in mass stored

\[
\frac{dM}{M} = \frac{dp_o}{\rho_o} = \frac{dp_o}{\gamma p_o}
\]

so that \( C_s = \frac{dM}{dp_o} = \frac{M}{\gamma p_o} = \frac{V_s}{R \cdot T_o \cdot \gamma} \).

This is, however, an incorrect assumption

There is, of course, no fundamental difficulty in accounting for a variable stagnation temperature \( (T_o) \) through the use of the conservation of energy principle but the solution of these more complex equations is best done numerically.

To do this \( k_t^*, k_v^*, \) and \( C_s^* \) are defined in the same ways as \( k_t, k_v, C_s \) but for \( T_o = 273.2^\circ K \). Then following the usual theories for flow through an orifice

\[
m_t = k_t^* \sqrt{\frac{273.2}{T_o}}
\]

\[
m_v = k_v^* p_o (1 - \cos \theta) \sqrt{\frac{273.2}{T_v}}
\]

\[
M = C_s^* p_o \cdot \frac{273.2}{T_o}
\]

The corresponding energy equations are:

\[
I_t = \gamma m_t T_o C_v
\]

\[
I_v = \gamma m_v T_v C_v
\]

\[
I_{\text{store}} = C_v M T_o
\]

To which must be added the heat transferred to the settling chamber walls (temp. \( T_w \))

\[
I_{\text{loss}} = h (T_o - T_w)
\]

Conservation of energy requires that

\[
C_v M \left( \frac{\partial T_o}{\partial t} \right) = \gamma C_v \left[ m_v T_v - m_t T_o \right] - h(T_o - T_w)
\]

Near the end of the run \( m_v \approx m_r \) and \( \frac{\partial T_o}{\partial at} = 0 \)
so that:

\[ \gamma c_v m_v \left( T_v - T_0 \right) = h \left( T_0 - T_w \right) \]

Thus

\[ T_0 = T_{of} = \frac{\left( \gamma c_v m_v T_v + h T_w \right)}{\left( h + \gamma c_v m_v \right)} \]

This equilibrium temperature is approached quickly, as noted in the main body of the text, so that the earlier, simple, "isothermal" equations normally give satisfactory accuracy, provided that the coefficients \( k_t \) and \( C_s \) are evaluated at this "final" value of stagnation temperature \( T_0 = T_{of} \). \( T_{of} \) can be computed from the above equation if the heat transfer factor \( h \) is known. Unless \( h \) is extremely large, as is the case for the NPL 15 in. x 10 in. tunnel at \( M = 7 \), then \( T_{of} \) may be taken as equal to \( T_v \) with little error since the response time of the settling chamber varies only slowly with \( T_0 \).

(b) Basic Control Characteristics

In this section the simplest forms of the basic equations are used to derive the fundamental features of the control problem as discussed in the main body of the text.

1. Low Mach number operation

Conservation of mass requires that

\[ m_v - m_t = \frac{\partial m}{\partial t} \]

i.e., \( k_v \rho_v \theta_v - k_t \rho_o = C_s \frac{\partial \rho_o}{\partial t} \)

During low Mach number operation \( \frac{\partial \rho_o}{\partial t} \) is small after the initial transient has decayed so that

\[ k_v \rho_v \theta_v = k_t \rho_o \]

and, as all quantities are changing slowly, terms in \( \frac{\partial \rho_o}{\partial t} \) in the feedback loop transfer function may be neglected so that the action of the controller may be written as:

\[ \theta_v = A + B \left( \rho_{od} - \rho_o \right) \]

i.e., \( k_v \rho_v [A + B \left( \rho_{od} - \rho_o \right)]^2 = k_t \rho_o \)
so \( k_v p_v A^2 + k_v p_v B^2 (p_{od} + p_o)(p_{od} - p_o) + 2k_v p_v AB (p_{od} - p_o) = k_t p_o \)

and if \((p_{od} - p_o) = \Delta p_o \ll p_o\)

\( k_v p_v A^2 + 2k_v p_v B^2 \Delta p_o + p_o + 2k_v p_v AB \Delta p_o = k_t p_o \)

Thus small errors are achieved only by the use of high controller gains (B) and correct choice of offset A. It may also be seen that as the run proceeds and \( p_v \) falls the error grows rapidly unless B is large.

Similar considerations apply to flow through two control valves in parallel.

2. **High Mach number operation**

The most important consideration in high Mach number operation is the long response time of the stagnation pressure to changes in control valve opening.

Consider a step change in valve opening from \( \theta_v = 0 \) to \( \theta_v = \theta_v^f \)

Conservation of mass gives

\[ C_s \frac{\partial p_o}{\partial t} = k_v p_v \theta^2 - k_t p_o \]

which may be solved to give

\[ p_o = p_{of} (1 - e^{-t/T}) \]

where \( p_{of} = \frac{k_v}{k_t} p_v \theta_v^2 \)

and \( T = C_s/k_t \)

Thus, the settling chamber acts like a simple low pass filter with a transfer gain \( (\partial p_o / \partial m_v) \) of \( 1/k_t \) and a characteristic time \( T = C_s/k_t \).

3. **High frequency characteristics of controller in feedback loop**

The discussions in Section 1 and 2 above have neglected the high frequency response of the controller. However, it must not be thought that this is unimportant. The high frequency response of the controller/valve actuator/control valve combination determines the stability of the Complete system and
hence the maximum useable gain (B) and the error Ap. It also directly influences the amplitude of the first overshoot in stagnation pressure during the initial transient of a high Mach number run and, as shown in the main body of the text, largely determines the minimum time required to attain a stable stagnation pressure.
Controller

-3dB at 20 c/s

Set point level generator

N.B. Safety interlocks, heaters, other (preset) valves, etc. omitted for clarity

Sketch of stagnation pressure control loop
NB. Individual data points were taken at approx 5 psio intervals in pv and lie on lines shown to within ±2 psi or better.

Typical results obtained during M = 3 runs with constant control valve openings. N.B. Valve openings denoted by : =

(Opening 12 in valve, opening 5 in valve)
FIG. 3

Installed flow characteristics of two butterfly valves

N.B. \((1 - \cos \theta)\) law curves are arbitrarily scaled to facilitate comparison with measured values. For relations used in analogue computer see Fig. 9.
Analysis of 20 runs

\[ M = 3 \quad \rho_{od} \approx 150 \text{ psia} \]

Duration of test = 45 secs

Typical stagnation pressure variation during \( M = 3 \) test
Schematic diagram illustrating nomenclature

- Air storage receivers
- Pneumatic capacitance $= C_b$
- Gate valve ("on-off")
- Butterfly valve
- Pressure shell wall temp. $= T_\omega$
- Heat loss to wall $= h \left( T_o - T_\omega \right)$
- Air pressure $= p_v$
- Air temperature $= T_v$
- NB. This is not a definitive diagram of control circuit.
  See Fig.1 for more complete sketch
FIG. 6

Typical stagnation pressure history, constant opening 5 in. valve.
No other valve in use. M=5
FIG. 7

+ Denote individual data points

Typical stagnation pressure history, constant opening 5 in valve. No other valve in use. $M = 7$

N.B. Intervals marked on abscissa are same as those used in Fig. 8 a
Stagnation pressure history

Automatic control of 5 inch butterfly valve (no other valve open)

\[ \rho_{_0} = 10.65 \text{ atm.} \]
Effect of stagnation pressure overshoot on settling time and comparison between wind tunnel and analogue computer data

\( M = 7 \)

10 atm \( \leq \rho_v \leq \) 25 atm. Sin. butterfly valve only used (automatic control)
Comparison of wind tunnel and analogue computer ($\theta^2$ law) results for stagnation pressure following initial transient ($p_{o_f}$) $M = 5$, in valve.
FIG. 10

Data taken from this part of run

Elapsed time after termination of flow (secs)

Delay of stagnation pressure after run
Comparison of wind tunnel data (W/T) with digital computer solution (D) and exponential curve with rise time as in analog computer model of settling chamber.
Run D \( p_o = 0.20 \text{ atm. at t=0} \)
Run C \( p_o = 0.82 \text{ atm. at t=0} \)

Digital computer solutions
(Response to step change of input flow rate)
FIG. 12b

Run B \( p_0 = 0.82 \) atm at \( t = 0 \)

Run E \( p_0 = 0.20 \) atm at \( t = 0 \)

\[ T_v = 581^\circ K \]

\[ m_v = 0, \ t < 0 \]

\[ m_v = 3.6 \text{ lb/sec}, \ t > 0 \]

Shell temp. \( = 285^\circ K \)

Settling chamber volume = 301.0 ft\(^3\)

M = 7

Digital computer solutions (Responses to step change in input flow rate)
NOTES ON THE AUTOMATIC CONTROL OF A BLOWDOWN WIND TUNNEL

This report briefly describes salient results obtained during commissioning of the stagnation pressure control system of the N.P.L. 15 in. x 10 in. (38.1 cm x 25.4 cm) blowdown wind tunnel. The very different problems of low and high Mach number operation are examined, and the representation of the various flow processes in a manner suitable for an analogue computer is discussed.