

C.P. No. 860

LIBRARY P. No. 860
ROYAL AIRCRAFT ESTABLISHMENT
BEDFORD.



MINISTRY OF AVIATION

AERONAUTICAL RESEARCH COUNCIL

CURRENT PAPERS

A Method for Correcting
Measurements of the Heat
Transfer Factor through the
Skin of a Wind Tunnel Model

by

B. A. M. Piggott, B.A.

LONDON: HER MAJESTY'S STATIONERY OFFICE

1966

FOUR SHILLINGS NET

A METHOD FOR CORRECTING MEASUREMENTS OF THE
HEAT TRANSFER FACTOR THROUGH THE SKIN OF A WIND TUNNEL MODEL

by

B. A. M. Piggott, B.A.

SUMMARY

In an experiment in which it is required to measure the heat transfer factor at the outer surface of the skin of a wind tunnel model, the temperature of the skin is measured at various points on this surface and the heat flow rate across the skin is measured at the corresponding points on the inner surface. Calculation of the heat flow rate across the outer surface of the skin involves the solution of Laplace's equation in a rectangular region with mixed boundary conditions. A numerical method of solution is described and illustrated by an example.

CONTENTS

| | <u>Page</u> |
|---|-------------|
| 1 INTRODUCTION | 3 |
| 2 THE HEAT EQUATION AND BOUNDARY CONDITIONS | 3 |
| 3 THE FINITE DIFFERENCE EQUATIONS | 4 |
| 4 THE METHOD OF SUCCESSIVE OVER-RELAXATION | 6 |
| 5 ACCURACY CHECKS | 8 |
| 5.1 Convergence of the iterative process | 8 |
| 5.2 Accuracy of the finite-difference approximation | 9 |
| 6 METHOD OF USE | 10 |
| SYMBOLS | 11 |
| REFERENCES | 12 |
| | |
| APPENDIX - Specimen data and results | 13-15 |
| ILLUSTRATIONS - Figs.1-5 | - |
| DETACHABLE ABSTRACT CARDS | - |

ILLUSTRATIONS

| | <u>Fig.</u> |
|--|-------------|
| Longitudinal section of wind tunnel model | 1 |
| Measurement station in skin of model | 2 |
| Heat equation and boundary conditions for the skin | 3 |
| The finite-difference mesh | 4 |
| Comparison of the observed and calculated heat flow rates in a special case | 5 |

1 INTRODUCTION

Fig.1 shows a longitudinal section of a wind tunnel model¹ in the form of a cylindrical shell with a conical forward section. The skin is heated by an airflow and heat is taken out by coolant flowing through the hollow interior of the model. When a steady state has been reached, the temperature T_W of the outer surface, and the heat flow rate Q_M across the inner surface of the skin of the model are measured by a series of thermocouples and heat flow meters² at various stations along the cylindrical section of the model (see Fig.2).

The heat transfer factor h across the outer surface of the skin at a given station is defined by the equation

$$Q = h(T_R - T_W) \quad (1)$$

where Q is the heat flow rate across the outer surface at that station and T_R is the recovery temperature - i.e. the temperature that would be reached there if no heat were allowed to escape into the model. As a first approximation it may be assumed that $Q = Q_M$; this is equivalent to neglecting any flow of heat along the skin, which is thin and of low thermal conductivity. By varying the amount of coolant flowing through the interior of the model, it is possible to obtain a series of values of Q_M and T_W for each station. From these, using the above assumption, h and T_R for that station may be calculated.

The method described in this paper, in which the assumption that $Q = Q_M$ is not made, solves the heat equation for a section of the skin, assuming only that there is no heat flow across the ends of the section. It thus provides an improved value of Q ; a comparison between the calculated values of Q and Q_M for a specific case is shown in Fig.5.

2 THE HEAT EQUATION AND BOUNDARY CONDITIONS

Since the skin of the cylindrical model is thin, its curvature is neglected and it is regarded as an infinite rectangular slab - see Fig.3; A and B are arbitrarily chosen points on the outer surface of the skin and O and P are the corresponding points on the inner surface. Taking OP and OA as x- and y-axis, respectively, with OP = a and OA = b, the temperature $T(x,y)$ of a point in the skin in the steady state satisfies Laplace's equation:

$$\nabla^2 T \equiv \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \quad (0 \leq x \leq a, 0 \leq y \leq b) . \quad (2)$$

The boundary conditions on AB and OP are known:

$$\left. \begin{aligned} T(x,b) &= T_W(x) \\ \frac{\partial T}{\partial y}(x,0) &= \frac{1}{k} Q_M(x) \end{aligned} \right\} \quad (0 \leq x \leq a) \quad \begin{matrix} (3) \\ (4) \end{matrix}$$

where k is the thermal conductivity of the skin, and Q_M is taken to be positive when the heat flow is directed towards the interior of the model. Boundary conditions are also needed along OA and PB; since the variation of T with x is observed to be small at the ends of the model, the boundary conditions are taken to be

$$\frac{\partial T}{\partial x}(0,y) = \frac{\partial T}{\partial x}(a,y) = 0 \quad (0 \leq y \leq b) \quad (5)$$

They are equivalent to the assumption that there is no longitudinal flow of heat across the ends of the section.

3 THE FINITE DIFFERENCE EQUATIONS

The rectangle OABP is covered by a uniform rectangular grid of points (x_i, y_j) with the mesh lengths δx and δy respectively in the x - and y -directions (see Fig.4) and grid-points are added along $x = -\delta x$ and $x = a + \delta x$. The equations of the previous section are then approximated by the finite-difference equations:

$$\frac{T(x_{i-1}, y_j) - 2T(x_i, y_j) + T(x_{i+1}, y_j)}{(\delta x)^2} + \frac{T(x_i, y_{j-1}) - 2T(x_i, y_j) + T(x_i, y_{j+1})}{(\delta y)^2} = 0 \quad (1 \leq i \leq m, 1 \leq j \leq n-1) \quad \dots (6)$$

$$T(x_i, b) = T_W(x_i) \quad (1 \leq i \leq m) \quad (7)$$

$$T(x_i, \delta y) - T(x_i, 0) = \frac{\delta y}{k} Q_M(x_i) \quad (1 \leq i \leq m) \quad (8)$$

$$T(-\delta x, y_j) = T(0, y_j) \quad (1 \leq j \leq n-1) \quad (9)$$

$$T(a + \delta x, y_j) = T(a, y_j) \quad (1 \leq j \leq n-1) \quad (10)$$

The heat flow rate across AB is approximated by the equations

$$Q(x_i) = \frac{k}{\delta y} \left\{ T_W(x_i) - T(x_i, y_{n-1}) \right\}. \quad (1 \leq i \leq m) \quad (11)$$

It is convenient at this stage to combine equations (6), (7), (9) and (10) in order to eliminate $T(x_i, b)$, $T(-\delta x, y_j)$, and $T(a+\delta x, y_j)$; the resulting equations, together with (8), provide mn linear equations for the unknowns $T(x_i, y_j)$, ($1 \leq i \leq m$, $0 \leq j \leq n-1$). These unknowns may be written as a column vector \underline{t} , where $T(x_i, y_j) = t_r$ ($1 \leq r \leq mn$) when $r = (i-m) + m(n-j)$, which corresponds to a numbering of the grid-points by rows, starting with the first row inside the outer surface. The equations then take the form

$$M\underline{t} = \underline{u} \quad (12)$$

where

$$\underline{u}_r = \begin{cases} -\left(\frac{\delta x}{\delta y}\right)^2 T_W(x_r) & 1 \leq r \leq m \\ 0 & m+1 \leq r \leq m(n-1) \\ -\frac{1}{k} \delta y Q_M(x_{r-m(n-1)}) & m(n-1) + 1 \leq r \leq mn \end{cases} \quad (13)$$

and M is the $mn \times mn$ block tridiagonal matrix

$$\left[\begin{array}{cccc} N & & & \\ & \left(\frac{\delta x}{\delta y}\right)^2 I_m & & \\ & & N & \\ \left(\frac{\delta x}{\delta y}\right)^2 I_m & & & \\ & & & \left(\frac{\delta x}{\delta y}\right)^2 I_m \\ & & & & N & \\ & & \left(\frac{\delta x}{\delta y}\right)^2 I_m & & & \left(\frac{\delta x}{\delta y}\right)^2 I_m \\ & & & & I_m & -I_m \end{array} \right] \quad (14)$$

I_m is the $m \times m$ unit matrix and N the $m \times m$ tridiagonal matrix

$$\begin{bmatrix} -\left\{1 + 2\left(\frac{\delta x}{\delta y}\right)^2\right\} & & & & & & & & & \\ & 1 & & & & & & & & \\ & & -2\left\{1 + \left(\frac{\delta x}{\delta y}\right)^2\right\} & & & & & & & \\ & & & 1 & & & & & & \\ & & & & -2\left\{1 + \left(\frac{\delta x}{\delta y}\right)^2\right\} & & & & & \\ & & & & & 1 & & & & \\ & & & & & & -2\left\{1 + \left(\frac{\delta x}{\delta y}\right)^2\right\} & & & \\ & & & & & & & 1 & & \\ & & & & & & & & -\left\{1 + 2\left(\frac{\delta x}{\delta y}\right)^2\right\} & \\ & & & & & & & & & \dots \end{bmatrix} \quad \dots (15)$$

4 THE METHOD OF SUCCESSIVE OVER-RELAXATION

Equation (12) corresponds to the mn scalar equations

$$\sum_{s=1}^{mn} m_{rs} t_s = u_r \quad (r = 1, 2, \dots, mn) \quad (16)$$

Given any approximate solution $t_r^{(p)}$, the method of (point) successive over-relaxation replaces it by a new approximation $t_r^{(p+1)}$ given by

$$t_r^{(p+1)} = t_r^{(p)} + \frac{\omega}{m_{rr}} \left\{ u_r - \sum_{s=1}^{r-1} m_{rs} t_s^{(p+1)} - \sum_{s=r}^{mn} m_{rs} t_s^{(p)} \right\} \quad (r = 1, 2, \dots, mn) \quad \dots (17)$$

where ω is a suitably chosen "relaxation factor".

In the present case, the process can be started by choosing $\omega = 1$ and assuming that the temperature gradient through the skin is $\frac{1}{k} Q_M(x_i)$, depending only on x , so that

$$\begin{aligned} t_r^{(0)} &= T^{(0)}(x_i, y_j) \\ &= T^{(0)}(x_i, y_{j+1}) - \frac{Q_M(x_i)}{k} \delta y \\ &= T_W(x_i) - \frac{n-j}{k} Q_M(x_i) \delta y \end{aligned} \quad (18)$$

where $r = (i-m) + m(n-j)$ ($1 \leq i \leq m$, $0 \leq j \leq n-1$). If $\mu = \lim_{p \rightarrow \infty} \mu_p$ where

$$\mu_p = \frac{\sum_{r=1}^{mn} (t_r^{(p)} - t_r^{(p-1)})^2}{\sum_{r=1}^{mn} (t_r^{(p+1)} - t_r^{(p)})^2} \quad (19)$$

then the best value of ω for use in (17) is given³ by

$$\omega(\mu) = 1 + \mu^{\frac{1}{2}} / \left(1 + \sqrt{1 - \mu^{\frac{1}{2}}} \right)^2 \quad (20)$$

The iterative process described by equation (17) is repeated with $\omega = 1$ until $|\mu_p - \mu_{p-1}| < f$ where f is a small positive number. An approximation μ' to μ is then calculated by Aitken's δ^2 -process⁴:

$$\mu' = \mu_p - \frac{(\mu_p - \mu_{p-1})^2}{\mu_p - 2\mu_{p-1} + \mu_{p-2}} \quad (21)$$

Repetition of (17) is resumed with $\omega = \omega(\mu')$ and continued until

$$\sum_{r=1}^{mn} (t_r^{(p+1)} - t_r^{(p)})^2 < \epsilon \quad (22)$$

where ϵ is a small positive number. Then $t_r^{(p+1)}$ is taken as the solution of (12) and substituted into (11) to give the required flow rate across the outer surface.

5 ACCURACY CHECKS

5.1 Convergence of the iterative process

Let $T(x_i, y_j) = t_r^{(p+1)}$ denote the final values of the temperatures computed as described above. Owing to the truncation of the process, equation (6) will not be satisfied exactly and, from it, new values $T_1(x_i, y_j)$ can be calculated:

$$\begin{aligned} T_1(x_i, y_j) &= \frac{1}{2} \left\{ \frac{1}{(\delta x)^2} + \frac{1}{(\delta y)^2} \right\}^{-1} \left\{ \frac{T(x_{i-1}, y_j) + T(x_{i+1}, y_j)}{(\delta x)^2} + \frac{T(x_i, y_{j-1}) + T(x_i, y_{j+1})}{(\delta y)^2} \right\} \\ &= T(x_i, y_j) + \frac{1}{2} \left\{ \frac{1}{(\delta x)^2} + \frac{1}{(\delta y)^2} \right\}^{-1} \left\{ \frac{\delta_x^2 T(x_i, y_j)}{(\delta x)^2} + \frac{\delta_y^2 T(x_i, y_j)}{(\delta y)^2} \right\} \quad (23) \end{aligned}$$

$$\div T(x_i, y_j) + \frac{1}{2} \left\{ \frac{1}{(\delta x)^2} + \frac{1}{(\delta y)^2} \right\}^{-1} \nabla^2 T(x_i, y_j) \quad (24)$$

where $\delta_x^2 T(x_i, y_j) = T(x_{i-1}, y_j) + T(x_{i+1}, y_j) - 2T(x_i, y_j)$ (25)

and $\delta_y^2 T(x_i, y_j) = T(x_i, y_{j-1}) + T(x_i, y_{j+1}) - 2T(x_i, y_j)$.

Equation (23) gives an indication of the error in $T(x_i, y_j)$ and, from equation (11), since $T_W(x_i)$ is fixed, the corresponding error in $Q(x_i)$ is

$$E_1[Q(x_i)] = \frac{k}{\delta y} \left\{ T_1(x_i, y_{n-1}) - T(x_i, y_{n-1}) \right\} \quad (1 \leq i \leq m) \quad (26)$$

5.2 Accuracy of the finite-difference approximation

A more accurate representation of the Laplacian operator $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ than has been used above is given by

$$\begin{aligned} \nabla^2 T(x_i, y_j) \doteq & \frac{1}{(\delta x)^2} \left\{ \delta_x^2 T(x_i, y_j) - \frac{1}{12} \delta_x^4 T(x_i, y_j) \right\} \\ & + \frac{1}{(\delta y)^2} \left\{ \delta_y^2 T(x_i, y_j) - \frac{1}{12} \delta_y^4 T(x_i, y_j) \right\} \quad (27) \end{aligned}$$

where

$$\delta_x^4 T(x_i, y_j) = T(x_{i-2}, y_j) + T(x_{i+2}, y_j) - 4\{T(x_{i-1}, y_j) + T(x_{i+1}, y_j)\} + 6T(x_i, y_j)$$

and

$$\begin{aligned} \delta_y^4 T(x_i, y_j) = & T(x_i, y_{j-2}) + T(x_i, y_{j+2}) - 4\{T(x_i, y_{j-1}) + T(x_i, y_{j+1})\} + 6T(x_i, y_j) \\ & \dots \quad (28) \end{aligned}$$

An improved value for $T(x_i, y_j)$ may be obtained by substituting (27) into (24); but to show separately the effects of the choice of step lengths in the two coordinate directions, it is convenient to calculate the following quantities:

$$\begin{aligned} T_2(x_i, y_j) &= T(x_i, y_j) - \frac{1}{24} \left\{ \frac{1}{(\delta x)^2} + \frac{1}{(\delta y)^2} \right\}^{-1} \delta_x^4 T(x_i, y_j) \\ T_3(x_i, y_j) &= T(x_i, y_j) - \frac{1}{24} \left\{ \frac{1}{(\delta x)^2} + \frac{1}{(\delta y)^2} \right\}^{-1} \delta_y^4 T(x_i, y_j) . \end{aligned} \quad (29)$$

When $j = n-1$, these formulae involve values of T at points outside the region ABPO; therefore it is assumed that the errors in $T(x_i, y_{n-1})$ have the same orders of magnitude as those in $T(x_i, y_{n-2})$ and the corresponding errors in $Q(x_i)$ are estimated by

$$E_2[Q(x_i)] = \frac{k}{\delta y} \left\{ T_2(x_i, y_{n-2}) - T(x_i, y_{n-2}) \right\} \quad (3 \leq i \leq m-2) \quad (30)$$

$$E_3[Q(x_i)] = \frac{k}{\delta y} \left\{ T_3(x_i, y_{n-2}) - T(x_i, y_{n-2}) \right\}$$

6 METHOD OF USE

A Mercury Autocode programme has been written which performs the calculations described in this paper; it requires the following quantities as data, which must be punched on a data tape in the order given:

- a length of the skin section, AB
- b skin thickness, OA
- m number of grid-points on AB
- 1/k reciprocal of the thermal conductivity of the skin
- $Q_M(x_i)$ for $1 \leq i \leq m$ measured heat flow rate at grid-points on OP
- $T_W(x_i)$ for $1 \leq i \leq m$ measured temperature at grid-points on AB
- n number of grid-points on OA, excluding O
- f, e see section 4.
- a distance of A from the beginning of the cylindrical section of the model

The units in which a, b, k, Q_M , T_W , and a are given must be consistent; if k and Q_M have been measured in units not consistent with those of the remaining quantities, the simplest procedure is to apply a suitable scaling factor to k, leaving Q_M unchanged; the programme will then calculate Q in the units in which Q_M is given. Thus if, as in the example mentioned below, a and b are given in inches, k in c.g.s. units, T_W in degrees k, and Q_M in CHU/ft²/hr, the data should include, in place of 1/k, the value of

$$\frac{9 \times 1055}{5 \times (12 \times 60)^2 \times 2.54k}$$

The appendix shows a specimen data sheet and the corresponding print-out of results, the computation of which occupied 95 seconds on the RAE Mercury computer. The first line of results gives the dimensions (AB by OA) of the

section of skin considered, the relaxation factor used, and the number of iterations made in each stage of the calculation; the succeeding table gives, for $i = 1, 2, \dots, m$, the values of $(\alpha + x_i)$, $Q(x_i)$, $T(x_i, 0)$, $E_1[Q(x_i)]$, $E_2[Q(x_i)]$, and $E_3[Q(x_i)]$.

Fig.5 shows Q and Q_M plotted against x_i for this example.

SYMBOLS

| | |
|-------------------------|--|
| A,B | initial and final points on outer surface of section of skin considered |
| a | length of skin section AB |
| b | skin thickness, OA |
| e,f | see section 4 |
| h | heat transfer factor across outer surface of skin |
| k | thermal conductivity of skin |
| m | number of grid-points on AB |
| $M = (m_{rs})$ | matrix of coefficients in finite-difference equations, see equations (14) and (15) |
| n | number of grid-points on OA, excluding 0 |
| O,P | initial and final points on section of skin considered |
| $Q(x_i)$ | computed heat flow rate across outer surface of skin at ith grid-point |
| $Q_M(x_i)$ | measured heat flow rate across inner surface of skin at ith grid-point |
| $\underline{t} = (t_r)$ | vector of temperatures at grid-points, see section 3 |
| $\underline{t}^{(p)}$ | pth approximation to \underline{t} according to equation (17) |
| $T(x_i, y_j)$ | temperature of skin at (i,j)th grid-point |
| T_R | recovery temperature at outer surface of skin |
| $T_W(x_i)$ | measured temperature at outer surface of skin at ith grid-point |

SYMBOLS (CONTD)

| | |
|-------------------------|---|
| $\underline{u} = (u_r)$ | vector of right-hand-sides in finite-difference equations, see equation (13) |
| x, y | coordinates measured along and into the skin from O, see Fig.3 |
| $\delta x, \delta y$ | mesh-lengths of finite-difference grid |
| a | distance of A from beginning of cylindrical section of model |
| ω | relaxation factor |

REFERENCES

| <u>No.</u> | <u>Author</u> | <u>Title, etc.</u> |
|------------|--|---|
| 1 | Naysmith, A. | Measurement of heat transfer in bubbles of separated flow in supersonic air streams International Developments in Heat Transfer, II, 378-381. (ASME, 1961) |
| 2 | Stafford Hatfield, H. Wilkins, F.J. | A new heat flow meter Journal of Scientific Instruments, <u>27</u> , 1-3. (January 1950) |
| 3 | Todd, J. (Editor) | Survey of numerical analysis, 389-394. (McGraw Hill, 1962) |
| 4 | N.P.L. | Modern Computing Methods, 123. (HMSO, 1961) |

APPENDIX
SPECIMEN DATA AND RESULTS

(a) Data

6.7
0.1875
68
0.25

a
b
m

$\frac{1}{k}$ x scaling factor

| | | | | | | |
|-------|-------|-------|-------|-------|-------|-------|
| 148.5 | 148.0 | 147.5 | 147.0 | 145.0 | 142.5 | 138.0 |
| 132.0 | 117.5 | 88.0 | 77.5 | 95.0 | 240.0 | 262.0 |
| 264.0 | 262.0 | 252.5 | 240.0 | 224.0 | 205.0 | 184.0 |
| 165.0 | 149.9 | 131.0 | 112.5 | 97.5 | 85.0 | 72.5 |
| 62.5 | 52.0 | 43.5 | 36.0 | 32.0 | 30.0 | 29.0 |
| 30.0 | 32.0 | 34.0 | 37.5 | 42.0 | 48.0 | 54.0 |
| 62.5 | 72.5 | 84.0 | 94.0 | 105.0 | 114.0 | 119.0 |
| 123.5 | 126.0 | 128.0 | 130.0 | 132.0 | 132.5 | 133.0 |
| 133.5 | 134.0 | 134.5 | 135.0 | 135.0 | 135.0 | 135.0 |
| 135.0 | 135.0 | 135.0 | 135.0 | 135.0 | | |
| 262.9 | 262.9 | 262.9 | 262.9 | 262.9 | 263.0 | 263.1 |
| 263.4 | 264.2 | 265.5 | 267.2 | 269.2 | 271.0 | 273.0 |
| 274.9 | 276.4 | 278.0 | 279.0 | 278.7 | 277.3 | 275.3 |
| 273.2 | 271.1 | 269.2 | 267.2 | 265.6 | 264.8 | 264.0 |
| 263.9 | 263.8 | 263.9 | 264.0 | 264.4 | 264.9 | 265.3 |
| 265.8 | 266.2 | 266.7 | 267.0 | 267.5 | 267.9 | 268.1 |
| 268.4 | 268.8 | 269.0 | 269.1 | 269.4 | 269.5 | 269.8 |
| 269.9 | 270.0 | 270.2 | 270.4 | 270.6 | 270.8 | 270.9 |
| 271.0 | 271.1 | 271.2 | 271.3 | 271.4 | 271.5 | 271.5 |
| 271.6 | 271.7 | 271.7 | 271.7 | 271.7 | | |

4
0.05
0.0001
0.1

n
f
e
a

Q_M

T_W

(b) Results

T DATE 14.4.64

B.A.M. FIGGOTT JOB NO.1655

6.70 BY 0.19 W = 1.62 5 + 18 ITERATIONS

| $\alpha + x_i$ | Q | T | E_1 | E_2 | E_3 |
|----------------|--------|--------|----------|----------|----------|
| 0.10 | 147.85 | 255.96 | | | |
| 0.20 | 147.34 | 255.98 | | | |
| 0.30 | 146.25 | 256.02 | 1.1, -3 | -1.2, -2 | -2.6, -3 |
| 0.40 | 144.02 | 256.09 | -2.6, -3 | -1.3, -2 | -2.9, -3 |
| 0.50 | 139.06 | 256.25 | -3.4, -3 | 4.4, -3 | 9.7, -4 |
| 0.60 | 134.54 | 256.53 | -4.9, -4 | -5.9, -2 | -1.3, -2 |
| 0.70 | 122.20 | 257.02 | 1.7, -3 | -1.9, -2 | -4.2, -3 |
| 0.80 | 105.87 | 257.84 | 1.1, -3 | 1.4, -1 | 3.1, -2 |
| 0.90 | 93.86 | 259.25 | 2.8, -4 | 2.2, -1 | 4.8, -2 |
| 1.00 | 88.24 | 261.28 | -3.3, -4 | -4.8, -3 | -1.1, -3 |
| 1.10 | 100.03 | 262.89 | -9.1, -4 | -4.8, -2 | -1.1, -2 |
| 1.20 | 141.92 | 263.41 | -2.0, -3 | -1.1, 0 | -2.4, -1 |
| 1.30 | 196.48 | 260.99 | -1.3, -3 | 1.0, 0 | 2.3, -1 |
| 1.40 | 239.87 | 261.35 | -5.7, -4 | 5.6, -2 | 1.2, -2 |
| 1.50 | 263.25 | 262.61 | -4.8, -4 | -1.6, -1 | -3.6, -2 |
| 1.60 | 264.71 | 264.06 | 1.3, -4 | 1.9, -1 | 4.1, -2 |
| 1.70 | 278.82 | 265.57 | 3.6, -4 | -3.3, -2 | -7.1, -3 |
| 1.80 | 286.96 | 266.71 | 5.5, -4 | -2.4, -1 | -5.2, -2 |
| 1.90 | 268.99 | 267.19 | 4.8, -4 | -1.5, -1 | -3.2, -2 |
| 2.00 | 235.01 | 267.00 | 7.0, -4 | -1.6, -2 | -3.3, -3 |
| 2.10 | 197.96 | 266.33 | 3.9, -4 | 6.4, -2 | 1.4, -2 |
| 2.20 | 169.47 | 265.34 | -2.6, -4 | 6.1, -3 | 1.2, -3 |
| 2.30 | 144.42 | 264.20 | -7.0, -4 | 1.0, -1 | 2.2, -2 |
| 2.40 | 126.45 | 263.18 | -5.6, -4 | -1.1, -1 | -2.5, -2 |
| 2.50 | 96.74 | 262.27 | -2.9, -4 | 2.9, -2 | 6.3, -3 |
| 2.60 | 73.00 | 261.56 | -2.9, -5 | 1.8, -1 | 4.0, -2 |
| 2.70 | 70.98 | 261.16 | -5.5, -4 | -1.5, -1 | -3.2, -2 |
| 2.80 | 51.95 | 261.05 | -6.4, -4 | 1.7, -1 | 3.6, -2 |
| 2.90 | 53.55 | 261.19 | -6.0, -4 | -8.9, -2 | -2.0, -2 |
| 3.00 | 44.54 | 261.53 | -2.3, -4 | 1.1, -2 | 2.4, -3 |
| 3.10 | 39.57 | 261.96 | -3.6, -4 | -5.7, -2 | -1.3, -2 |
| 3.20 | 29.37 | 262.44 | -1.4, -4 | 6.3, -2 | 1.4, -2 |
| 3.30 | 29.39 | 262.95 | -2.3, -4 | 4.1, -3 | 8.8, -4 |
| 3.40 | 31.78 | 263.46 | -3.8, -4 | -5.7, -2 | -1.2, -2 |
| 3.50 | 29.23 | 263.92 | -3.7, -4 | 3.4, -2 | 7.5, -3 |
| 3.60 | 32.73 | 264.33 | -2.0, -4 | -4.3, -2 | -9.3, -3 |
| 3.70 | 32.35 | 264.68 | -5.1, -4 | 5.8, -2 | 1.3, -2 |
| 3.80 | 38.67 | 265.00 | -4.4, -4 | -8.6, -2 | -1.9, -2 |
| 3.90 | 36.81 | 265.24 | 2.3, -4 | 9.6, -2 | 2.1, -2 |

| | | | | | |
|------|--------|--------|----------|----------|----------|
| 4.00 | 46.35 | 265.43 | 2.7, -4 | -3.2, -2 | -7.0, -3 |
| 4.10 | 53.92 | 265.52 | 9.0, -6 | -6.1, -2 | -1.3, -2 |
| 4.20 | 55.27 | 265.52 | -1.8, -4 | 4.2, -2 | 9.2, -3 |
| 4.30 | 63.43 | 265.43 | 1.1, -4 | 4.8, -2 | 1.1, -2 |
| 4.40 | 78.19 | 265.28 | 1.8, -5 | -7.8, -2 | -1.7, -2 |
| 4.50 | 86.39 | 265.01 | -3.5, -4 | -1.8, -2 | -4.1, -3 |
| 4.60 | 91.25 | 264.74 | 6.7, -5 | 8.0, -2 | 1.8, -2 |
| 4.70 | 105.78 | 264.48 | -4.0, -4 | -8.0, -2 | -1.8, -2 |
| 4.80 | 108.96 | 264.27 | -6.9, -4 | 1.1, -1 | 2.5, -2 |
| 4.90 | 120.66 | 264.20 | -5.6, -4 | -8.9, -2 | -2.0, -2 |
| 5.00 | 122.15 | 264.15 | -3.1, -4 | 9.8, -3 | 2.1, -3 |
| 5.10 | 122.96 | 264.16 | 7.1, -5 | 3.6, -2 | 8.0, -3 |
| 5.20 | 126.89 | 264.23 | -4.7, -4 | -1.0, -2 | -2.3, -3 |
| 5.30 | 129.58 | 264.32 | -2.1, -4 | -3.0, -3 | -6.8, -4 |
| 5.40 | 131.92 | 264.42 | -2.0, -4 | 1.7, -2 | 3.7, -3 |
| 5.50 | 134.68 | 264.54 | -1.7, -4 | -3.5, -2 | -7.7, -3 |
| 5.60 | 133.75 | 264.65 | -8.1, -5 | 7.5, -3 | 1.6, -3 |
| 5.70 | 133.79 | 264.73 | -3.7, -4 | 3.4, -3 | 7.5, -4 |
| 5.80 | 134.10 | 264.82 | -3.0, -4 | 1.3, -3 | 1.7, -4 |
| 5.90 | 134.52 | 264.89 | -2.4, -4 | 8.2, -4 | 1.2, -4 |
| 6.00 | 134.98 | 264.97 | -2.9, -4 | 6.1, -3 | 1.4, -3 |
| 6.10 | 135.51 | 265.06 | -3.4, -5 | 5.0, -3 | 1.1, -3 |
| 6.20 | 136.77 | 265.14 | -8.8, -5 | -4.0, -2 | -8.8, -3 |
| 6.30 | 133.66 | 265.20 | -1.1, -5 | 4.5, -2 | 1.0, -2 |
| 6.40 | 135.35 | 265.26 | -1.5, -4 | 4.1, -3 | 9.7, -4 |
| 6.50 | 137.38 | 265.32 | -1.4, -4 | -3.5, -2 | -7.7, -3 |
| 6.60 | 135.84 | 265.35 | 3.7, -5 | 8.3, -3 | 1.9, -3 |
| 6.70 | 135.34 | 265.36 | | | |
| 6.80 | 135.18 | 265.37 | | | |

3

4

5

6

7

8

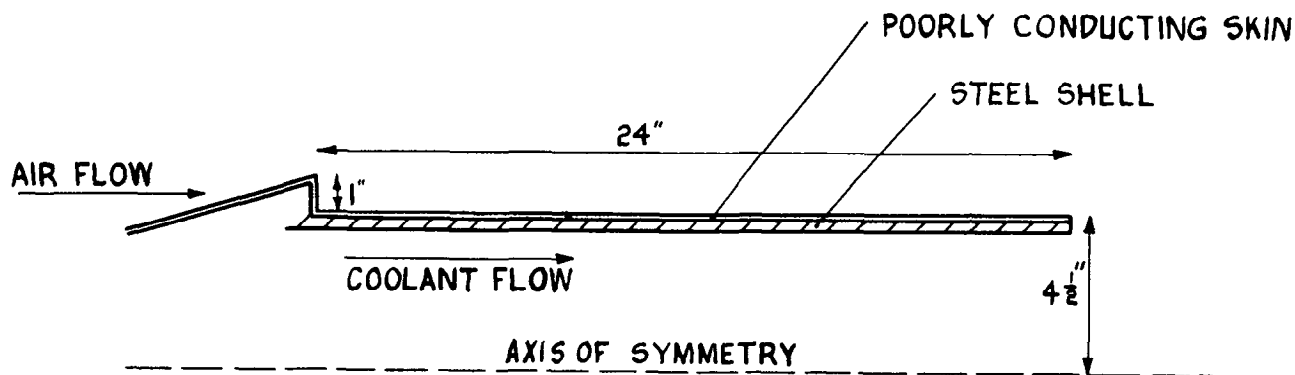


FIG. 1. LONGITUDINAL SECTION OF WIND-TUNNEL MODEL.

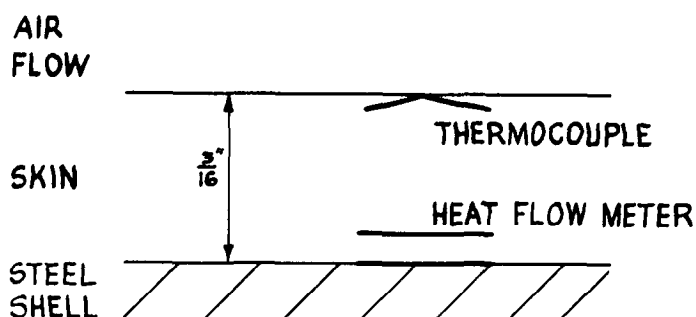


FIG. 2. MEASUREMENT STATION IN SKIN OF MODEL.

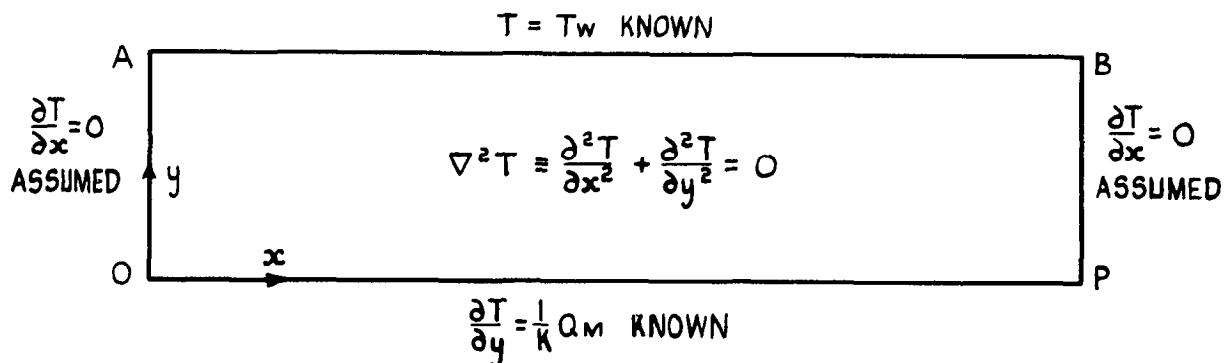


FIG. 3. HEAT EQUATION AND BOUNDARY CONDITIONS FOR THE SKIN.

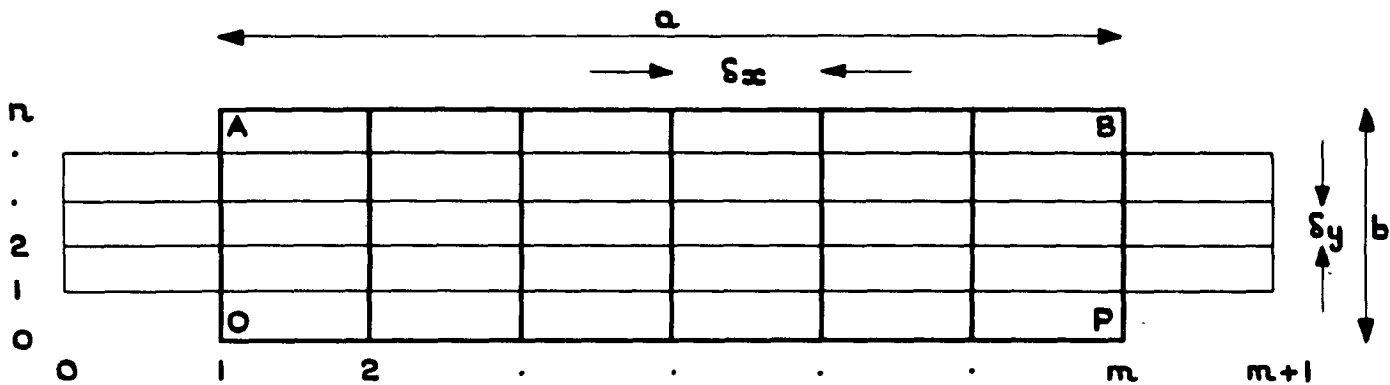


FIG. 4. THE FINITE-DIFFERENCE MESH.

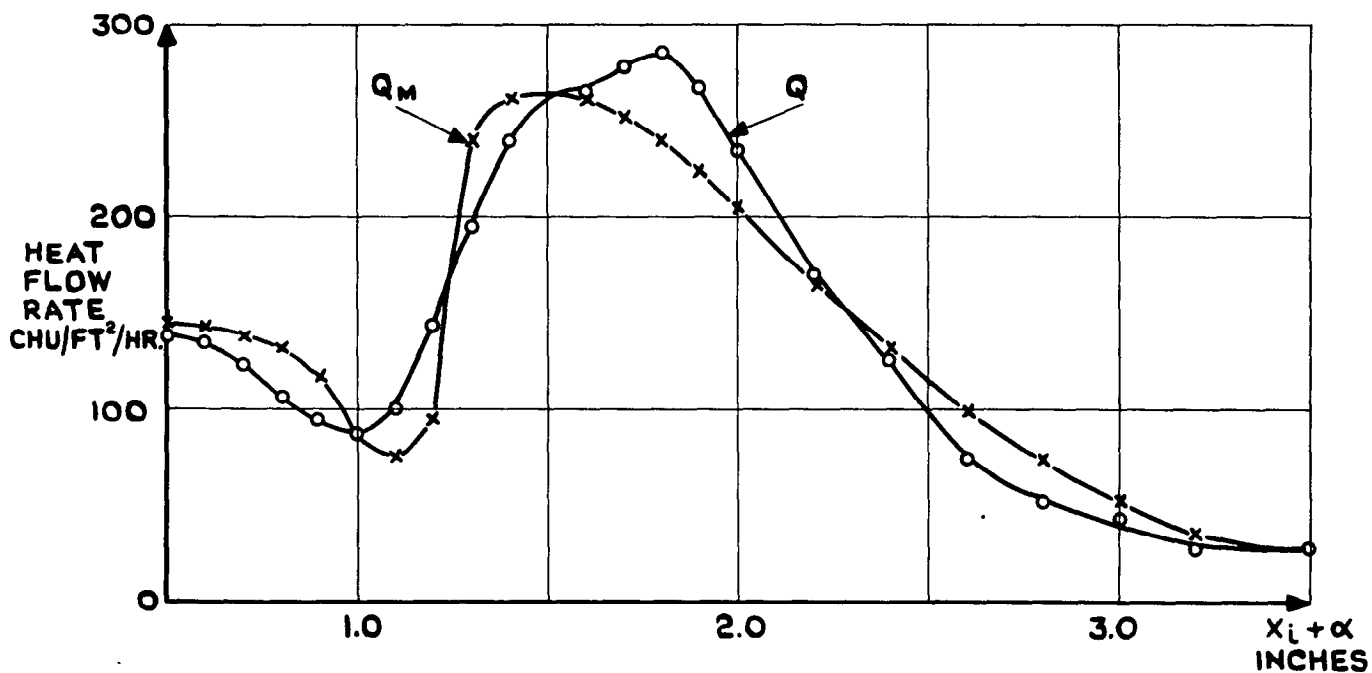


FIG. 5. COMPARISON OF THE OBSERVED AND CALCULATED HEAT FLOW RATES IN A SPECIAL CASE.

A.R.C. C.P. No. 860

536.24:
517.949

A METHOD FOR CORRECTING MEASUREMENTS OF THE HEAT TRANSFER FACTOR THROUGH THE SKIN OF A WIND TUNNEL MODEL. Piggott, B.A.M. May 1964.

In an experiment in which it is required to measure the heat transfer factor at the outer surface of the skin of a wind tunnel model, the temperature of the skin is measured at various points on this surface and the heat flow rate across the skin is measured at the corresponding points on the inner surface. Calculation of the heat flow rate across the outer surface of the skin involves the solution of Laplace's equation in a rectangular region with mixed boundary conditions. A numerical method of solution is described and illustrated by an example.

A.R.C. C.P. No. 860

536.24:
517.949

A METHOD FOR CORRECTING MEASUREMENTS OF THE HEAT TRANSFER FACTOR THROUGH THE SKIN OF A WIND TUNNEL MODEL. Piggott, B.A.M. May 1964.

In an experiment in which it is required to measure the heat transfer factor at the outer surface of the skin of a wind tunnel model, the temperature of the skin is measured at various points on this surface and the heat flow rate across the skin is measured at the corresponding points on the inner surface. Calculation of the heat flow rate across the outer surface of the skin involves the solution of Laplace's equation in a rectangular region with mixed boundary conditions. A numerical method of solution is described and illustrated by an example.

A.R.C. C.P. No. 860

536.24:
517.949

A METHOD FOR CORRECTING MEASUREMENTS OF THE HEAT TRANSFER FACTOR THROUGH THE SKIN OF A WIND TUNNEL MODEL. Piggott, B.A.M. May 1964.

In an experiment in which it is required to measure the heat transfer factor at the outer surface of the skin of a wind tunnel model, the temperature of the skin is measured at various points on this surface and the heat flow rate across the skin is measured at the corresponding points on the inner surface. Calculation of the heat flow rate across the outer surface of the skin involves the solution of Laplace's equation in a rectangular region with mixed boundary conditions. A numerical method of solution is described and illustrated by an example.

C.P. No. 860

© *Crown Copyright 1966*

Published by

HER MAJESTY'S STATIONERY OFFICE

To be purchased from

49 High Holborn, London W.C.1

423 Oxford Street, London W.1

13A Castle Street, Edinburgh 2

109 St. Mary Street, Cardiff

Brazenose Street, Manchester 2

50 Fairfax Street, Bristol 1

35 Smallbrook, Ringway, Birmingham 5

80 Chichester Street, Belfast 1

or through any bookseller

C.P. No. 860

S.O. CODE No. 23-9016-60