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On the Variation of Profile Drag Coefficient below the Critical Mach Number

By

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On the Variation of Profile Drag Coefficient below the Critical Mach Number

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November, 1963

SUMMARY

An expression is indicated for the estimation of the drag coefficient of two-dimensional aerofoil sections at subsonic speeds up to the critical Mach number.

Calculations for the 12% thick R.A.E.103 section indicate a rise of about 10% in the value of the profile drag coefficient between low speeds and the critical Mach number, for the same Reynolds number and transition position. The predicted drag variation is in good agreement with measurement.

Contents

List/

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List of Symbols

c	aerofoil chord
C_D	profile drag coefficient
M	Mach number
Re	Reynolds number
T	absolute temperature
u	velocity
x, y	Cartesian co-ordinates
γ	ratio of specific heats
θ	boundary-layer momentum thickness
ν	kinematic viscosity

Suffices:

∞	free stream condition
o	stagnation condition
T	trailing edge condition
t	conditions at boundary-layer transition point
$1, 2$	upper and lower surface conditions

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1. Introduction

The classical assumption that the drag coefficient of an aerofoil section at a constant incidence is insensitive to compressibility effects below the critical Mach number has recently been called in question. Such doubt has in turn led to the suspicion that increases in profile drag coefficient with increasing Mach number might have contributed to the increases of drag coefficient exhibited by a number of particular aircraft. The object of the present study is to check whether, and to what extent, sub-critical drag variations can be predicted by modern boundary-layer theory for the simple case of a two-dimensional aerofoil.

The drag of a two-dimensional section can be expressed as the sum of the skin-friction drag, and of the form drag which is generated by the modification of the pressure distribution brought about by the boundary-layer displacement effect. The effect of compressibility on the structure of the boundary layer can be shown to be associated in general

with/

with a reduction of the skin friction drag coefficient (see, e.g., Ref.1). On the other hand the pressure coefficients tend to be numerically increased by increase of Mach number, and the loss of pressure recovery accentuated, indicating a progressively larger form drag coefficient. Moreover, at a given Reynolds number, the increase of Mach number itself and the effect of the more pronounced adverse pressure gradients could be expected in many cases to lead to an increase in the boundary-layer displacement thickness and a further increase in form drag. Thus the total drag coefficient is determined by a balance between components which have a Mach-number dependence of opposite sign. In the case of a particular section, therefore, there is no clear indication from these considerations as to whether the drag coefficient is likely to increase or decrease with increase of Mach number, and if it does remain constant this cannot be regarded as anything but fortuitous.

In deriving the variation of profile drag coefficient for a constant lift coefficient as distinct from constant incidence, the position is complicated further by the effect of Mach number on lift-curve slope. Thus, for a given lift coefficient, the incidence would be reduced with increasing Mach number with a consequent effect on the pressure distribution and in turn on both factors that influence the balance on profile drag.

In the light of this situation, it would seem highly desirable to drive a method of calculating profile drag for compressible sub-critical flow that can be applied readily to specific section shapes or pressure distributions at specific conditions of Reynolds number, transition position, etc., as a supplement to existing methods for incompressible flow. Obviously the two-dimensional profile drag is only the first step to the drag estimates needed for accurate performance calculations on modern swept-wing aircraft (more complete methods are urgently required, for example, for estimating the effect of sweep itself on profile drag) but it is a very important one. Moreover, the method to be selected should also form the basis for a systematic investigation of the influence of such factors as section shape and pressure distribution on the variation of profile drag below the critical Mach number.

For low-speed flow standard methods exist for the estimation of the drag of two-dimensional sections, one of the most simple to apply being that presented in Ref.2. The method is based on that proposed originally by Squire and Young[3] in which an appeal was made to the fundamental result that in subsonic flow the momentum thickness of the wake tends to a constant value far downstream, this value being proportional to the total drag of the section. The asymptotic wake momentum thickness was then related to conditions at the trailing edge by an approximate integration along the wake of the momentum-integral equation. Thus the drag coefficient could be expressed in terms of the product of the boundary-layer momentum thickness at the trailing edge and a power of the local velocity at the edge of the boundary layer. In the formulation of Ref.2 it emerges that this product is of the appropriate form to be given directly by an available method for the computation of the boundary-layer development. This is fortunate since the estimated drag is then relatively insensitive to the precise values of either the velocity or the momentum thickness at the trailing edge. Thus the potential-flow pressure distribution can be used without incurring any great embarrassment on account of the rear stagnation point.

The/

The final expression given in Ref.2 for the estimation of the drag coefficient is of a convenient form for the investigation of scale effect and the position of the transition point on each surface, the latter appearing explicitly. In the present note an expression is derived which represents the equivalent form for compressible flow*. The expression involves simple quadratures of flow parameters round the section and would appear to be in a form which would lend itself readily to computer techniques.

In deriving the expression for the drag in subsonic compressible flow use will be made of the Stewartson-Illingworth transformation[11,12]. This transformation is rigorous for laminar flow under conditions of unit Prandtl number, viscosity proportional to temperature and zero heat transfer. For turbulent flow, however, it is not rigorous but depends on an assumption about the effect of compressibility on the turbulent shear stresses[14]. For the turbulent boundary layer an alternative transformation has been derived by Spence[17] on the basis of the "intermediate enthalpy" method. At the higher Mach numbers the latter transformation appears to give better results than the Stewartson-Illingworth transformation, but at subsonic speeds the differences are likely to be negligible. Thus there seems little to be gained by the use of the transformation due to Spence, particularly as it could not be applied to the wake and the methods of treating the boundary layer and wake would then lack consistency. For these reasons the use of the Stewartson-Illingworth transformation throughout is felt to be justified.

2. Analysis

It is a fundamental result that in subsonic flow the total drag of the section appears as a momentum loss in the wake at infinity downstream (see, e.g., Woods[5]). This is expressed analytically as

$$C_D = \frac{2}{c}\,\theta_\infty , \qquad \dots (1)$$

where C_D is the drag coefficient, c the aerofoil chord and θ_∞ the asymptotic momentum thickness of the wake. If a second relation can now be derived between θ_∞ and known conditions on the section, the formal elimination of θ_∞ will represent a solution for C_D.

The result obtained by the approximate integration of the momentum-integral equation along the wake in incompressible flow will be quoted here without proof. (For the derivation see, e.g., Thwaites[2].) It is found that

$$\theta_\infty = (\theta_{T_1} + \theta_{T_2})\left(\frac{u_T}{u_\infty}\right)^{\frac{7}{2}}, \qquad \dots (2)$$

where/

*The method of Squire and Young was itself extended to compressible flow by Young and Winterbottom[4] but aside from being easier to apply the present method is based on more modern boundary-layer techniques.

where θ_{T_1} and θ_{T_2} are the momentum thicknesses of each boundary layer at separation, and u_T is the velocity at the edge of the layer, see Fig.1. Equation (2) is valid so long as the trailing edge is sharp and premature separation does not take place. In compressible flow the equivalent relation is given by (see Appendix)

$$\theta_\infty = (\theta_{T_1} + \theta_{T_2}) \left(\frac{u_T}{u_\infty} \right)^{\frac{7}{2}} \left(\frac{T_T}{T_\infty} \right)^{\frac{5}{4}} . \qquad \ldots (3)$$

Alternatively equation (3) may be written as

$$\theta_\infty = \sum \theta_T \left(\frac{u_T}{u_\infty} \right)^{\frac{7}{2}} \left(\frac{T_T}{T_\infty} \right)^{\frac{5}{4}} , \qquad \ldots (4)$$

the summation referring to the upper and lower surfaces of the aerofoil.

An appeal is now made to boundary-layer theory for the determination of θ_T. For the laminar boundary layer upstream of transition Thwaites[6] gives an expression for θ in incompressible flow of the form

$$\theta^2 u^6 = 0{\cdot}45\, \nu \int_0^x u^5\, dx , \qquad \ldots (5)$$

and for the turbulent boundary layer in incompressible flow we have from Spence[7]

$$\theta^{\frac{6}{5}} u^{\frac{21}{5}} = \theta_t^{\frac{6}{5}} u_t^{\frac{21}{5}} + 0{\cdot}0106\, \nu^{\frac{1}{5}} \int_{x_t}^{x} u^4\, dx , \qquad \ldots (6)$$

where the subscript t indicates conditions at the transition point. Hence, if θ_t is determined from equation (5), the value of θ at the trailing edge, θ_T, is given by

$$\theta_T^{\frac{6}{5}} u_T^{\frac{21}{5}} = \left\{ 0{\cdot}45\, \nu\, u_t \int_0^{x_t} u^5\, dx \right\}^{\frac{3}{5}} + 0{\cdot}0106\, \nu^{\frac{1}{5}} \int_{x_t}^{c} u^4\, dx . \qquad \ldots (7)$$

Applying again the Stewartson-Illingworth transformation (see Appendix), we obtain the corresponding expression in compressible flow:-

$\theta_T^{\frac{6}{5}}$ /

$$\theta_T^{\frac{6}{5}} u_T^{\frac{21}{5}} \left(\frac{T_T}{T_o}\right)^{\frac{3}{2}} = \left\{0{\cdot}45\ \nu_\infty \left(\frac{T_\infty}{T_o}\right)^{\frac{3}{2}} u_t \left(\frac{T_o}{T_t}\right)^{\frac{1}{2}} \int_o^{x_t} u^5 \left(\frac{T}{T_o}\right)^{\frac{3}{2}} dx\right\}^{\frac{3}{5}}$$

$$+\ 0{\cdot}0106\ \nu_\infty^{\frac{1}{5}} \left(\frac{T}{T_o}\right)^{\frac{3}{10}} \int_{x_t}^{c} u^4 \left(\frac{T}{T_o}\right)^2 dx\ . \qquad \ldots (8)$$

Hence

$$\left(\frac{\theta_T}{c}\right)^{\frac{6}{5}} \left(\frac{u_T}{u_\infty}\right)^{\frac{21}{5}} \left(\frac{T_T}{T_\infty}\right)^{\frac{3}{2}} = \left\{\frac{\theta_T}{c}\left(\frac{u_T}{u_\infty}\right)^{\frac{7}{2}} \left(\frac{T_T}{T_\infty}\right)^{\frac{5}{4}}\right\}^{\frac{6}{5}}$$

$$= \left\{0{\cdot}45 \left(\frac{\nu_\infty}{u_\infty c}\right) \frac{u_t}{u_\infty}\left(\frac{T_\infty}{T_t}\right)^{\frac{1}{2}} \int_o^{\frac{x_t}{c}} \left(\frac{u}{u_\infty}\right)^5 \left(\frac{T}{T_\infty}\right)^{\frac{3}{2}} d\left(\frac{x}{c}\right)\right\}^{\frac{3}{5}}$$

$$+\ 0{\cdot}0106 \left(\frac{\nu_\infty}{u_\infty c}\right)^{\frac{1}{5}} \left(\frac{T}{T_o}\right)^{\frac{4}{5}} \int_{\frac{x_t}{c}}^{1} \left(\frac{u}{u_\infty}\right)^4 \left(\frac{T}{T_\infty}\right)^2 d\left(\frac{x}{c}\right) . \qquad \ldots (9)$$

Substituting now for θ_T in terms of C_D (equations (1), (4)) we have

$$C_D = \sum \left[\frac{1{\cdot}422}{Re^{\frac{3}{5}}} \left\{ \frac{u_t}{u_\infty}\left(\frac{T}{T_t}\right)^{\frac{1}{2}} \int_o^{\frac{x_t}{c}} \left(\frac{u}{u_\infty}\right)^5 \left(\frac{T}{T_\infty}\right)^{\frac{3}{2}} d\left(\frac{x}{c}\right)\right\}^{\frac{3}{5}}\right.$$

$$\left. +\ \frac{0{\cdot}02429}{Re^{\frac{1}{5}}} \left(\frac{T_\infty}{T_o}\right)^{\frac{4}{5}} \int_{\frac{x_t}{c}}^{1} \left(\frac{u}{u_\infty}\right)^4 \left(\frac{T}{T_\infty}\right)^2 d\left(\frac{x}{c}\right)\right]^{\frac{5}{6}} . \qquad \ldots (10)$$

Alternatively equation (10) may be written in terms of the local Mach number on the section:-

$$C_D = \sum \left[\frac{1{\cdot}422}{Re^{\frac{3}{5}}} \left\{ \frac{M_t}{M_\infty} \int_o^{\frac{x_t}{c}} \left(\frac{M}{M_\infty}\right)^5 \left(\frac{T}{T_\infty}\right)^4 d\left(\frac{x}{c}\right)\right\}^{\frac{3}{5}}\right.$$

$$\left. +\ \frac{0{\cdot}02429}{Re^{\frac{1}{5}}} \left(\frac{T_\infty}{T_o}\right)^{\frac{4}{5}} \int_{\frac{x_t}{c}}^{1} \left(\frac{MT}{M_\infty T_\infty}\right)^4 d\left(\frac{x}{c}\right)\right]^{\frac{5}{6}} . \qquad \ldots (11)$$

3./

3. Example

Equation (11) has been used to estimate the variation of drag coefficient with Mach number for an RAE 103 section of 12% thickness-chord ratio, at zero incidence. The potential-flow pressure distributions were derived by the method of Weber[9] extended to compressible flow as indicated in Ref.10. Calculations have been made of the drag coefficient over the Mach number range $0 < M_\infty < 0{\cdot}75$ assuming that the transition point lies (a) at the leading edge, and (b) at 10% chord on each surface.

At a constant Reynolds number (Fig.2) the drag coefficient varies little at low subsonic speeds - increasing by about 2% from $M_\infty = 0$ to $M_\infty = 0{\cdot}6$. At higher subsonic speeds, however, the drag coefficient increases more rapidly reaching, at a Mach number of 0·75, a value some 10% above its level at low speeds.

The drag of the 12%-thick RAE 103 section has been determined experimentally in tests in the N.P.L. 20 in. x 8 in. wind tunnel by the wake-traverse method*. The stagnation pressure of the tunnel is held constant giving an increase of Reynolds number with Mach number (Fig.3). Transition was fixed on the model by a roughness band (320 grade carborundum) extending from the leading edge to 8% chord. Using the shadowgraph technique[16] the transition point was found to lie just downstream of the end of the roughness band. The measured drag coefficients are shown in Fig.3 in comparison with the values predicted from equation (11) with the appropriate Reynolds number variation. It is seen that the drag level and the variation of C_D with Mach number are in agreement with the theory to within about 2% of the value of C_D (assuming transition at 10% chord). The critical Mach number of the section is just under 0·75 but the tests indicated that the onset of shock-wave drag does not occur below a Mach number of about 0·78.

Also shown in Fig.3 is the variation of C_D expected in view of the Reynolds number variation alone. These values have been computed using the expression for C_D given in Ref.2, which is valid for incompressible flow, and the compressible-flow pressure distribution. The difference between these values and those indicated by equation (11) is thus a measure of the effect of compressibility.

An important feature of the present method is the implication that the potential-flow pressure distribution can be used in the estimation of the viscous drag. A check on this point has been made by computing the value of C_D from equation (11) using the measured pressure distributions. Fig.4 illustrates the results of this exercise. The values obtained in this way are seen to be in agreement with the measured values of C_D to within a fraction of a percent+, and indeed within the accuracy to which the drags can be measured. The 2% by which the values of C_D were underestimated by the predictions made on the basis of the inviscid pressure distributions is thus an indication of the error involved in ignoring the precise details of the pressure distribution as it is affected by the growth of the boundary

layer./

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*The tests were carried out in the tunnel fitted with slotted walls, the slot configuration being appropriate to negligible tunnel interference[15].

+This order of accuracy is probably fortuitous. In view of the nature of the assumptions made in the method, one would not normally expect an accuracy of better than about 2%.

layer. On this evidence it would seem, therefore, that the use of a higher approximation to the pressure distribution can lead to a measurable improvement in the predicted values of C_D. Nevertheless calculations based on the potential-flow pressure distribution yield drag estimates which would be sufficiently reliable for most purposes.

4. Concluding Remarks

A simple expression is indicated for the estimation of the profile drag of a two-dimensional aerofoil section at subsonic speeds. The expression has been derived from that given by Thwaites for incompressible flow by the application of the Stewartson-Illingworth transformation. The method is valid for any sharp-trailing-edge section so long as (a) significant boundary-layer separation does not occur ahead of the trailing edge, and (b) the flow is shock-free.

The method is illustrated by some calculations for an RAE 103 section of 12% thickness-chord ratio. For this aerofoil it is seen that at a constant Reynolds number the drag coefficient remains nearly constant at low subsonic speeds but exhibits a fairly rapid increase between Mach numbers of 0·6 and 0·75 (which is approximately equal to the critical Mach number), when the value of C_D is some 10% above its level at low speeds.

Some calculations are also presented for the same section assuming a Reynolds-number variation with Mach number appropriate to constant stagnation pressure. The indicated variation of C_D with Mach number is seen to be in good agreement with wind-tunnel measurements.

That a substantial increase of profile-drag coefficient with Mach number, below the critical Mach number, has been predicted and accurately reproduced in experiments on a two-dimensional aerofoil under conditions where movements of the transition point were effectively suppressed is highly significant. In view of the importance of this result and the simplicity of the method of drag estimation, it would seem that the method could be of immediate value in performance estimates. Moreover, further research is clearly justified to study the effects of section shape, pressure distribution, etc., and also to calculate skin-friction drag explicitly in order to elucidate more precisely the reasons for the Mach number effect.

5. Acknowledgement

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APPENDIX /

APPENDIX

Compressibility Transformations

(See, e.g., Refs.11, 12, 14)

Incompressible	Compressible
dx	$\left(\frac{T}{T_o}\right)^{\frac{3\gamma-1}{2(\gamma-1)}} . dx = \left(\frac{T}{T_o}\right)^{4} . dx$ $(\gamma = 1{\cdot}4)$
u	$\left(\frac{T_o}{T}\right)^{\frac{1}{2}} . u$
θ	$\left(\frac{T}{T_o}\right)^{\frac{\gamma+1}{2(\gamma-1)}} . \theta = \left(\frac{T}{T_o}\right)^{3} . \theta$ $(\gamma = 1{\cdot}4)$
ν	$\left(\frac{T}{T_o}\right)^{\frac{2-\gamma}{\gamma-1}} . \nu = \left(\frac{T}{T_o}\right)^{\frac{3}{2}} . \nu$ $(\gamma = 1{\cdot}4)$

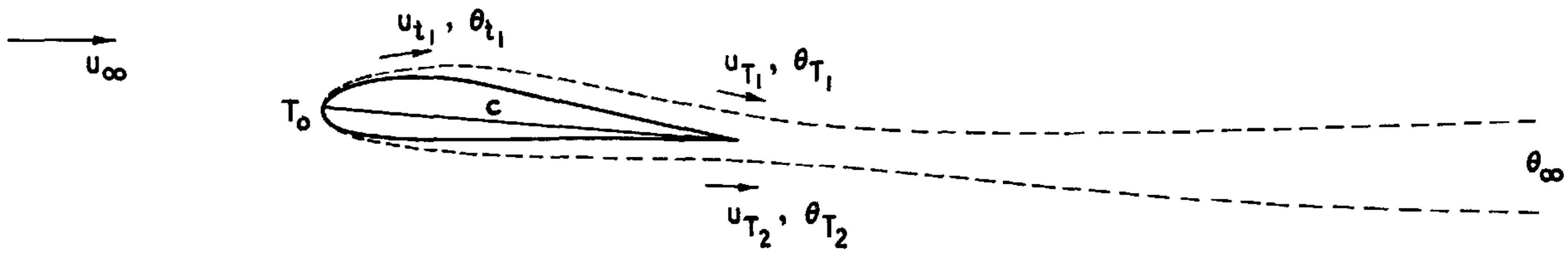
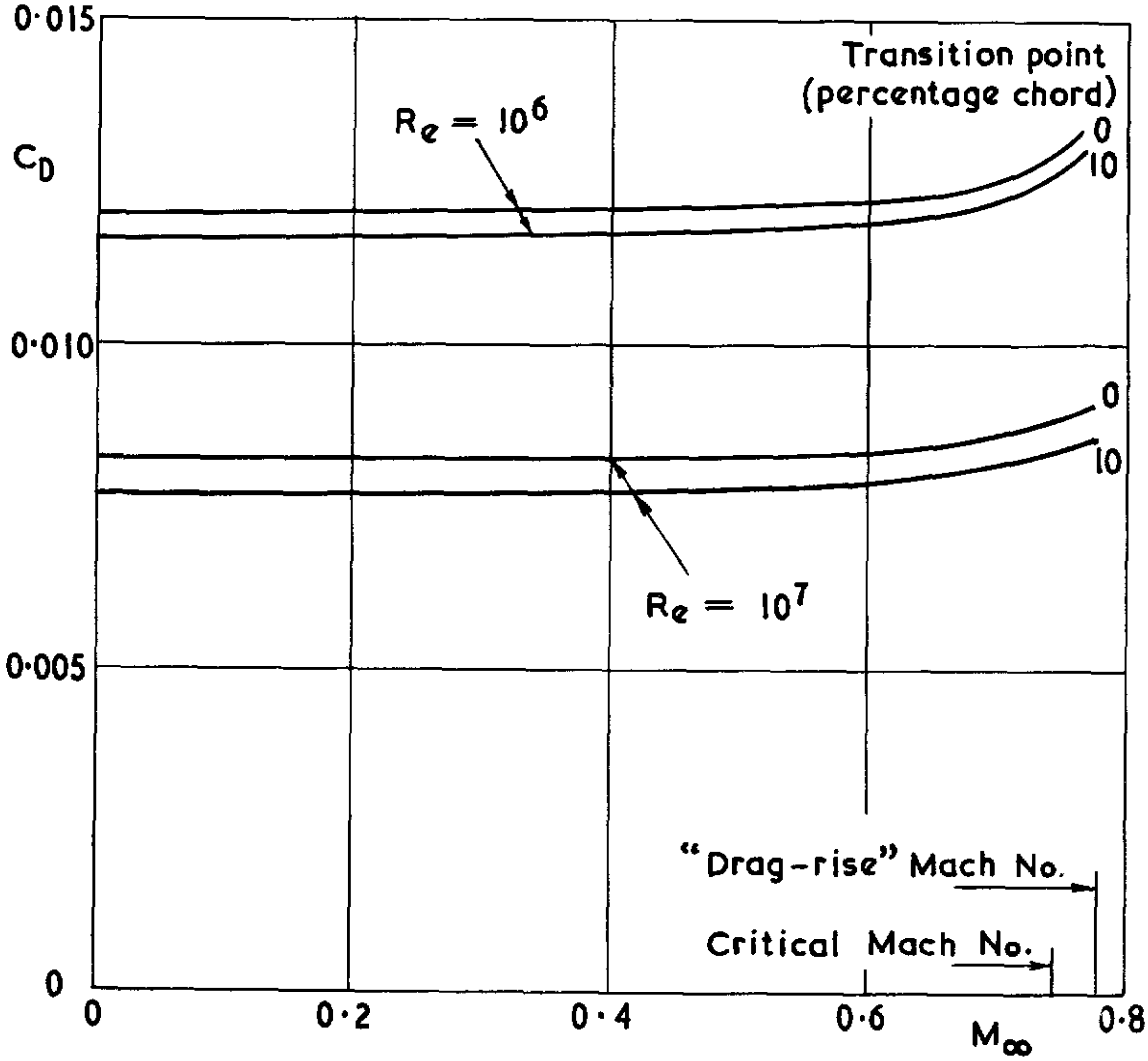


FIG. 1

Definition of symbols

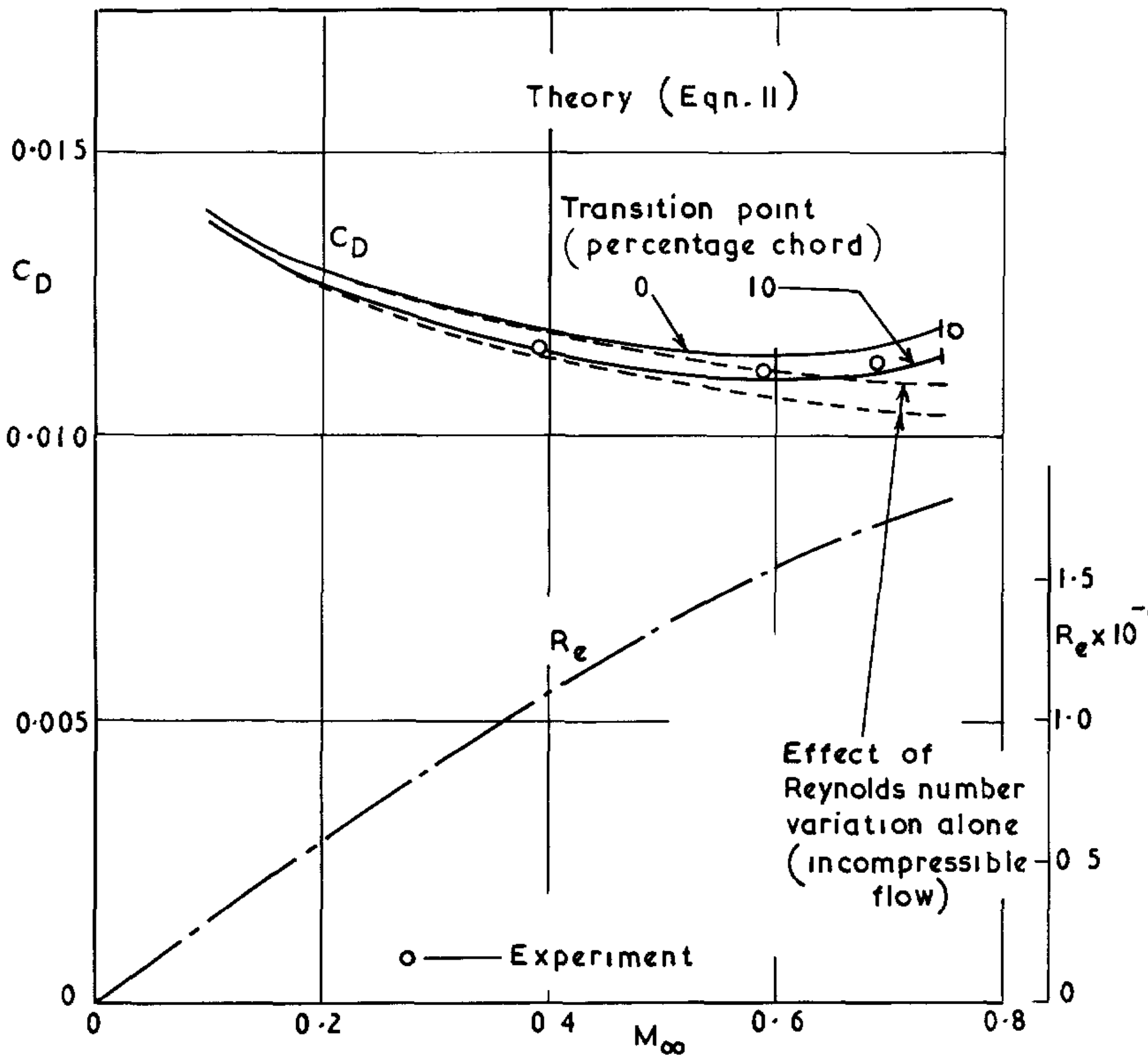
FIG. 2



Variation of C_D with Mach number, RAE 103 section.

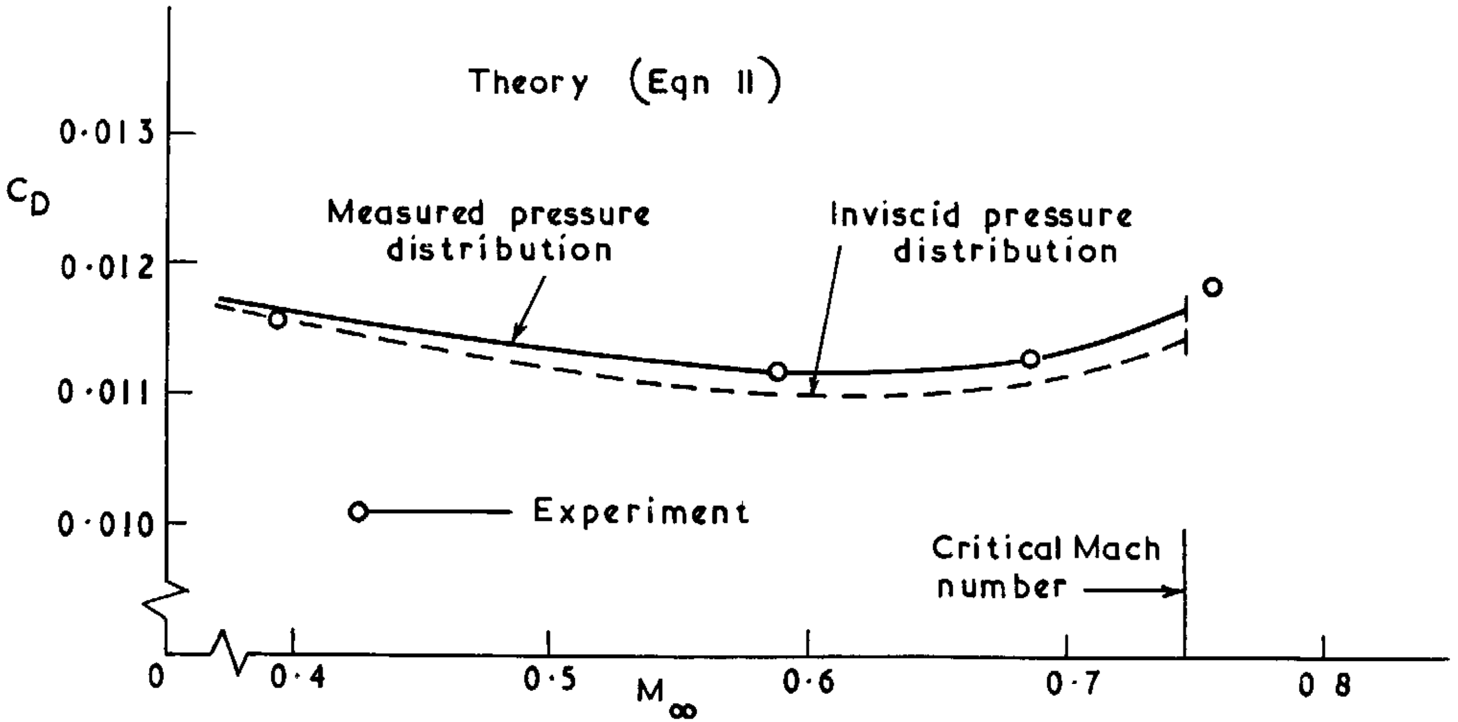
(Constant Reynolds number)

FIG 3



Variation of C_D with Mach number, RAE 103 section
(Constant stagnation pressure)

FIG. 4



Effect on predicted C_D of using measured pressure distribution

(transition at 10 percent chord)

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