The Response Times of Typical Transducer-Tube Configurations for the Measurement of Pressures in High-Speed Wind Tunnels

By

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SUMMARY

The theories of Kendall and Davis are extended to obtain the response time for low pressures with slip flow in a compound tube system. The theory provides a useful estimate of the response time, even when the volume of the measuring instrument falls to zero, and predicts the correct trends for the effects of varying the applied pressure and the tube geometry.

However, the detail design of a pressure-measuring system remains dependent on experimentally-determined response times. Comprehensive data are therefore presented from the present experiment covering a range of tube configurations, incorporating a flush-diaphragm pressure transducer as the measuring instrument, suitable for the measurement of model pressures in high-speed wind tunnels.

Consideration is given to the special conditions that apply to the case in which the tube systems are connected to a pressure-scanning switch.
1. Introduction

The response times in pressure-measuring systems increase as the pressures to be measured decrease and thus become of increasing importance in high Mach number wind tunnels. The low test section and model static pressures could be raised in principle by increasing the stagnation pressure, but this is usually not practicable because of the limitation in tunnel design and because of the need to simulate high altitude flight.

An investigation of the response time problem is therefore necessary in order to derive the optimum measuring and tubing systems to exploit the capabilities of new wind-tunnel facilities and, in the limit, to ensure that the pressure in the measuring instrument reaches equilibrium with the orifice pressure within the tunnel operating time. Such an investigation has been undertaken with the new wind-tunnel facilities in the Aerodynamics Division of the N.P.L. in mind. The operating conditions of one of these is shown in Fig. 1 to provide an example of the typical range of test section static pressures and operating times to be encountered; the model static pressures can, of course, be even lower than the test section values shown.

The present investigation covers one aspect of the pressure measuring and data recording systems on the new N.P.L. high speed wind tunnels. Other aspects will be reported in subsequent papers.
The finite volume of the measuring instrument, the length, the diameter and the volume of the connecting tubing, and the small size of the model orifice all affect the response time.

Fig. 2 shows experimental results of the variation of response time with instrument volume for various applied pressures and a fixed tube configuration, illustrating that the response time is approximately proportional to the volume and inversely proportional to the measured pressure. The first step in reducing the response time is therefore the use of a flush-diaphragm transducer, for which the added volume can be very small (about 0.0004 cu in.).

For this reason and because earlier theories appeared to be inadequate for systems involving near-zero instrument volumes, emphasis in the present investigation has been placed on the use of a flush-diaphragm pressure transducer.

The most frequent application of the pressure-measuring systems to be developed will be to the measurement of pressure distributions on models inserted in the test sections of the wind tunnels. The sizes of the models used in the N.P.L. tunnels are necessarily small, so that, at present, it is not possible to mount a multiplicity of transducers inside each model. The model pressures must therefore be tubed to transducers situated either in the model support housing or external to the tunnel test section. The number of pressure tappings and the size and shape of the model dictate the possible tube and orifice geometry. The present investigation has therefore covered a range of orifice-tube configurations embracing the needs of the various installations.

In the present report theoretical methods of predicting response times have been extended to low pressures and to the case of measuring instruments with zero volume, but in the detailed design of tubing systems are found to provide a guide only. Extensive experimental data are therefore presented also.

A feature of the N.P.L. systems for the measurement of model pressures is a specially designed pressure-scanning switch which is used for the following:

1) to protect each transducer from tunnel starting and shut-down overloads;
2) to provide a method of applying calibration and zero reference pressures before and after each tunnel run;
3) to connect a number of model pressures to each transducer sequentially during a given run when the response times permit.

The overall time required to record a number of pressures sequentially with a pressure-scanning switch can be minimised by reconsidering the design of the tubing systems. An Appendix provides some notes on the use of a pressure-scanning switch and the design of the associated tubing systems.

A miniature pressure-scanning switch that can be inserted inside a small model is now under development in N.P.L. Aerodynamics Division.
2. Notation

- $a_0$: speed of sound, ft/sec
- $d$: inside diameter of tube, ft
- $d_e$: equivalent diameter of tube, ft
- $f$: fraction of gas molecules that is diffusely reflected on striking the walls of the tube
- $k$: fraction of tubing volume added to instrument volume

\[
K = 8 \left( \frac{\mu}{d} \right) \left( \frac{2}{f} - 1 \right) \frac{\mu}{d(\rho/p)^2}, \text{lb/ft}^2
\]

- $\ell$: tube length, ft
- $\ell_e$: equivalent length of tube, ft (see equation (3))
- $p$: pressure, lb/ft$^2$
- $p_a$: pressure applied to open end of tube, lb/ft$^2$
- $p_0$: initial pressure in tube and instrument, lb/ft$^2$
- $\Delta p$: initial pressure difference at the open end of the tube, $|p_i - p_o|$, lb/ft$^2$
- $p_m$: mean pressure in tube at any instant

\[
\frac{p_i^2 - p^2}{2(p_i - p)} = \frac{(p_i + p)}{2}, \text{lb/ft}^2
\]

- $Q$: mass flow, slugs/sec
- $t$: time, sec
- $t_P$: response time for Poiseuille flow, sec
- $t_s$: response time for slip flow, sec
- $V$: volume of instrument, cu ft
- $v$: volume of tubing, cu ft

- $\lambda$: time constant, defined as $\frac{\Delta p}{(d\rho/\rho_t)_{t=0}}$, sec (see equation (1))
- $\mu$: coefficient of viscosity: for air at 20°C, $3.8 \times 10^{-7}$ slug/ft·sec
- $\rho$: density, slug/ft$^3$. 

3./
3. Theory

Consider a simplified pressure-measuring system consisting of a tube of small bore connected to a measuring instrument of constant volume. If the open end of the tube is subjected to a sudden pressure change, the pressure in the measuring instrument theoretically approaches the new value asymptotically with time.

For most practical applications the response time can be taken as the time for the instrument pressure to reach a value corresponding to 99% or 99.9% of the initial pressure difference. Since this is difficult to measure in practice a number of authors have checked the validity of their theories by measuring a 'time constant'. This time constant can be defined as the time that would be taken to reach the measured pressure at the maximum response rate. Fig. 3 illustrates the methods for determining the response time and time constant.

3.1 Poiseuille flow

The equations for the response time and time constant, based on the Poiseuille equation for low Reynolds number flow are derived in Refs. 2, 3 and 4.

Time constant, \( \lambda = \frac{(p_a - p)}{(dp/dt)_{t=0}} = \frac{128\mu(V + kv)\ell}{\pi d^4 p_m} \) \( \ldots(1) \)

Response time, \( t_p = \frac{128\mu(V + kv)\ell}{\pi d^4 p_a} \left\{ \log \left( \frac{P_a + P_0}{P_a - P} \right) - \log \left( \frac{P_a + P_0}{P_a - P_0} \right) \right\} \), \( \ldots(2) \)

where \( p = 0.999P_a + 0.001P_0 \) for the time to 99.9% of the initial pressure difference,

and \( p = 0.99P_a + 0.01P_0 \) for the time to 99% of the initial pressure difference.

In Ref. 4 the problem is analysed more exactly with the result that a term, \( \ell/a_0 \), is added to the right hand side of equation (2); this term has little influence on the response time for configurations involving short tube lengths and long response times.

The above equations can be extended to the case of a compound tubing system consisting of various tube diameters and lengths. This is effected by considering an equivalent system of length \( \ell_e \) of constant diameter tubing \( d \), where \( \ell_e \), derived from equation (2), is given by

\[ \ell_e = \ell + d \sum_{i=1}^{n} \frac{\ell_i}{d_i^4} \] \( \ldots(3) \)

and where \( \ell_i \) and \( d_i \) are the length and diameter respectively of the \( i \)th tube.

An orifice is treated in the same manner as any other tube in the system.

3.2/
3.2 Slip flow

Equations (1), (2) and (3) are not valid for very low pressures and tubes of small diameter in combination, because of the occurrence of slip flow or even of free molecular flow.

In Ref. 5 a solution for the time constant is obtained using an equation for the gas flow rate derived in Ref. 6:

\[
\text{time constant, } \lambda = \frac{128\mu (V+kv) \ell}{\pi d^3 \left[ p_m \cdot d + 2 \left( \frac{2}{f} - 1 \right) \mu \left( \frac{\pi p}{2 \rho} \right) \right]} \quad \ldots (4)
\]

In the present investigation we need the response time and we therefore return to the gas flow rate equation derived in Ref. 6,

\[
Q = \frac{\pi d^4 (p_f^2 - p_p^2)}{256 \mu \ell \cdot RT} \left( 1 + 4 \left( \frac{\pi^{1/2}}{2} \right) \left( \frac{2}{f} - 1 \right) \frac{2\mu}{p_m \cdot d \left( \frac{\rho}{p} \right)} \right), \quad \ldots (5)
\]

where \( R \) is the gas constant and \( T \) is the temperature. The rate of change of mass of air in the instrument and tube, \( \frac{\partial}{\partial t} [(V+kv)\rho] \), must equal the mass flow in the tube, giving

\[
Q = \frac{(V+kv) dp}{RT} \quad \frac{\partial}{\partial t};
\]

therefore

\[
\frac{dp}{dt} = \frac{\pi d^4 (p_f^2 - p_p^2)}{256(V+kv) \mu \ell \cdot RT} \left( 1 + 4 \left( \frac{\pi^{1/2}}{2} \right) \left( \frac{2}{f} - 1 \right) \frac{\mu}{p_m \cdot d \left( \frac{\rho}{p} \right)} \right). \quad \ldots (6)
\]

Integrating equation (6) and setting \( p = p_o \) when \( t = 0 \) gives the response time for slip flow,

\[
t_s = \frac{128\mu (V+kv) \ell \cdot \log \left( \frac{P_a + P}{P_a - P} \right) - \log \left( \frac{P_a + P_o}{P_a - P_o} \right) + \log \left( 1 + \frac{2K}{P_a + P} \right)}{\pi d^4 (P_a + K)}, \quad \ldots (7)
\]

where/
where

\[ K = 8 \left( \frac{\pi}{2} \right)^{\frac{1}{2}} \left( \frac{2}{f} \right)^{\frac{2}{f}} \left( \frac{\mu}{d} \right)^{\frac{2}{f}}. \]

Equation (7) reduces to equation (2) at higher pressures when \( p_1 \gg K \).

With a value of \( f = 0.8 \) (suggested in Ref. 5), and air at 20°C

\[ K \cdot d = 0.0054 \text{ lb/ft}. \]

These equations can be extended to the case of a compound tubing system in a manner similar to that used for Poiseuille flow. The expression for equivalent length becomes

\[ \ell_e = \ell + \sum_{i=1}^{n} \frac{-K_d}{d_i} \left( p_{1i} + K \right) \log \left\{ \frac{199p_{1i} + p_o + 200K_i}{p_{1i} + p_o + 2K_{1i}} \right\}, \ldots (8) \]

where \( K_d = 0.0054 \cdot K_d_i \).

The above equivalent length has been determined, assuming the response time to 99% of the pressure difference is required, by substituting \( p = 0.99p_1 + 0.01p_o \). It should be noted that the equivalent length is not constant for a given tubing configuration (as was the case with Poiseuille flow) but is now pressure-dependent. When \( p_1 \) becomes large in comparison with \( K \) and \( K_1 \), the above expression reduces to the Poiseuille form of \( \ell_e \) (equation (3)). For example, with \( p_1 = 50 \text{ mm Hg} \) and \( p_o = 0 \), the values of \( \ell_e \) given by equations (3) and (8) differ by less than 1%.

3.3 Comparison with experiment

In Figs. 4 and 5 the theoretical predictions for the response times of typical tube configurations are compared with experimental results. The predictions are shown for both Poiseuille flow and slip flow with the value of the empirical constant \( k \) taken as unity (i.e., taking into account the total volume of the tube system), and, for slip flow only with \( k = \frac{3}{5} \). These comparisons show that the slip-flow theory as derived in Section 3.2 predicts the correct trends for the variation of response time with applied pressure difference, but that in some cases (e.g., Fig. 5) the absolute values may be in error by as much as 100%.

There is an inconsistency between previous authors on the value of the empirical constant \( k \) to be used, especially when the instrument volume \( (V) \) is reduced to zero. Ref. 3 states that the full volume \( (k = 1) \) of a tube should be added to the total system volume only if the diameter of the tube is greater than three times the diameter of the first tube, or orifice. This leads to a trivial solution if the instrument volume is zero and the diameters of the tubes comprising the compound system are not greater than three times the diameter of the first tube. Ref. 5 proposes that \( k = \frac{1}{3} \) for all tube and orifice configurations whereas Ref. 2 neglects the effects of tube volume.
Figs. 4 and 5 and other comparisons between theory and experiment shown in Figs. 15-32 suggest that the best overall fit for the tube configurations considered in this investigation is obtained with a value of the empirical constant, \( k \), of unity.

### 3.4 Effect of the initial pressure level and of the sign of the applied pressure difference

It is convenient to draw a theoretical comparison between response times for the same pressure difference applied at opposite directions and at various initial pressure levels (Fig. 6).

We use as a basis for comparison the response time, \( t_s' \), obtained with an initial instrument pressure of zero \( (p_0 = 0) \) and an applied pressure difference \( \Delta p = p_1' \). If the response time for the same pressure difference, \( \Delta p = p_1' = |p_1 - p_0| \), starting at an elevated pressure level, is \( t_s \), then the ratio

\[
\frac{t_s}{t_s'} = \left( \frac{p_1' + K}{p_1 + K} \right) \frac{\log \left( \frac{199p_1 + 200K + p_0}{p_1 + 2K + p_0} \right)}{\log \left( \frac{199p_1' + 200K}{p_1' + 2K} \right)}
\]

Curves of \( t_s/t_s' \) derived from this equation are plotted in Fig. 7 for a range of pressure differences, \( \Delta p (= 1 \text{ mm Hg}, 10 \text{ mm Hg and } 100 \text{ mm Hg}) \) for \( p_1 > p_0 \) and \( p_1 < p_0 \). The value of \( K \) chosen corresponds to a constant tube diameter of 0.036 in,

\[
i.e., \quad K = \frac{0.0054}{d} = 1.8 \text{ lb/ft}^2.
\]

It can be seen from Fig. 7 that for a constant pressure difference a pressure drop in the instrument leads to a longer response time than the corresponding pressure rise. This is due to the fact that the equilibrium process takes a longer time near the lower final pressure level. However, the direction of the pressure step has little effect on the response time when the overall pressure level is increased, \( |p_1 + p_0| \gg |p_1 - p_0| \).

To evaluate \( t_s/t_s' \) for a compound tube system it is necessary to obtain the equivalent lengths from equation (8) for the pressure levels under consideration.

Using the notation \( \xi (p_0, p_1) \) as a general expression for the equivalent length of a system at initial and final pressure levels of \( p_0 \) and \( p_1 \) respectively, and \( \xi_e (0, p_1') \) as a general expression for the equivalent length at initial and final pressure levels of \( p_0 = 0 \) and \( p_1 = p_1' \), then

\[
\frac{t_s}{t_s'}
\]
Therefore \( t/t' \) for a compound tube system can be determined by multiplying the values from Fig. 7 by a factor

\[
\frac{\zeta_e(p_0, p_1)}{\zeta_e(0, p_1')}
\]

It should be noted that Fig. 7 has been plotted for a tube diameter of 0.036 in. and to use this graph an equivalent compound system must be derived with an equivalent diameter of 0.036 in.

### 3.5 Optimisation of the tube system

In the majority of pressure-measuring systems the tube diameter in the model, and possibly the model support, is governed by the size of the model. The end of each model tube is connected via a larger diameter tube to the measuring instrument. The diameter of the latter tube may be optimised since the response time depends on the total flow resistance and the volume of the system, for Poiseuille flow (Ref. 3).

Consider an orifice of diameter and length \((d_o, \ell_o)\), first tube \((d_1, \ell_1)\) and second tube \((d_2, \ell_2)\) connected to a zero-volume transducer.

For a given pressure, \( t_p \propto \frac{\nu \ell_o}{d^4} \),

\[
\nu = - \frac{\pi}{4} \left[ d_o^2 \ell_o + d_1^2 \ell_1 + d_2^2 \ell_2 \right].
\]

From equation (3) using \( d_1 = d \) (the basic tube diameter)

\[
\ell_e = \ell_o \left( \frac{d_1}{d_0} \right)^4 + \ell_1 + \ell_2 \left( \frac{d_2}{d_1} \right)^4.
\]

Thus, for a minimum value of \( d_2 \),

\[
\frac{d}{dd_2} \left\{ \frac{\pi}{4d_2^4} \left[ d_0^2 \ell_o + d_1^2 \ell_1 + d_2^2 \ell_2 \right] \left[ \ell_o \left( \frac{d_1}{d_0} \right)^4 + \ell_1 + \ell_2 \left( \frac{d_1}{d_2} \right)^4 \right] \right\} = 0.
\]

This/
This leads to a cubic equation in \( d^2 \),

\[
2a^2 \varepsilon_0 \left[ \frac{d_4}{d_0} \right]^{\frac{4}{3}} + \varepsilon_0 = \frac{1}{2} \varepsilon_0 \frac{d_1}{d_0} \left[ a_1^2 \varepsilon_0 + a_2^2 \varepsilon_2 \right] - 2a^2 a_1^2 \varepsilon_0 = 0, \quad \ldots (11)
\]

which can be solved graphically or numerically for \( d_4 \).

An investigation into optimisation for slip flow for a number of tube configurations revealed that the optimum diameter of the second tube was less than 5% different from that for Poiseuille flow at pressures above 0.5 mm Hg. Therefore in this range the Poiseuille cubic (equation (11)) gives a good approximation. Furthermore a fairly broad minimum is obtained and the variation of optimum second-tube diameter is relatively small for a large range of tube configurations. In Fig. 8 the variation of response time for Poiseuille and slip flow is shown for a compound tubing configuration in which only the second tube diameter is varied. Figs. 9 and 10 compare the slip flow theory with experimental results, and show reasonable agreement for the value of \( d_4 \) at which the response time is a minimum.

4. Details of the Experimental Investigation

The tubing configurations used in the experimental investigation are tabulated in Fig. 11. The choice was based on a survey of existing wind-tunnel models and possible positions for mounting the pressure transducers.

A pressure range of 1 mm Hg to 100 mm Hg was used throughout the experiments since model pressures in this region lead to response times of the same order as the tunnel operating times. The initial pressure in the tubing system was held below 0.02 mm Hg and all the response times quoted are therefore appropriate to the rise from vacuum to the applied pressure.

A Statham PA 2084C, 0-5 psi absolute, \( \frac{1}{2} \) in. flush-diaphragm, unbonded resistance strain-gauge transducer was used as the pressure-measuring instrument. The full-scale output of the transducer with a 7V supply was 76 mV, giving approximately 300 \( \mu \)V/mm Hg.

Fig. 12 shows the transducer mounting block bolted to a brass face plate and tube adaptor. The transducer was fitted to the mounting block with the diaphragm approximately 0.002 in. below the surface, thus reducing the internal volume to a minimum (\( < 0.0004 \) cu in.). A number of face plates were constructed for connection to the various tube diameters used in the tests. All the tubes were made of nickel or copper so that outgassing and porosity effects prevalent in some plastic tubes could be minimised. However, all butt joints in the system were sealed with short lengths of plastic tube.

The complete pneumatic system, shown in Fig. 13, enabled a pressure step to be applied to the tube system via a quick-acting manual stop-valve or an electromagnetic stop-valve. The bore of these valves compared with the orifice diameter was large enough to have negligible effect on the response times. The control for operating the electromagnetic valve included a single-shot pulse generator to trigger an oscilloscope (Fig. 14).


4.1 Experimental method

The pressure in the transducer-tube system was initially reduced to less than 0.02 mm Hg, as indicated on a Leybold Alphatron ionisation gauge. The stop valves $V_1$ and $V_3$ were then closed and $V_2$ opened to set the required pressure level, recorded on a Wallace and Tiernan dial gauge. $V_3$ was then opened rapidly to simulate a step pressure change at the inlet to the tube system. The ratio of transducer-tube volume to cylinder volume was small so that a negligible change of the pressure level in the cylinder was observed after application of the pressure step.

The output of the transducer was recorded on various instruments depending on the response time. For short times the output was fed, via a Redcor 361 D.C. differential amplifier, into a Tektronix type 502 oscilloscope fitted with a Polaroid camera. The electromagnetic valve and associated oscilloscope trigger were used for these runs. The longer response times were recorded on an S.E.I., type 2000, ultra-violet recorder, using a C25 low frequency galvanometer. The Redcor 361 was used to amplify the transducer signal with a switched C-R filter-attenuator on the output. The filter-attenuator was arranged so that, with the amplifier on a fixed gain of 1000, the resulting transducer output at 1, 5, 10, 20 and 100 mm Hg gave practically full-scale output on the ultra-violet recorder. (The reason for this approach was that the output signal to noise ratio was considerably better at the maximum gain setting of the amplifier.)

4.2 Results

The response times for the tube configurations tabulated in Fig. 11 were measured at a number of pressure levels from the recorder or oscilloscope traces, and are presented in Figs. 15-32. The response times to 99% of the applied pressure difference have been determined. The time to 99% represents 0.010 in. from equilibrium for a full-scale deflection of 4 in. on the recorder. This deviation from equilibrium was relatively easy to discriminate on the records. The times to 99.9% were not measured since difficulty was encountered owing to the trace thickness (0.015 in.) and also the uncertainty in determining the final trace deflection.

The overall errors in the measured response times are estimated to be of the order of 15%. Inaccuracies arose due to the following:

1. non-uniformity of tube bore and inaccuracies of bore measurement - these effects were prominent for the 0.120 in. and 0.1875 in. diameter copper tubes;
2. variation in measuring the response times from the trace records - the repeat accuracy was approximately 10%;
3. change of initial pressure in the measuring instrument - the pressure was held below 0.02 mm Hg for all the tests, but changes below this level affected response times at the low pressure levels;
4. leaks and outgassing effects in the transducer-tube system - this led to a gradient on the final pressure level and made the measurement of the 99% response time uncertain.
5. Discussion

In Figs. 4 and 5 and other comparisons with experiment in a number of Figures from 15-32, the theory predicts the correct trends for the variation of response time with applied pressure difference but in some cases the absolute values may be in error by as much as 100%. Some of the differences must be attributed to the inaccuracy of the experimental points. Furthermore, the theory has been derived on the assumption that the measuring instrument has a finite volume and that the complete tube volume can be considered to be concentrated at the end of the tube adjacent to the instrument; some doubt must be expressed on the validity of the equations when the actual instrument volume approaches zero.

As indicated above, it is expected that the theory will be used to give an approximate estimate of the response time in the design of tubing configurations and that a closer estimate will be possible from the experimental data presented. However, it should be noted that the ideal conditions associated with the theory and the experimental investigation may not always obtain in a practical wind tunnel. For example, a step-function pressure change is assumed, but in a wind tunnel of the blowdown type the pressure history of a typical test section orifice may be as shown in Fig. 33. The high-pressure pulse produced as the tunnel starting shock passes the orifice will affect the measuring instrument since the response time will be extremely short compared with the response time at the low pressure to be measured. The effect of this could be favourable or unfavourable depending on whether the initial pressure in the tube is above or below the final pressure to be measured.

The actual form of the starting pulse will vary with the type of tunnel, model, and test conditions and the differences between the actual and assumed pressure changes should be borne in mind in the application of the results of the investigation.

The optimum second tube diameters for the majority of the configurations considered in this investigation vary between 0.04 in. and 0.080 in. In many wind-tunnel experiments it may be necessary to use a fixed diameter for the second tube in conjunction with orifices and first tubes of different geometry, and various lengths of the second tube. If a compromise has to be made for different configurations, then it is clearly advisable to bias this towards the larger diameter because the rate of increase of response time is greater as \( d_2 \) decreases from the optimum value than when it increases (Figs. 8, 9 and 10). Under these conditions a diameter of 0.060 in. is suggested, as a compromise, for configurations similar to those tabulated in Fig. 11.

For the measurement of model pressures it is advisable to hold the pressure in the test section, and hence the initial pressure in all the tube systems, at a value just less than the lowest pressure to be measured. This ensures that the pressure difference applied to each tube during the run is as small as possible and therefore that the response times are as short as possible. Furthermore, the fact that the applied pressure differences are positive also helps to minimise the response times (Fig. 7).

The experimental response times are presented for a pressure difference applied from vacuum to the measured pressure. Fig. 7 can be used, in conjunction with the experimental data, to obtain an estimate of the response time for the same pressure difference applied in either a positive or negative sense at an elevated pressure level.
6. **Conclusions**

1. The extension of existing theories predicts the correct trends for the effects of varying the applied pressure and tube geometry for slip flow in a compound tube system, even when the volume of the measuring instrument is reduced to zero. A useful estimate of the response time can thus be obtained for the initial design of a pressure-measuring system.

2. The detail design will still depend to some extent on experimental data for the response time because the theory cannot be relied on for an accurate absolute prediction; comprehensive data from the present investigation will therefore be useful in this respect.

3. Slip flow effects make little difference to the theoretical value for the optimum diameter of the second tube in a compound tube system. The optimum diameter can therefore be calculated to sufficient accuracy from the less complex Poiseuille flow equation.

4. The experimental values for the optimum diameter of the second tube are in good agreement with the theoretical predictions.

5. The pressure in the tunnel test-section prior to a run should, if possible, be held close to the lowest pressure to be measured and preferably below it rather than above.

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APPENDIX
The Design of Tube Systems when Connected to a Pressure Scanning Switch

As mentioned in the Introduction, a pressure scanning switch may be used to measure a number of pressures sequentially if the tunnel operating time is long compared with the response times of the individual tubes. Special consideration of the response times then applies for minimizing the overall time required to record a given number of pressures.

Fig. 34 shows a tube system connected to a port of an N.P.L. pressure scanning switch with the transducer registered with the port in the sampling position. This indicates how the transducer is sealed to the port by means of the inner O-ring. During a scan of the pressure switch the transducer is stepped from port to port and in the interport position is vented to the switch case. The case is continuously evacuated to prevent a carry-over of pressure from one port to the next. This procedure also eliminates hysteresis effects in the pressure transducer since the transducer output always has to rise to record the measured pressure. The volume in front of the transducer is kept to a minimum to reduce the response time of the switch (travelling volume approximately 0.0004 cu in.). This response time is the time required for the pressure in front of the transducer to reach equilibrium with the measured pressure after the transducer has been moved to the appropriate port. The effect of the travelling volume of the transducer has been isolated by connecting a large volume, at various pressure levels, on to a port and then stepping the switch from an interport position on to the port. The addition of a tube system between the port and volume has a radical effect on the match response time (Fig. 35b). Although the tube volume may be 1000 times the travelling volume of the transducer, the response time is increased due to the flow resistance of the tube system. Fig. 35b represents the case where the tube systems from the model are connected directly to the ports of the pressure scanning switch.

Consider the total time, T, required to scan n ports from the initiation of a tunnel run. The transducer will be at rest on port 1 during the time, t₀, required for all the tube systems to reach equilibrium with the measured model pressures. Ports 2 to n remain to be scanned and each will have a response time, tᵣ, once the transducer is registered with it. There will be (n-1) travels between ports, each taking time, tᵣ, and n recordings, each taking time, tᵣ. Thus,

\[ T = t₀ + \sum_{2}^{n} (tᵣ + (n-1)tᵣ) + ntᵣ \]  \( \ldots (1) \)

Fig. 36a shows the pressure-time profile of the transducer during a tunnel run.

The total time, T, required to scan n ports can be reduced if n is large by placing volumes immediately in front of the switch ports (Fig. 37) to isolate the switch from the effect of the tubing resistance. If the added volumes are made 1000 times the travelling volume of the transducer, then the error due to sampling by the transducer will be approximately 0.1% after the response time of the switch in isolation as defined in Fig. 35a. The addition
of a volume to the tube system increases the time, \( t_0 \), for the pressure in the tube to reach equilibrium with the measured pressure, but the time, \( t_v \), required to sample this pressure with the scanning switch is decreased due to the isolation from flow resistance effects in the tube. The pressure-time profile of the transducer for tube systems with volumes attached to the switch is shown in Fig. 36b which can be compared with Fig. 36a for similar tube systems connected directly to the switch.

Equation (1) is modified to

\[
T_v = t_{ov} + \sum_{2}^{n} t_{tv} + (n-1)t_{i} + nt_{r}, \quad \ldots (2)
\]

where suffix \( v \) indicates the addition of volumes to the tube systems, and time, \( t_{tv} \), is the same as the response time of the switch in isolation.

Thus, by setting \( T = T_v \) we can find a value of \( n \) beyond which the total time will be reduced by the addition of the volumes; for smaller numbers of tubes the time will be increased. It is important to take advantage of the added volumes for large values of \( n \) for both intermittent tunnels (to increase the number of readings that can be made in a given operating time) and for continuous tunnels (to decrease the operating time for a given number of readings and hence to economise on power).

As an example, consider an actual pressure measuring system, the characteristics of which are summarised below.

**Characteristics of the N.P.L. 12 Port Scanning Switch**

(i) Travelling volume of the transducer, \( 0.0004 \text{ cu in.} \)

(ii) Interport time, \( t_i = 400 \text{ milliseconds} \)
     (using a 13.5 r.p.m. motor).

(iii) Time required to record the output of the pressure transducer, \( t_r = 100 \text{ milliseconds} \) (including a dwell time to ensure that the transducer does not move off a port before recording is completed).

**Characteristics of the Tube System**

(i) Orifice I \( (0.010 \text{ in.} \times 0.040 \text{ in.}) + (15 \text{ in.} \times 0.036 \text{ in.}) \)
    + (60 \text{ in.} \times 0.080 \text{ in.}) \)

(ii) Tube capacity \( 0.331 \text{ cu ins.} \)

(iii) Capacity of added volume, \( 0.3 \text{ cu in.} \) (i.e., 750 times the transducer travelling volume).

**Response Time Characteristics for a Measured Pressure of 10 mm Hg**

(i) From Fig. 35a the response time of the switch alone and hence the sampling time for the switch with tube system and added volume, \( t_{tv} = 35 \text{ milliseconds} \).
(ii) From Fig. 35b the response time of the switch with tube system, $t_t = 4.7$ seconds.

(iii) From Fig. 17 the time for tube system to reach equilibrium with the measured pressure, $t_o = 20$ seconds.

(iv) Time for tube system with added volume to reach equilibrium with the measured pressure, $t_{ov} = 36$ seconds.

Substituting the above values in equations (1) and (2) gives

$$T = (n-1)(0.4 + 4.7) + n(0.1) + 20$$

and

$$T_v = (n-1)(0.4 + 0.035) + n(0.1) + 36.$$  

Solving for $n$ when $T = T_v$ gives $n \approx 4.4$.

Thus, for minimum tunnel operating time, if the number of pressures to be measured is greater than 5 it is advisable to use tube systems with added volumes. If the number of pressures is less than 4 then the tube systems should be connected directly to the pressure-scanning switch.

When calculations for an optimum second tube diameter are performed the capacity of the added volumes should be taken into account; this tends to increase the optimum diameter of the second tube.
FIGURE 1

Predicted operating time for various static pressures, stagnation pressures, and Mach numbers for the helium tunnel.
RESPONSE TIME TO 99% OF APPLIED PRESSURE ($p_1$) (SECONDS)
(INITIAL PRESSURE IN TUBE, $p_0 = 0$)

EXPERIMENTAL VARIATION OF RESPONSE TIME WITH MEASURING INSTRUMENT VOLUME

TUBE LENGTH 60"
TUBE BORE 0.060"

FIG. 2
FIGURE 3

METHODS OF RESPONSE TIME MEASUREMENT
Comparison of Theoretical and Experimental Results

- Experimental Results
- Poiseuille ($k = 1$)
- Slip ($k = 1$)
- Slip ($k = \frac{1}{2}$)
- Orifice III ($\theta = 60^\circ \times 0.080^\circ$)

Response Time (Seconds) vs. Applied Pressure (mm Hg)

(initial tube pressure, $p_0 = 0$)
COMPARISON OF THEORETICAL AND EXPERIMENTAL RESULTS
COMPARISON OF RESPONSE TIMES FOR A CONSTANT APPLIED PRESSURE DIFFERENCE AT VARIOUS INITIAL INSTRUMENT PRESSURES
DIFFERENCES ($\Delta p$) AT VARIOUS MEAN PRESSURES (FOR 0.036" BORE TUBE) FOR CONSTANT PRESSURE THEORETICAL CURVES OF RESPONSE TIME RATIO

\[\frac{t_2}{t_1} = \frac{2d}{p_1 + p_0}\]

$\Delta p = 0$ of $1$ mm Hg

$\Delta p = 10$ mm Hg

$\Delta p = 100$ mm Hg

FIGURE 7
FIG. 8

VARIATION OF RESPONSE TIME WITH SECOND TUBE DIAMETER FOR POISEUILLE FLOW AND SLIP FLOW
FIG. 9

Comparison of slip flow theory with experimental results for optimum second tube diameter.
FIG. 10

COMPARISON OF SLIP FLOW THEORY WITH EXPERIMENTAL RESULTS FOR OPTIMUM SECOND TUBE DIAMETER
### ORIFICE NOTATION:

| I | 0.040 LONG x 0.010 DIAMETER |
| II | 0.040 LONG x 0.020 DIAMETER |
| III | 1/400 LONG x 0.036 DIAMETER |

**FIGURE 11**

**TUBE SYSTEMS FOR EXPERIMENTAL INVESTIGATION**

(all dimensions in inches)

<table>
<thead>
<tr>
<th>Orifice</th>
<th>1st Tube</th>
<th>2nd Tube</th>
<th>Figure Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>Diameter</td>
<td>Length</td>
<td>Diameter</td>
</tr>
<tr>
<td>0 in/0 in</td>
<td>0.001</td>
<td>1</td>
<td>0.036</td>
</tr>
<tr>
<td>0.040</td>
<td>0.010</td>
<td>1</td>
<td>0.036</td>
</tr>
<tr>
<td>0.040</td>
<td>0.010</td>
<td>15</td>
<td>0.036</td>
</tr>
<tr>
<td>0.040</td>
<td>0.010</td>
<td>15</td>
<td>0.036</td>
</tr>
<tr>
<td>0.040</td>
<td>0.010</td>
<td>15</td>
<td>0.080</td>
</tr>
<tr>
<td>0.040</td>
<td>0.010</td>
<td>15</td>
<td>0.120</td>
</tr>
<tr>
<td>0.040</td>
<td>0.020</td>
<td>1</td>
<td>0.036</td>
</tr>
<tr>
<td>0.040</td>
<td>0.020</td>
<td>15</td>
<td>0.036</td>
</tr>
<tr>
<td>0.040</td>
<td>0.020</td>
<td>15</td>
<td>0.080</td>
</tr>
<tr>
<td>0.040</td>
<td>0.020</td>
<td>15</td>
<td>0.120</td>
</tr>
</tbody>
</table>

**FIGURE 11**
FACE PLATE AND TUBE ADAPTOR

PLASTIC TUBE SEALS

ORIFICE

FLANGE FOR VALVE

TRANSDUCER MOUNT AND TUBE CONNECTIONS.
DIAGRAM OF PNEUMATIC SYSTEM

WALLACE AND TIERNAN DIAL GAGE 0-100mmHg

LEYBOLD IONISATION GAUGE HEAD

VENT

NEEDLE VALVE

MANUAL OR ELECTROMAGNETIC STOP VALVE

VACUUM PUMP

TRANSDUCER MOUNT

TUBE SYSTEM UNDER TEST

CYLINDER CAPACITY 574cu.ins
ELECTROMAGNETIC VALVE CONTROL & OSCILLOSCOPE TRIGGER.
Figures 15 & 16

**Figure 15**

**Applied Pressure**

$p_1 \text{ mm Hg}$

**Response Time (seconds)**

**1st Tube**

1" x 0.036"

2nd Tube

○ 60° x 0.036"

△ 60° x 0.080"

□ 60° x 0.120"

× 60° x 0.1875"

**Figure 16**

**Applied Pressure**

$p_1 \text{ mm Hg}$

**Response Time (seconds)**

**1st Tube**

1" x 0.036"

**2nd Tube**

○ 120° x 0.036"

△ 120° x 0.080"

□ 120° x 0.120"

× 120° x 0.1875"

Response Time to 99% of Applied Pressure (with a Pressure Drop of 0)
Figures 17 & 18

**Figure 17**

- **APPLIED PRESSURE** $p_1$ mm Hg
- **RESPONSE TIME** (seconds)
- Flagged symbols indicate slip flow
- Theory for $K = 1$

- Orifice I
- 1st Tube: $15'' \times 0.036''$
- 2nd Tube: $60'' \times 0.036''$
- $60'' \times 0.080''$
- $60'' \times 0.120''$
- $60'' \times 0.1875''$

**Figure 18**

- **APPLIED PRESSURE** $p_1$ mm Hg
- **RESPONSE TIME** (seconds)
- Flagged symbols indicate slip flow
- Theory for $K = 1$

- Orifice I
- 1st Tube: $15'' \times 0.036''$
- 2nd Tube: $120'' \times 0.036''$
- $120'' \times 0.080''$
- $120'' \times 0.120''$
- $120'' \times 0.1875''$

Response time to 99% of applied pressure

(initial pressure, $p_0 = 0$)
RESPONSE TIME TO 99% OF APPLIED PRESSURE
INITIAL PRESSURE, \( p_0 = 0 \)
FIG. 21

ORIFICE II
1st TUBE
1" x 0.036"
2nd TUBE
O 60° x 0.036"
△ 60° x 0.080"
□ 60° x 0.120"
x 60° x 0.1875"

APPLIED
PRESSURE
$p_1$, mm Hg

RESPONSE TIME (seconds)

0.1 0.2 0.5 1 2 5 10 20 50 100 200 500 1000

FIG. 22

ORIFICE II
1st TUBE
1" x 0.036"
2nd TUBE
0.120" x 0.036"
△ 120° x 0.080"
□ 120° x 0.120"
x 120° x 0.1875"

APPLIED
PRESSURE
$p_1$, mm Hg

RESPONSE TIME (seconds)

0.1 0.2 0.5 1 2 5 10 20 50 100 200 500 1000

RESPONSE TIME TO 99% OF APPLIED PRESSURE

INITIAL PRESSURE, $p_0 = 0$
FIGURES 23 & 24

FIGURE 23

APPLIED PRESSURE
\( p_1 \) mm Hg

ORIFICE II
1st TUBE
15' x 0.036'
2nd TUBE
\( \bigcirc \) 120' x 0.036'
\( \bigtriangleup \) 120' x 0.080'
\( \square \) 120' x 0.120'
\( \times \) 120' x 0.1875'

FLAGGED SYMBOLS INDICATE SLIP FLOW
THEORY FOR \( k=1 \)

RESPONSE TIME (seconds)

FIGURE 24

APPLIED PRESSURE
\( p_1 \) mm Hg

ORIFICE II
1st TUBE
15' x 0.036'
2nd TUBE
\( \bigcirc \) 60' x 0.036'
\( \bigtriangleup \) 60' x 0.080'
\( \square \) 60' x 0.120'
\( \times \) 60' x 0.1875'

FLAGGED SYMBOLS INDICATE SLIP FLOW
THEORY FOR \( k=1 \)

RESPONSE TIME (seconds)

RESPONSE TIME TO 99% OF APPLIED PRESSURE

INITIAL PRESSURE, \( p_0 = 0 \)
FIGS. 25 & 26

**FIG. 25**

ORIFICE II
1st TUBE
15" X 0.080"
2nd TUBE
O 60" X 0.120"
\(\Delta\) 120" X 0.120"
\(\Box\) 120" X 0.1875"
X 60" X 0.1875"

**FIG. 26**

APPLIED PRESSURE
\(p_i\) mm Hg

RESPONSE TIME (seconds)

RESPONSE TIME TO 99\% OF APPLIED PRESSURE

INITIAL PRESSURE, \(p_0 = 0\)
FIGS 27 & 28

FIGURE 27

APPLIED PRESSURE $p_1$ mm Hg

FLAGGED SYMBOLS INDICATE SLIP FLOW THEORY FOR $K = 1$

RESPONSE TIME (seconds)

FIGURE 28

RESPONSE TIME TO 99% OF APPLIED PRESSURE (INITIAL PRESSURE, $p_0 = 0$)

RESPONSE TIME

APPLIED PRESSURE $p_1$ mm Hg

RESPONSE TIME (seconds)
**FIGURES 29 & 30**

**FIGURE 29**

- **ORIFICE III**
  - 1st TUBE
    - 15\" x 0.036\"
  - 2nd TUBE
    - • 120\" x 0.036\"
    - △ 120\" x 0.080\"
    - □ 120\" x 0.120\"
    - X 120\" x 0.1875\"

Flagged symbols indicate slip flow. Theory for $K = 1$.

**APPLIED PRESSURE** $p_1$ mm Hg

**RESPONSE TIME** (seconds)

**FIGURE 30**

- **ORIFICE III**
  - 1st TUBE
    - 15\" x 0.036\"
  - 2nd TUBE
    - • 60\" x 0.036\"
    - △ 60\" x 0.080\"
    - □ 60\" x 0.120\"
    - X 60\" x 0.1875\"

Flagged symbols indicate slip flow. Theory for $K = 1$.

**RESPONSE TIME** (seconds)

**RESPONSE TIME TO 99% OF APPLIED PRESSURE**

(Initial pressure, $p_0 = 0$)
Figure 31

**Applied Pressure** $p_1$ mm Hg

**Response Time** (seconds)

Figure 32

**Applied Pressure** mm Hg

**Response Time** (seconds)

Response time to 99% of applied pressure

Initial pressure, $p_0 = 0$
FIGURE 33

TYPICAL PRESSURE HISTORY OF TEST SECTION ORIFICE.

PRESSURE (p)

\(P_0\)

Pre run test-section pressure

\(P_1\)

Final orifice pressure

TIME (t)

TUNNEL SHOCK PASSING ORIFICE

PRESSURE HISTORY OF ORIFICE FOR THEORETICAL AND EXPERIMENTAL INVESTIGATION

PRESSURE (p)

\(P_0\)

\(P_1\)

TIME (t)

COMPARISON OF ACTUAL AND ASSUMED PRESSURE HISTORY OF TEST-SECTION ORIFICE.
7-H IS SPACE CONNECTED TO SWITCH CASE AND EVACUATED.

TRANSDUCER DIAPHRAGM

O-RING SEALS

TRANSUDER BODY MOUNTED IN ROTOR.

TRAVELLING VOLUME OF TRANSUDER

MODEL ORIFICE.

TUBE SYSTEM

DIAGRAM OF TRANSUDER REGISTERED WITH A PORT IN THE NPL PRESSURE SCANNING SWITCH
PRESSURE APPLIED TO PORT $p_1$, mm Hg

RESPONSE TIME (seconds)

(a) RESPONSE TIME OF SCANNING SWITCH

(b) RESPONSE TIME OF SCANNING SWITCH WITH TUBE SYSTEM (ORIFICE + 15° x 0.036° + 60° x 0.080°)

FIG. 35
(a) PRESSURE-TIME PROFILE OF TRANSDUCER IN SCANNING SWITCH WITH SIMILAR TUBE SYSTEMS CONNECTED TO TUNNEL

(b) PRESSURE-TIME PROFILE OF TRANSDUCER IN SCANNING SWITCH WITH SIMILAR TUBES SYSTEMS + VOLUMES CONNECTED TO TUNNEL
METHOD OF ATTACHING ADDED VOLUME TO SWITCH PORT AND TUBE SYSTEM.
A.R.C. C.P. No. 913
July, 1965
Larcombe, M. J. and Peto, J. W.

THE RESPONSE TIMES OF TYPICAL TRANSDUCER-TUBE CONFIGURATIONS FOR THE MEASUREMENT OF PRESSURES IN HIGH-SPEED WIND TUNNELS

Existing theories are extended to obtain the response time for low pressures with slip flow in a compound tube system. The theory provides a useful estimate of the response time even when the volume of the measuring instrument falls to zero.

Comprehensive experimental data are presented for a range of tube configurations, including a flush-diaphragm pressure transducer, suitable for applications in high-speed wind tunnels.

An Appendix provides some notes on the design of a pressure-measuring system incorporating a pressure-scanning switch.