Factors Affecting the Performance of the Nozzle of a Hypersonic Shock Tube

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SUMMARY

An analysis of the starting process of a hypersonic shock tube is made and the subsequent flow conditions in the working section are evaluated for various static temperatures and Mach numbers of the flow. A discussion of the simulation of free-flight conditions leads to recommendations for the optimum operation of hypersonic shock tubes; performance charts are given.

2. Introduction

The ease with which very high Mach number flows - with stagnation temperatures approximating to those of full-scale flight - may be generated in hypersonic shock tubes has led to the rapid development of this latest type of intermittent wind tunnel. However, the ultra-short duration of the steady hypersonic flow poses formidable instrumentation problems.

Reference 1 reviews the aerodynamic principles involved in the production of very strong shock waves and very high Mach number flows; a brief recapitulation of the main features of the flow pattern occurring in the channel of a hypersonic shock tube is given below. Let us consider the distance-time diagram of Figure 1. The supersonic flow behind the strong shock wave $M_{ss}$ is expanded in a divergent nozzle. If $p_0 = p_1$, then the strength of the primary shock wave will decrease during its passage through the divergent section - in an analogous manner to the decay of a cylindrical blast wave - and a secondary shock wave of increasing strength will be formed by the continuous diffraction of the primary shock through the divergent channel. (For a complete account of this flow process see Ref.7.) In general, a secondary contact surface $C_4$ or contact region of entropy gradients will also be formed in the diffraction process; both this and the secondary upstream facing shock wave $M_{s}$ will be swept downstream after the primary downstream - facing shock wave $M_{ss}$. A region of steady flow at a high Mach number $M_4$ is established in the hypersonic working section until the arrival of the primary contact surface $C_4$. It is possible, if the expansion ratio of the nozzle is sufficiently large, for the strength of the shock $M_{ss}$ to be such that it becomes stationary with respect to the walls of the shock tube, that is $|M_{ss}| = M_4$: it is then equivalent to the standing shock observed in a steady nozzle flow with too high a back pressure.

*It is convenient to 'label' shock waves with their shock Mach number $M_\text{sh}$ - see Appendix for Notation.

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Let us consider the primary contact surface $C_0$ reaches the upstream end of the hypersonic nozzle at a time $\tau$ after the incident shock $M_{S_0}$ has reached it. Reference 1 shows that this time $\tau$ is extremely short and the time $\tau_R$ for which there is steady hypersonic flow in the nozzle working section (the running time) is even shorter.

Let the starting time $\tau_s$ be the time taken for the secondary shock wave $M_{S_4}$ to travel through the divergent nozzle and let $\tau_c$ be the time taken by the primary contact surface $C_0$ to do likewise. Then the running time $\tau_R$ is given by

$$\tau_R = \tau - (\tau_s - \tau_c). \tag{2.1}$$

For a given shock tube $\tau$ is a function of $M_{S_0}$ only, and $(\tau_s - \tau_c)$ is a function of the pressure ratio $p_0/p_4$ across the nozzle diaphragm.

3. The Starting Process of a Hypersonic Shock Tunnel

3.1 Analysis of the Problem

In order to investigate the starting process in greater detail, it is necessary to determine the shock Mach numbers of the transmitted and secondary shock waves for given conditions at the entry to the nozzle and in the working section. Thus, referring to Fig. 1, $M_{S_4}$ and $M_{S_4}$ must be determined in terms of $M_{S_0}$ and $p_0/p_4$ for various values of $M_4$ and $T_4$. It is assumed that the temperature on both sides of the nozzle diaphragm is initially atmospheric ($T_0 = T_s = 288^\circ K$).

3.1.1. Variation of the area ratio $A_4/A_0$ with working section Mach number $M_4$ and temperature $T_4$

The flow at Mach number $M_4$, which is generated behind the primary shock in the straight tube, becomes accelerated to a Mach number $M_4$ after passage through the divergent nozzle. For a given area ratio $A_4/A_0$, $M_4$ depends only on $M_0$ and is independent of $T_0$ as with conventional high speed wind tunnels. If the primary shock $M_{S_0}$ is strong, $M_4$ asymptotically approaches the limiting value 1.99 for infinite shock strength, and large variations in shock strength produce only small variations in $M_4$ (although large variations of $T_0$ occur). Accordingly if the area ratio $A_4/A_0$ is fixed, the working section Mach number is approximately independent of shock strength if the shock strength is large. This is also true with shocks of moderate strength as the following analysis shows:—

From/
From isentropic flow theory, if \( \gamma = 1.4 \),

\[
T_3 = 1 + 0.266 T_4 = \left( \frac{p_3}{p_4} \right)^{\frac{\gamma}{\gamma-1}} = \left( \frac{p_0}{p_4} \right)^{\frac{1}{\gamma-1}} \quad \ldots (3.1)
\]

and

\[
A_4 = \frac{\rho_0}{\rho_4} T_4 = \frac{M_4}{M_3} \left( \frac{T_0}{T_4} \right)^{\gamma} \quad \ldots (3.2)
\]

If \( T_0 = 288^\circ K \), \( M_4 \) is given by

\[
M_4 = \left[ \frac{5}{3} \left( \frac{288}{T_0} \right) \left( 1 + 0.266 \frac{T_0}{T_4} - 1 \right) \right]^{\frac{1}{2}} \quad \ldots (3.3)
\]

From simple shock tube theory, \( M_4 \) and \( T_3/T_0 \) are functions of the primary shock Mach number \( M_{30} \) and are given by the equations

\[
M_4 = \frac{5(\gamma^3 - 1)}{\sqrt{(7\gamma^3 - 1)(5 + M_{30}^2)}} \quad \ldots (3.4)
\]

\[
T_0 = \frac{7\gamma^3 - 1}{36 M_{30}^2} \quad \ldots (3.5)
\]

Table I presents values of \( M_4 \) and \( A_4/A_0 \) for various values of \( M_{30} \) and \( T_4 \), calculated from (3.1) to (3.5). The numerical suffixes to \( (M_4) \) and \( (A_4/A_0) \) are the values of \( T_4 \) in \( ^\circ K \), and thus \( (M_4)_{150} \) denotes \( M_4 \) when \( T_4 = 150^\circ K \). Note that if \( M_4 < 1 \), a nozzle which first contracts to a throat would be required to produce the supersonic flow at a Mach number \( M_4 \). The corresponding tabular values are not of practical interest, but are useful in plotting \( M_4 \) as a function of \( A_4/A_0 \) as in Figure 2. As stated above the relation between the test section Mach number \( M_4 \) and the area ratio \( A_4/A_0 \) is approximately unique and independent of the working section temperature \( T_4 \). This is a most useful result, since it means that if a particular Mach number \( M_4 \) is desired and the appropriate area ratio \( A_4/A_0 \) for the nozzle is chosen from Fig. 2, substantially the same Mach number \( M_4 \) will be obtained even if, due to experimental imperfections, the primary shock Mach number \( M_{30} \) is considerably different from its expected value.

The values of the temperature \( T_4 \) and pressure \( p_4 \) in the working section will, however, be approximately proportional to \( T_3 \) and \( p_3 \) and thus will be affected by errors in the shock Mach number \( M_{30} \).

Hence, for a given shock tube with \( A_4/A_0 \) fixed, \( M_4 \) is fixed; however, \( T_4 \) may be varied by alteration of \( M_{30} \).

Figures 3 and 4 present results calculated from equations (3.3), (3.4) and (3.5) and illustrate the variation of Mach number \( M_4 \) and static temperature \( T_4 \) in the working section with incident shock Mach number \( M_{30} \).

3.1.2. Determination of shock velocities

The solution of the equations governing the passage of the shocks through the expanding nozzle presents considerable difficulties, but when the shocks have passed out of the expansion into the parallel working section their velocities are readily calculable. Let us refer to Fig. 1, where both shocks are proceeding with uniform velocities
along the hypersonic channel. From the Rankine-Hugoniot equations for the downstream-facing shock $M_{S_1}$

$$
\frac{p_1}{p_3} = \frac{2y}{y+1} \frac{\gamma^2}{\gamma-1},
$$
\hspace{1cm} \ldots \ldots (3.6)

and

$$
\frac{u_2}{u_4} = \frac{2}{y+1} \left[ M_{S_1} - \frac{1}{Y_{s_1}} \right],
$$
\hspace{1cm} \ldots \ldots (3.7)

Also, for the upstream-facing shock $M_{S_4}$, where $M_{S_4}$ is defined by

$$
M_{S_4} = \frac{u_4 - u_3}{u_4}
$$
\hspace{1cm} \ldots \ldots (3.8)

and $U_4$ is the shock velocity relative to the tube,

$$
\frac{p_4}{p_3} = \frac{2y}{y+1} \frac{\gamma^2}{\gamma-1},
$$
\hspace{1cm} \ldots \ldots (3.9)

and

$$
\frac{u_4-u_3}{u_4} = \frac{2}{y+1} \left[ M_{S_4} - \frac{1}{Y_{s_4}} \right].
$$
\hspace{1cm} \ldots \ldots (3.10)

Note that in the regions 2 and 3 (between the shocks $M_{S_1}$ and $M_{S_4}$), $p_2 = p_3$ and $u_2 = u_3$.

From (3.6) and (3.9), with $\gamma = 1.4$, we have

$$
\frac{p_2}{p_3} = \begin{cases} \frac{7M_{S_1}^2 - 1}{7M_{S_4}^2 - 1} \frac{p_3}{p_4} \\ \frac{7M_{S_1}^2 - 1}{7M_{S_4}^2 - 1} \frac{p_4}{p_3} \end{cases}
$$
\hspace{1cm} \ldots \ldots (3.11)

and from (3.7) and (3.10), with $\gamma = 1.4$,

$$
\frac{a_4}{a_1} = \left( \frac{M_{S_1}^2 - 1}{M_{S_4}^2 - 1} \right) \left[ 1.2 M_4 - \left( \frac{M_{S_4}^2 - 1}{M_{S_1}^2 - 1} \right) \right].
$$
\hspace{1cm} \ldots \ldots (3.12)

From (3.12) it follows that

$$
M_{S_1} = \frac{1}{8} \left( B + \sqrt{B^2 + 4} \right)
$$
\hspace{1cm} \ldots \ldots (3.13)

where

$$
B = \frac{a_4}{a_4} \left[ 1.2 M_4 - \left( \frac{M_{S_4}^2 - 1}{M_{S_1}^2 - 1} \right) \right].
$$
\hspace{1cm} \ldots \ldots (3.14)

Now

$$
a_4 = \frac{T_4}{288}
$$
\hspace{1cm} \ldots \ldots (3.15)

and
It follows that
\[
\frac{\rho_s}{\rho_t} = \frac{\frac{\rho_s}{\rho_t}}{\frac{\rho_s}{\rho_t}} = \left(\frac{\frac{\rho_s}{\rho_t}}{\frac{\rho_s}{\rho_t}}\right) - \left(\frac{1 + 0.2 M_s^2}{1 + 0.2 M_s^2}\right)^{3.5}
\]

The variation of \(N_{s1}\) and \(\rho_s/\rho_t\) may now be determined for any combination of the working section variables \(M_4\) and \(T_4\). Given \(M_4\) and \(T_4\), \(B\) is calculated from (3.14) and hence \(N_{s1}\) from (3.13) for a range of values of \(M_{s4}\). Then from Figure 3 or Figure 4 the value of \(M_{s4}\) which corresponds to the given values of \(M_4\) and \(T_4\) is read off and furthermore, the variation of \(N_{s4}\) with \(M_{s4}\) is known from equation (3.4). Hence \(\rho_s/\rho_t\) is calculable from (3.16) for the given \(M_4\) and \(T_4\) and the corresponding \(M_{s1}\), \(M_{s4}\) and \(N_{s4}\) values. Calculations were made for a series of hypersonic flow Mach numbers \(M_4\) up to 30 with free stream static temperatures \(T_4\) between 500 K and 550 K, and the results are presented as Figures 5 to 9 inclusive.

3.2 Discussion of the solution

The starting time \(t_s\), which is the time taken for the upstream-facing secondary shock wave \(M_{s4}\) to travel through the divergent portion of the nozzle, will be a minimum when the shock \(M_{s4}\) becomes a sound wave, \(M_{s4} = 1\), and is swept through the nozzle with a variable velocity \((u - a)\), where \(u\) and \(a\) are local values. When \(\rho_s = \rho_t\), shock \(M_{s4}\) will have a finite strength; however if \(\rho_s \neq \rho_t\), it is possible to choose the value of \(\rho_s/\rho_t\) for any given working section conditions \(M_4\) and \(T_4\) in order to make \(N_{s4} = 1\). Figure 7 illustrates how \(\rho_s/\rho_t\) varies with \(M_4\) and \(T_4\) for the special case \(M_{s4} = 1\); these curves for \(M_{s4} = 1\) are defined as 'perfect' start curves, since they correspond to the minimum starting time. If \(\rho_s/\rho_t\) is reduced below the value required for perfect starting \(M_{s4}\) becomes greater than unity; specimen calculations for the case \(M_4 = 15\) and variable \(T_4\) are presented in Figure 8. When \(M_{s4} = M_4\), the shock becomes stationary with respect to the walls of the shock tube - see equation (3.6) - and if \(\rho_s/\rho_t\) is still further reduced a stationary shock will occur in the divergent portion of the nozzle.

If \(\rho_s/\rho_t\) is greater than the value required for perfect starting, an expansion wave will occur instead of the shock wave \(M_{s4}\). This feature does not improve the starting process since the head of an expansion wave travels at sonic speed in any case. Figures 5 and 6 give the corresponding values of the transmitted shock Mach number \(M_{s4}\) for the \(\rho_s/\rho_t\) results given in Figures 7 and 8. It will be noted from Figure 9 that the transmitted shock \(M_{s4}\) travels faster than the corresponding incident shock \(M_{s4}\) for perfect starting approximately according to the formula, \(M_{s4} = 1.54 M_{s8}\) for all \(M_4\) and \(T_4\) values considered.

Figures/
Figures 7 and 8 show that the pressure ratio across the nozzle diaphragm $p_s/p_a$ is not affected significantly by changes of $T_4$; it follows from Figure 2 that, for given values of $A_a/A_4$ and $p_s/p_a$, $M_{4a}$ (like $M_4$) is approximately independent of $M_{as}$.

3.3 Evaluation of the testing time losses

3.3.1. Perfect starting

It has been noted that the shock $M_{4a} (= 1)$ travels through the nozzle with a speed $(u - a)$ where $u$ and $a$ are local values. Then, approximately, if $N$ is the nozzle length,

$$r_s = \frac{N}{(u-a)_{\text{mean}}} = \frac{N}{\dot{a}(\dot{M}-1)} \quad \ldots (3.17)$$

Further, any disturbance due to the arrival of the primary contact surface $C_e$ at the nozzle will travel through the nozzle with velocity $(u + a)$ where $u$ and $a$ are local values. Then, approximately

$$r_e = \frac{N}{(u+a)_{\text{mean}}} = \frac{N}{\dot{a}(\dot{M}+1)} \quad \ldots (3.18)$$

From equation (2.1), the net total loss in testing time $r_L = (r_s - r_e)$.

Hence

$$r_L = \frac{2N}{\dot{a}} \left( \frac{1}{\dot{M}^2-1} \right) \quad \ldots (3.19)$$

From specimen calculations for any 6 straight walled nozzles it is reasonable to put

$$\dot{a} = \frac{5}{4} a_4 \quad \text{and} \quad \frac{3}{4} M_4$$

where $a_4$ and $M_4$ describe conditions in the working section.

It follows that

$$r_L = \frac{648 N}{(9M_4^2-16)\sqrt{T_4}} \quad \ldots (3.20)$$

or, if $M_4 >> 1$

$$r_L = \frac{72 N}{M_4^3\sqrt{T_4}} \quad \ldots (3.21)$$

where $N$ is in feet and $T_4$ in $^\circ$K.

3.3.2. Non-perfect starting

By a similar analysis to the above section, and the use of the assumption that

$$r_s = \frac{N}{\dot{a}(\dot{M}-M_4)} = \frac{N}{\dot{a}(3M_4-4M_4)}$$

where $M_4$ and $M_4$ describe conditions in the working section.
It follows that

\[ \tau_L = \frac{324N}{\sqrt{T_4}} \left[ \frac{1 + M_{S_4}}{(M_4-M_{S_4})(M_4+4)} \right] \text{milliseconds} \quad \ldots(3.22) \]

Note that this is probably an overestimate of \( \tau_L \) since \( M_{S_4} = \frac{3}{4} M_4 \) gives \( \tau_L = \infty \) whereas \( M_{S_4} = M_4 \) should correspond to \( \tau_L = \infty \). If \( M_4 \gg 1 \), then \((3.22)\) becomes

\[ \tau_L = \frac{108N}{M_4\sqrt{T_4}} \left[ \frac{1 + M_{S_4}}{3M_4-M_{S_4}} \right] \text{milliseconds} \quad \ldots(3.23) \]

### 3.3.3. Nozzle length

For given entry conditions the length \( N \) of a nozzle designed to produce uniform flow at the exit increases as the exit flow Mach number is raised.

An empirical relation, based on an extrapolation of results obtained at N.P.L., is \((M_4 \gg 1)\)

\[ \frac{N}{A_4} = \frac{3M_4}{10} \quad \ldots(3.24) \]

where for a two-dimensional nozzle \( A_4 \) is the height of the working section.

Hence, for hypersonic Mach numbers, \((3.21)\) and \((3.23)\) become

\[ \tau_L = \frac{64.8}{M_4\sqrt{T_4}} \cdot A_4 \text{ milliseconds} \quad \ldots(3.25) \]

for a 'perfect start' and

\[ \tau_L = \frac{97.2}{\sqrt{T_4}} \left[ \frac{1 + M_{S_4}}{3M_4-M_{S_4}} \right] A_4 \text{ milliseconds} \quad \ldots(3.26) \]

for a non-perfect start.

Since the slope of the characteristics at hypersonic speeds is very small, the model would normally be placed approximately halfway along the nozzle. Thus \((3.25)\) and \((3.26)\) probably overestimate \( \tau_L \) by a factor of about 2 and more realistic estimates of the net total loss in testing time \( \tau_L \) per foot of working section height would be

(Perfect start) \( \tau_L = \frac{32.4}{M_4\sqrt{T_4}} \cdot A_4 \text{ milliseconds} \quad \ldots(3.27) \)

(Non-perfect start) \( \tau_L = \frac{48.6}{\sqrt{T_4}} \left[ \frac{1 + M_{S_4}}{3M_4-M_{S_4}} \right] A_4 \text{ milliseconds} \quad \ldots(3.28) \)

where \( A_4 \) is in feet and \( T_4 \) in °K.

Calculations/
Calculations made from equations (3.27) and (3.28) are presented as Figures 10 and 11; the former gives the net total loss of testing time per foot of working section height for perfect start conditions at various Mach numbers \( M_4 \) and temperatures \( T_4 \), whilst the latter illustrates the losses due to imperfect starting for the special case \( M_4 = 15 \) with variable \( T_4 \). For a fixed \( A_3 \) and a variable \( A_2 \), the testing time losses are greatest for low \( T_4 \) and low \( M_4 \).

4. Simulation of Free Flight Conditions in Hypersonic Shock Tube

In the following sections the factors which affect the simulation in a shock tube of hypersonic flight at extreme altitudes are discussed.

4.1 The Reynolds number

The Reynolds number based on the length \( L_m \) of the model is

\[ R_m = \frac{\rho_4 U_4 L_m}{\mu_4}. \]

If \( T_4 \) is in °K and \( \mu \propto T \), then if \( T_0 = 288°K \),

\[ \rho_4 M_4 L_m \frac{1120}{(T_4/288)^{3/2}} \]

\[ R_m = \frac{3.72 \times 10^{-7}}{(T_4/288)^{3/2}}. \]

But, if \( \gamma = 1.4 \), equation (3.1) yields

\[ \rho_4 = \rho_\infty \left( \frac{T_4}{T_0} \right)^{3.5} = 0.00238 \frac{\rho_\infty}{\rho_0} \left( \frac{T_4}{288} \right)^{3.5} \]

where \( P = \rho_\infty \) (measured in atmospheres)

and \( \rho_\infty = 0.00238 \frac{P_\infty}{P_0} \) slugs/cu ft.

Now \( \frac{P_\infty}{\rho_\infty} = \frac{T_0}{\rho_0} \) and using (3.1) we get

\[ R_m = \frac{7.15 \times 10^6}{(1+0.2h^2)^2} \cdot P_\infty M_4 L_m \cdot G \]

where

\[ G = \left( 1 + 0.2 \frac{M_4^2}{M_4} \right)^2 \left( \frac{T_4}{T_0} \right) \]

and \( L_m \) is measured in foot.

Clearly \( G \) is a function of the primary shock Mach number \( M_{58} \) only and specimen values are given below.

<table>
<thead>
<tr>
<th>( M_{58} )</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
<th>14</th>
<th>16</th>
<th>18</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>( G )</td>
<td>0.752</td>
<td>0.272</td>
<td>0.115</td>
<td>0.055</td>
<td>0.030</td>
<td>0.018</td>
<td>0.017</td>
<td>0.0078</td>
<td>0.0056</td>
<td>0.0042</td>
</tr>
<tr>
<td>( P_\infty )</td>
<td>3.38</td>
<td>5.02</td>
<td>4.82</td>
<td>4.10</td>
<td>3.50</td>
<td>3.06</td>
<td>2.68</td>
<td>2.32</td>
<td>2.11</td>
<td>1.95</td>
</tr>
</tbody>
</table>

Let/
Let us assume that the maximum pressure in the channel \( p_s = P \) atmospheres, independent of \( M_{\infty} \), and governed only by the tube construction. Then for a given \( M_4 \), the Reynolds number \( R' \) is proportional to \( G \) and the value of \( p_s \) is so chosen that \( \frac{p_s}{p_e} = P \) atmospheres for each \( M_{\infty} \).

Hence for maximum Reynolds numbers the tube must be operated so that \( GP \) is a maximum; that is, the lowest possible shock Mach number \( M_{\infty} \) must be used. Now suppose that the actual \( M_{\infty} \) achieved in an experiment is less than the value expected; then \( R' \) is proportional to \( \frac{p_s}{p_e} \). Table 2 shows that \( G \frac{p_2}{p_e} \) decreases slowly as \( M_{\infty} \) increases; hence if \( M_{\infty} \) is less than expected, the Reynolds number will be slightly greater than expected.

This important result - that the lowest values of \( M_{\infty} \) give the highest Reynolds numbers - is subject to the limitation that the static temperature \( T_4 \) in the working section must not be allowed to become less than the temperature at which air liquefies. This critical temperature, a function of the pressure \( p_4 \) in the working section, is given by the following approximate relation obtained by extrapolation of the data given in Ref. 3c, P.406.

\[
\log_{10} \frac{p_4}{p_e} = \log_{10} \frac{P}{p_e} - \frac{360}{T_4} \quad \ldots (1.5)
\]

where \( T_4 \) is measured in °K and \( P \) in atmospheres.

Now

\[
\frac{p_4}{p_e} = \left( \frac{1 + 0.2 M_{\infty}^2}{1 + 0.224} \right)^{2.5}
\]

and since for any

\[ M_{\infty} > 4 \] we have \( 1.48 < 1 + 0.2 M_{\infty}^2 < 1.71 \), the ratio \( \frac{p_4}{p_e} \) is approximately a function of \( M_4 \) only.

Thus, to avoid liquefaction,

\[
T_4 > \frac{360}{4.4 + 3.5 \log_{10} \left( \frac{1 + 0.2 M_{\infty}^2}{1 + 0.224} \right)} \log_{10} P \quad \ldots (4.4)
\]

Table 3 gives values of the liquefaction temperature calculated from equation \((4.4)\) for the two extreme cases of \( P = 1 \) and \( P = 500 \) atmospheres. (The factor \( (1 + 0.2 M_{\infty}^2) \) in equation \((4.4)\) is replaced by \( 1.71 \), its approximate value for all large values of \( M_{\infty} \)).

<table>
<thead>
<tr>
<th>( M_{\infty} )</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_4 ) °K ( (P = 500 \text{ atm}) )</td>
<td>65</td>
<td>89</td>
<td>102</td>
<td>115</td>
<td>128</td>
</tr>
<tr>
<td>( T_4 ) °K ( (P = 1 \text{ atm}) )</td>
<td>44</td>
<td>40</td>
<td>36</td>
<td>33</td>
<td>31</td>
</tr>
</tbody>
</table>

Hence, to avoid liquefaction, \( T_4 \) must not fall below about 50°K for any hypersonic Mach number \( M_4 \).
4.2 The equivalent altitude

The altitude at which a full-scale aircraft must fly if it is to have the same Reynolds number as a scale model in a hypersonic shock tube is now determined. The temperature in the stratosphere between 40,000 and 100,000 feet is usually assumed constant and equal to 216.5°K. At extreme altitudes the temperature is greater than 216.5°K; but this increase appears to be small, particularly during winter and during night time in summer. Hence it is assumed that \( T = 216.5°K \) for all altitudes greater than 40,000 feet. The density of the atmosphere then decreases exponentially with the altitude \( (h \text{ feet}) \) according to the approximate relation

\[
\rho = 0.00406 \times 10^{-5} \text{slug/ft}^3. \quad (4.5)
\]

Now if the full-scale aircraft is \( n \) times as large as the model it follows that, for equal Reynolds numbers

\[
\left( \frac{\mu U}{\mu} \right)_{\text{shock tube}} = n \left( \frac{\mu U}{\mu} \right)_{n, \text{flight}} \quad (4.6)
\]

If equal model and full-scale Mach numbers are assumed it can be shown, by an analysis similar to that leading to equation (4.1), that

\[
h + 4.8 \times 10^4 \log_e \frac{n}{P} = 4.8 \times 10^4 \log_e \frac{1.6 \times 10^6 (1 + 0.2M_e^2)}{G} \quad (4.7)
\]

where \( P = P_e \) (atmospheres) - shock tube and the variation of \( G \) with \( M_e \) is given by Table 2. An alternative form for (4.7) is

\[
h + 2.1 \times 10^4 \log_e \frac{n}{P} = 2.1 \times 10^4 \log_e \frac{1.6 \times 10^6 (T_e)}{T_e^{a+b}} \quad (4.8)
\]

Equations (4.7) and (4.8) show that the equivalent altitude \( h \) can be reduced by making \( P \) large or \( n \) small. Nevertheless, it would be necessary to use impractically large pressures and large models in the shock tube in order to simulate a large aircraft moving (say) at a Mach number of 20 at a relatively low altitude - say 50,000 feet. Fortunately such aircraft are likely to operate only at extreme altitudes. It may be useful to have an expression for the full-scale Reynolds number \( R \), at Mach number \( M \), under the atmospheric conditions assumed in this section. Then \( R \) is given by

\[
R = \frac{1.31 \times 10^7}{M L \sqrt{\frac{0.00406 \times 10^{-5} h}{\rho}}} \quad (4.9)
\]

where \( L \) is the length of the full-scale aircraft in feet.

4.3 Slip-flow conditions

At large equivalent altitudes and very low air densities the mean free path \( \delta \) of the air molecules becomes large. If the ratio of \( \delta \) to the length of the body becomes too great the air can no longer be considered as a continuum and slip flow occurs. It is of interest to examine the conditions which must be achieved in the model experiment if the onset of slip flow under full-scale conditions is to be reproduced.
Consider the equivalent altitude, $H$ feet, at which
\[
\frac{\text{Mean free path}}{\text{length of model, shock tube}} = \frac{\text{mean free path}}{\text{length of aircraft}, H, \text{flight}}
\]
\[.....(4.10)\]

The mean free path is inversely proportional to the density and is independent of temperature. Thus, for an aircraft $n$ times as large as the shock tube model, similarity is achieved if $n$ is given by
\[
\frac{\rho_e, \text{shock tube}}{\rho_H, \text{flight}} = n
\]

By an analogous analysis to that leading to equation (4.8) it follows that
\[
H + 2.1 \times 10^4 \log_e \frac{\rho}{\rho_0} = \frac{2.35 \times 10^6 (T_0^{3.5})}{T_0^{4.5}}
\]
\[.....(4.11)\]

If equations (4.8) and (4.11) are considered, it is noted that the right hand sides are functions of $M_\infty$ and $T_4$ only (see Table 1). Calculated values of $H + 2.1 \times 10^4 \log_e \frac{\rho}{\rho_0}$ for ranges of $M_\infty$ and $T_4$ are presented in Figure 12.

The simple kinetic theory of gases\(^4\) shows that the Knudsen number
\[
\frac{\rho_e}{M} = 1.5
\]
\[.....(4.12)\]

and hence for Reynolds number and Mach number similarity between full scale and shock tube, $\epsilon/L$ in flight at height $h$ should be the same as in the shock tube. It follows that on this basis $h$ and $H$ should be identical. The reason for the small discrepancy in Fig. 12 between $h$ and $H$ is that $h$ has been calculated by assuming viscosity proportional to absolute temperature. This is roughly correct, but is at variance with simple kinetic theory which predicts too small a variation of viscosity with temperature. However, these small differences between $h$ and $H$ may be ignored.

Following Tsien\(^4\), it is assumed that if slip flow is not to occur
\[
\frac{\epsilon}{\delta^*} < 0.01
\]
\[.....(4.13)\]

where $\epsilon$ is the mean free path and $\delta^*$ is the displacement thickness of the boundary layer. Since aircraft flying at very high Mach numbers are likely to be travelling at extreme altitudes, the Reynolds numbers will be low and the boundary layers laminar. If there is zero heat transfer between the free-stream air and the body (that is, if there is no radiation and the body has become heated to its equilibrium temperature) $\delta^*$ will be of the same order as the theoretical result\(^5\) for a flat plate with zero heat transfer.

Hence
\[
\frac{\delta^*}{X} = \frac{1.72 (1 + 0.277 X^2)}{X} \sqrt{\frac{X}{X}}
\]

which becomes, when $X >> 1$
\[
\delta^* \approx \frac{X^2}{2 \sqrt{\frac{X}{X}}}
\]
\[.....(4.14)\]

where/
where $R_x$ is based on $x$, the distance from the leading edge to the boundary-layer station under consideration. For an average value of the boundary-layer thickness along the body it is convenient to take the value at $x = \frac{L}{4}$; this is given by

$$
\delta^* = \frac{M^2 L}{4\sqrt{R}} \quad \ldots \ldots (4.15)
$$

In (4.13) it is not appropriate to use $\xi$ as given by (4.12), since (4.12) refers to the mean free path outside the boundary layer. Within the boundary layer the air is heated and consequently the density decreases and the mean free path increases. Hence the maximum value, $\xi_{\text{max}}$, of $\xi$ from (4.13) should be used. This is given by

$$
\xi_{\text{max}} = \frac{\xi_{\text{max}}}{\xi_{\infty}} = 1 + 0.2 M^2 \leq 1.2 M^2
$$

where $\xi_{\infty}$ and $T_{\infty}$ are the mean free path and temperature in the free stream. Hence (4.12), (4.13) and (4.15) require that, if slip flow is to be avoided,

$$
R > 10^4 M^2 \quad \ldots \ldots (4.16)
$$

It is interesting to note that (4.16) is also obtained if it is assumed that the wall temperature is much less than the zero heat transfer value. Thus if the wall temperature were equal to the free stream temperature (which would require drastic cooling) the boundary layer thickness would be reduced to about 0.25 of the unheated value, but so would the maximum temperature; hence $\xi_{\text{max}}$ would be reduced in the same proportion as $\delta^*$, and the net result would be to leave $\xi_{\text{max}}/\delta^*$ unchanged.

By substituting in (4.9) from (4.16) it can be shown that slip flow will be avoided with the full scale aircraft if

$$
h < 2.1 \times 10^6 \log_e \left( 1.31 \times 10^3 \frac{L}{M} \right) \quad \ldots \ldots (4.17)
$$

where $h$, the altitude, and $L$, the aircraft length, are measured in feet. In the shock tube, for a model of length $L_m$ feet, the corresponding condition is, from (4.1) and (4.16),

$$
\frac{715 \text{ GPL}_{L_m}}{M_4 (1 + 0.2 M^2)^3} > 1. \quad \ldots \ldots (4.18)
$$

As an example, suppose an aircraft of length 50 feet is travelling at $M = 15$. Slip flow will occur if it flies above 175,000 feet. In the shock tube, for a model of length 1 foot and $P = 500$ atmospheres, we find that $G$ must be > 0.068 (which corresponds to $T_{\text{wall}} < 7$ and $L_4 < 100^9K$ approximately) to avoid slip flow.
4.4 Simulation of full-scale temperature conditions

In Sections 4.1 and 4.2 the problem of attaining full-scale Reynolds numbers was discussed. However, even if the Mach number and Reynolds number in the shock tube have their full-scale values, it does not necessarily follow that similarity of flow will be achieved. It is necessary also that the ratio of the body surface temperature $T_s$ to the free-stream temperature $T_\infty$ should be the same for the model as for the full-scale aircraft. For the aircraft, except near the nose, $T_s/T_\infty$ is likely to be much less than the zero heat transfer value, which is approximately 0.2. The extent to which $T_s/T_\infty$ falls below this value will depend on the time for which the aircraft has been flying and its heat capacity, on radiation conditions, etc. However in all conditions $T_s/T_\infty$ is likely to be large, although much less than 0.2. In the shock tube, if the heat capacity and thermal conductivity of the model are both fairly large, $T_s$ is likely to remain quite near to its value before the flow starts, except near the nose, because the flow duration is so short. Normally the initial value of $T_s$ will be room temperature, so to get realistically large values of $T_s/T_\infty$ it is necessary to use low free-stream temperatures $T_\infty$ (or $T_a$ in our usual notation). In fact it is probably necessary to use the lowest possible value of $T_a$, roughly 50°K. Fortunately this will also give the highest possible Reynolds number. However it is desirable to investigate in the shock tube the effects of varying $T_s/T_\infty$ ($= T_s/T_a$); this can be done by preheating (or less conveniently by pre-cooling) the model or by the use of various free-stream temperatures $T_\infty$. Thus for certain experiments, it will probably be desirable to work at values of $T_a$ much higher than the minimum*. This will of course entail a considerable sacrifice of Reynolds number.

There is a further barrier to the attainment of complete flow similarity between the shock tube and full scale. Even if the temperature ratio $T_s/T_\infty$ is the same as in flight the flow will not be similar if the absolute temperatures in the full-scale condition are much greater than in the shock tube; particularly if they are so great that serious dissociation and ionisation effects occur. Such effects certainly will occur in flight at sufficiently high Mach numbers, and under these conditions complete simulation of the full-scale flow by shock tube testing is virtually impossible. However, a similar limitation has always attended the testing of models at relatively low speeds in wind tunnels. It is pertinent to point out that studies of ionisation and dissociation effects in real gas flows may be made in shock tubes and hence a further type of flow simulation is possible. In this latter case the simulation of the radiation from a heated aircraft flying at hypersonic speeds would be a very difficult problem.

5. Conclusions and Recommendations

Performance charts for the operation of hypersonic shock tubes have been developed from an evaluation of the simulation of free-flight conditions, and an analysis of the factors affecting the starting process of the shock tunnel. The use of a diaphragm at the entrance to the nozzle is shown to be imperative in order to minimise the loss of testing time due to tunnel starting; this diaphragm would normally have a pressure ratio of at least $10^4$ across it. This implies that the working

*Values greater than room temperature are, however, unlikely to be wanted since these would make $T_s/T_\infty < 1$ if $T_s$ is equal to room temperature, and such small ratios would only be of academic interest.
section must be evacuable to an initial pressure of 1 to 0.1 microns of mercury if Reynolds number simulation of flight conditions is to be obtained in the working section during the hypersonic flow test period.

A complete calculation of the design performance of the N.P.L. hypersonic shock tube installation is deferred until a later paper; however, specimen results indicate that the testing time losses should not normally exceed 25% of the theoretical testing time under normal working conditions.

From the present calculations it is clear that, to obtain Mach number and Reynolds number simulation of free flight conditions, the shock tube must be operated at low incident shock Mach numbers $M_{\infty}$; this implies that the free-stream temperature in the working section will be less than atmospheric (i.e., the model will be hotter than the free stream as under full-scale conditions) but the stagnation temperature of the flow will be less than that of free flight. Alternatively, the correct stagnation temperature and Reynolds number conditions may be obtained in a shock tube without a nozzle; this implies that $M_{\infty}$ should be very large, with a consequent reduction of theoretical testing time and an elimination of the nozzle starting losses.

Thus hypersonic shock tubes should have two working sections; one in the constant area channel for temperature simulation and one following a divergent nozzle for Mach number simulation.

It is possible to obtain simultaneous Mach number and temperature simulation in the expanded hypersonic flow but the static temperature of the flow will be greater than the temperature of the model—an unreal condition—and furthermore it is very difficult to avoid slip flow for this case.

Thus it appears likely that the simulation of flight at hypersonic Mach numbers and extreme altitudes will be studied in the following stages in shock tube installations:

1. Reynolds number and Mach number simulation with variable $T_e/T_{\text{model}}$ in a divergent nozzle working section.

and (2) Reynolds number and stagnation temperature simulation (with the associated real gas effects) in a constant area working section in the channel of the shock tube.

APPENDIX/
APPENDIX

Notation

a velocity of sound
A cross-sectional area of shock tube
G a function of shock Mach number - see eqn. (4,2)
h altitude - see eqn. (4,8)
H altitude - see eqn. (4,11)
ℓ mean free path of a molecule
L characteristic length
M Mach number

\[ M_L = \text{flow Mach number with respect to the local speed of sound (e.g. } \frac{M_3}{a_3} \) \]

\[ M_{S_1} = \text{shock wave propagation Mach number with respect to the speed of sound in the flow ahead of the shock front (e.g. } \frac{M_{S_1}}{a_1} \) \]

n scale ratio between model and full size aircraft
N length of nozzle section of hypersonic shock tube
p absolute pressure

\[ P = \frac{p_0}{\rho_0} \text{, see Figure 1) absolute pressure, measured in atmospheres} \]

\[ R_x = \text{Reynolds number based on length } x \]

T absolute temperature
u flow velocity

\[ U = \text{shock velocity; subscript corresponds to flow region ahead of the shock} \]

x distance

\[ γ = \frac{C_p}{C_v} \text{, the ratio of the specific heats} \]
ρ absolute density

\[ μ = \text{coefficient of viscosity} \]

\[ τ = \text{theoretical testing time} \]

\[ τ_R = \text{run time} \]

\[ τ_L = \text{net loss of testing time due to starting process of hypersonic shock tube} \]

δ* displacement thickness of boundary layer

Subscripts/
Subscripts

- $o$ stagnation conditions
- $1, 2, 3, 4 \ldots$ etc identify quantities related to gas in corresponding region of shock tube flow diagram (Figure 1)
- $\infty$ free stream conditions in full-scale flight
- $m$ model
- $s$ surface conditions

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**References**

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The flow in the channel of a hypersonic shock tube
FIG 2.

Legend

Symbol | $T_A^oK$
---|---
+ | 50
Δ | 150
□ | 250
• | 350
▽ | 450
○ | 550

$T_1 = 288^oK$
$\gamma = 1.4$

Mach number $M_4$ of hypersonic flow

Hypersonic shock tube:

Nozzle area ratio required to produce hypersonic Mach number flows.
Fig. 3.

$T_b = T_i = 288^\circ K$

$\gamma = 1.4$

Hypersonic shock tube:
Variation of working section conditions with incident shock
Mach number $M_{s8}$
Static temperature of hypersonic flow \(T_4\) °K

Hypersonic shock tube:
Variation of working section conditions with incident shock Mach number \(M_{sb}\)

\[T_{b} - T_{1} = 288^\circ K\]
\[\gamma = 1.4\]
Hypersonic shock tube:
Nozzle starting process:
Transmitted shock Mach number $M_{s1}$
'Perfect start' with $M_{s1} = 1.0$
Fig. 6.

Hyper sonic shock tube:
Nozzle starting process; imperfect start: M₄ = 15.
Transmitted shock Mach number Mₛ₁.

M₄ = 12
Mₛ₄ = 8
Mₛ₄ = 4
Mₛ₄ = 2
Mₛ₄ = 1

Stationary shock limit Mₛ₄ = 15 = M₄

Transmitted shock Mach number Mₛ₁

Tₛ₄ = T₄ = 288 K

γ = 1.4

T₄ = 500 K
Asymptotic values for large $T_4$

- $M_4 = 5$
- $M_4 = 7$
- $M_4 = 10$
- $M_4 = 15$
- $M_4 = 20$
- $M_4 = 25$
- $M_4 = 30$

Pressure ratio across nozzle diaphragm

Perfect starting $M_4 = 1$

Hi4 = 7

Limiting $P_4$

FIG. 7

$T = T = 288^\circ K$

1. Static temperature of hypersonic flow
2. Static pressure ratio $P_4/P_0$
FIG. 8.

Region where expansion wave is present instead of secondary shock $M_{s4}$

Perfect start $M_{s4} = 1.0$

$M_{s4} = 2$

$M_{s4} = 4$

$M_{s4} = 8$

$M_{s4} = 12$

Stationary shock limit $M_{s4} = M_4 = 15$

Region where standing shock is present in divergent portion of nozzle.

Hypersonic shock tube:
Nozzle starting process: pressure ratio $P_0 / P_1$
Imperfect starting $M_4 = 15$
Fig. 9.

Hypersonic shock tube:
Nozzle starting process. Incident and transmitted shock Mach numbers
Perfect starting \( M_s = 1 \).
FIG. 10.

Hypersonic shock tube:
Loss of testing time due to starting process
Perfect start $M_s = 1.0$
(Two-dimensional nozzle).

$T_1 = 288\degree K$
$\gamma = 1.4$

Nett total loss of testing time per foot of working section height $T_2/A_4$ milliseonds/foot

Static temperature of hypersonic flow $T_4\degree K$

$M_4 = 5$
$M_4 = 10$
$M_4 = 15$
$M_4 = 20$
$M_4 = 30$
Hypersonic shock tube

Loss of testing time due to starting process. Imperfect start $M_5 = 16$
FIG 12

\[ \frac{\dot{h}}{2.1 \times 10^4 \log_e \frac{p}{n}} \quad (Eqn \ 4.8) \]

\[ H + 2.1 \times 10^4 \log_e \frac{p}{n} \quad (Eqn \ 4.11) \]

M₈₈ values

Altitude in thousands of feet

Static temperature of supersonic flow T₄ °K

Hypersonic shock tube: Reynolds number chart