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The Influence of Gas Streams and Magnetic Fields on Electric Discharges
Parts I and II

by

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THE INFLUENCE OF GAS STREAMS AND MAGNETIC FIELDS ON ELECTRIC DISCHARGES
PART 1 ARCS AT ATMOSPHERIC PRESSURE IN ANNUAL GAPS

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SUMMARY

This paper describes an experimental study of the motion and electric properties of a d.c. arc moving round an annular gap under the action of a magnetic field.

The dependence of the motion on the arc current, magnetic field and gap width is given for values up to 750 amps, 940 gauss and 3.2 cm respectively. The electrical characteristics of a fixed current arc are given for fields up to 940 gauss. Photographic evidence of the arc rotation together with an indication of the shape of the conducting path are also included.

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DETACHABLE ABSTRACT CARDS

LIST OF SYMBOLS

B flux density of applied magnetic field (gauss)
C_D drag coefficient
d width of annular gap (cm)
D projected width of arc perpendicular to the gas flow (cm)
f rotation frequency of arc with respect to the electrodes (cycles/sec)
I mean arc current (amps)
e length of arc column (cm)
r mean radius of annular gap (cm)
LIST OF SYMBOLS (CONT'D)

\( r_1 \) radius of inner electrode (cathode) (cms)

\( r_2 \) radius of outer electrode (anode) (cms)

\( U \) mean velocity of arc with respect to the electrodes (cms/sec)

\( U_a \) cathode root velocity of arc with respect to the gas in the annulus (cms/sec)

\( U_c \) cathode root velocity of arc with respect to the electrodes (cms/sec)

\( V \) mean total arc voltage (volts)

\( v \) sum of voltages in the arc near the electrodes (volts)

\( X \) column voltage gradient of arc (volts/cm)

\( \rho \) gas density (grm/cm\(^3\))

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f as function of \( B \) at constant \( I \) and \( d \) for different values of \( r \)

f as a function of \( r \) at constant \( I \), \( d \) and \( B \)

f as a function of \( \frac{1}{r} \) at constant \( d \), \( I \) and \( B \)

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Examples of the film records of arc rotating on 5.7 cms dia disc

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1.1 General

This paper is the first of a series which will report the results of experiments on the behaviour of various kinds of electric discharges under the influence of gas streams and magnetic fields. The main incentive for the proposed work is the current interest in research on electric arcs and magneto-hydrodynamics and one application of such work would be the use of electric discharges to heat the test gas in wind tunnels.

Shawl deduces that three types of hypersonic wind tunnel are practicable, i.e. high-density, low-density and high enthalpy, all requiring a heated, high pressure air supply and in view of the temperatures and pressures involved he suggests that an electric arc heater should be developed. To facilitate this there is a need to extend our knowledge of electric discharges to gas pressures of about 1000 atmospheres. In tunnel applications the electrodes would have to withstand high current arcs for fairly long periods and, to ensure uniform erosion, it would be advantageous to cause the arc to move over them under the action of an applied magnetic field. In addition, at the high pressures involved, the gas flow past the arc may have to be allow and heat transfer enhanced by causing rapid local motion of the arc in the gas.

Apparatus is being made so that discharges may be studied in fields up to 10,000 gauss in gas pressures up to one or two hundred atmospheres (later to be increased to 1000 atmospheres). At least three important practical questions need answering:

(i) whether conditions such that the electromagnetic driving force is in equilibrium with the aerodynamic drag force at all points along the arc, can be fulfilled,

(ii) whether discharges can be maintained at such pressures,

(iii) what proportion of the power consumed by the arc and its circuits can be transferred to the gas stream.

The motion of an arc in a magnetic field is governed by many factors, and is not solely due to electrodynamic and aerodynamic forces acting on the arc column. In general the factors influencing its motion, apart from the arc current and the applied magnetic field, are the conditions at the cathode, the nature of the electrode material and the arc dimensions. Several papers have been published on the motion of an arc in a magnetic field at atmospheric pressure\(^2,3,4,5,6,7,8\), and these have some relevance to the present work. Refs.2 and 3 deal with a rotating arc at low currents and magnetic fields, and Refs.4 to 8 deal with the motion of arcs along straight electrodes under the action of a transverse magnetic field:

(a) Nicol\(^2\) has reported on arcs at low currents (15 amps maximum) which were caused to rotate between the ends of two hollow cylindrical copper electrodes (2 cms dia) under the action of a radial magnetic field. The arc velocities were measured using a stroboscopic method. He came to the following conclusions. The speed of motion was found to be:-
(i) independent of the arc length from 0.272 to 0.360 cm,
(ii) proportional to the magnetic field strength and
(iii) to increase linearly with the arc current.

(b) Stolt$^3$ used a similar electrode configuration to that of Nicol with arc currents up to 12 amps and magnetic fields up to 145 gauss. He used electrodes made of the following materials:

- copper, silver, gold, aluminium and carbon

in various anode-cathode combinations with electrode spacing from 0.121 to 0.67 cm. The arc velocities were measured using a stroboscopic method and in all cases his results show that the rotational velocity was directly proportional to the arc current from about 3 to 12 amps. It would appear from his plotted results that the rotational velocity was proportional to the square root of the field from about 30 to 145 gauss.

(c) Babakov$^4$ has described experiments on arcs moving between flat copper plates in a magnetic field. He measured the arc velocity photographically at the rate of $10^4$ exposures per second for electrode spacing from 0.01 to 0.3 cm, arc currents of 100 and 400 amps and magnetic fields from 100 to 930 gauss. In addition he measured the arc velocity for electrode widths from 0.01 to 0.4 cm for arc currents of 100 and 400 amps and magnetic fields of 200 and 800 gauss. His results were divided into three groups:

(i) electrode spacing from 0.3 to 0.17 cm (where the velocity decreases slowly with increasing gap) for which the arc column only was considered,

(ii) spacing from 0.17 to 0.03 cm (where the velocity decreases more steeply with decreasing gap) for which the electrode effects were considered and

(iii) spacing below 0.03 cm (where the velocity is very small) for which it was concluded that it was not an arc, but a bridge of molten metal that was moving.

(d) Winsor and Lee$^5$ used straight electrodes of square cross section mounted horizontally $\frac{3}{8}$ in. apart, arc currents from 3 to 109 amps, fields up to 150 gauss and electrode materials of silver, copper, aluminium, tungsten, molybdenum, nickel or titanium. They report that for a continuous arc movement the results were independent of electrode material but were only consistent after the electrodes were conditioned by running arcs along them several times. Seeker and Guile$^7$ point out that the photographic technique used by Winsor and Lee was insensitive and that errors up to 50% in the field were obtained due to current flow in the electrodes.

(e) Eidingen and Rieder$^6$ investigated arc velocities on straight cylindrical copper electrodes mounted vertically with current connections at the bottom, and they report that the arc velocity depended on the current and field according to the relation $U = C I^{0.61} B^{0.74}$. Seeker and Guile$^7$ point out that the work suffers from similar limitations to that of Winsor and Lee.
(f) Becker and Guile\textsuperscript{7} used straight cylindrical electrodes of mild steel, brass, aluminium or carbon mounted horizontally, with current feed from both ends. They report having found several modes of arc movement which refer specifically to the cathode root. These were:

1. discontinuous,
2. continuous with some surface melting,
3. slow and continuous with severe melting and
4. high speed and continuous with little surface marking.

For arc currents from 40 to 600 amps and magnetic fields up to 500 gauss they reported that for mode (i) the values of velocity were independent of arc current and proportional to $B^n$ where $n < 1$. For mode (iv) the values of velocity were independent of current and linearly dependent on the magnetic field. They also found that consistent results were obtained by using clean, polished electrodes and considerable scatter was obtained on oxidised surfaces.

(g) Guile, Lewis and Seeker\textsuperscript{8} have considered previously published experimental results on arcs moving in magnetic fields in the light of Ecker's proposed mechanisms for the cathode fall region.

1.2 Scope of the present work

The experiments reported here were confined to studies of a d.c. electric arc in air at one atmosphere moving round the annular gap formed between graphite electrodes having the forms shown in Fig. 1 (a and b). These two forms of central electrode were used in order to study the effect of changing the circumference of the annular gap and making it large compared with the gap width. (An experiment in which an arc is driven magnetically along straight electrodes under the action of an applied magnetic field is also being carried out.) The arc was caused to move round the annulus by an axial magnetic field, the arc current being fed into the central rod equally from both ends to eliminate as far as possible the effect on the arc of the self-magnetic field due to the current flow in this electrode. The magnetic field was applied by mounting the apparatus on the axis of a solenoid 45 cms in diameter and 45 cms long (Fig. 2). The arc was initiated by moving the central electrode until it touched the outer electrode and then "drawing out" the arc by restoring it to its concentric position (the central electrode had a pivot and stop to facilitate this operation). The magnetic field was applied before the arc was struck in order to avoid local erosion of the electrodes by a stationary arc at the striking point. The electrodes were made of "horganite" carbon grades EY\textsubscript{1} and EY\textsubscript{9}.

These experiments were started in order to gain experience in this type of work, and graphite, being a refractory material, is a natural choice for preliminary experiments. It should be noted however, that although the arc in this case moves through its own wake (no applied gas flow) and the electrode material (graphite) is unlikely to be used in a wind tunnel heater, the electrode configuration described above if used with a high pressure, low velocity axial
flow of gas through the annular space, would then have the basic features of one
form of heater for a wind tunnel. In addition it may well be that an electro-
magnetic force opposing the axial gas flow will have to be applied in order to
maintain a steadily rotating arc.

The mean arc currents and voltages were measured using Sangamo Weston 582
moving coil meters, and the magnetic flux densities were measured by means of a
search coil used in conjunction with a Cambridge Insta. Fluxmeter. A high speed
Wollensak Fastax rotating prism camera was used for recording the rotational
movement and to obtain photographic records of the moving arc and the conditions
at the electrode surfaces. The frequency of rotation and hence the average
velocity of the arc was measured by means of a small search coil mounted in a
protective alumina tube placed near the edge of the outer electrode. As the arc
rotates, the magnetic field through the coil varies and an alternating voltage
is induced in the coil in phase with the arc rotation. This voltage was
recorded and photographed on a cathode ray oscilloscope.

The range of variables used was:

(a) Width of annular gap \(- d\) (0.3 to 3.2 cm)
(b) Mean radius of annular gap \(- r\) (0.95 to 5.2 cm)
(c) Mean arc current \(- I\) (100 to 750 amps)
(d) Supply voltage \(\) (Giving rise to mean arc
terminal voltages, \(V\), from
30 volts to 200 volts)
(e) Ballast voltage drop \(\)
(f) Magnetic field \(- B\) (60 to 940 gauss)
(g) Rotational frequency of arc \(- f\) (up to 4,700 c/s)
(h) Cathode root velocity \(- U_c\) (up to \(18.7 \times 10^3\) cm/sec or
\(64,5\) ft/sec).

\(U_c\) is defined as the velocity of the arc at the inner electrode
radius \(r_i\), and is calculated from \(U_c = 2\pi r_i f\).

N.B. The quantities \(f\) and \(U_c\) are both relative to the electrodes, and it must be
pointed out that these quantities may be different if considered relative to the
gas in the annular space.

2 EXPERIMENTAL RESULTS

2.1 Arc motion

The experimental results for arc motion are given in three parts:

(1) A general qualitative description of the observed phenomena for both
types of inner electrode.
(2) Quantitative results for a constant value of annular gap width which are presented in two parts dealing with:

(a) results for constant gap and electrode radii where the gap width was made small and equal to the radius of the inner electrode \((d = r_1 = 0.65 \text{ cm})\),

(b) results for the same gap width but for different values of electrode radii \((r_1 \text{ up to } 5.1 \text{ cm})\).

(3) Quantitative results for various values of gap width with a constant size of inner electrode \((d \text{ up to } 3.2 \text{ cm and } r_1 = 0.65 \text{ cm})\).

2.1.1 General description

At an early stage and for small diameter central electrodes it was found that by making the central electrode the cathode the arc motion was steadier, and this configuration was kept throughout the experiments. In general, it was found that a steady arc rotation was not always immediately established, but took a short time (up to a few seconds for the larger gaps) to reach a steady frequency. During this initial period the arc voltage and current were unstable and the audible noise associated with the arc's rotation was either increasing in frequency or erratic. The point at which a steady rotational frequency was reached could easily be detected by observing when the arc current and voltage were steady, and the noise settled to a steady pitch (sometimes a well-defined note could be distinguished). It is thought that the initial variation is due either to the acceleration of the air in the gap resulting in a flow pattern which helps to stabilise the arc motion, or some conditioning period for the electrodes which could be accounted for by a necessity for heating up the cathode track before a smooth motion of the arc root is possible. The measurements were made after the arc had reached a steady rotational frequency.

The two forms of central electrode (rod and disc, Figs. 1a and b) gave rise to two conditions of the cathode track. For a rod electrode (diameters of 1.3 and 2.9 cm only were available) the cathode root appeared to traverse the electrode over a track in the form of a continuously luminous line positioned centrally with respect to the edges of the anode ring. For a disc electrode (diameters from 2.5 to 10 cm and of same thickness as the outer electrode) the cathode root appeared to travel over a series of hot spots (on one edge of the disc) which remained luminous after the arc had traversed them. Those two conditions were observed both visually and photographically, (see Figs. 17 to 20). The frequency of arc rotation did not seem to be affected by this change of the cathode track conditions.

2.1.2 Results for a constant gap

(a) The experimental results for a constant annular gap \((d = 0.65 \text{ cm})\) and cathode radius \((r_1 = 0.65 \text{ cm})\) are shown in Fig. 3, in which the frequency of rotation \((f)\) and cathode root velocity \((v_c)\) are plotted as functions of arc current \((I)\) for different values of magnetic field \((B)\). It is seen that the variation of frequency and cathode root velocity with arc current rises fairly steeply at first and then tends to become less dependent on \(I\). This is more pronounced as the magnetic field is increased. Fig. 4, being a plot of \(f\) and \(v_c\) as functions of \(B\) is derived from Fig. 3. These two families of curves indicate that,
\[ f \text{ or } U_c \propto I^m \text{ for constant } B \]
\[ f \text{ or } U_c \propto B^n \text{ for constant } I \]

where \( 0 < m < 1 \)

where \( 0 < n < 1 \)

and \( m < n \).

The indices \( m \) and \( n \) and the constants of proportionality were deduced from the experimental results by plotting \( \log_{10} U_c \) versus \( \log_{10} I \) and \( \log_{10} U_c \) versus \( \log_{10} B \), respectively. This gave the following relations:

\[ U_c = e_B I^{0.35} \tag{1} \]

for constant \( B \) of 118, 235, 470 and 940 gauss and \( I \) varying from 100 to 540 amps

where \( e_B = 63.8^{0.58} \), obtained by plotting \( \log_{10} e_B \) versus \( \log_{10} B \).

\[ U_c = e_I B^{0.62} \tag{2} \]

for constant \( I \) of 150, 270 and 500 amps and \( B \) varying from 118 to 940 gauss

where \( e_I = 50.1^{0.31} \), obtained by plotting \( \log_{10} e_I \) versus \( \log_{10} I \).  

i.e.

\[ U_c = 57.6 B^{0.60} I^{0.33} \tag{3} \]

for \( U_c \) in cm/sec, \( B \) in gauss and \( I \) in amps.

The standard deviations about the mean straight lines were less than 0.1 for the log/log plots.

(b) The experimental results for a constant annular gap \( (d = 0.65 \text{ cm}) \) and magnetic field \( (B = 4.70 \text{ gauss}) \) but for different values of electrode radii are shown in Fig.5, in which the frequency of rotation \( (f) \) is plotted as a function of arc current \( (I) \) for different values of mean radius \( (r) \). Results for the same \( d \) but at a constant arc current \( (I = 550 \text{ amps}) \) and for different values of electrode radii are shown in Fig.6, in which \( f \) is plotted as a function of \( B \) for different values of \( r \). Plotting \( f \) as a function of \( r \) (Fig.7) suggested that \( f \) may be approximately proportional to \( \frac{1}{r} \), but a plot of \( f \) against \( \frac{1}{r} \) (Fig.8) shows some departure from a straight line indicating that the arc velocity decreases slowly with increasing \( r \). Fig.9 shows how the mean arc velocity \( (U) \) changes with mean radius \( r \), and by plotting \( \log_{10} U \) versus \( \log_{10} r \), it was found that \( U \) was approximately proportional to \( r^{-0.3} \). A similar relation will also apply for \( U_c \).
2.1.3 Results for various values of annular gap

Experiments were made, using a 1.3 cm diameter rod as the central cathode and a constant arc current of 360 amps, for values of \( d \) from 0.3 cm to 3.2 cm. Fig. 10 is a plot of \( f \) and \( U \) as a function of \( r \) for several different values of \( B \). At the fixed value of arc current used for these experiments it was not possible to obtain a smoothly rotating arc at fields below a certain minimum value which increased with the gap width. For fields of 60 and 95 gauss it was only possible to obtain results for gaps up to about 1.5 cm. Fig. 11 is a plot of \( f \) as a function of \( \frac{1}{r} \) (\( r = \) mean radius) for the results at 470 gauss showing that \( U \) (mean arc velocity) is approximately equal to \( 2\pi rf \) (i.e. \( U \) is approximately independent of \( d \)) for a small diameter inner electrode.

2.2 Electrical characteristics

2.2.1 Results for a constant gap

The arc terminal voltage \((V)\) was measured across the electrodes as shown in Fig. 1, and corrected for the voltage drop in the central rod. The experimental results for a constant annular gap \((d = 0.65 \text{ cm})\) and mean radius \((r = 0.95 \text{ cm})\) are shown in Fig. 12 in which \( V \) is plotted as a function of \( I \) for different values of \( B \). These curves have the falling characteristic typical of arcs, but the arc voltage at any particular value in general increased with the magnetic field. The results for \( B = 470 \text{ gauss} \) do not follow the general trend, and it is thought that this variation is due to an increase in \( d \) as the result of severe erosion during these particular tests. Care was taken in all other tests to avoid this by frequent changing of the electrodes.

The results for the same annular gap width and at a constant value of \( B \) (470 gauss) but for various values of electrode radii are shown in Fig. 13. These curves again exhibit a falling characteristic, with the arc voltage at constant \( I \) increasing with the mean radius. The change in \( V \) with \( I \) is rather greater than that shown in Fig. 12 and the reason for this is not clear. The increase in voltage with increasing \( r \) may be due to changes in the electrode processes since the cathode arc root traverses a path whose length increases with \( r \), but further investigation is needed to confirm this.

2.2.2 Results for various values of annular gap

Experiments were made using a 1.3 cm diameter rod as the central cathode and a constant arc current of 360 amps, for various values of \( d \) from 0.3 cm to 3.2 cm. The variation of \( V \) with \( d \) (Fig. 14) is a straight line for each value of \( B \), the lines converging to a common intercept of approximately 15 volts. The slopes of these lines are plotted as a function of \( B \) in Fig. 15 on logarithmic scales, and it is seen that these results lie approximately on a straight line whose slope gives:

\[
\frac{AV}{Ad} \propto B^{0.27} \tag{4}
\]

for a fixed arc current of 360 amps and magnetic fields from 60 to 940 gauss where the voltage gradient, \( \frac{AV}{Ad} \) is proportional to the column voltage gradient, \( X \). These results are discussed in a later section.

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2.3 Accuracy of measurements

Examples of the oscilloscope records of the alternating voltage induced in the search coil used for the measurement of the rotational frequency are shown in Fig. 16 and examples of the cine film records of the arc are shown in Figs. 17, 18, 19 and 20. The arc frequency of rotation as inferred from the frame speed of the cine camera (only a 100 c/s timing marker for film speeds of up to 16,000 frames/sec was available) agreed quite well with those obtained from the oscilloscope records. The frame speed changes over the entire length of the film reaching a maximum somewhere near its mid point and as there is only a 100 c/s timing marker available on the film the measure of arc frequency by this means is inaccurate.

The accuracy of the measuring techniques may be assessed from the following. The oscilloscope used for the measurement of frequency of rotation had a calibrated time base accurate to within ±3% and the scale on which the period of the alternating signal was measured could be read with an accuracy of ±1% (1 mm in 10 cm), giving an overall accuracy of ±2%. This assumes that the calibration of the time base remained within limits throughout the series of experiments. The permanent magnet moving coil meters used for measuring the mean voltage and current of the arc were previously calibrated and found to be accurate to within ±1% of full scale deflection (±10 amps for the current measurements, ±1 volt for voltages below 100 volts and ±4 volts for voltages above 100 volts). The voltage drop in the central rod was taken into account. It is estimated that the current scale of the ammeter was read to within ±5 amps, and the voltage scales (100 volts and 400 volts) were read to within ±0.5 volt and ±2.5 volts respectively. This gives an overall accuracy of ±15 amps for current, ±1.5 volts for voltages below 100 volts and ±3.5 volts for voltages above 100 volts. The meters were so placed that there was negligible interference caused by stray magnetic fields associated with the arc apparatus. The magnetic fields were checked in the central region of the coil and found to be essentially uniform over a diameter of 15 cm. All the tests were made in this central region. A search coil of 50 turns and cross sectional area 15 cm² was used in conjunction with a calibrated fluxmeter (5000 flux linkages per division) accurate to within ±2%. The scale could be read to within 1/2 division (2500 flux linkages or 3.4 gauss), giving an overall accuracy of not more than ±4% for the lowest value of field used, and ±2.5% for the largest value. The annular gaps were measured by means of a slide gauge to within 0.05 cm.

3 DISCUSSION

3.1 Arc motion

The experimental arrangement was such that the frequency of rotation and hence the velocities were measured relative to the electrodes. Since there will also be a flow of gas in the annulus induced by the arc's motion, the arc was in fact moving in a moving gas stream. In addition, it is possible that there was a gas flow into and out of the annular space, which may have been an important factor in stabilising the motion of the arc.
The following forces act on an arc rotating steadily in an annular gap under the action of an applied axial magnetic field:

1. The electromagnetic force of interaction of the arc current \( I \) with the magnetic field \( B \), which is always mutually perpendicular to both the direction of the magnetic field and the direction of the conducting path in the arc:

\[
P_1 = B I \text{ per unit length of the conducting path} \quad (5)
\]

2. The resistive force of the gas in the annulus to the movement of the arc, which will depend upon the arc shape and dimensions. For the purpose of discussion of the results using constant values of annular gap width and cathode radius an element of the arc near the surface of the inner electrode will be considered. This element is assumed to be radial and moving round the inner electrode at a velocity of \( U_\alpha = 2\pi r_1 \) relative to the electrode, and the aerodynamic drag force may be written:

\[
P_2 = C_D \frac{1}{2} \rho U_\alpha^2 \text{ per unit length of the arc near the inner electrode} \quad (6)
\]

where \( C_D \) is a drag coefficient which depends upon the arc shape and the Reynolds number,

\( D \) is the "width" of the arc perpendicular to the flow,

\( \rho \) is the density of the gas through which the arc is moving,

\( U_\alpha \) is the inner arc root velocity relative to the gas in the annulus.

(Note that from physical considerations it is possible that the arc is a slightly porous body so that it is only approximately represented by a solid body of "width" \( D \).)

The electromagnetic and aerodynamic forces \( P_1 \) and \( P_2 \) may be equated:

\[
BI = \frac{1}{2} \rho U_\alpha C_D D
\]

i.e.

\[
U_\alpha = \left( \frac{2}{C_D D \rho} \right)^{1/2} (BI)^{1/2} \quad (7)
\]

However, the arc "width" \( D \) may depend on \( B \) and \( I \) in the following way:

(a) An increase in \( B \) or \( I \) would be expected to increase the arc velocity, and hence the energy losses to the gas by increased convection, giving a tendency to reduce the arc "width" and to increase the total arc power. (Reference to Fig. 12 will show that an increase in \( B \) or \( I \) (above 200 amps) increases the input power to the arc at constant \( I \) or \( B \) respectively.)
(b) In the absence of arc motion an increase in $I$ would tend to increase the arc width.

It follows from these two considerations that the arc velocity increases more slowly with increasing $I$ than with increasing $B$.

Hence, this would lead to an expression of the form

$$U_a = C B^m I^n$$  \hspace{1cm} (8)

where $m > \frac{1}{2}$, $n < \frac{1}{2}$ and $C$ is a constant.

### 3.1.1 Motion in a constant gap

The experimental relation for $U_o$, the cathode root velocity relative to the electrodes given by equation (3) in section 2.1.2,

i.e.

$$U_o = 57.6 B^{0.60} I^{0.33},$$

is in accordance with equation (8) and for comparison the following notes are included on work reported on arcs moving under very different conditions:

(a) Nicol$^2$ and Stolt$^3$ have both reported that for an arc moving round the ends of two hollow cylindrical electrodes, its speed of motion was linearly dependent on the current up to 15 amps, and Stolt reports that the speed was proportional to the square root of the field up to 145 gauss.

(b) Endering and Rieder$^6$ have reported that for straight copper electrodes mounted vertically, with $I$ up to 100 amps and transverse magnetic fields up to 320 gauss, the arc velocity is proportional to $I^x B^y$ where $x < y$.

(c) Babakov$^4$ has described experiments using flat copper electrodes with arc currents of 100 and 400 amps and magnetic fields between 100 and 1000 gauss. From his plotted curves of arc velocity against arc gap for different values of $B$ it would appear that the velocity varied in an approximately linear manner with $B$ and was only slightly dependent on $I$.

(d) Winsor and Lee$^5$ have described experiments using straight copper electrodes mounted horizontally with a $\frac{1}{2}$ in. electrode spacing, arc currents up to 109 amps and transverse magnetic fields up to 150 gauss. From their plotted results of arc velocity against flux density for different values of $I$ it would appear that the arc velocity was approximately proportional to $B$ and was less dependent upon $I$ although no simple relation between arc velocity and current could be determined.

(e) Secker and Guile$^7$ using straight electrodes of mild steel, brass, aluminium or carbon mounted horizontally reported that, for both continuous and discontinuous movement of the cathode root, the arc velocity was independent of the current from about 40 to 600 amps for fields up to 500 gauss. The dependence
of the velocity on $B$ was reported to be linear for the continuous movement and proportional to $B^n$ where $n < 1$ ($U_0$ for carbon electrodes) for the discontinuous movement. Subsequent work by Spink and Guile using brass electrodes, fields up to 1280 gauss and arc currents up to 10,000 amps derived from an a.c. source, showed a marked increase in arc velocity with arc currents from a few hundred to 10,000 amps. An examination of their results has shown that the arc velocity is proportional to $I^n$ at constant $B$ where $n < 1$, but increases for increasing values of $B$. A similar relation also holds for $B$ at constant $I$.

(f) Theoretical work by Lord considers an arc held stationary against an imposed gas flow ($U$) by an applied magnetic field ($B$). The arc is assumed to be in thermal equilibrium and is treated using continuum magnetohydrodynamics. One of his results for the case which assumes energy loss by conduction inside the arc and convection outside is of interest here:

$$U = \text{(constant)} \frac{10^{0.15} E^{0.58}}{B^{0.42}}$$

This equation agrees with the general relation given by equation (8).

The experimental results for a constant annular gap ($d = 0.65$ cm) but for different values of electrode radius showed that the arc velocity was approximately proportional to $r^{-0.3}$ where $r$ was the mean radius of the annulus. The motion of the arc induces a flow of gas in the annular space so that the arc is rotating in gas which is itself moving, and it is possible that as the electrode diameters are increased the effect of this flow is diminished resulting in a slower motion with respect to the electrodes. For the case when the gap is small compared to the mean radius it may be possible to assume that the arc is moving through still air, in which case a direct comparison can be made with an arc caused to move along two parallel straight electrodes. (Such an experiment will be made using carbon electrodes in the near future.)

3.1.2 Motion for various values of annular gap

The electromagnetic and aerodynamic forces per unit length of the arc, would be unchanged for a change in the annular gap width if we assume that the arc length is proportional to the gap width. Hence, the arc velocity would be expected to be independent of the gap width.

The experimental results for various values of annular gap (Figs. 10 and 11) show that the mean arc velocity at 360 amperes and 470 gauss is approximately independent of the gap for values from 0.3 to 3.2 cm. The following are given for comparison:

(a) Babakov reports results which indicate that for a 400 amp arc moving between two flat copper electrodes under the influence of an applied magnetic field of 500 gauss, the arc velocity decreases at first for increasing electrode spacing above 0.15 cm and then tends to become approximately independent of the electrode spacing above about 0.2 cm.
(b) Spink and Guile\textsuperscript{9} using brass electrodes and currents up to 10,000 amps (constant field of 320 gauss) have obtained a similar result, the dependence of arc velocity on the electrode spacing becoming small above gaps of about 0.4 cm.

(c) Nicol\textsuperscript{2} reports having found no dependence of arc velocity with arc length from 0.272 to 0.360 cm, for a discharge moving round the ends of two hollow cylindrical electrodes.

3.2 Arc voltage

Under conditions of free convection cooling it has been shown\textsuperscript{11,12} that for a fixed arc current the total arc voltage is given by:

\[ V = v + \ell X \]  

where \( X \) is the column gradient
\( \ell \) is the arc length
\( v \) is the sum of the voltages near the electrodes.

King\textsuperscript{11} reports that \( X \) depends on arc current up to 100 amps and then remains constant at 10 volts/cm for very long arcs up to 10,000 amps. He plots total arc voltage as a function of arc length for currents up to 100 amps and from extrapolations of these it would appear that \( v \) also depends on the arc current, but tends to a limiting value. (Plots for currents greater than 100 amps are not given in his report.)

The present experimental results for a constant cathode diameter of 1.3 cm and gaps for 0.3 to 3.2 cm are shown in Fig.14, together with a part of King's plot for 100 amps (taken from Fig.5 of Ref.11, where total arc voltage is plotted as a function of arc length from 2 to 15 cm). It is seen that these results may also be represented by equation (10) if it is assumed that the length of the rotating arc is proportional to the annular gap, \( d \), so that the column voltage gradient is proportional to the measured gradient, \( \frac{AV}{Ad} \).

The following points should be noted:-

(i) \( \frac{AV}{Ad} \) (from Fig.14) is not only the arc column gradient but contains the gradient of the induced voltage due to the motion of the arc in the magnetic field; however Table 1 shows that the gradient of the induced voltage (8. 10\textsuperscript{-3} volts/cm of the gap width) makes a negligible contribution to \( X \) at the fields and velocities used.

(ii) As can be seen in Fig.12 the value of \( V \) does not change much with \( I \) over the range 250 to 500 amps, i.e. variations in the motion of the arc when caused by changes in \( I \) above 250 amps do not affect \( V \). It follows that the curves in Fig.14 would also apply to currents in the range 250 to 500 amps.

The extrapolated limit of King's results for \( v \) is very near to the value of the common intercept of the present results (15 volts in Fig.14). It is also interesting to note that the value of this intercept (15 volts) is approximately
the same as estimates for the sum of the anode and cathode fall voltages for free-burning arcs in air at one atmosphere\textsuperscript{12,13}.

If, as might be expected, the effects of $B$ (including any effect it may have on the arc conductivity as well as the motion it induces) are dominant, then we should expect that as $B$ tends to zero, $\frac{AV}{Ad}$ also tends to zero.

The variation of $\frac{AV}{Ad}$ with $B$ (Fig. 15) gives the approximate relation (4) given in section 2.2.2,

$$x = b^{0.27}$$

and an extrapolation of the experimental results to zero $B$, by using the above relation, would pass through the origin. This variation of $X$ with $B$ agrees fairly well with the theoretical work by Lord\textsuperscript{10} referred to in section 3.1.1(f), which gives a relation of the form:-

$$X = \text{(constant)} \frac{b^{0.21}}{1^{0.58}}$$

where $P$ is the ambient pressure.

It is suggested that the increase in the column voltage gradient with increasing $B$ could be accounted for as follows:-

(a) As has already been suggested in section 3.1(a), the motion of a constant current arc under the action of a magnetic field increases the energy losses from the arc by forced convection, so that the input energy is increased. The results presented in Fig. 12 demonstrate that the input energy increases with increasing $B$, and it has been shown that this increase takes place in the column and not as a change in $v$, the voltage due to the electrode end effects.

(b) The arc length may be increased during its motion under the influence of the field, so that to maintain a constant current the column voltage is increased, but this may not necessarily result in an increased voltage gradient.

(c) The application of the magnetic field may decrease the conductivity of the arc so that to maintain a constant current, the arc voltage is increased.

It is likely that the predominant factor is that due to forced convection, (a) above, and an experiment in which extra convection independent of a magnetic field is imposed by a small axial gas flow through the annular gap may help to confirm this. Note that the increase in energy to the arc is transferred to the surrounding gas rather than the electrodes, and if this is also true for metal electrodes it could be an important factor in wind tunnel applications.
3.3 **Arc shape**

Nothing very definite can be said about the arc dimensions except that it is roughly spiral in form along the length of the conducting path, this spiral becoming longer as the annular gap is increased (Figs. 17 and 18).

A spectrum violet filter was used in conjunction with the cine camera so that the film records indicate the location of radiation associated with the arc in the spectral region defined by this filter. Such a filter only transmits light between 3800 Å and 4800 Å with a peak transmission of 29% at 4300 Å. Hence, the conducting path referred to is a region emitting radiation between the spectral limits of the filter. For a carbon arc in air there is a region of intense radiation emitted from a CN band beginning at 416 Å and falling off in intensity towards the shorter wavelengths, so that the cine records are, strictly speaking, mainly sites of CN band radiation. However, it is likely that the conducting path is contained within such sites.

A careful study of the film records has revealed the following points about the arc shape and motion.

(i) The cathode root is usually in advance of the anode root, but a small part of the column near the cathode curves towards the direction of motion.

(ii) The conducting column can assume shapes varying from an almost radial spoke to a rough spiral, which, for the largest value of annular gap, has occasionally been observed to cover a total sweep of 180°.

(iii) The anode root usually proceeds via a series of discontinuous movements and the cathode root motion is usually smooth, although this has also been observed to move discontinuously.

(iv) The arc sometimes has more than one anode root, particularly with the larger gaps.

Preliminary considerations of the elements of arc heating for wind tunnels have shown that a straight radial arc cannot rotate steadily round a circular electrode under the action of an axial magnetic field unless some attempt is made to match the magnetic and velocity fields, and if this is not done one might expect the arc to be curved, as shown by Figs. 17 and 18.

It is possible that an equation for the balance of the electromagnetic and aerodynamic forces at all points along a rotating electric arc not considered to be radial may yield the correct form for the dependence of the rotational frequency on the various parameters, but this is left for consideration elsewhere.

4 **CONCLUSIONS**

It appears from the foregoing results that when an arc is driven round an annular gap under the action of an axial magnetic field the main consequence is that the forced convection increases the power loss from the arc column. The experimental results indicate that changes in electrode effects are associated
only with changes in cathode diameter. (Preliminary experiments using copper electrodes have shown that electrode material has an influence on arc velocity, a result in accordance with that reported by Secker and Guille7).

The investigation has given an approximate form for the dependence of the steady motion of an arc on the arc current and the magnetic field for values up to 750 amps and 940 gauss respectively. It has been shown that this is given by the relation:

\[ U_c \approx 57.6B^{0.60}I^{0.33} \text{ cm/sec} \]

where \( B \) is in gauss and \( I \) in amps.

There is also some evidence to suppose that the mean arc velocity is independent of the gap width from 0.3 to 3.2 cms (a factor of 10 to 1) for a constant inner electrode (cathode) radius.

The measurements of the arc terminal voltage for different gap values have been shown to concur with King’s results11, and indicate that a column whose length is proportional to the gap width exists in the rotating arc and that this column has a voltage gradient which is proportional to \( B^{0.27} \); the photographic records show that the column has a roughly spiral shape, the length depending on the gap.

5 FUTURE WORK

Further work on rotating arcs will include:-

(1) **Studies at one atmosphere**

(a) Investigation of the effect of increasing the gap by varying the cathode diameter.

(b) The effect of introducing an imposed axial gas flow through the annular gap, with, if necessary, a balancing force derived by allowing a net current to flow along the inner electrode.

(c) A time resolved study of the arc voltage, current and motion.

(d) Investigation of the effect of different electrode materials.

(2) **Studies at high pressures**

As far as possible it is intended to repeat the foregoing experimental work, beginning with the range 1 - 10 atmospheres.

(3) **Additional studies**

It is proposed to use spectroscopic and photographic techniques in connection with measurement of:
(i) temperature,
(ii) heat transfer to an imposed gas flow,
(iii) impurity content in imposed gas flow,
(iv) radiation from the arc, and
(v) arc diameter. (It is thought that a measurement of the conducting diameter could be achieved by using a chosen interference filter in conjunction with a cine camera so that only radiation from a line in an ionization spectrum excited in the conducting path of the arc is photographed.)

ACKNOWLEDGMENTS

I wish to thank Mr. A.H. Mitchell for helpful criticisms of the manuscript, and Mr. J.W.T. Palmer for help in translating the foreign references.

LIST OF REFERENCES

Ref. No.  Author Title, etc.
1  Shaw, J.M. The air supply and exhaust services required by hypersonic wind tunnels and the limitations these impose on tunnel performance. Unpublished M.O.A. Report.
3  Stolt, H. Rotation of electric arcs at atmospheric pressure. Annalen der Physik, Vol. 74, 80, 1924 (in German).
<table>
<thead>
<tr>
<th>Ref. No.</th>
<th>Author</th>
<th>Title, etc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>Secker, P.E.</td>
<td>Arc movement in a transverse magnetic field at atmospheric pressure.</td>
</tr>
<tr>
<td>8</td>
<td>Guile, A.E.</td>
<td>The motion of cold-cathode arcs in magnetic fields.</td>
</tr>
<tr>
<td>9</td>
<td>Spink, H.C.</td>
<td>Unpublished work.</td>
</tr>
<tr>
<td>10</td>
<td>Lord, W.T.</td>
<td>Unpublished work.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Aerodynamics Department, R.A.E.</td>
</tr>
<tr>
<td>11</td>
<td>King, L.A.</td>
<td>The voltage gradient of the free-burning arc in air or nitrogen.</td>
</tr>
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<td>12</td>
<td>Engel, A. von</td>
<td>Ionised gases.</td>
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<tr>
<td>13</td>
<td>Finkelburg, H.</td>
<td>Electric arcs and thermal plasma.</td>
</tr>
<tr>
<td>14</td>
<td>Kiechemann, D.</td>
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### Table 1

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<tr>
<th>B (gauss)</th>
<th>U (cms/sec)</th>
<th>X (volts/cm)</th>
<th>B x 10^{-8} (volts/cm)</th>
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<tbody>
<tr>
<td>60</td>
<td>4.7 x 10^3</td>
<td>28</td>
<td>0.0028</td>
</tr>
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<td>95</td>
<td>7.9 x 10^3</td>
<td>32</td>
<td>0.0075</td>
</tr>
<tr>
<td>190</td>
<td>10^4</td>
<td>35</td>
<td>0.019</td>
</tr>
<tr>
<td>235</td>
<td>1.1 x 10^4</td>
<td>43</td>
<td>0.026</td>
</tr>
<tr>
<td>470</td>
<td>1.7 x 10^4</td>
<td>55</td>
<td>0.08</td>
</tr>
<tr>
<td>940</td>
<td>2.45 x 10^4</td>
<td>60</td>
<td>0.23</td>
</tr>
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</table>
FIG. 1. ELECTRODE CONFIGURATION AND ELECTRICAL CIRCUIT.
FIG 2. ARRANGEMENT OF APPARATUS.
FIG. 3. $f$ AND $u_c$ AS FUNCTIONS OF $I$ AT CONSTANT $d$ AND $r$ FOR DIFFERENT VALUES OF $B$.

FIG. 4. $f$ AND $u_c$ AS FUNCTIONS OF $B$ AT CONSTANT $d$ AND $r$ FOR DIFFERENT VALUES OF $I$. 
FIG. 5. \( f \) AS A FUNCTION OF \( I \) AT CONSTANT \( B \) AND \( d \) FOR DIFFERENT VALUES OF \( r \).

FIG. 6. \( f \) AS A FUNCTION OF \( B \) AT CONSTANT \( I \) AND \( d \) FOR DIFFERENT VALUES OF \( r \).
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FIG. 13. V AS A FUNCTION OF I AT CONSTANT d AND B FOR DIFFERENT VALUES OF r.
FIG. 15. $\frac{\Delta V}{\Delta d}$ AS A FUNCTION OF B AT CONSTANT I.
I = 360 amps
B = 95 gauss
f = 1470 c/s
Time base = 2 msec/cm
Voltage scale = 2 mV/cm
Arc gap = 1.0 cm
Cathode dia. = 1.3 cm

I = 400 amps
B = 470 gauss
f = 1100 c/s
Time base = 5 msec/cm
Voltage scale = 1 mV/cm
Arc gap = 0.65 cm
Cathode dia. = 2.9 cm

I = 260 amps
B = 825 gauss
f = 3370 c/s
Time base = 1 msec/cm
Voltage scale = 1 mV/cm
Arc gap = 0.65 cm
Cathode dia. = 1.3 cm

FIG. 16. EXAMPLES OF THE OSCILLOSCOPE RECORDS OF ARC ROTATIONAL FREQUENCY
(a) Gap width $d = 0.65$ cm. (counter clockwise rotation)
$I = 650$ amps. $B = 135$ gauss. $f = 1330$ c/s
Frame speed $\approx 7000$/sec

(b) Gap width $d = 1$ cm. (clockwise rotation)
$I = 360$ amps. $B = 470$ gauss. $f = 2480$ c/s
Frame speed $\approx 12,000$/sec

(c) Gap width $d = 1.9$ cm. (clockwise rotation)
$I = 380$ amps. $B = 470$ gauss. $f = 1500$ c/s
Frame speed $\approx 8000$/sec

(d) Gap width $d = 3.2$ cm. (clockwise rotation)
$I = 380$ amps $f = 1270$ c/s
Frame speed $\approx 12,000$/sec

FIG. 17. FILM RECORDS OF ARC ROTATING ON 1 3cms dia. CATHODE ROD
FIG. 18. FILM RECORDS OF ARC ROTATING ON 1.3 cms dia. CATHODE ROD
Disc inner electrode (5.7 cm. dia.)

d = 0.8 cm. (clockwise rotation)
I = 400 amps  B = 470 gauss  f = 600 c/s
Frame speed ≈ 15,000/sec

The regular luminous dots are on one edge of the inner electrode (disc), see Fig. 20.

FIG 19. FILM RECORDS OF ARC ROTATING ON 5.7 cms dia. DISC
FIG. 20. FILM RECORDS OF ARC ROTATING ON 5.7cms dia. DISC
This paper describes an experimental study of the motion and electric properties of a d.c. arc moving round an annular gap under the action of a magnetic field.

The dependence of the motion on the arc current, magnetic field and gap width is given for values up to 750 amps, 940 gauss and 3.2 cm respectively. The electrical characteristics of a fixed current arc are given for fields up to 940 gauss. Photographic evidence of the arc rotation together with an indication of the shape of the conducting path are also included.
PART 2. THE SHAPE OF AN ARC ROTATING ROUND AN ANNULAR GAP

by

V. W. Adams

SUMMARY

The shape of the column of a d.c. electric arc rotating in an annular gap between carbon electrodes is derived using a simple model for the arc, which is based on the concept of a solid conductor with uniform current density in a transverse uniform magnetic field and an opposing uniform flow field.

It is shown that the shape for steady rotation is the involute of a circle, if the electromagnetic and aerodynamic forces are in equilibrium for all points along the arc. This shape is independent of the form of the expressions used for these two forces.

The production of a plasma jet from the inner electrode arc root can, however, make the arc column straight for certain conditions.

The derived shapes are compared with experimental results, and the simple model is confirmed for large gaps when the electrode effects are small.
ILLUSTRATIONS

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INTRODUCTION

The first paper in this series described experiments in which a direct-current electric arc in air at one atmosphere pressure was driven round an annular gap between graphite electrodes under the action of an axial magnetic field. The present paper extends and discusses some of the results of these experiments, namely the shape of the conducting column and the electrical properties of the arc.

The apparatus and experimental procedure have been described in Ref. and for the results discussed here the annular gaps were formed between graphite rings having various internal diameters and a graphite rod 1.3 cm in diameter.

The maximum value of the annular gap width used in the previous experiments was 3.2 cm and the inner electrode was always made the cathode. The results have since been extended up to a gap width of 5.1 cm, some experiments have been made with eccentric electrodes and further experiments with the inner electrode as the anode.

In order to obtain a steadily rotating arc the magnetic driving field and the resulting velocity field must be matched, which for the simple electrode system considered here requires a curved arc. It will be shown that for certain conditions the column approximates to a theoretically derived involute shape.

A SIMPLE MODEL OF AN ARC MOTION

In proposing a model of the motion, relative to a surrounding gas, of an arc column under the influence of a superimposed magnetic field, we wish to represent in the simplest possible way only what we consider to be the most important features. In the absence of the arc, the superimposed magnetic field is assumed to be uniform. The electric discharge is assumed to be contained in an infinitely long straight column of approximately but not necessarily circular cross-section; electrode effects are thus not considered. At large distances from the arc the flow field is considered to be uniform.

The combined flow and magnetic fields of the moving arc are thought to be such that the gas can be divided into the conducting column of the arc surrounded by a continuum of non-conducting gas, the two regions being separated by an unspecified hot layer. These two separate regions are each governed by their own set of equations, and the scales involved (for instance of time and velocity) may differ by orders of magnitude. It is assumed that the inner and outer solutions in the separate regions can be matched through the intermediate layer without disturbing the concept of either the conducting column or the external continuum. We are concerned here only with the forces exerted on an element of such a moving arc column and not with the details of the fields.

Consider first the distortion of the applied magnetic field, H, caused by the presence of the conducting column when a current, I, is flowing along it. Because of the assumption that the gas external to the column is non-conducting, the field lines of an element of the column are circular and combine with the applied uniform field, the occurrence of singular points...
depending on the relative strengths of $H$ and the current density, $j$. If axes are drawn so that the conductor with uniform current density, $j$, is directed along the $Z$-axis and the applied field, $H$, is parallel to the $X$-axis (Fig. 1(a)), then the field lines within the conductor are the family of circles,

$$x^2 + (y - H/2nj)^2 = k^2$$

and exterior to the current region the lines are represented by

$$r = k_1 \exp(yH/2\pi a^2 j)$$

where $r^2 = x^2 + y^2$ and $a$ is the radius of the conductor.

Three cases are represented in Figs. 1(b), (c) and (d) and the permeability of the conductor is assumed to be approximately the same as that of the external gas.

1(b) - the field is characterized by

$$H/2\pi j < a$$

and singularities occur at the points $(0, H/2nj)$ and $(0, 2\pi a^2 j/H)$.

In 1(c)

$$H/2\pi j = a$$

and the two singular points are coincident at $(0, a)$.

In 1(d)

$$H/2\pi j > a$$

and there are no singularities.

We are concerned with the latter case characterized by equation (5). Body forces on the gas in the arc result in an overall force, $F$, exerted on an element of unit length of the column, of magnitude,

$$F = Bi$$

where $B$ is the flux density of the superimposed magnetic field and $i$ the arc current in e.m.u.'s. The force is perpendicular to both the magnetic field and the direction of the column (along the $Y$-axis in Fig. 1).
Consider next the external gas stream relative to the arc. If the conducting column is taken to be a body of finite (i.e., non-zero) extent placed across a gas stream of velocity \( U \) relative to the arc column, so that a characteristic thickness or diameter, \( d \), can be assigned to it, then the arc causes a displacement of the external flow, as indicated by the full lines in Fig. 2. This displacement flow is assumed to be of a type which does not close downstream of the column, that is to say, part of the hot intermediate layer is swept downstream under the action of the gas flow to form a wake. In this way, heat is added to the gas stream but this heat addition is taken to be of the kind which does not lead to any force acting on the column. However, the existence of a wake signifies the existence of an aerodynamic drag force on the arc. This may be assumed to depend on the density, \( \rho \), of the gas and on \( U \), \( d \), and \( \nu \), the kinematic viscosity. Momentum considerations on dimensional arguments show that this drag force per unit length can be written as

\[
D = \rho U^2 d f\left(\frac{14}{\nu}\right) = \frac{1}{2} \rho U^2 d C_D
\]  

(7)

where \( C_D \) is a non-dimensional drag coefficient which is a function of the Reynolds number \( Re = U d / \nu \). If the relative velocity of the arc were comparable with the speed of sound, \( a \), the drag coefficient would also be a function of the Mach number, \( M = U/a \). In the simple model proposed here, the drag force \( D \) from equation (7) is taken to be normal to the column, i.e., the ("friction") component of the drag force along the column is neglected. Thus, if the element does not cross the gas stream at right angles, the drag force corresponds to that due to the velocity component in the direction normal to the direction of the element, so that in this case \( U \) is this resolved component.

Consider now how the two fields must be combined in order to produce a uniform motion of the arc column. In that case, the forces acting on the arc column must be in equilibrium, i.e., the drag \( D \) from equation (7) must be equal and opposite to the magnetic force \( F \) from equation (6). Hence their magnitudes obey the relation

\[
\frac{1}{2} \rho U^2 d C_D = B_i
\]  

(8)

and the velocity field normal to the arc at large distances from the element must be perpendicular to the magnetic field normal to the arc at large distances, i.e., \( U \) is perpendicular to \( B \). It must be noted that both \( d \) and \( C_D \) are functions of \( B \) and \( i \). The two fields are combined as indicated in Fig. 2, where the curve A is the boundary of the interface which (on a time average) contains the conducting gas, whereas, the curve B indicates (again on a time average) the outer edge of the viscous hot layer surrounding the column, which is then continued to form the wake.

This physical model may be interpreted by stating that the arc column is propagated uniformly by an electrodynamic driving force in a direction normal to itself and to the direction of the magnetic field at a speed of propagation...
which is finite because of the existence of an aerodynamic drag force. It should be noted that these relations are of general validity, provided only that a column of conducting gas and a viscous wake behind it exist, because both equations (6) and (7) are integral relations obtainable from momentum considerations applied to surfaces of integration at large distances from the column. Detailed solutions in the two regions of the field are required only when the values of the current, i, and of the "drag area" (per unit length) dC_D are to be determined for given external conditions. In the absence of theoretical solutions, an experiment can be used to check the applicability of the model proposed and, if its usefulness can be confirmed, to determine empirical values for these parameters.

3 APPLICATION TO A ROTATING ARC

The simple model outlined in the previous section will now be applied to an arc rotating round an annular gap under the action of a uniform magnetic field directed along the axis of the annulus. In applying this simple model it will be assumed that the arc is rotating uniformly and that any unsteadiness due to the arc moving through its own wake may be ignored. In this section the effects due to the electrodes and non-uniformities in the arc column near the electrodes will also be ignored.

What interests us here are the conditions which must be fulfilled in order to produce a steady rotation of the arc where the electromagnetic driving force is in equilibrium with the aerodynamic retarding force for all elements of the arc.

Consider first the simple case of a radial arc of uniform cross-section. If the applied magnetic field is uniform then the electromagnetic driving force is constant for all elements of the arc and its magnitude per unit length of column is given by equation (6). The aerodynamic drag force per unit length is given by equation (7) and its magnitude would depend upon the radial distance, r, of the element from the axis of the annulus because its velocity would increase with the radius, r, in the following way:

\[ U = \sqrt{\frac{Bl}{2\rho dC_D}} \]  

where \( U \) is the steady frequency of rotation of the arc with respect to the surrounding gas. Hence, in order to obtain a uniformly rotating radial arc either the magnetic field would have to vary radially across the annulus, or the arc would have to vary in cross-section.

We will now consider the case where the arc column assumes such a curved shape that it can rotate uniformly, and that all elements satisfy the equilibrium condition given by equation (8). We make the additional assumptions that the column has no tension forces along it and no inertia. Let the
conducting column be of a fixed shape, as yet unspecified, moving round the annular gap with a constant angular velocity, \( \omega \) radians per sec. If each element of the column is considered to behave like an element of an infinitely long straight conductor, then the magnitude of the electromagnetic force per unit length of the arc may be approximated to the value given by equation (6), and in the same way the aerodynamic drag force may be approximated to the value given by equation (7), so that the equilibrium condition given by equation (8) may be applied

i.e.

\[
\frac{1}{2} \rho U^2 d C_D = Bi,
\]

where \( U \) is the velocity of the element resolved in the direction normal to the element (Fig. 3), and is constant for all elements if \( C_D, d \) and \( \rho \) are assumed to be independent of \( r \).

Now,

\[
U = rw \cos \phi
\]

so that by equation (8)

\[
\cos \phi = \frac{c}{r},
\]

where \( c \) is a constant given by

\[
c = \frac{1}{w} \left( \frac{Bi}{\frac{1}{2} \rho d C_D} \right)^{\frac{1}{2}}.
\]

Note that at this point the aerodynamic and electromagnetic laws of force are contained in the constant \( c \) so that what follows is independent of the form of these two functions (i.e., equations (6) and (7)) provided only that they are equated for each element of the arc (equation (8)).

The equation of the arc shape can be deduced by integrating the expression

\[
\frac{d\theta}{dr} = \tan \frac{\phi}{r} = \frac{1}{r} \left( \frac{r^2}{\omega^2} - 1 \right)^{\frac{1}{2}}
\]

obtained from the geometry of Fig. 3, giving,

\[
\theta = \left( \frac{r^2}{\omega^2} - 1 \right)^{\frac{1}{2}} - \cos^{-1} \frac{c}{r}.
\]
which, if $\theta$ is measured from a point where $r = 0$ and $\phi = 0$, is the equation of the involute of a circle of radius $c$.

Let us now consider the condition that an element of the arc moves at right angles to itself and the magnetic driving field at a velocity whose magnitude is given by equation (9). To satisfy both this condition and equation (15) each element moves in a straight line along a tangent to the generating circle of radius $c$. It follows that the element adjacent to this circle is radial and if we postulate that the arc is at right angles to the inner electrode surface the constant $c$ is seen to be equal to $r_1$, the radius of the inner electrode, so that

$$U = r_1 w \quad \text{(see Fig.4)} \quad (16)$$

and if $C_D$, $d$ and $\rho$ are independent of $U$, equation (8) may be written

$$w = \frac{1}{r_1} \left( \frac{B_1}{\frac{1}{2} \rho d C_D} \right)^{\frac{1}{2}} \quad (17)$$

Hence, for an involute-shaped arc the frequency of rotation is independent of the annular gap width and varies inversely as $r_1$, provided that $B$ and $I$ are kept constant.

The length, $s$, of the arc up to the radius $r_2$ (i.e. the outer radius of the annulus) is obtained by integrating the expression

$$ds = \frac{dr}{\cos \phi} = \frac{r \, dr}{r_1}$$

between the limits $r_1$ and $r_2$ giving,

$$s = \frac{1}{2} r_1 \left( \frac{r_2^2}{r_1^2} - 1 \right) \quad (18)$$

and, since the annular gap width

$$g = r_2 - r_1 \quad (19)$$

the arc length

$$s = g \left( 1 + \frac{g}{2 r_1} \right) \quad (20)$$
4  EFFECTS OF ELECTRODES

Consideration will now be given to the behaviour of the end points of the arc and the way in which these may affect the results obtained above.

We consider first the angle between the end points of the arc and the electrode surfaces. There are two ways in which this angle might affect the arc motion:

(i) The arc may have to join the electrodes at right angles, or the discontinuity in the current distribution which would otherwise be formed would produce a large local magnetic field and hence a force on this part of the arc.

(ii) If we refer to Fig.3 it is seen that the angle $\phi$ increases as $r$ increases so that the angle between the outer electrode and the involute becomes more acute as the gap is increased.

This could have two consequences, the first being, that due to the finite thickness of the column the arc terminates at the outer electrode at a point such as c in Fig.4. The second consequence is that the velocity component of the arc track along an element is no longer negligible and equation (8) cannot be applied to the outer part of the column. However on the physical interpretation of the motion illustrated in Fig.4 (which is also consistent with the involute shape) there would be no velocity relative to the surrounding gas of the arc contents along its track. Either of these effects would cause the involute shape to be distorted at the outer electrode and hence cause the motion of the outer arc root to be unsteady. However, if the gap width, and consequently the arc length, is great enough, it is reasonable to assume that the arc forms the involute shape over most of its length. There is probably a limit to the gap width for which a fixed current arc column can sustain the involute shape since, as $\theta$ is made greater than $2\pi$, the involute makes more than one revolution and due to the finite width of the arc column the discharge may become diffuse in form.

Another way in which electrode effects could influence the arc motion is the possibility of the formation of plasma jets due to the constriction of the end points of a high current arc. These jets arise from compressive forces exerted in an arc by its own magnetic field. If a constriction forms in the arc, such as at the cathode root, then the self-magnetic compressive force is greater at this constriction due to the local high current density, than in the uniform arc column. Hence, as one goes from the cathode spot to the column, the current density, the self-magnetic field and consequently the self-magnetic force, decrease, so that a negative pressure gradient is produced along the axis away from the spot. Since the self-induced magnetic field vanishes along the axis of the arc, the pressure gradient produces a jet flow from the electrode cut along the column. Consequently, gas or electrode vapour is drawn into the arc near the electrode surface, and the plasma jet so formed can extend for some distance along the arc and tend to add "stiffness" to it. As a result the aerodynamic drag force near the cathode root is modified (see later). The phenomenon of plasma jets has been observed and explained by Maeker for stationary free-burning arcs and, if the cathode root constriction is maintained in an arc moving over the electrode surfaces, then in this case one would also expect a plasma jet to be formed.
Maecker has shown that the jet is perpendicular to the electrode surface and has a maximum value of velocity at the surface, given by

\[
\frac{1}{2}\bar{\rho} v^2 \simeq ij
\]  

(21)

where \(\bar{\rho}\) is the mean density of the gas or vapour between the electrode and the column, in g/cm,

\(v\) is the maximum value of jet velocity in cm/sec,

\(i\) is the arc current in electromagnetic units

and \(j\) is the arc current density in e.m.u./cm².

If \(b\) is the diameter of the arc cathode spot the current density at the electrode surface is given by

\[
J = \frac{4I}{\pi b^2}
\]

(22)

and by substitution in equation (21) the maximum jet velocity

\[
v = \left(\frac{8I}{\pi \bar{\rho} b^2}\right)^{1/2}
\]

(23)

An estimate for the value of the spot diameter of the rotating arc at 360 amps is 0.23 cm (see Appendix 1). Hence for a 360 ampere arc with \(b \approx 0.2\) cm and \(\bar{\rho} \approx 1.5 \times 10^{-5}\) g/cm³, equation (23) gives a value \(v \approx 7.4 \times 10^4\) cm/sec. This velocity is about 10 times greater than the cathode root velocities obtained (see Fig.5) so that at the inner electrode the arc might be expected to be approximately perpendicular to the surface. The jet velocity decreases rapidly in a direction away from the electrode surface due to the distribution of momentum to nearby gas particles by means of friction. Wienecke has measured the axial velocity of the plasma jet from the cathode of a vertical 200 ampere arc in air at one atmosphere between carbon electrodes, and his results support Maecker's theory. He has shown that at a distance of 2.5 cm above the cathode the axial velocity is reduced to less than 1/3 of its value at the electrode surface, and at 3.5 cm above it is reduced to about 1/10 of its value at the surface. Thus, for a vertical free-burning arc the axial plasma jet velocity is, at a distance between 3 and 3.5 cm above the cathode surface, of the same order of magnitude as the cathode root velocity of a rotating arc. Hence, if a cathode plasma jet is formed from the cathode of a rotating arc, one would expect the involute shape to be distorted near the inner electrode. If the annular gap is sufficiently small the influence of this jet would tend to make the arc straight in form, and the frequency of rotation would then depend upon the gap width.
where \( f(g) \) is a function of the gap width.

On these grounds, a transition from an involute to a straight arc would be expected as the gap is decreased, this transition taking place over a range of gap widths for a fixed arc current. The effect of the plasma jet would still be evident when the gap is large and the arc column nearly an involute, but would only affect the inner portion of the column, so that the arc consists of a small straight part near the inner electrode followed by an involute shaped part. This involute part would be generated by a circle of radius \( r_1 \) depending on the extent of the straight part of the arc.

We consider next the effect of entraining gas into the arc near the cathode surface as a result of the negative pressure gradient produced along the axis away from the cathode spot. In a stationary arc this gas motion is in dynamic equilibrium so that there is no net force on the arc root. However if the arc is moving rapidly over the electrode surface, as in the rotating arc, then one would expect this dynamic equilibrium to be destroyed with the result that a force, opposing the amperian motion of the root, is exerted on the non-uniform region of the arc near the electrode. This could constitute yet another factor influencing the shape of the arc, with the result that the cathode spot would tend to lag behind the arc column, and the plasma jet tend to point towards the arc's direction of motion.

Finally, we consider effects due to the curvature of the arc. If the column is a perfect involute then the curvature approaches infinity at the inner arc root. Therefore, there may be regions near the central electrode at which the arc curvature is sufficient to produce significant self-magnetic forces. The self-magnetic radial force for a circle of current \( i \) is \( i^2/R \) dynes/cm, where \( R \) is the radius of the circle. An approximate calculation based on this expression shows that in our experiments (\( I = 360 \) amps, \( B = 470 \) gauss and inner electrode radius = 0.65 cm) the self-magnetic radial force is about 10% of the driving force due to the applied magnetic field at a distance of about 0.3 cm from the inner electrode. Since, in this region of the arc the shape is affected by electrode effects such as plasma jets, then the influence of arc curvature would not be visible in our experimental records.

Let us now summarise briefly the predicted shapes of an arc rotating round an annulus so that if possible, we may compare them with experimental observations.

When the inner electrode is made the cathode the arc is expected to consist of a part near the cathode surface inclined towards the direction of motion followed by an approximate involute which is unsteady at the anode.

When the gap is reduced so that the electrode effects predominate then the arc is expected to be approximately straight in form, with some inclination towards the direction of motion.

When the outer electrode is made the cathode the general arc shape would still be approximately an involute for large gaps.
DISCUSSION OF DIFFERENCES BETWEEN THE SIMPLE MODEL AND THE EXPERIMENTAL CONDITIONS

Before discussing the experimental results in the light of the simple model discussed hitherto, we will compare some of the theoretical conditions with the experimental conditions.

(1) The superimposed magnetic field, $B$, and the external fluid density, $\rho$, were assumed to be everywhere constant outside the arc. The experimental arrangement was such that the applied magnetic field was constant to within ±2% over a diameter of 15 cm in the plane of the annulus. The distribution of the arc current in the outer electrode would also disturb the uniformity of the field but this was not measured. The density of the surrounding gas could not be measured, but due to convection currents and turbulence caused by the arc's motion it is certain that pressure and density gradients existed in the neighbourhood of the annulus.

(2) The arc column was represented as a solid conductor with uniform current density having both a drag coefficient and diameter invariant with arc length. An electric arc is a gaseous discharge and can thus entrain gas from its surroundings. However, on a time average the conducting gas may be assumed to be contained within a fixed boundary so that a diameter can be assigned to it. In practice, since the arc terminates at the surfaces of two electrodes this diameter would change at the end points and is only constant for the uniform part of the column.

(3) It was assumed that there are no inertial forces on the arc due to its motion (i.e. the arc has zero mass). This assumption is justified because for the experimental conditions used the forces involved, except in the vicinity of a plasma jet, are negligible compared with the electromagnetic forces on the column.

(4) In proposing the simple model and deriving the shape of the column for steady rotation, the effect of the arc rotating in its own turbulent, hot wake was ignored. This assumption is completely unjustified and it is suggested later that the effect of this wake is partly responsible for the inconsistencies in the shape of the arc column (see section 6).

(5) The frequency of rotation, $n$, of an involute shaped arc column has been shown to be steady relative to the surrounding gas, whereas the experimental results for $n$, which are to be discussed and used to determine a value for the term, $\rho D C_p$, (per unit length), were measured relative to the electrodes. However, since any induced motion of the gas in the annulus may be assumed to be governed by the motion of the arc, then this induced flow may also be taken to be uniform. Thus, it is reasonable to compare the experimental variation of $n$ with the results obtained in sections 3 and 4, (i.e. $n$ is independent of $g$ for an involute arc and dependent on $g$ for a straight arc).

6 DISCUSSION OF EXPERIMENTAL RESULTS

The results of experiments where a d.c. arc is rotated round the annular gap formed between a 1.3 cm diameter carbon rod and one of a range of carbon
rings, 15 cm outside diameter, will now be discussed. The magnetic driving field and arc current were kept constant at 470 gauss and 360 amperes respectively, the annular gap width was varied between 0.3 and 5.1 cm and the inner electrode (rod) was made the cathode. In addition, results of some experiments with the inner electrode as the anode will be discussed.

The experimental apparatus and methods of measurement have been described in Part I which reported results for (a) a constant current and magnetic field of 360 amps and 470 gauss respectively, with gaps up to 3 cm, and (b) a constant gap of 0.65 cm with arc currents and magnetic fields up to 750 amps and 940 gauss, respectively.

We shall consider first the experimental results in the light of equations (15), (17), (20) and (24).

Equation (15) is the equation for the curve which the arc column must form in order to rotate steadily; i.e. an involute. The experimental results show that, on a time average, the arc rotates steadily and that the arc column can, for certain conditions be curved. This curve has been found to approximate to the involute of a circle (see later discussion on photography of arc).

Equation (17) shows that the frequency of rotation for an involute arc is independent of the gap width. On the other hand, if, in a uniform magnetic field the arc is not an involute one would expect that there is some dependence of the frequency on the gap width (equation (24)). Fig. 5 shows the variation of the frequency, $n$, for gap widths, $g$, from 0.3 to 5.1 cm; it is seen that $n$ decreases as $g$ increases from 0.3 up to about 2.5 cm and then remains approximately constant at 1200 c/s for values of $g$ above about 2.5 cm. Fig. 6 is a plot of $n$ against the reciprocal of the mean gap radius, and shows that for values of $g$ below about 2.5 cm $n$ is approximately inversely proportional to the mean gap radius.

These two figures suggest that the arc is approximately straight for gaps up to 2.5 cm and approximately involute shaped for larger gaps.

Equation (20) enables the length of an involute arc to be calculated, and if the arc voltage is plotted as a function of this length it would be linear for an involute arc with a uniform column voltage gradient. Conversely, a plot of arc voltage against the radial gap width would be linear for a straight arc with a uniform voltage gradient, (provided that the effect of the cathode plasma jet on the voltage gradient is small). Fig. 7 shows the variation of the arc voltage, $V$, with $g$ and this is seen to be linear for values of $g$ up to about 3 cm. Fig. 8 shows the variation of $V$ with the calculated arc length, $s$, based on equation (20); from this it is seen that the gradient is constant (12 volts/cm) above $s = 6$ cm, which corresponds to a value for $g$ between 2 and 2.5 cm.

The above results indicate that the transition from a straight to an involute arc, suggested in section 4, takes place between gaps of 2 and 3 cm for the particular experimental conditions being considered, and that the discontinuities in Figs. 5, 7 and 8 are due to this transition.

We will now consider the photographic evidence and compare the experimental observations with the arc shapes suggested in section 4. The arc was
photographed with a rotating prism camera capable of operating up to speeds of 16,000 frames/sec for a short period. Examples of the photographic records are shown in Figs. 9, 10, 11, 12 and 14.

It was found that, by comparing the filmed arc shapes against the theoretical involute, the arc column could form an approximate involute shape for gap widths of 2 cm and above (e.g. see Fig. 12 and Ref. 1, Fig. 18), and approximated to a straight arc for gaps of 1 cm and below. The following points should be noted: — (a) For the larger gaps the arc shape alternated between the involute form and an approximately straight form; however, over a time average, the most consistent shape formed was an approximate involute. (b) For the smaller gaps the arc was not a well-defined linear column; the photographic evidence simply shows an indeterminate shape with a trailing hot wake moving round the annulus, the leading edge inclined towards the direction of motion (in accordance with the shape suggested at the end of section 4).

In order to obtain a record of the growth of the involute shape and the transition from a small to a large gap, the arc was photographed when rotating in an eccentric electrode system (see Appendix 2 and Fig. 14).

Consider now the photographic evidence for gaps of 2 cm and above, so that we can examine how closely the arc column conforms to the shape suggested in section 4 (i.e. effects of electrodes and a plasma jet).

It can be seen from Figs. 9, 10 and 11 that the shape of the arc column is continually changing in detail as it rotates with a steady frequency, so that the time-averaged involute is often destroyed from one frame to the next. It is possible that several factors are responsible for these departures from the curved form:

(i) The arc is rotating in its own turbulent, hot wake so that any unsteadiness in the wake is transferred to the arc. (Note: — In an attempt to reduce this effect an experiment in which a flow of air is forced through the annulus at sufficient speed for the arc to rotate through cold air and not its own wake is being planned. Preliminary attempts have resulted in the arc voltage rising to a value too great for the arc to be maintained, and air flows of about 100 cm/sec can only be used at the present time without too great a rise in the voltage occurring.)

(ii) The plasma jet emitted from the cathode may be intermittent so that the arc is at times distorted into an approximately straight form.

(iii) The effects discussed at the beginning of section 4 (unsteadiness at the outer electrode) may cause the motion of the outer arc root to be discontinuous so that the column shape is altered. (A careful study of the film records has shown that the arc motion over the inner electrode is more uniform than the motion over the outer electrode.)

Consider next the production of a cathode plasma jet and its effect on the involute shape (section 4). Although the photographic evidence shows that, on a time average, the arc column is approximately an involute, it does not clearly indicate the presence of a plasma jet. The only visual evidence of such a jet is that a part of the arc near the inner electrode is curved
towards the direction of motion, this part extending for distances of up to about 0.5 cm from the electrode surface. This type of behaviour was attributed in section 4 to the production of a plasma jet and the effect can be seen in Fig. 12. (Some visual evidence for the support of the postulate that a plasma jet can be produced when an arc moves over the electrode surfaces is given in Appendix 3 which describes experiments where an arc moves along a pair of rod electrodes).

We next discuss very briefly some experiments in which the inner electrode was made the anode. For the same arc current and magnetic driving field (360 amps and 470 gauss) used for the experiments already discussed, it was found more difficult to obtain a steady rotation. However, when this was achieved (with a 2 cm gap) the frequency of rotation was the same as before (inner electrode the cathode) and the photographic record showed an arc column of the same approximate involute shape; i.e. the overall shape of the column was not appreciably affected by a reversal of the electrode polarity.

Finally, we use equation (17) which is based on the simple model of sections 2 and 3, to determine a value for the Reynolds number of the arc column, $C_D$. This value only of course applies for the experimental conditions ($B = 470$ gauss and $I = 360$ amps) because both $d$ and $C_D$ are functions of $B$ and $I$. In Ref. 1 this dependence of $dC_D$ on $B$ and $I$ was demonstrated when an experimentally determined relation for the dependence of $U$ on $B$ and $I$ was given,

\[ U \propto B^{0.6} I^{0.3} \]  
\[ (\text{and not}) \quad U \propto B^{0.5} I^{0.5}. \]

We substitute into equation (17) the results plotted in Fig. 5 for gaps above 2.5 cm (i.e. an involute shaped arc column) to determine the product $p \cdot d \cdot C_D$.

Equation (17) may be rewritten

\[ p \cdot d \cdot C_D = \frac{2BI}{(2\pi nr_1)^2}. \]

The values of $B$, $I$, $n$ and $r_1$ for Fig. 5 are 470 gauss, 36 e.m.u., 1200 c/s and 0.65 cm respectively. Substitution in equation (26) gives

\[ p \cdot d \cdot C_D = 1.4 \times 10^{-3} \text{ g/cm}^2. \]
It is not possible to go further with any certainty since d and ρ have not been measured. However, from the photographic evidence, the arc diameter is about 0.5 cm, so that

\[ \rho C_D \approx 2 \cdot 3 \times 10^{-3} \text{ g/cm}^3 \tag{28} \]

where \( \rho \) is the density of the gas through which the arc is rotating (i.e. its own hot wake). Thus, if we assume perfect gas laws apply to the surrounding gas, \( C_D \) is between 1 and 5 depending on the temperature and constituents of the gas.

7 CONCLUSIONS

The simple model proposed in section 2 and applied to the rotating arc in section 3 has been confirmed as a useful one provided that large gaps are considered so that electrode effects can be neglected. The deductions from the model that a d.c. arc in a uniform magnetic field:-(1) rotates uniformly round on annular gap, (2) has a column of approximately involute shape, have been confirmed experimentally.

It has also been shown that the shape of the column (an involute) is independent of the form of the expressions for the aerodynamic and electromagnetic forces on each element of the arc, provided that these functions are the same for all elements, and that the arc has a constant diameter. Further, for the experimental conditions used, such an arc can only assume an approximate involute shape for gap widths greater than about 2 cm, and is approximately straight for smaller gaps. It is not possible to specify the critical gap width accurately since the transition appears to take place between 2 and 3 cm. However, as the gap width is made smaller than about 2 cm electrode effects dominate and the arc can no longer be considered as a uniform column.

A numerical value for the product \( \rho C_D d \) has been obtained for a 360 amp arc in a magnetic field of 470 gauss, i.e. \( \rho C_D d = 1.4 \times 10^{-3} \text{ g/cm}^3 \).

8 FURTHER WORK

In addition to experiments with an axial flow of gas through the annulus, it is proposed to compare the results obtained with those for arcs driven magnetically along a pair of straight, parallel electrodes. This will remove the complication of the arc moving in a heated, disturbed gas stream, and if it is found that the simple model described above can be applied, then values of \( C_D \) or a lumped constant containing \( C_D \) can be determined.

The work proposed in Part 1 of this series is being planned.
ACKNOWLEDGEMENTS

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### SYMBOLS

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<td>$\nu$</td>
<td>kinematic viscosity</td>
<td>c.g.s. units</td>
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REFERENCES

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The high-speed photographs were, in the initial stages of the work, taken through various coloured or neutral high-density filters, until a point was reached where the smallest diameter image of the arc was obtained, i.e. the least amount of halation due to over-exposure was obtained. The filter finally decided upon was a spectrum violet which transmits light between 3,800 Å and 4,300 Å. Thus the photographic records are of the region of the arc emitting radiation between the above limits. For a carbon arc in air there is a region of intense radiation emitted from a CN band beginning at 4246 Å and falling off in intensity towards the shorter wavelengths, so that the cine records are of the location of CN band radiation. It is likely that the CN bands arise from the conducting path.

Measurements of the cathode end of the arc column in the photographic records were made using a vernier travelling microscope.

The width of the photographic image could not be taken as the actual cathode root width because of its motion across the film. To overcome this the diameter of the image, compared with the size of the inner electrode image, was measured for four different exposure times, a mean of ten measurements being taken each time. If \( b' \) is the apparent width as measured and \( t \) the exposure time, the actual arc spot diameter \( b \) is given by:

\[
b' = b + Ut
\]

where \( U \) is the cathode root velocity.

Hence if \( b' \) is plotted as a function of \( t \) the intercept equals \( b \) and the slope the velocity, \( U \).

The results for a 360 amp arc rotating round a 0.65 cm annular gap under the action of an axial magnetic field of 470 gauss are shown in Fig. 13. The intercept gives a value for \( b \) of 0.23 cm and the slope a value for \( U \) of 11,500 cms/sec which corresponds to a frequency of rotation of 2800 c/s.

These results agree quite well with the measured frequency of rotation (see Fig. 5), and a value for the cathode spot diameter of a vertical free-burning arc given in Ref. 3 (0.3 cm for a 360 amp arc).
APPENDIX 2

ARC ROTATING IN THE GAP FORMED BETWEEN ECCENTRIC ELECTRODES

In the main text it has been suggested that a transition from a straight to an involute arc occurs as the width of the annular gap is increased.

In order to obtain a record of such a transition the arc was photographed when rotating in an eccentric electrode system so that the gap varied between 1 cm and 6.7 cm. Part of this record together with an oscilloscope record of the arc voltage and current is shown in Fig. 14, which demonstrates that the arc expands into an approximate involute shape and shows that this expansion is independent of the shape of the outer electrode. In addition, this illustration demonstrates two effects, which are consistent with the condition that an element of the arc moves at right angles to itself and the applied magnetic field as represented for the ideal case in Fig. 4. These effects are:

(1) When a part of the column is approximately straight this part continues to move, at right angles to itself and the applied field, without change of length.

(2) In the same way when a "bulge" or curve is developed in the column due to some instability, this curved part expands in length so that the "bulge" increases in size.

Both of the above effects can be seen in frames 9, 10 and 11 of Fig. 14, in which an approximately straight portion of the column, between two curved parts is moving from left to right while the curved parts are expanding in length.

It should be noted however that the arc always expands, from the short to the large gap, in an approximately overall involute form.
APPENDIX 3
PLASMA JETS ARISING FROM A MOVING/CATHODE SPOT

Any constriction of an arc discharge can give rise to a plasma jet as described in section 4, and such a constriction exists at the cathode of a carbon arc. The validity of the assumption that the cathode spot remains constricted when the arc moves over the surface of the electrodes is not positively shown from the photographic evidence, and neither is the presence of a cathode plasma jet.

However, the phenomenon of the formation of plasma jets when the arc is in motion has been observed visually for an arc moving along two parallel carbon rods (1.3 cm O.D.) under the action of the magnetic field produced by the current flowing in the electrodes, i.e., under the action of the "self-magnetic" field. A photograph of part of the cathode track is shown in Fig. 15 and illustrates the constricted size of the moving cathode spot, which was circular in shape. During these experiments the motion, observed visually, was slow (2 to 10 cm/sec) but at some places the arc jumped and the motion became more rapid (2 to 100 cm/sec). It was possible to observe the arc shape when its motion was slow and this is illustrated in Fig. 16. Two points are offered in evidence for the presence of plasma jets:

(i) The arc curved away from the cathode spot towards the direction of motion (a result also observed in the rotating arc, and attributed to the entrainment of air forming a plasma jet, section 4).

(ii) A large "flame" was observed, similar in shape to those obtained for a stationary arc where collision took place between jets from the anode and cathode.

The above effects were observed for electrode spacings of 0.5, 1.3, 2.3 and 4 cm, the arc currents ranging from 110 to 250 amps with a series resistance of 1.2 ohms.

The track on the anode rod was about 1 cm in width and the carbon was only slightly marked. The appearance of the anode end of the arc gave the impression that the arc root was moving rapidly to and fro across the carbon rod and appeared as an intense line on the surface which moved bodily with the motion of the column.
FIG. I. COMBINED MAGNETIC FIELD OF INFINITE STRAIGHT CONDUCTOR WITH UNIFORM CURRENT DENSITY AND A UNIFORM MAGNETIC FIELD,
FIG. 2. SCHEMATIC DIAGRAM OF COMBINED FLOW AND MAGNETIC FIELDS FOR ARC MOTION.
FIG. 3. NOTATION FOR ROTATING ARC.
FIG. 4. MOTION OF ARC IN ANNULAR GAP.
FREQUENCY OF ARC ROTATION 2 KC/S.

\[ V = 470 \text{ GAUSS} \]
\[ I = 360 \text{ AMPS.} \]
\[ r = 0.65 \text{ CM.} \]

FIG. 5. FREQUENCY OF ROTATION AS FUNCTION OF GAP WIDTH.
FIG. 6. FREQUENCY OF ROTATION AS FUNCTION OF RECIPROCAL OF MEAN GAP RADIUS.
FIG. 7. TOTAL ARC VOLTAGE AS A FUNCTION OF GAP WIDTH.
FIG. 8. TOTAL ARC VOLTAGE AS A FUNCTION OF CALCULATED ARC LENGTH.
FIG. 9 ARC ROTATING IN 1 cm. ANNULAR GAP (CLOCKWISE ROTATION)

I = 360 amps \quad B = 470 \, \text{gauss} \quad g \approx 1 \, \text{cm} \quad n = 2500 \, \text{c/s}

FIG. 10 ARC ROTATING IN 2 cm. ANNULAR GAP (CLOCKWISE ROTATION)

I = 360 \, \text{amps} \quad B = 470 \, \text{gauss} \quad g \approx 2 \, \text{cm} \quad n = 1500 \, \text{c/s}
FIG. II. ARC ROTATING IN 3.2 cm. ANNULAR GAP (CLOCKWISE ROTATION)

$I = 360$ amps  $B = 470$ gauss  $g = 3.2$ cm  $n = 1250$ c/s
FRAME 21 FROM FIG. II

DIAGRAMMATIC EXPLANATION OF ABOVE PHOTOGRAPH.

FIG. 12. DETAIL OF FRAME 21 (FIG. II)
FIG. 13. MEASURED DIAMETER \((b')\) OF CATHODE ROOT AS FUNCTION OF FILM EXPOSURE TIME \((t)\). (APPENDIXI)

\(I = 360\) AMPS. \(B = 470\) GAUSS. \(r_i = 0.65\) CM.
FIG 14 ARC ROTATING IN GAP BETWEEN ECCENTRIC ELECTRODES. (CLOCKWISE ROTATION) (APPENDIX 2)
FIG. 15. TRACK OF CATHODE SPOT OF 200 amp. ARC MOVING ALONG CARBON RODS (APPENDIX 3)

FIG. 16. ARC MOVING ALONG TWO CARBON RODS (APPENDIX 3)
The influence of gas streams and magnetic fields on electric discharges. Part 2, The shape of an arc rotating round an annular gap.

The shape of the column of a d.c. electric arc rotating in an annular gap between carbon electrodes is derived using a simple model for the arc, which is based on the concept of a solid conductor with uniform current density in a transverse uniform magnetic field and an opposing uniform flow field.

It is shown that the shape for steady rotation is the involute of a circle, if the electromagnetic and aerodynamic forces are in equilibrium.
for all points along the arc. This shape is independent of the form of the expressions used for these two forces.

The production of a plasma jet from the inner electrode arc root can, however, make the arc column straight for certain conditions.

The derived shapes are compared with experimental results, and the simple model is confirmed for large gaps when the electrode effects are small.