C.P. No. 331
(18,495)
A.R.C Technical Report

C.P. No. 331
(18,495)
A.R.C Technical Report

MINISTRY OF SUPPLY

AERONAUTICAL RESEARCH COUNCIL

CURRENT PAPERS

A Review of Theoretical Work Relevant to the Problem of Heat Transfer Effects on Laminar Separation

By

G. E Gadd, B.A., Ph.D.,
of the Aerodynamics Division, N P L.

LONDON HER MAJESTY'S STATIONERY OFFICE

1957

Price 1s 3d net

C.P. No. 331

A Review of Theoretical Work Relevant to the Problem of Heat Transfer Effects on Laminar Separation

- By -
G. E. Gadd, B.A., Ph.D.
of the Aerodynamics Division, N.P.L.

13th June, 1956

SUMMARY

The results of various theories concerning the effects of heat transfer on laminar separation are briefly discussed. All the theories agree in predicting that wall temperature has an important influence on separation conditions.

Introduction

The effects of heat transfer on laminar boundary-layer separation have been discussed in several theoretical papers. Most of them postulate arbitrarily fixed pressure distributions which are unrelated to the rate of growth of the boundary layer. In this respect the theories are unrealistic[1]. However one would expect there to be a qualitative agreement between experiment and the theoretical findings concerning the effects of wall temperature on separation conditions. Hence, when the experiments reported in Ref.1 were planned, a survey was made of the relevant theoretical work, and this brief note is the outcome.

1. The case with linear adverse velocity gradient

Exact integration of the boundary layer equations for cases with external velocity distributions of the form

$$U_1 = U_0 - \beta x = U_0 (1 - \xi)$$

where x is the distance from the leading edge, would probably give results for the effects of heat transfer and Mach number on ξ_s, the value of ξ at separation, as shown in Fig.1. Here curves are drawn of ξ_s as a function of M_0, the free-stream Mach number, for various uniform wall temperatures T_w. T_0 is the free-stream temperature and $T_z = (1 + 0.2\, M_0^2)T_0$ is the wall temperature for zero heat transfer, all temperatures being measured on the absolute scale. The figure shows the general shape of the curves with a very rough indication of the numerical values. Secondary effects, such as those due to the difference between the Prandtl number σ and 1, are ignored.

Notes (a) Fig.1 shows a bigger variation of ξ_s with wall temperature at high Mach numbers than is predicted by Ref.2, according to which ξ_s depends primarily on the ratio T_w/T_0, and is almost independent of Mach number. In support of this theoretical result, it is argued in Ref.2 that the density of the fluid near the wall is the primary relevant variable. However there are other important factors.

Thus/

Published with permission of the Director, National Physical Laboratory

Thus with a cooled wall at a high Mach number, the temperature in the middle of the boundary layer is high, and the consequent difference between the viscosity at the wall and that in the middle of the boundary layer has an important effect on the profile shape upstream[1].

(b) Refs. 3 and 4 agree in predicting that at moderate Mach numbers there is a considerable increase of ξ_s with M_o for fixed ratios of T_w/T_o. The result of Ref.3 is obtained by numerical integration and is exact apart from any computational errors.

(c) The asymptotic limit of ξ_s at high Mach numbers for finite wall temperatures is estimated from Ref.5. The whole field of Fig.1 could be covered by the method of Ref.5, though the computation would be somewhat tedious. However it can be simplified for very large Mach numbers. Relations are presented in Ref.5 between various shape parameters of the boundary layer profiles as functions of, effectively, T_w/T_z. One of the shape parameters is related to the skin friction and another is related to the pressure gradient. For infinite Mach numbers $T_w/T_z \to 0$ for all finite values of T_w, and for $T_w/T_z = 0$ the relation between the skin friction parameter* and the pressure gradient parameter* is not single-valued. The limit $\xi_s = 0.34$ in Fig.1 was determined on the assumption that separation occurs when the pressure gradient parameter reaches its maximum. At this point, according to the solution, the skin friction c_f would be small and decreasing with infinite dc_f/dx. However the solution would break down before c_f actually became zero, presumably because of the approximate method of integration employed and the nature of the assumed velocity profiles. Thus the numerical value of the upper limit of ξ_s in Fig.1 may be rather inaccurate.

(d) The method of Ref.5 can also be used to show that $\xi_s \to 0$ as $M_o \to \infty$ if the wall temperature is maintained at a sufficiently large fraction of the zero heat transfer value T_z. This limiting fraction is somewhere between 1 and 0.6, and for smaller values of T_w/T_z, ξ_s does not tend to zero as $M_o \to \infty$.

(e) For wall temperatures in the region of the zero heat transfer value T_z, ξ_s is roughly proportional to $T_w^{-\frac{1}{2}}$ at $M_o = 0$. (c.f. Ref.4.) However at higher Mach numbers it is clear from Fig.1 that the variation with temperature is bigger. Thus ξ_s probably varies roughly as $T_w^{-0.8}$ at $M_o = 3$ for T_w in the region of T_z.

2. The case with velocity profiles of constant shape and zero skin friction everywhere

It is shown in Ref.6 that with suitable pressure distributions and uniform wall temperatures similar solutions can be obtained for the boundary-layer velocity profiles. These solutions are related to those for incompressible flow with zero heat transfer and external velocity distributions of the form $U_1 \propto x^m$. For the solutions of Ref.6, it is of interest to consider how the non-dimensional pressure gradient

$\frac{x}{p}\frac{dp}{dx}$ varies with Mach number and wall temperature T_w for the boundary

layer that is everywhere on the point of separation. When the Mach

number is small $\frac{x}{p}\frac{dp}{dx}$ varies roughly as $T_w^{-\frac{1}{2}}$ for T_w in the region of

the zero heat transfer value T_z. At large Mach numbers, it follows from the analysis of Ref.6 that

$\frac{x}{p}$

---

*As defined in Ref.5.

$$\frac{x}{p}\frac{dp}{dx} = -\frac{2\gamma m}{\gamma-1}\frac{x}{M_e^{\frac{3\gamma-1}{\gamma-1}}\int_o^x \frac{dx}{M_e^{\frac{3\gamma-1}{\gamma-1}}}} \qquad \ldots(1)$$

where γ is the ratio of the specific heats, and M_e, the local Mach number at the edge of the boundary layer, satisfies the equation

$$M_e\frac{d^2M_e}{dx^2} + \left(\frac{2\gamma}{\gamma-1} + \frac{1}{m}\right)\left(\frac{dM_e}{dx}\right)^2 = 0. \qquad \ldots(2)$$

The constant m, which is related to the exponent in the incompressible $U_1 \propto x^m$ solutions, takes different values at different wall temperatures to give separation everywhere. From (2), $M_e \propto x^{-K}$ where $K = \frac{-m(\gamma-1)}{(\gamma-1)+(3\gamma-1)m}$, provided that $0 > m > -\left(\frac{\gamma-1}{3\gamma-1}\right)$. This proviso is satisfied for values of T_W greater than about $0.4\ T_Z$, and under these conditions at large Mach numbers M_e, $\frac{x}{p}\frac{dp}{dx} = \frac{-2\gamma m}{(\gamma-1)+(3\gamma-1)m}$ from (1). Hence from the figures given in Ref.6 for the separation values of m in terms of T_W/T_Z, $\frac{x}{p}\frac{dp}{dx}$ varies at large Mach numbers roughly as T_W^{-n}, where $1.5 < n < 2$, for T_W in the region of T_Z.

Thus the variation of $\frac{x}{p}\frac{dp}{dx}$ with temperature increases as the Mach number increases, in much the same way that the variation of ξ_s with temperature in Fig.1 increases.

3. Cases with an abrupt pressure gradient provoking separation without any pressure increase, according to a Pohlhausen-type analysis

In incompressible flow, methods of the well known Pohlhausen type, based on the momentum integral equation and a single parameter family of assumed velocity profiles, indicate that an abrupt adverse gradient, such as is sketched in Fig.2, causes immediate separation with no pressure increase if the gradient exceeds a certain critical finite value. This result is probably untrue and is due to the false assumption that the velocity profile at separation is always of the same shape. However separation would probably occur with only a small increase of pressure under the conditions which are critical according to the theory.

A similar situation is predicted by Ref.4 for compressible flow with heat transfer. The case of a stagnation point flow with a favourable pressure gradient followed by an abrupt adverse gradient is considered. It is found that the adverse gradient needed to provoke immediate separation is somewhat decreased by increasing wall temperature T_W. However the effect is not large because contrary effects operate in the regions of favourable and unfavourable gradients. For the case as sketched in Fig.2 where the pressure is constant for a certain distance from the leading edge, and there is then a constant adverse gradient, it may be shown that according to the theory of Ref.4 immediate separation occurs if

$$x\frac{dC_p}{dx} \geqslant \frac{6.41}{(1+0.2M^2)\left(11\frac{T_W}{T_Z}+4\right)} \qquad \ldots(3)$$

where/

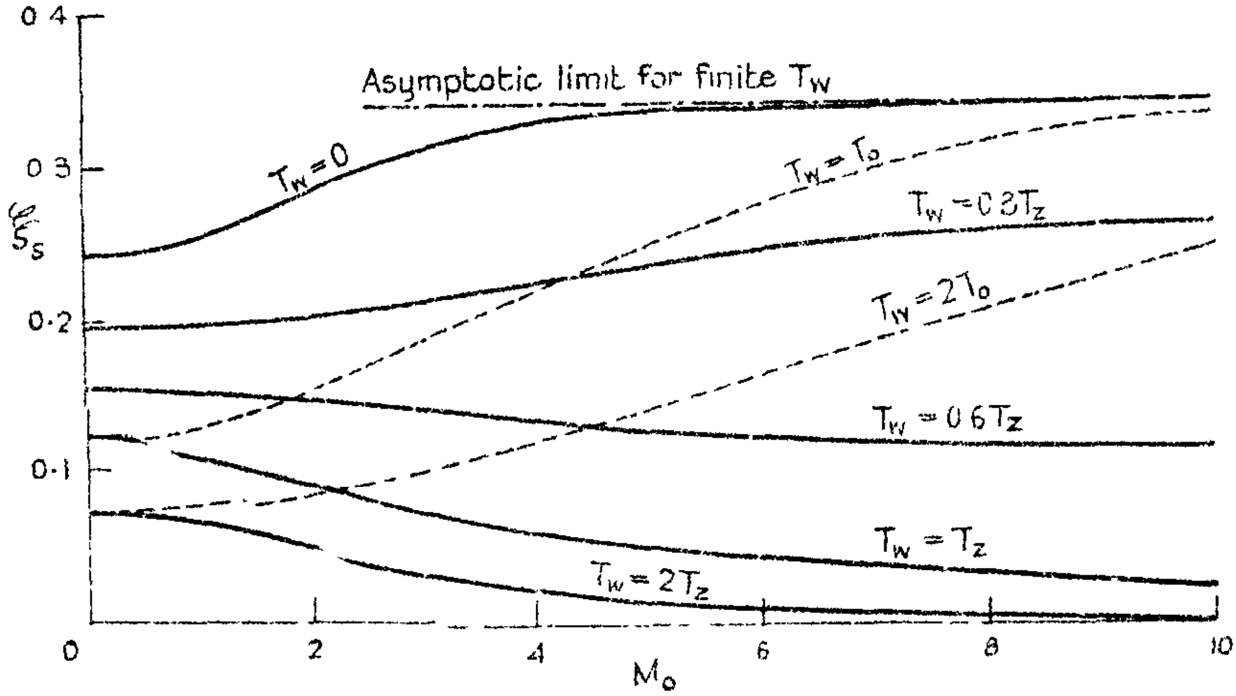
where C_p is the pressure coefficient and M is the free-stream Mach number. Thus approximately $x \frac{dC_p}{dx} \propto \left(\frac{T_W}{T_Z}\right)^{-0.73}$ for T_W in the region of T_Z. For zero heat transfer, when $T_W = T_Z$, $x \frac{dC_p}{dx}$ according to (3) takes the values 0.291, 0.239, 0.155 and 0.102 at Mach numbers of 1.5, 2,3 and 4 respectively. These are fairly close to the experimental data for the maximum value of $x \frac{dC_p}{dx}$ reached at the upstream end of the laminar foot for separation from a flat wall in supersonic flow. (c.f. Ref.7.) The variation with T_W/T_Z indicated by (3) is less than that predicted for $\left(x \frac{dC_p}{dx}\right)_{max}$ in Ref.7, but nevertheless it is greater than is observed experimentally[1,7] for $\left(x \frac{dC_p}{dx}\right)_{max}$.

References

No.	Author(s)	Title, etc.
1	G. E. Gadd	An experimental investigation of heat transfer effects on boundary layer separation in supersonic flow. A.R.C.18,397. 25th April, 1956.
2	C. R. Illingworth	The effect of heat transfer on the separation of a compressible laminar boundary layer. Quart. J. Mech. & Appl. Maths. Vol.VII, Part 1, p.8, (1954).
3	G. E. Gadd	The numerical integration of the laminar compressible boundary layer equations, with special reference to the position of separation when the wall is cooled. Current Paper No. 312. 1st August, 1952.
4	M. Morduchow and R. G. Grape	Separation, stability, and other properties of compressible laminar boundary layer with pressure gradient and heat transfer. N.A.C.A. T.N.3296. May, 1955.
5	C. B. Cohen and E. Reshotko	The compressible laminar boundary layer with heat transfer and arbitrary pressure gradient. N.A.C.A. T.N.3326. April, 1955.
6	C. B. Cohen and E. Reshotko	Similar solutions for the compressible laminar boundary layer with heat transfer and pressure gradient. N.A.C.A. T.N.3325. February, 1955.
7	G. E. Gadd	A theoretical investigation of the effects of Mach number, Reynolds number, wall temperature, and surface curvature on laminar separation in supersonic flow. A.R.C.18,494. 13th June, 1956.

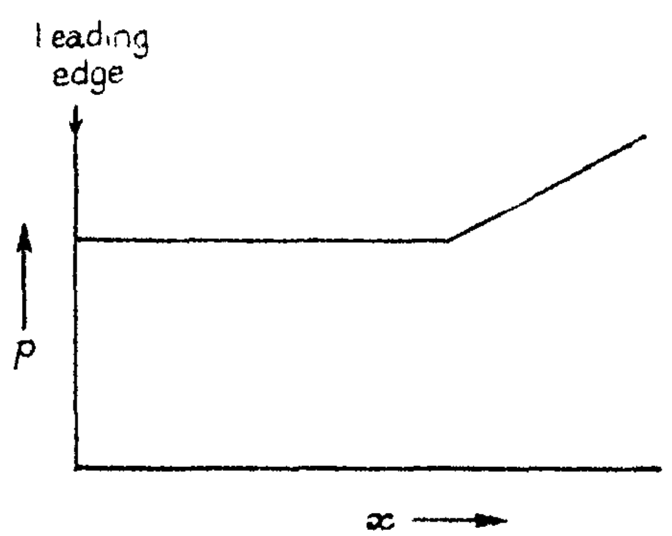
HBF

Fig 1.



The variation of $\xi_s \equiv (U_0 - U_1)/U_0$ at separation with Mach number and wall temperature for the linear adverse velocity gradient case

Fig 2.



Type of pressure distribution which can lead to separation with no pressure increase, according to methods of the Pohlhausen type

C.P. No. 331
(18,495)
A.R.C. Technical Report

Printed and published by
HER MAJESTY'S STATIONERY OFFICE

To be purchased from
York House, Kingsway, London W.C.2
423 Oxford Street, London W.1
13A Castle Street, Edinburgh 2
109 St Mary Street, Cardiff
39 King Street, Manchester 2
Tower Lane, Bristol 1
2 Edmund Street, Birmingham 3
80 Chichester Street, Belfast
or through any bookseller

Printed in Great Britain

S.O. Code No. 23-9010-31

C.P. No. 331