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By

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Some Notes on the Flow Durations occurring in Hypersonic Shock Tubes

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11th August, 1955

1. Introduction and Summary

Interest in hypersonic shock tubes is increasing both in the United States of America [1,2,3] and the United Kingdom [4,5]; principally because the shock tube offers the only simple method whereby very high Mach number flows - with stagnation temperatures approximating to those of full-scale flight - may be generated. The present paper gives a résumé of the aerodynamic principles involved in the production of very strong shock waves and very high Mach number flows; subsequently we consider the effects of the deviations from perfect gas theory which occur when strong shock waves are generated. Finally, some calculations of the flow durations occurring in hypersonic shock tubes are presented in tabular and graphical form. It is found that the influence of the gaseous imperfections is such that the flow durations are reduced; this reduction is of the order of 50% for very strong shock conditions.

2. Hypersonic Shock Tube Flow

2.1 The Production of Very Strong Shock Waves

Fig.1 is a diagram of the flow patterns which occur in a simple conventional shock tube and it illustrates the notation used below and listed in the Appendix. The ultimate attainable pressure ratio, $P = \hat{p}_o/p_o$, across the diaphragm is often governed by considerations of chamber strength and the sensitivity of the optical system used in the working section and will not normally exceed 10^4; but, regardless of the precise value of P, the shock Mach number V_+ and shock strength p_s rapidly approach limiting values as $P \to \infty$. On the basis of simple theory (see, for example, Ref.6) these maximum values as $P \to \infty$ are given by

$$p_s = \frac{2\gamma}{\gamma+1} V_+^2 - \frac{\gamma-1}{\gamma+1} \qquad \text{.......(2.1)}$$

and $$V_+ = \left(\frac{\gamma+1}{\hat{\gamma}-1}\right) \frac{\hat{a}_o}{a_o} \qquad \text{.......(2.2)}$$

where the superscript ^ refers to quantities related to the gas initially at the higher pressure in the shock tube. Thus, if air at room temperature $(\gamma = 1.4)$ is used on both sides of the diaphragm,

$$p_s \to 42 \text{ and } V_+ \to 6 \text{ as } P \to \infty .$$

Clearly, for large V_+, $\hat{a}_o$ must be as large as possible; hence the gas in the chamber must have a low molecular weight and as high a temperature as practicable. It is dangerous to heat pure hydrogen and the requirements of high temperature, high pressure and low molecular weight are best achieved by

(i)/

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(i) heating helium

or (ii) exploding a combustible mixture of oxygen and hydrogen

in the chamber. In either case, the molecular weight of the gas in the chamber may be varied by the addition of nitrogen; hence, although the tube operates at a fixed diaphragm pressure ratio P - thus enabling a standard procedure to be followed - the shock Mach number V_+ is controllable.

With the above techniques, it is possible[2] to operate the constant-volume⁺ combustion type shock tube with $\hat{a}_0/a_0 \ngtr 6{\cdot}6$; this corresponds to $V_+ = 16$ when $P = 10^4$. If the diaphragm is ruptured at the instant the mixture in the chamber is ignited - the so-called 'constant-pressure' process in which the shock pressure ratio p_s is equal to the diaphragm pressure ratio P - results[2] are obtained which indicate that $\hat{a}_0$ becomes very large and $\hat{a}_0/a_0 \to \infty$. In this manner it is feasible to generate, in air, shock Mach numbers $V_+ \doteq 25$ and shock strengths $p_s \doteq 700$ with practicable pressure ratios across the diaphragm. The thermodynamics and aerodynamics of combustion shock tubes will be discussed in a later paper.

Another method[7,8] whereby the shock Mach number V_+ may be increased is known as the double-diaphragm step-type shock tube and is illustrated in Fig.2. If the area ratio $\hat{A}/A$ is very large or the diaphragm D_2 does not shatter under the impact, shock S_1 undergoes normal reflection at diaphragm D_2 and leaves the gas there at rest at an increased temperature and pressure. Subsequently when diaphragm D_2 is ruptured, either mechanically ($\hat{A} = A$) or automatically by the impact of shock S_1 ($\hat{A} > A$), the ensuing flow produces a shock S_3 whose Mach number is greater than that of shock S_1. We have noted that when the diaphragm pressure ratio P is very large, a large increase in P does not alter the attainable shock Mach number V_+ appreciably. However, equation (2.2) shows that V_+ is very sensitive to changes in the speed of sound ratio across the diaphragm $\hat{a}_0/a_0$ when P is large.

For a given diaphragm pressure ratio P and a given initial temperature ratio $\hat{T}_0/T_0$ the double-diaphragm technique essentially sacrifices pressure ratio in favour of increased temperature ratio. The reservoir behind diaphragm D_2 has a pressure $\hat{p}_0'$ ($< \hat{p}_0$) and a temperature $\hat{T}_0'$ ($> \hat{T}_0$); the increase of temperature has a much greater effect than the decrease of pressure and hence the shock S_3 is stronger than the shock S_1.

If $\hat{A} = A$ and the shock S_1 shatters the diaphragm D_2 on impact, a simple rarefaction wave only will arise. The full gain obtainable from the double-diaphragm method will not then be realised.

Experiments are proceeding[1] in the United States of America to develop a diaphragmless shock tube where strong shock waves are produced by the direct discharge of high voltage electrical energy through the gas in the chamber.

2.2 The Production of very high Mach Number Flows

In the above section we have considered the generation of very strong shock waves; but the Mach number M of the flow behind a shock does not increase without limit as the shock Mach number V_+ is increased. From simple shock tube theory[5], the relevant equations connecting V_+ and M are:-

$\frac{u}{a_0}$ /

⁺The diaphragm is ruptured when the combustion of the gases in the chamber is complete.

$$\frac{u}{a_0} = \frac{2}{\gamma+1}\left[V_+ - \frac{1}{V_+}\right] \qquad \ldots\ldots\ (2.3)$$

$$\frac{a}{a_0} = \frac{2}{(\gamma+1)\,V_+}\left[\left(\gamma V_+^2 - \frac{\gamma-1}{2}\right)\left(\frac{\gamma-1}{2}V_+^2 + 1\right)\right]^{\frac{1}{2}} \qquad \ldots\ldots\ (2.4)$$

and $$M = \frac{u}{a_0}\cdot\frac{a_0}{a} = \left[V_+^2 - 1\right]\left\{\left(\gamma V_+^2 - \frac{\gamma-1}{2}\right)\left(\frac{\gamma-1}{2}V_+^2 + 1\right)\right\}^{-\frac{1}{2}} \qquad \ldots\ (2.5)$$

Hence as the shock strength is increased and V_+ becomes $\gg 1$ we have

$$\frac{u}{a_0} \rightarrow \frac{2}{\gamma+1}V_+ = 0.833\,V_+ \text{ for air} \qquad \ldots\ldots\ (2.6)$$

$$\frac{a}{a_0} \rightarrow \left[\frac{2\gamma(\gamma-1)}{(\gamma+1)^2}\right]^{\frac{1}{2}} V_+ = 0.44\,V_+ \text{ for air} \qquad \ldots\ldots\ (2.7)$$

and $$M = \frac{u}{a} \rightarrow \left[\frac{2}{\gamma(\gamma-1)}\right]^{\frac{1}{2}} = 1.89 \text{ for air} \qquad \ldots\ldots\ (2.8)$$

Further, we define a strong shock wave as one which generates supersonic flow behind it when it propagates into air at rest; that is, for a strong shock wave, the shock Mach number V_+ must be greater than 2.07 - see equation (2.5) with $M = 1$.

Hypersonic flow (defined in the present context by $M > 5$) may be generated in a shock tube if the supersonic flow behind a strong shock wave is expanded in a divergent nozzle. However, the strength of the primary shock wave will decrease during its passage through the divergent section - in an analogous manner to the decay of a cylindrical blast wave - and hence, to maintain flow equilibrium, a secondary shock wave of increasing strength is formed by the continuous diffraction of the primary shock through the divergent channel. In general, a secondary contact surface or contact region of entropy gradients will also be formed in the diffraction process; both this and the secondary upstream-facing shock wave will be swept downstream after the primary downstream-facing shock wave. A region of steady flow at high Mach number is established in the hypersonic working section until the arrival of the primary contact surface. Ultimately the strength of the secondary shock is such that it becomes stationary with respect to the walls of the shock tube; it is then equivalent to the standing shock observed in underexpanded flow in a nozzle.

3. The Duration of Hypersonic Flow in a Shock Tube

3.1 Ideal Shock Tube Flow

Fig.3 illustrates those features discussed above of the flow in the channel of a hypersonic shock tube; the extent of the hypersonic flow region is shaded. Clearly the maximum duration τ_H of hypersonic flow occurs at the position+ x_H; however, we assume $\tau_H \simeq \tau$ where τ is given by

$$\tau = \left(\frac{1}{u} - \frac{1}{U}\right)L \qquad \ldots\ldots\ (3.1)$$

and/

+ The model under test would be placed at position x_M where $x_M > x_H$, but we assume

$$\tau_M \lessapprox \tau_H \simeq \tau .$$

and where L is the length of the constant area channel from the diaphragm D to the entrance of the divergent nozzle. It should be noted that the length of the chamber must be such that the reflected head of the rarefaction wave does not overtake the primary contact surface before the entrance to the nozzle section; and, further, the length of the 'hypersonic' channel H must be chosen so that the reflected shock wave from the end of the channel does not interfere with the hypersonic flow region. For the latter consideration, when $V_+ >> 1$, the reflected shock velocity tends to one third of the corresponding incident shock velocity; hence we determine that the length of the 'hypersonic' channel H must be greater than $\frac{1}{4}U\tau$.

That is,

$$H > \frac{1}{4} U \tau \qquad \ldots\ldots\ (3.2)$$

where U is primary shock wave velocity and τ is the duration of hypersonic flow. The length of the chamber may be determined by standard methods[5,8,9]; specimen calculations indicate that this dimension is usually less than 20 feet. For strong shock waves ($V_+ >> 1$), see equation (2.6),

$$u \simeq \frac{5}{6} U ,$$

and hence, from equation (3.1), we have

$$\tau = \frac{L}{5U} = \frac{L}{5V_+ a_0}$$

If we assume $a_0 = 1120$ ft/sec, the testing time per foot of channel is given approximately by

$$\frac{\tau}{L} = \frac{0.179}{V_+} \text{ milliseconds} \qquad \ldots\ldots\ (3.3)$$

A more accurate relation for the ideal testing time may be obtained by the use of equation (2.3) instead of (2.6); in this case

$$\frac{\tau}{L} = \frac{1}{a_0}\left[\frac{1}{\frac{5}{6}\left[V_+ - \frac{1}{V_+}\right]} - \frac{1}{V_+}\right] \qquad \ldots\ldots\ (3.4)$$

Relations (3.3) and (3.4) are plotted in Fig.4 for a range of values of V_+.

3.2 The Effects of the Non-Perfect Gas Parameters: Variable Specific Heat; Dissociation and Ionisation

The effects of variable specific heat on shocks propagated into air at atmospheric temperatures and pressures have been considered by Becker, Burkhardt and Bethe and Teller. Their results are summarised in Ref.9 and plotted, using shock tube notation, in Fig.5. The shock characteristics begin to deviate appreciably from ideal theory when the shock Mach number V_+ exceeds 4. Dissociation and ionisation become appreciable when $V_+ > 8$ and $V_+ > 11$ respectively; and they were taken into account by Burkhardt in the calculations presented in Fig.5. We denote by T^*, u^*, p^*, γ^* the values of the parameters of state behind strong shock waves when the effects of gaseous imperfections are present.

3.3 Experimental Results

Few observations of the flow behind very strong shock waves have been made; measurements of flow Mach number M by Hertzberg, quoted by Dodge in Ref.1, are given in Table I for a range of values of V_+ and for two absolute channel pressures p_0.

3.4 Correlation between Experiment and Theory

In the present section an attempt is made to correlate the experimental results of Hertzberg with an approximate non-perfect gas theory. For real imperfect gases, equation (3.1) must be replaced by

$$\frac{T}{L} = \frac{1}{u^*} - \frac{1}{U} \qquad \text{.......... (3.5)}$$

where the value of u^* is, as yet, undetermined. Suppose the relation (2.3) holds for real gases; that is

$$\frac{u^*}{a_0} = \frac{2}{\gamma^*+1}\left[V_+ - \frac{1}{V_+}\right] \qquad \text{.......... (3.6)}$$

Then we may write

$$\frac{u^*}{u} = \frac{\gamma+1}{\gamma^*+1} \qquad \text{.......... (3.7)}$$

where γ^* is the effective γ of the real gas. Further, let

$$\frac{U - u}{U - u^*} = k \qquad \text{.......... (3.8)}$$

where the variation of k with V_+ is given by Fig.5. Then, from (3.8)

$$\frac{u^*}{u} = \left[\frac{U(k - 1) + u}{ku}\right] \qquad \text{.......... (3.9)}$$

Equate (3.7) to (3.9), put $\gamma = 1.4$ and put $u = \frac{5}{6}U$. [A good approximation only if $V_+ > 10$ - see Section 3.1.] Hence we obtain

$$\gamma^* = \frac{6k + 1}{6k - 1} \qquad \text{.......... (3.10)}$$

The variation of γ^* with T^* and V_+ is shown by Fig.6, where equation (3.10) is compared with standard data[8].

We may now compute an approximate non-ideal theoretical Mach number M_T^* of the quasi-steady flow behind the shock wave.

Now

$$M^* = M \cdot \frac{u^*}{u} \cdot \frac{a}{a^*} \simeq M \cdot \frac{u^*}{u}\sqrt{\frac{\gamma T}{\gamma^* T^*}} \qquad \text{.......... (3.11)}$$

or, using (3.7)

$$M_T^* = M_T \frac{\gamma+1}{\gamma^*+1}\sqrt{\frac{\gamma T}{\gamma^* T^*}} \qquad \text{.......... (3.12)}$$

where M_T values are given in Table I for specified V_+, and corresponding values of T^*/T and γ^* are obtained from Figs. 5 and 6.

The calculated values of flow Mach number M_T^* are compared with the experimental values M_E in Table I, it is clear that the values are in fair agreement. However, two factors, which are not takeninto account in the theoretical analysis, will have a considerable effect on the experiment results. The growth of boundary layers on the walls of the shock tube may be shown[5] to cause an acceleration of the contact surface (velocity u) as it progresses down the tube; hence one might expect that the observed+ Mach number would be larger than the theoretical value. Moreover, when the shock wave is very strong, considerable heat transfer must occur from the intensely hot gas behind the shock to the cold walls of the shock tube; this effect would cause a smaller Mach number M to be observed than predicted by theory. It might be expected that for small V_+ the acceleration effect is predominant; whilst for large V_+ the heat transfer criteria is of prime importance. [Table I shows that the variation of M_E is consistent with the above remarks.]

Thus/

+ At some distance from the diaphragm section.

Thus, if it were possible to correct the M_T^* values for the above effects, we might find good agreement between theory and experiment.

3.5 Calculation of Approximate Non-Ideal Flow Duration

We may now evaluate (3.5) using the value of u^* defined by (3.9); this should give an approximate upper limit for the flow duration (see Section 3.1).

From (3.9) $$u^* = \left[\frac{U(k - 1) + u}{k}\right]$$ and hence, evaluating

(3.5) we obtain $$\frac{\tau}{L} = \frac{1}{a_0}\left[\frac{k}{V_+(k - 1) + \frac{5}{8}(V_+ - \frac{1}{V_+})} - \frac{1}{V_+}\right] \quad \ldots\ldots (3.13)$$

where k is defined by (3.8) and is related to V_+ as shown in Fig.5.

Calculated flow durations from equation (3.13) are given in Fig.4 and compared with values obtained from equations (3.3) and (3.4).

4. Conclusions

The double-diaphragm, combustion-type shock tube may be used to generate shock Mach numbers $V_+ \simeq 20$; subsequently the flow behind the shock may be expanded to yield a short duration hypersonic flow with a stagnation temperature approximating to that of full-scale flight.

The influences of deviations from ideal gas theory, which occur when strong shock waves are generated, are considered and these are shown to reduce appreciably the available testing times; this reduction is of the order of 50% for very strong shock conditions.

APPENDIX/

APPENDIX

Notation

a	velocity of sound
A	cross-sectional area of shock tube
H	length of 'hypersonic' channel (see Fig.3)
k	function of shock Mach number V_+ (see Fig.5)
L	length of constant area channel preceding the entrance to the divergent nozzle (see Fig.3)
M	Mach number
p	absolute pressure
R	the Gas Constant
t	time
T	absolute temperature
u	flow velocity
U	shock velocity
x	distance measured along longitudinal axis of shock tube
$\gamma = C_p/C_v$	the ratio of the specific heats
τ	duration of hypersonic flow

Non-dimensional quantities

$V_+ = \frac{U}{a_o}$	shock Mach number
$P = \hat{p}_o/p_o$	pressure ratio across the diaphragm
$p_s = p/p_o$	pressure ratio across the shock wave

Superscripts

$\wedge$	quantities related to gas initially at highest pressure in shock tube
'	quantities related to gas initially at intermediate pressure in double-diaphragm shock tube (Fig.2)
*	denotes the values of the parameters of state behind strong shock waves when the effects of gaseous imperfections are present

Subscripts/

Subscripts

o	initial state of gas (at rest) or stagnation state
E	experimental value
I	ideal gas theory value
T	non-ideal gas theory value

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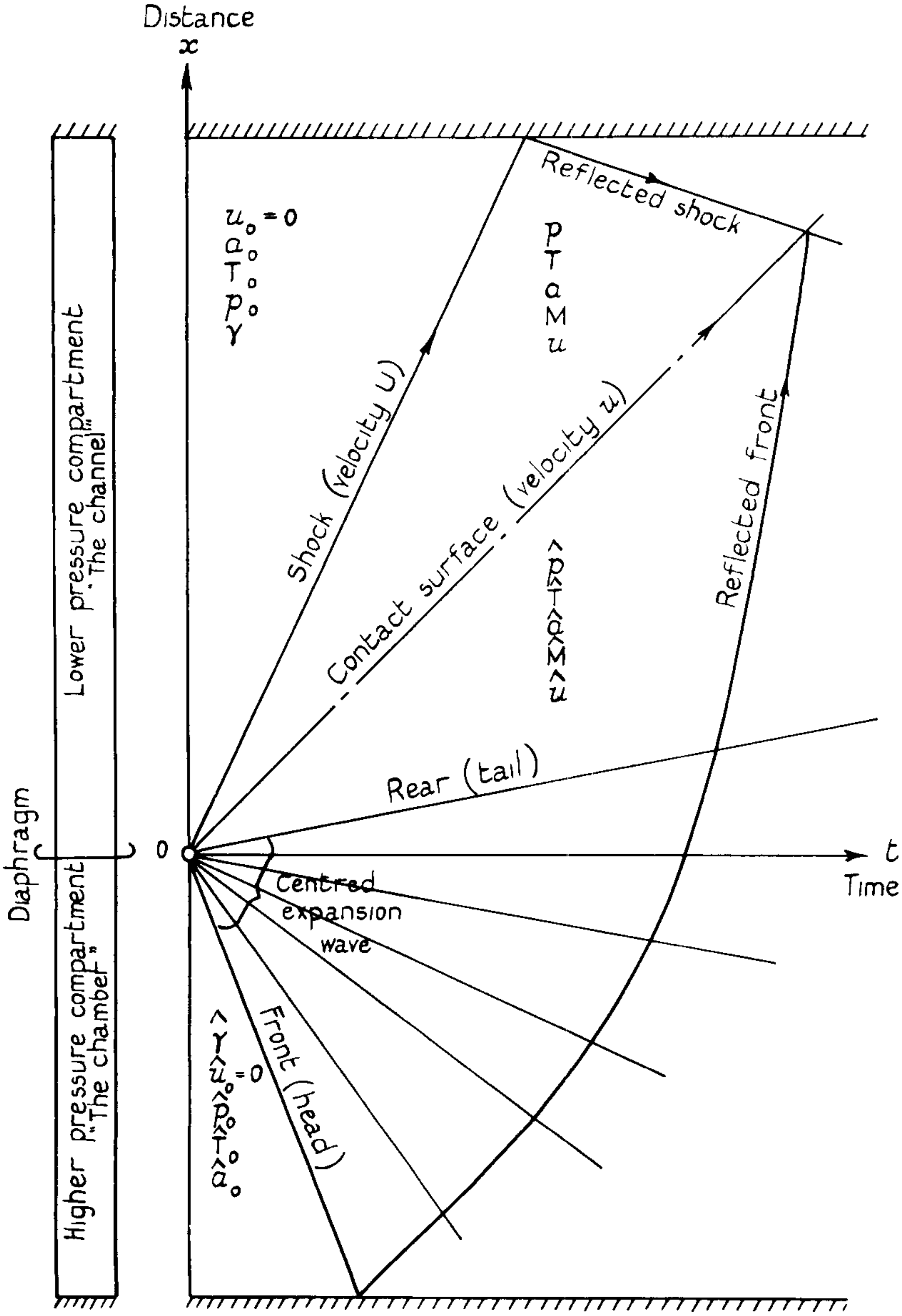
TABLE I/

TABLE I

Shock Mach Number V_+	Flow Mach Number (Ideal) M_I	Flow Mach Number M_E Channel Pressure Po 76 mm Hg	Flow Mach Number M_E Channel Pressure Po 7·6 mm Hg	Flow Mach Number (Non-ideal) Gas M_I^*
4	1.54	1.64	1.64	—
5	1.66	1.80	1.80	1.74
6	1.74	1.96	1.98	1.88
7	1.78	2.10	2.16	1.97
8	1.80	2.24	2.32	2.06
9	1.82	2.38	2.48	2.17
10	1.83	2.50	2.62	2.30
11	1.84	2.61	2.76	2.43
12	1.84	2.73	2.87	2.56
13	1.85	2.82	2.98	2.72
14	1.86	2.91	3.06	2.88
15	1.87	2.98	3.14	3.07
16	1.87	3.04	3.20	3.21
17	1.88	3.08	3.24	3.35
18	1.88	3.12	3.28	3.48
19	1.88	3.13	3.30	3.59
20	1.89	3.14	3.32	3.69
21	1.89	3.13	3.32	3.78
22	1.89	3.12	3.32	3.85
23	1.89	—	3.27	3.89
24	1.89	—	3.17	3.91

HBD

FIG. 1

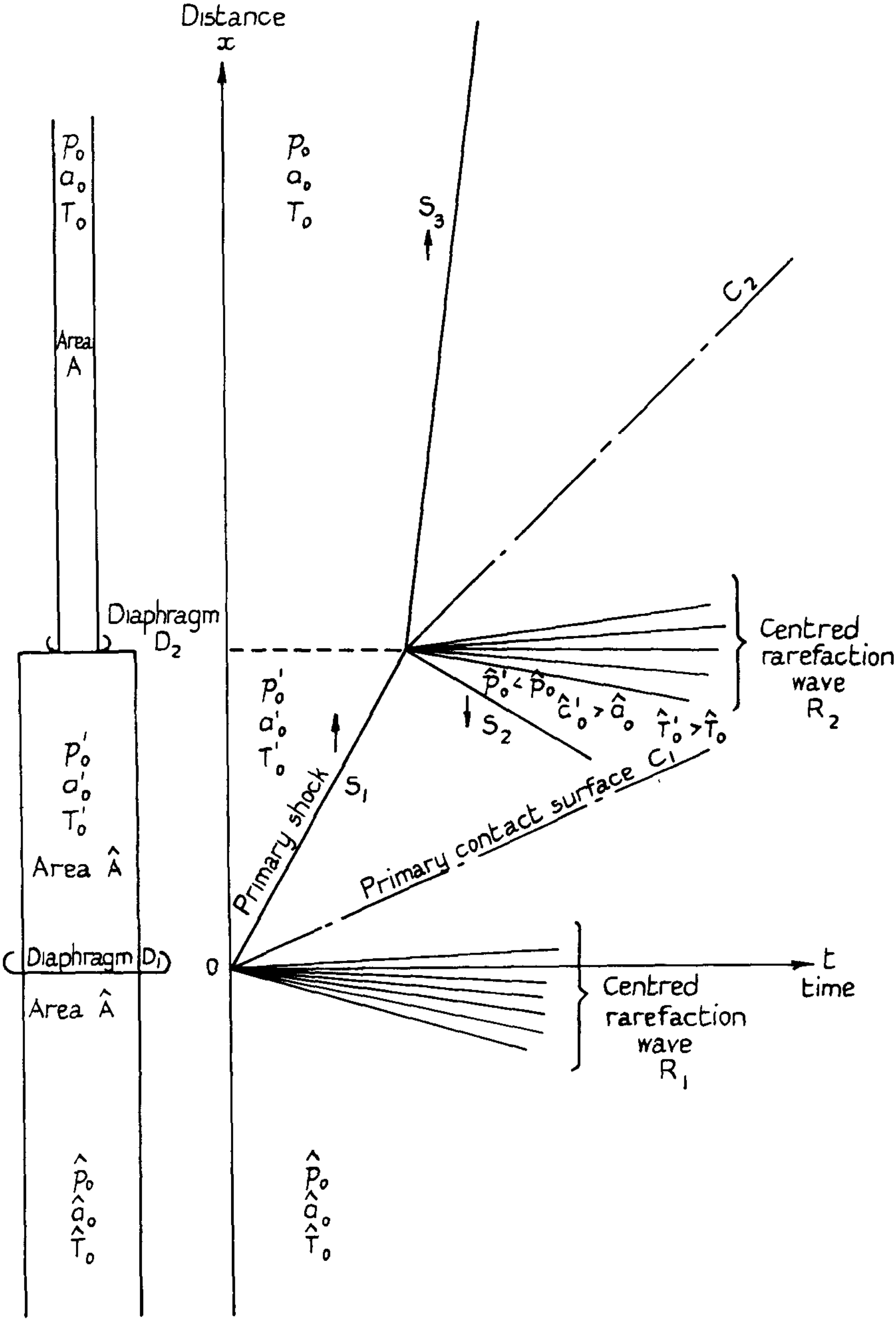


Non-dimensional notation -

$$V_+ = \frac{U}{a_0} \; ; \quad P = \frac{\hat{p}_0}{p_0} \; ; \quad p_s = \frac{p}{p_0} \; , \quad M = \frac{u}{a}$$

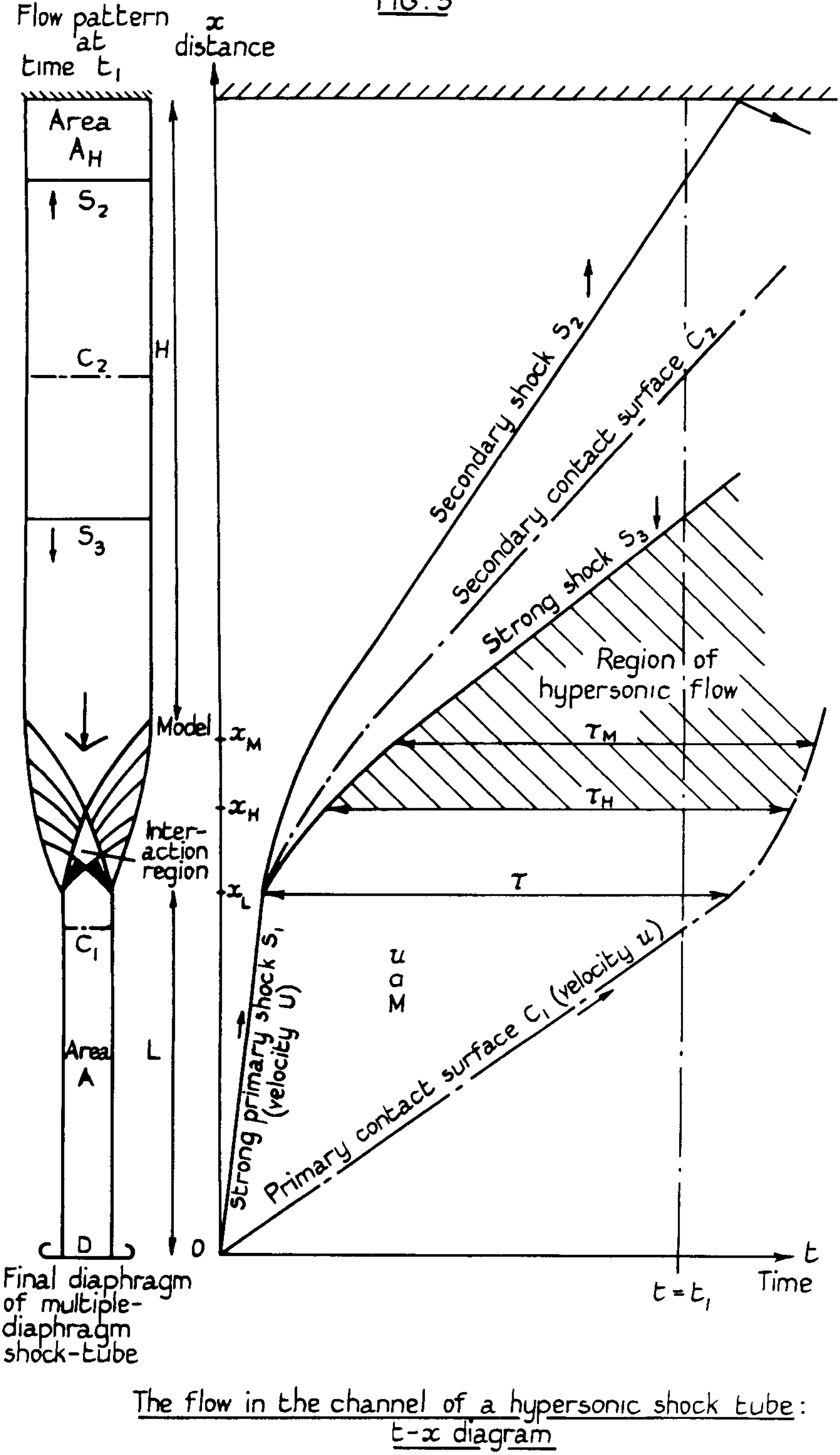
Simple shock tube flow · $t - x$ diagram

FIG. 2.



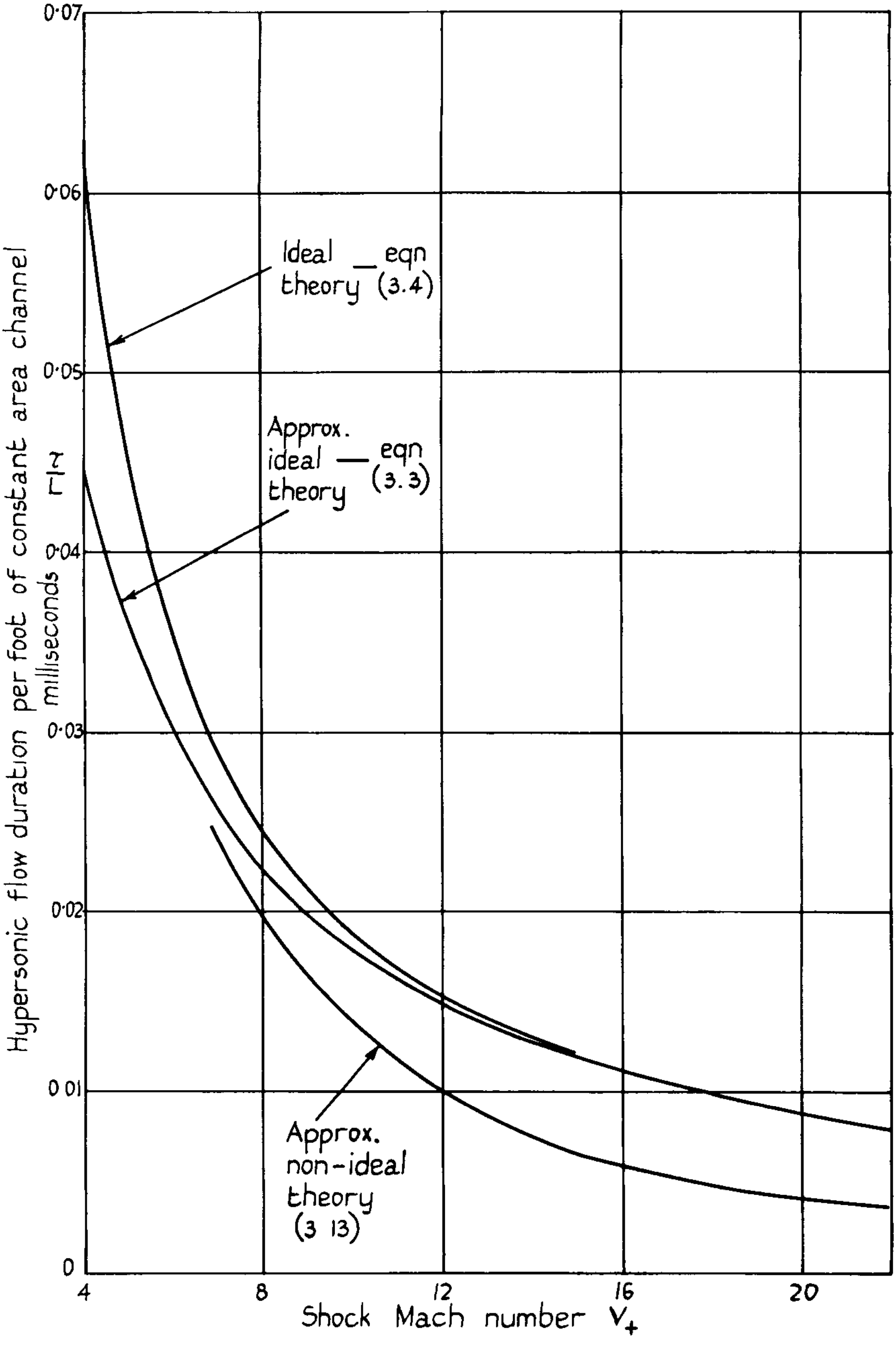
The flow in a double-diaphragm step-type shock tube
t-x diagram.

FIG. 3



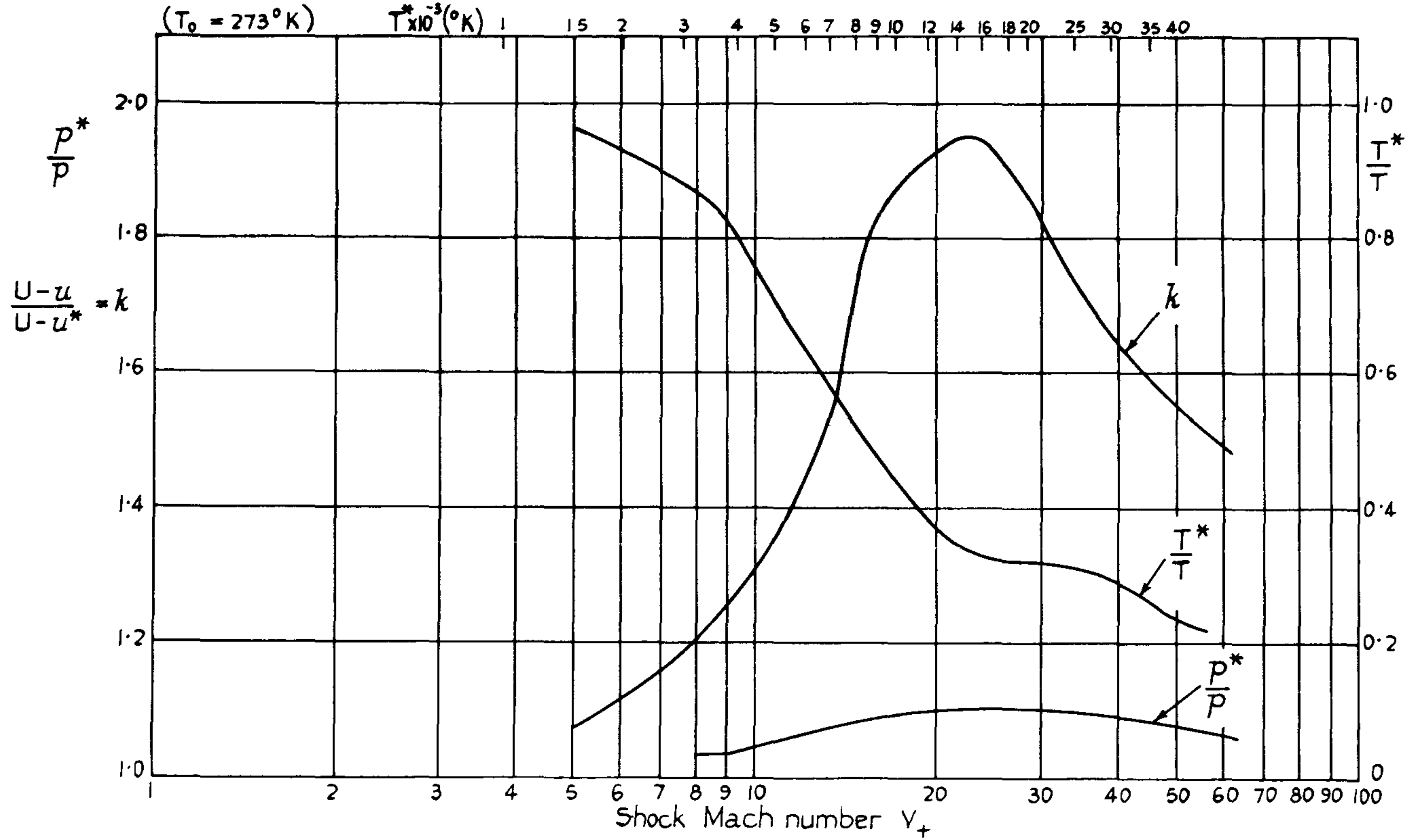
The flow in the channel of a hypersonic shock tube: t-x diagram

FIG. 4.



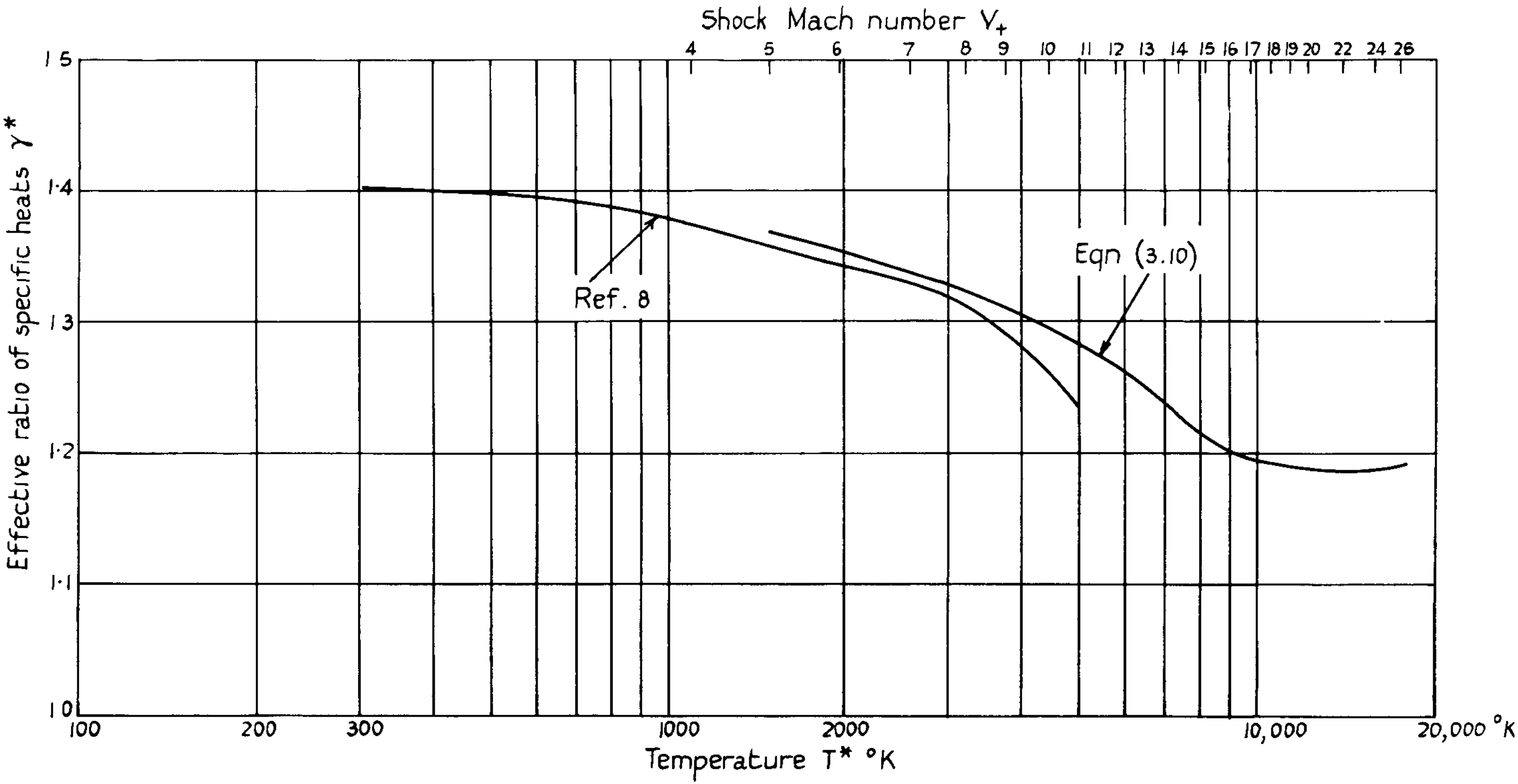
Hypersonic shock tube: flow durations

FIG. 5.



Comparison of ideal and actual flow parameters behind strong shock waves – from ref. 9

FIG 6



The variation of the effective ratio of specific heats γ^* with temperature T^* behind strong shock waves

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