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Diffraction of a Plane Straight Shock Wave

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SUMMARY

In the present paper Lighthill's problem on the diffraction of a straight shock wave has been extended to general value of γ, γ being the ratio of specific heats. The numerical computation for the pressure distribution along the wall has been carried out for $\gamma = \frac{5}{3}$ and has been compared with those of Lighthill. Also the curvature of the diffracted shock has been found out for $\gamma = \frac{5}{3}$.

1. Introduction

Lighthill in the year 1949 has investigated the diffraction of a straight plane shock wave past a small bend. In this paper the same problem has been treated but the equations have been derived for general value of γ. After the equations have been obtained the numerical computation for the pressure distribution along the wall has been carried out for $\gamma = \frac{5}{3}$. Also the curvature of the diffracted shock wave has been obtained for the same value of γ. In order to bring out a comparison for $\gamma = \frac{7}{5}$ and $\frac{5}{3}$ the same Mach numbers of the shock wave has been taken as those of Lighthill. When the flow behind the shock wave before diffraction is sonic, there is a change in the Mach number of the shock wave. As in Lighthill's paper, the only physical constants defining the problem will be U, the original shock velocity, p_0 and ρ_0, the pressure and density is still air and δ. Lighthill's method for the pressure computation along the wall and my method of computation are different. I have integrated the differential equation for the pressure computation along the wall in exact terms. The principal value of the integral has been considered at the corner when the Mach number of the uniform flow behind the shock wave is less than one.

2. Formulation of the Problem

Let the velocity, pressure and density behind the shock before diffraction takes place be q_1, p_1 and ρ_1 then by conservation of mass, momentum and energy across the shock wave

$$\rho_1 (U - q_1) = \rho_0 U \qquad \ldots (1)$$

$$\rho_0 U q_1 = p_1 - p_0 \qquad \ldots (2)$$

$$p_1 q_1 = \rho_0 U \left[\frac{q_1^2}{2} + \frac{1}{(\gamma-1)} \left(\frac{p_1}{\rho_1} - \frac{p_0}{\rho_0} \right) \right] \qquad \ldots (3)$$

From/

Replaces A.R.C.23,301.

From these equations we get

$$q_1 = \frac{2U}{(\gamma+1)}\left(1 - \frac{a_o^2}{U^2}\right) \qquad \dots (4)$$

$$\rho_1 = \frac{\rho_o\,(\gamma+1)}{(\gamma-1) + \dfrac{2a_o^2}{U^2}} \qquad \dots (5)$$

$$p_1 = \frac{\rho_o}{(\gamma+1)}\left[2U^2 - a_o^2\,\frac{(\gamma-1)}{\gamma}\right] \qquad \dots (6)$$

where $a_o = \left(\dfrac{\gamma p_o}{\rho_o}\right)^{\frac{1}{2}}$ is the velocity of sound in still air.

The Mach number of the shock wave is $M = \dfrac{U}{a_o}$. The Mach number of the uniform flow behind the shock is

$$M_1 = \frac{q_1}{a_1} = \frac{q_1}{\sqrt{\dfrac{\gamma p_1}{\rho_1}}} = \frac{2U\left(1 - \dfrac{a_o^2}{U^2}\right)}{\sqrt{\gamma}}\sqrt{\frac{1}{\left\{(\gamma-1) + \dfrac{2a_o^2}{U^2}\right\}}\;\frac{1}{\left\{2U^2 - a_o^2\,\dfrac{(\gamma-1)}{\gamma}\right\}}} \qquad \dots (7)$$

The generalised result of Lighthill can be written down now.

The equation of the straight portion of the shock wave is

$$x = \frac{U - q_1}{a_1} = (\text{Function of } M \text{ and } \gamma) = k < 1 \qquad \dots (8)$$

The co-ordinates of the corner is $(-M_1, 0)$

where $M_1 = q_1/a_1$

$$u = \frac{a_1}{U}\,\frac{(M^2 + 1)}{(M^2 - 1)}\,[\,f(y) - yf'(y)\,] \qquad \dots (9)$$

$$p = \frac{p_1}{q_1\,\rho_1}\,[\,f(y) - yf'(y)\,]\,\frac{4U}{\left[2U^2 - a_o^2\,\dfrac{(\gamma-1)}{\gamma}\right]} \qquad \dots (10)$$

This/

This gives

$$u = \frac{a_1}{U}\frac{(M^2+1)}{(M^2-1)}\frac{q_1\rho_1}{4p_1U}\left[2U^2 - a_o^2\frac{(\gamma-1)}{\gamma}\right]p$$

$$= Ap \qquad \ldots (11)$$

where

$$A = \frac{a_1}{U}\frac{(M^2+1)}{(M^2-1)}\frac{q_1\rho_1}{4p_1U}\left[2U^2 - a_o^2\frac{(\gamma-1)}{\gamma}\right] \qquad \ldots (12)$$

Also

$$v = -f'(y) \qquad \ldots (13)$$

giving

$$y\frac{\partial v}{\partial y} = \frac{q_1\rho_1\left[2U^2 - a_o^2\frac{(\gamma-1)}{\gamma}\right]}{4p_1U}\frac{\partial p}{\partial y}$$

$$= B\frac{\partial p}{\partial y} \qquad \ldots (14)$$

where

$$B = \frac{q_1\rho_1}{4p_1U}\left[2U^2 - a_o^2\frac{(\gamma-1)}{\gamma}\right] \qquad \ldots (15)$$

After giving the expressions for general value of γ, we come to the case when $\gamma = \frac{5}{3}$. When $\gamma = \frac{5}{3}$ we have the following results:

$$q_1 = \tfrac{3}{4}U\left(1 - \frac{a_o^2}{U^2}\right) \qquad \ldots (16)$$

$$\rho_1 = \frac{4\rho_o}{1 + \frac{3a_o^2}{U^2}} \qquad \ldots (17)$$

$$p_1 = \tfrac{3}{4}\rho_o\left(U^2 - \frac{a_o^2}{5}\right) \qquad \ldots (18)$$

$$M_1 = \frac{3(M^2-1)}{\{(5M^2-1)(M^2+3)\}^{\frac{1}{2}}} \qquad \ldots (19)$$

$$x = k = \left(\frac{M^2+3}{5M^2-1}\right)^{\frac{1}{2}} < 1 \qquad \ldots (20)$$

$$A = \frac{1}{2M^2}\left(\frac{5M^2-1}{M^2+3}\right)^{\frac{1}{2}}(M^2+1) \qquad \ldots (21)$$

$$B = 2\frac{(M^2-1)}{(M^2+3)} \qquad \ldots (22)$$

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The general expression from which the expression for computing the pressure distribution along the wall is obtained is given by

$$w(z_1) = \frac{C\,\delta\,[D(z_1 - x_0) - 1]}{(z_1^2 - 1)^{\frac{1}{2}}\,(z_1 - x_0)\,[\alpha - i\,(z_1 - 1)^{\frac{1}{2}}]\,[\beta - i\,(z_1 - 1)^{\frac{1}{2}}]} \quad \ldots (23)$$

where α and β are functions of M and k. The relation is $\alpha\beta = 2M^2$ and $\alpha + \beta = 2M^2 k\sqrt{2}$. This equation is similar to equation (43) of Lighthill but here k is different.

Also

$$\gamma^2 = 1 - x_0 = \frac{2(M_1 + k)^2}{(M_1 k + 1)^2} \quad \ldots (24)$$

$$\frac{C}{(\alpha + \gamma)(\beta + \gamma)} = \frac{2M_1 k'\,(M_1 + k)}{\pi\,(M_1 k + 1)^2} \quad \ldots (25)$$

$$\frac{(M_1 k + 1)^2}{2BM_1\,(M_1 + k)} = \frac{D(\gamma + \alpha)(\gamma + \beta)}{\alpha\beta} - \frac{\alpha + \beta + \gamma}{\alpha\beta\gamma} \quad \ldots (26)$$

Now these are the expressions of Lighthill but for the present problem the values of $\alpha, \beta, \gamma, M_1$, k and B will be different from those of Lighthill for different values of M.

Now the table is being given for $\gamma = \frac{5}{3}$ which has been utilised for computing the pressure along the wall and also for finding the curvature of the diffracted shock.

Table 1

M	1	1.36277	2.75782	2.95200	∞
p_1/p_0	1	2.07142	9.25690	10.64288	∞
M_1	0	0.40534	1	1.03634	1.34164
k	1	0.76564	0.53518	0.52456	0.447213

When $M \to 1$ the situation is similar to that of Lighthill and so it has not been discussed in this paper. When $M \to \infty$, $\alpha \to \infty$, $\beta \to \frac{1}{k\sqrt{2}}$ and

$$w(z_1) = \frac{2M_1 k'(M_1 + k)(\beta + \gamma)}{\pi(M_1 k + 1)^2}\;\frac{\delta\,[D(z_1 - x_0) - 1]}{(z_1^2 - 1)^{\frac{1}{2}}(z_1 - x_0)\,[\beta - i\,(z_1 - 1)^{\frac{1}{2}}]} \quad \ldots (27)$$

3. The Shape of the Diffracted Shock

Now the curvature of the diffracted shock wave at any point is given by

K/

$$K = \frac{BC\delta(\alpha + \beta)(x_1 + 1)^{\frac{3}{2}}}{(1 - k^2)(x_1 - x_0)} \frac{[D(x_1 - x_0) - 1]}{[\alpha^2 + x_1 - 1][\beta^2 + x_1 - 1]} \quad \ldots (28)$$

K/δ has been graphed against y/k' which runs from 0 to 1 on the curved part of the shock $\left(\frac{y}{k'} = \left(\frac{x_1 - 1}{(x_1 + 1)}\right)^{\frac{1}{2}}\right)$. When $M \to \infty$

$$K = \delta \frac{2M_1 k' (M_1 + k)}{\pi (M_1 k + 1)^2} \frac{(\beta + \gamma)}{(1 - k^2)} \frac{B(x_1 + 1)^{\frac{3}{2}}}{(x_1 - x_0)} \frac{[D(x_1 - x_0) - 1]}{[\beta^2 + x_1 - 1]} \quad \ldots (29)$$

The curvature of the diffracted shocks are plotted in Fig.5.

Now when M has the value 1.36277 and 2.75782 the nature of the curve is similar as for the case when $\gamma = \frac{7}{5}$. When $M = 2.95200$ then there arises quite a good deal of difference. For this Mach number in the case when $\gamma = \frac{7}{5}$, there is a point of inflexion, i.e., the shock which is concave to the still air near the wall changes to convex but for the case when $\gamma = \frac{5}{3}$, there is no point of inflexion. The case when $M \to \infty$, the nature of the curve is similar to that of Lighthill. Here also $\frac{K}{\delta} \to \infty$ as $\frac{y}{k'} \to 1$.

4. Pressure Distribution Along the Wall

Now the expression for pressure distribution along the wall is given by

$$\frac{\partial p}{\partial x_1} = - \frac{C\delta\,[D(x_1 - x_0) - 1]}{(1 - x_1^2)^{\frac{1}{2}}(x_1 - x_0)\,[\alpha + (1 - x_1)^{\frac{1}{2}}]\,[\beta + (1 - x_1)^{\frac{1}{2}}]}$$

where $-1 < x_1 < 1$. ... (30)

This equation has been integrated in exact terms. The principal value of the integral has been considered at $x_1 = x_0$ when $M_1 < 1$.

As in Lighthill $\frac{p_1 - p_2}{\delta(p_1 - p_0)}$ is graphed against x. In the subsonic case $M_1 < 1$ the deficiency is zero at the boundary but attains a logarithmic infinite value at the corner. From infinity it again decreases and finally rises. The graph is shown in Fig.1. In this case the final value at the shock wave is nearly the same as in the case when $\gamma = \frac{7}{5}$. In the sonic case $M_1 = 1$, the graph comes out to be similar as for the case when $\gamma = \frac{7}{5}$ (Fig.2). In the supersonic case ($M = 2.95200$) there appears to be major difference between the two cases (Fig.3). Firstly when $\gamma = \frac{5}{3}$, the value of $\frac{p_1 - p_2}{\delta(p_1 - p_0)}$ which maintains a constant value from the corner to the point of intersection of the unit circle and wall is much higher than in the case when $\gamma = \frac{7}{5}$. Secondly for this value of M_1 for $\gamma = \frac{7}{5}$, there is monotonic decrease in the value of $\frac{p_1 - p_2}{\delta(p_1 - p_0)}$ which has a minimum at the shock wave.

In/

In the case when $\gamma = \frac{5}{3}$ for the same Mach number this is not so. Here after maintaining a constant value from the corner to the point of intersection of the unit circle and wall the value of

$$\frac{p_1 - p_2}{\delta(p_1 - p_0)}$$

decreases, attains a minimum and then slightly rises. For the case when $M_1 = 1.34164$ $(M \to \infty)$ there is a monotonic decrease in the value of $\frac{p_1 - p_2}{\delta(p_1 - p_0)}$ from the point of intersection of the wall and unit circle to the shock wave (Fig.4).

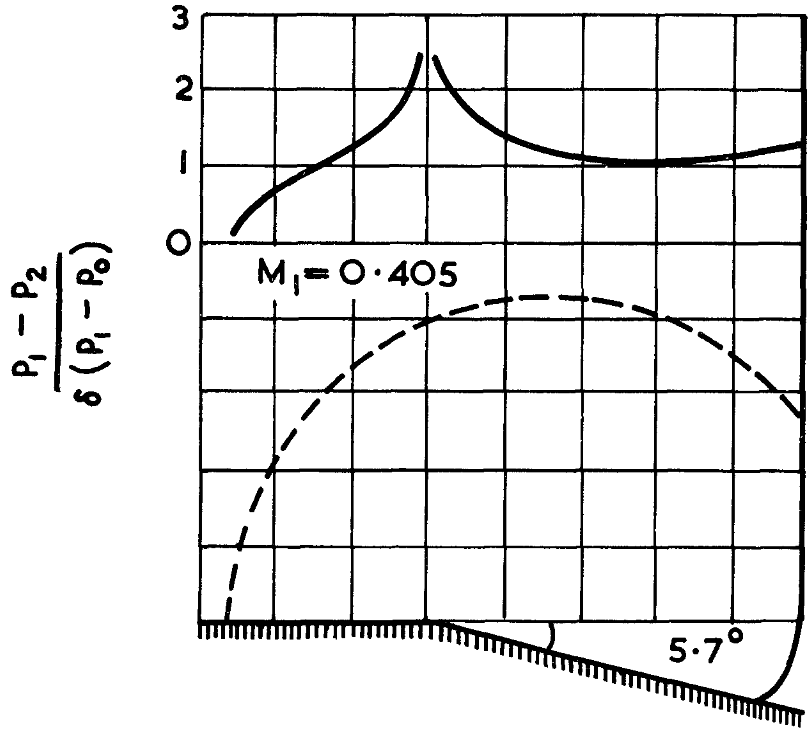
The graphs have been sketched for the four cases which can be compared with those of Lighthill given on pages 465 and 466 in his paper.

I thank Prof. Ram Ballabh for his keen interest in the preparation of the paper.

Reference

Lighthill, M. J. The diffraction of blast. Proc. Roy. Soc., London, Vol.198, Series A (1949), p.454.

FIG. 1

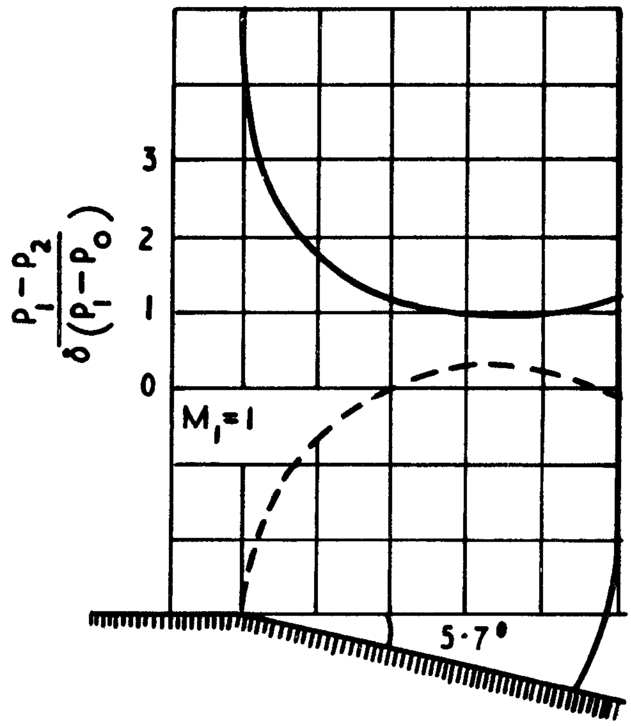


Wall pressure distribution and shape of disturbed region

($\delta = 0 \cdot 1$ radian $p_1 / p_0 = 2 \cdot 07$)

FIGS. 2 & 3

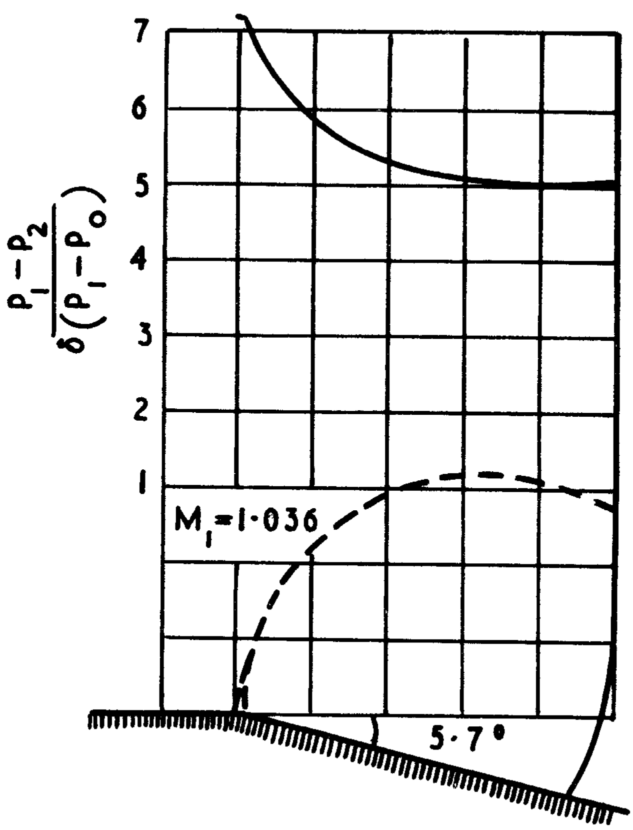
FIG. 2



Wall pressure distribution and shape of disturbed region

($\delta = 0\cdot1$ radian $p_1/p_0 = 9\cdot26$)

FIG. 3



Wall pressure distribution and shape of disturbed region

($\delta = 0\cdot1$ radian $p_1/p_0 = 10\cdot64$)

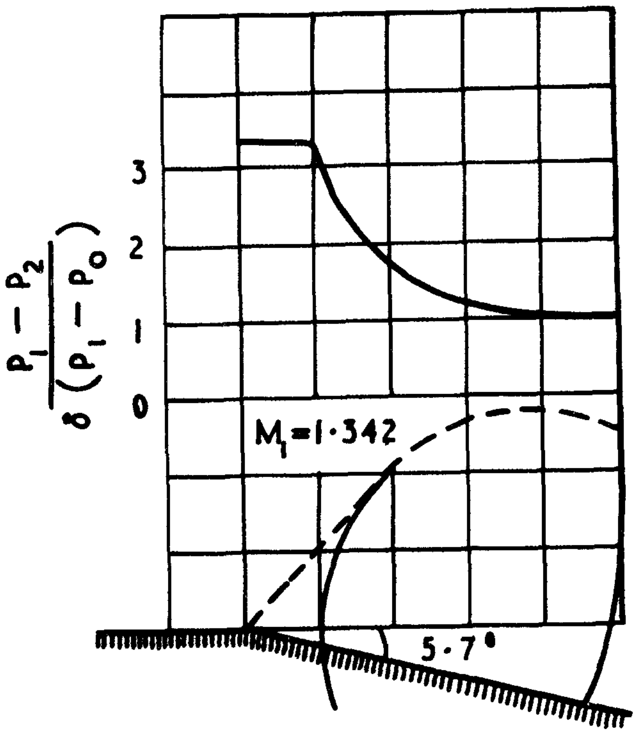
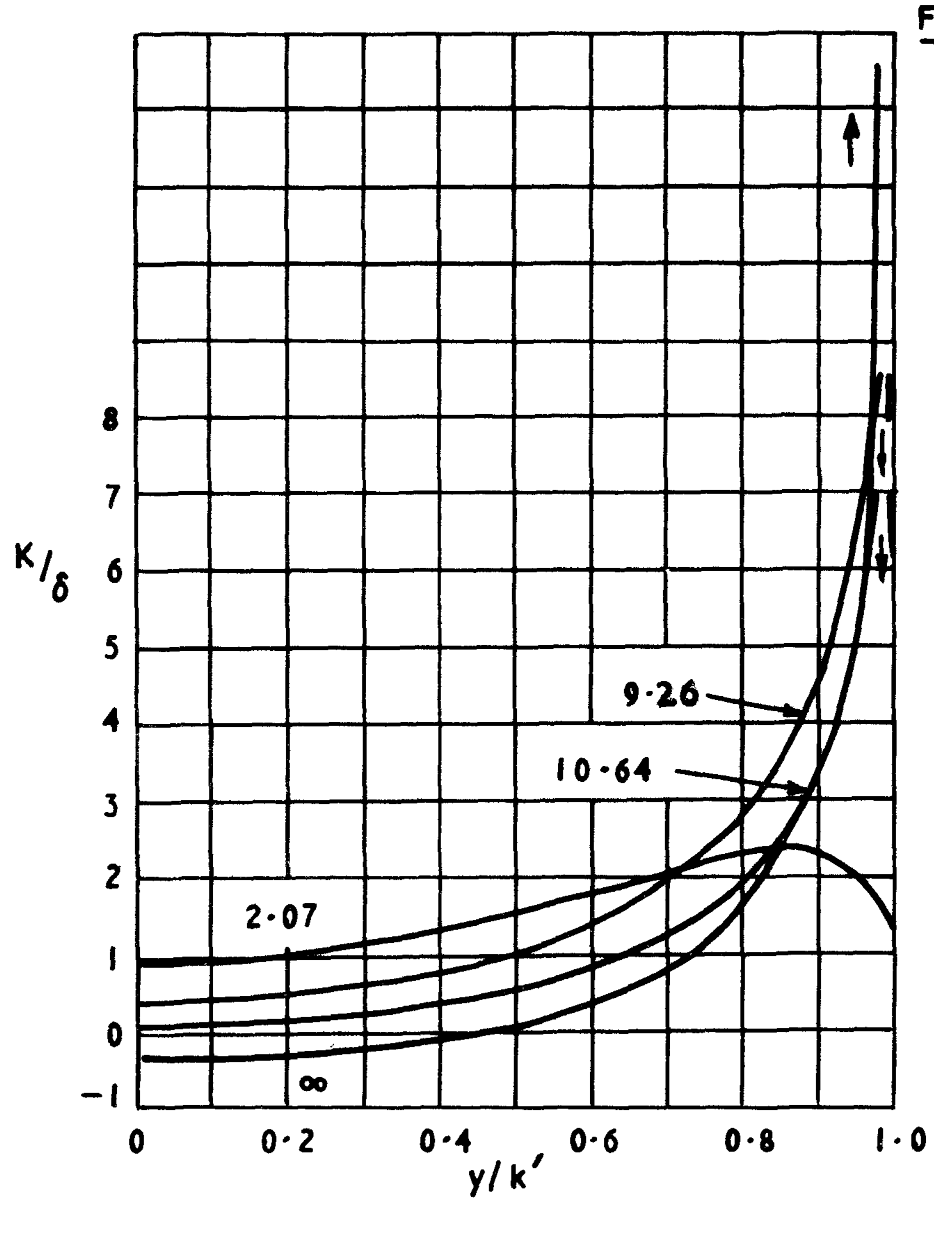


FIG. 4

Wall pressure distribution and shape of disturbed region ($\delta = 0{\cdot}1$ radian $p_1/p_0 = \infty$)

FIG. 5



Curvature of the diffracted shock. The numbers on the curves are the values of p_1/p_0

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DIFFRACTION OF A PLANE STRAIGHT SHOCK WAVE

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