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A Constructional Method for Minimising the Hazard of Catastrophic Failure in a Pressure-Cabin

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ROYAL AIRCRAFT ESTABLISHMENT

A constructional method for minimising the hazard of catastrophic failure in a pressure-cabin

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SUMMARY

A method is put forward for substantially reducing the chances of a local failure in the shell of a pressure-cabin from developing into catastrophic failure of the cabin. The increased safety is achieved without weight penalty, and consists essentially in using closely-spaced (10 inches or thereabouts) transverse flat bands, the material for which is obtained by reducing the sheet thickness normally available for the shell walls.

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1 Introduction

In a recent report[1] the writer has considered the problem of reducing to a minimum the stress concentrations that are the inevitable consequence of discontinuities in a pressure-cabin shell such as are caused by windows, doors, canopies and transverse frames. The recommendations made in that report were all designed to preserve, as far as possible, the smooth flow of the membrane forces characteristic of the unbroken shell, and so to discourage the development of cracks in the stressed skin walls. The problem of how to stop a crack, once started, from rapidly spreading and leading to catastrophic failure of the cabin, was not specifically faced. It is therefore with the purpose of considering this special aspect of the structural design of pressure-cabins that the present note is written.

2 Ways in which Catastrophic Failure can occur

Catastrophic failure can occur in two ways, but one is far more likely than the other. The less likely way is for the shell to split open circumferentially between two adjacent transverse frames. This is unlikely for two reasons: the longitudinal tension that would propagate the initial crack is only half the hoop tension and, moreover, would have to overcome the tough resistance offered by the closely spaced stringers. In contrast to this the shell is excessively vulnerable to the propagation of a crack running longitudinally between two adjacent stringers. For, in conventional construction, the comparatively flimsy and widely spaced former rings alone offer any resistance to the splitting of the cabin wall from end to end once a crack has started. This ineffective resistance, combined with the severity of the hoop, in relation to the longitudinal stress, leaves the relative likelihood of the two types of failure in little doubt.

3 Type of Construction Advocated

If this argument is accepted as valid, the problem is reduced to that of choosing the best way of providing an effective barrier to the propagation of a longitudinal crack, once such a crack has started.

The method of construction advocated here is based on the argument developed in Appendix D of Ref.1 (where the mathematical analysis is presented in full). It was there shown that the effectiveness of the transverse former frames or rings in limiting the maximum hoop stress in the skin to that in the frames themselves depends not so much on their stiffness as on their spacing. Rings spaced more than some 30 inches apart (in the case of conventional stringer reinforcement), while locally restricting the radial expansion of the skin, allow unrestricted expansion in the region midway between the rings. At the conventional spacing of some 18 to 20 inches the maximum inter-ring expansion is still about 50% of unrestricted expansion. With 10 inch spacing, however, the radial expansion of the skin nowhere exceeds that of the rings by more than a negligible percentage.

We are thus led to the important conclusion that, for ring-spacing of 10 inches or less (assuming the conventional type of stringer reinforcement) the maximum hoop-stress in the skin is equally reduced by material added to the rings as by the same weight of material added to the skin.

This suggests at once that if it is desired to strengthen a conventional design, the most effective way of doing so, is not to use a heavier gauge of skin but to halve the pitch of the rings and increase their cross-sectional area. The beneficial effect of strengthening the rings and closing up their pitch is five-fold:-

(i) The stronger ring will not itself split open in the path of a crack to allow the disruptive forces to multiply their leverage on the sheet each side of it.

(ii) The stronger ring will tend to stop the propagation of the split into the adjacent inter-ring area of sheet.

(iii) The closing up of the ring spacing will reduce the distance the crack (or split) can spread before meeting an effective barrier to its further progress.

(iv) The material in the rings will be fully effective in reducing the hoop stress in the skin so that, with the same total amount of material in skin and frames, the hoop stress in the thinner skin and stronger frames is no different from that in the thicker skin and weaker frames.

(v) The bending stresses in the stringers, due to the radial swelling of the skin between rings, drop to about 1/8 of their original value as a result of halving the ring pitch.

A word or two on each of these points will perhaps clarify what one has in mind.

Point (i) is very important, because it is highly desirable that the rings adjacent to the initial crack - and by crack is meant a local split right through the sheet thickness - should not open up in its path, even should they fail to stop the crack from crossing into the next inter-ring area. By remaining intact, the rings, even though the initial crack is propagated across a whole series of them, continue to play a dominant role in converting into a mere mishap what would otherwise be a catastrophe. They are able to do this by limiting the disruptive pressure forces in the shell to what they would be if the split were confined to the inter-ring area where it started.

It is however highly unlikely, as stated in point (ii), that a split can immediately jump across an unbroken ring. For, so long as the ring holds, the tearing force exercised by the pressure on the sheet each side of the split cannot affect the sheet beyond the ring, because the ring itself provides the necessary reaction. The confinement in this way of an initial split to its own ring 'compartment' is an important factor in saving the cabin from rapid deflation.

Regarding point (iii), the main purpose of closing up the pitch of the rings was, as explained above, and listed in point (iv), to ensure that the skin should have the same radial expansion midway between two adjacent rings as at the rings. By a stroke of good fortune, however, the closer spacing of the rings provides the further advantage stated at point (iii) above. The shorter the length of the split before it meets a barrier the easier it is for that barrier to stop it. This follows at once from the readily acceptable proposition that, for moderate distances of a foot or a yard, the disruptive effect of the internal pressure is roughly proportional to the length of the split.

Regarding point (v), it should be appreciated that it is only by virtue of the stiffness of the conventional type of stringers that a pitch of some 10 inches is adequate to prevent radial expansion of the skin beyond that of the rings. It will readily be understood that to increase the critical pitch much above 10 inches would require an altogether disproportionate increase in stringer stiffness and therefore in stringer weight.

4 Type of Rings advocated for Suggested Method of Construction

It is well to be clear on what kind of rings are necessary for giving effect to the constructional suggestion made in the above paragraphs. It is, of course, not necessary to increase the capacity of the rings for stabilising the fuselage walls against elastic instability by buckling due to orthodox end loads, because the size, stiffness, and spacing of conventional type rings are quite adequate for that. What is wanted for the present purpose is an increase in the number of rings on the one hand and, on the other, an increase in the capacity of each ring for resisting radial expansion - in other words, an increase in cross-sectional area.

The argument perhaps may be better developed if it is associated with a numerical example.

4.1 Numerical Example

Suppose we have a 10 ft diameter cabin, with an operational pressure of 9 lb/in^2, in which the shell wall is 0.028 inches (22 s.w.g.) thick and the rings are of conventional Z section with a cross-sectional area of 0.16 in^2 and a pitch of 20 inches. Suppose further that it is desired to make the structure substantially safer.

If the argument developed in this paper is to be accepted, the safest and most economical way of doing so is to leave the shell wall thickness unaltered and to use all the extra material available for halving the pitch and increasing the cross-sectional area of the rings. The reasoning, reduced to numerical values, is roughly as follows.

The hoop tension in the cabin skin under the assumed operational pressure of 9 lb/in^2 is 540 lb/in^2 which represents a stress of 540/0.028 or 19,500 lb/in^2, i.e. about 1/3 of the ultimate stress in the sheet.

If a longitudinal slit suddenly develops between two adjacent rings at the operational pressure, each of the two will be called upon to take the hoop tension previously taken by half the inter-ring skin width. If the split jumps across the ring, each ring must take the hoop tension of a full ring pitch, i.e. 10 × 540 = 5400 lb. Since this is a single static load, a factor of 2 seems adequate, which brings the ring load up to 10,800 lb, and the ring cross-sectional area (on the basis of an ultimate stress of 60,000 lb/in^2) up to

$$A = 10,800/60,000 = 0.18 \text{ in}^2.$$

Having regard to the existing rings of 0.16 in^2 at 20 inches pitch, we see that the extra material in the rings is equivalent to increasing the skin thickness from 0.028 to 0.038 inches (or from 22 to not quite 19 s.w.g.). This is on the basis that the assumed existing rings are increased in area from 0.16 to 0.18 and that intermediate rings of 0.18 are fitted between each pair of the original rings.

The proper form for the new intermediate rings of 0.18 cross-sectional area to take is that of plain bands of sheet - say $1\frac{1}{2}$ inches wide and 0.12 inches thick - the important factor being to have enough thickness in the band to discourage the formation of a crack in line with the crack already developed in the sheet.

The proper way to deal with the existing stabilising rings is not so obvious. For one thing the cross-sectional area of the ring, if of Z section, is not wholly effective because the top lip, and indeed the whole section is too thin, so inviting a split to run down the Z. To overcome this disability and feel satisfied with the efficacy of the stabilising rings, the Z section should be discarded and a catenary section (of the type discussed in Ref.1) of the same area substituted. In addition a band of sheet 1 inch (say) wide and 0.1 inch thick should be used in conjunction with the catenary section in order to present a thickness more effective than that of the lip of the Z section to act as a barrier to crack propagation. This extra band is equivalent to increasing the skin thickness by a further 0.005 inches to make the total equivalent skin thickness (0.038 + 0.005) or 0.043, i.e. 0.003 thicker than 19 s.w.g.

Since the material in the rings is equally effective with that in the skin, the total increase of 50% in material increases the ratio of the ultimate stress to the operational stress in the sheet from 3 to 4.5.

The ideal place, were it not for aerodynamic drag considerations, to fit the reinforcing bands is on the outside of the skin, for they would then be able to perform their function of limiting the radial expansion of the skin directly and not indirectly through rivets in tension. That would also be by far the easiest way of introducing the extra rings as a modification to a cabin already built. It is therefore worth serious consideration whether the extra drag - particularly if the bands were chamfered at the edges - may not be small enough to be accepted.

5 Experimental Work

It is highly probable that a cabin pressurised to a fairly high operational pressure - 9 to 10 lb/in^2 - would, if fitted with conventional former rings, split right open if a sudden longitudinal split were made in the shell. In this paper it is argued that such a sudden split would be comparatively innocuous in a cabin shell fitted with rings of the type advocated.

The question whether, in any cabin design, a split (from whatever cause) is to be the forerunner of a mild deflation or of a catastrophic failure can only be determined with certainty by experiment. It is therefore suggested that such an experiment would be well worth making.

6 Conclusions

It is concluded that the way to make a pressure cabin safe against catastrophic failure in the air, without paying any weight penalty, is to apportion the weight of material normally available for the skin between closely spaced (10 inch) transverse bands and skin proper, and to have roughly half (or more than half) as much material in these bands as in the skin. The conventional former rings, at conventional pitch, are retained but their section is changed from a Z shape to a deep catenary. With a skin of 0.028 (22 s.w.g.) the bands would take up material equivalent to 0.014 (28 s.w.g.) of skin.

By doing this, a longitudinal split (from whatever cause) in the skin - and a longitudinal split is by far the most dangerous and the most likely form for an initial failure to develop into, in whatever form it starts, - will be stopped within 5 inches from its source instead of running the whole length of the cabin, as it probably would with the conventional type of construction. The contrast between these two results, it need hardly be pointed out, is that between a mild deflation of the pressure and catastrophic failure.

It is further concluded that, having regard to the importance of the implications of the argument put forward, some experimental work to test its validity should be urgently undertaken.

REFERENCE

No.	Author	Title, etc.
1	D. Williams	"Pressure-cabin Design - A discussion of some of the Structural Problems Involved, with Suggestions for their Solutions." C.P.226 March, 1955.

APPENDIX I

Effect of inter-stringer swelling of pressure-cabin skin

In the constructional method outlined in the text of this note, inter-ring skin-swelling is prevented by spacing the reinforcing bands close enough together and relying on the stringers to hold the skin down between adjacent rings. The possibility of the skin swelling between stringers was not specifically considered but, as pointed out to the writer by Mr. H. B. Howard, that is a point that ought to be examined.

This is done in the following analysis, where it is assumed in accordance to a main feature of the original scheme, that the stringers have the same radial displacement under pressure as the rings or bands that hold them down. This allows one to imagine the rings replaced by a continuous but completely porous skin, having the same resistance to radial expansion as the uniformly distributed resistance of the rings. The radius of curvature of this skin will be constant and equal to the nominal radius of the cabin. Carried by this skin, and uniformly spaced around its circumference will be the stringers, to which are also attached the skin proper.

The simplified representation of the actual structure will perhaps be better understood by means of the diagram of Fig. 1.

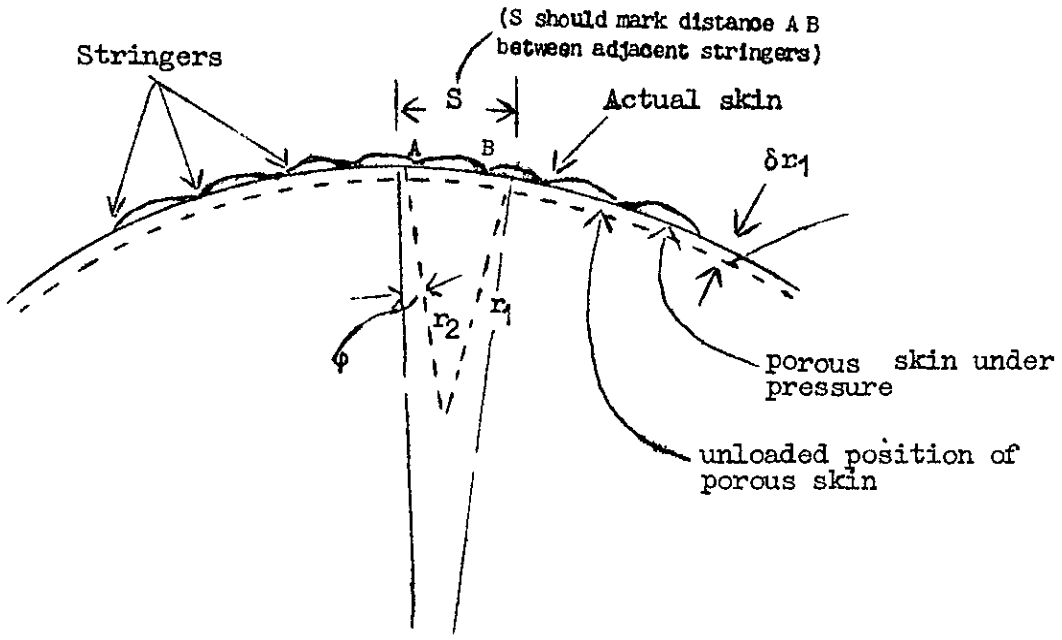
(S should mark distance A B between adjacent stringers)

Fig. 1

Here the porous inner skin - skin 1 call it - is represented in its unexpanded position by the dotted line. It is also shown after a radial expansion from r_1 to $(r_1 + \delta r_1)$. The actual skin - skin 2 - tends to swell out between stringers and so take up a radius r_2 smaller than r_1. Thus let

r_1 = original radius before pressurisation (same for both skins)

r_2 = radius of skin 2 under pressure

δr_1 = radial expansion of skin 1

T_1 = hoop tension per unit length for skin 1

T_2 = hoop tension per unit length for skin 2

t_1 = thickness of skin 1

t_2 = thickness of skin 2

s = pitch of stringers

$\phi = \frac{s}{2}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$ = angle between the two skins at a stringer

p = internal pressure

P = reaction between the two skins at a stringer

$p_1 = \frac{P}{s}$ = effective distributed radial load applied by skin 2 to skin 1.

We consider the case where the skin resists the pressure by membrane tension alone, the bending resistance of the skin being neglected. Thus the hoop tension T_2 in skin 2 becomes

$$T_2 = pr_2 . \tag{1}$$

The hoop tension T_1 in skin 1 is

$$T_1 = \frac{\delta r_1}{r_1} t_1 E . \tag{2}$$

The reaction P between the two skins (per unit width) where, at the stringers, they have the same radial displacement is

$$P = 2T_2 \sin\phi . \tag{3}$$

Regarded as a uniformly distributed radial load p_1 this gives

$$\begin{aligned} p_1 &= \frac{P}{s} \\ &= \frac{2T_2}{s}\sin\phi = \frac{2pr_2}{s}\sin\phi . \end{aligned} \tag{4}$$

Now
$$\frac{\delta r_1}{r_1} = \frac{p_1 r}{t_1 E} = \frac{2r}{t_1 E} \cdot \frac{pr_2}{s}\sin\phi , \tag{5}$$

by (1) and (4).

Putting $\sin\phi = \phi = \frac{s}{2}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$, we write the hoop strain ε_1 in skin 1 as

$$\varepsilon_1 = \frac{\delta r_1}{r_1} = \frac{pr_1 r_2}{t_1 E}\left(\frac{1}{r_2} - \frac{1}{r_1}\right) . \tag{6}$$

It is easy to show that the hoop strain produced in skin 2 by swelling it out beyond skin 1 between stringers to a radius r_2 is

$$\varepsilon_2 = \frac{s^2}{24}\left(\frac{1}{r_2^{\,2}} - \frac{1}{r_1^{\,2}}\right) . \tag{7}$$

The total hoop strain of skin 2 is therefore

$$\varepsilon_1 + \varepsilon_2 = \frac{s^2}{24}\left(\frac{1}{r_2^{\,2}} - \frac{1}{r_1^{\,2}}\right) + \frac{p}{t_1E}\,(r_1 - r_2) . \tag{8}$$

But the total hoop strain in skin 2 can also be written in the form

$$\varepsilon_2 = \frac{pr_2}{t_2E} . \tag{9}$$

Equating (8) and (9), we therefore have

$$\frac{s^2}{24}\left(\frac{1}{r_2^{\,2}} - \frac{1}{r_1^{\,2}}\right) + \frac{p}{t_1E}\,(r_1 - r_2) = \frac{pr_2}{t_2E} \tag{10}$$

or

$$r_2^{\,3}\, p\left(\frac{1}{t_2} + \frac{1}{t_1}\right) - r_2^{\,2}\left(\frac{pr}{t_1} - \frac{s^2E}{24r_1^{\,2}}\right) = \frac{s^2E}{24} . \tag{11}$$

This cubic gives r_2 in terms of the physical constants of the system and the pressure p.

The extreme cases where $p = 0$, or $t_1 = 0$ are satisfied, as they should be by physical reasoning, by the solution $r_2 = r_1$. It may also be noted that the overall condition $(T_1 + T_2) = pr_1$ is identically satisfied by equations (1) and (6).

A typical example for which

$$\left.\begin{aligned} r_1 &= 60 \text{ in.} \\ t_1 &= t_2 = 0.022 \text{ in.} \\ s &= 4 \text{ in.} \\ p &= 8 \text{ lb/in}^2 \end{aligned}\right\} \tag{12}$$

gives $r_2 = 34$ in. (13)

It follows that

$$\left.\begin{aligned} T_2 &= pr_2 = 8 \times 34 = 272 \text{ lb/in} \\ T_1 &= p(r_1 - r_2) = 208 \text{ lb/in} \end{aligned}\right\} \tag{14}$$

Thus with equal amounts of material in skin and bands, the skin takes some 13% more of the total hoop tension than the rings. This, however, takes no account of the pressure-containing effect of the accompanying longitudinal curvature of the skin in between the rings. This wipes out part of the 13% difference and so again brings the skin and ring hoop-tensions into approximate equality.

The effect of including the bending resistance of the skin can be easily determined for the particular numerical example above considered by comparing the strain energies of bending and stretching. Thus

Bending moment per unit width of skin

$$M = EI\left(\frac{1}{r_2} - \frac{1}{r_1}\right) \tag{15}$$

where I = moment of inertia of cross section of skin per unit width.

$$\text{Strain energy } U_1 = M^2/2EI \times s \tag{16}$$

and, for the numerical values above quoted, this gives

$$U_1 = 0.0025 \text{ lb in.}$$

The membrane strain energy is given in terms of the hoop stress σ by

$$U_2 = \frac{\sigma^2 s t_2}{2E} \tag{17}$$

$$= 0.67 \text{ lb in.}$$

It is seen from this that the bending stiffness of the skin contributes less than $\frac{1}{2}$ per cent of the total resistance to change of curvature. The bending stiffness does, however, introduce a bending stress of 1,300 lb/in^2 which is therefore to be added to the membrane stress of T_2/t_2 or 12,400 lb/in^2.

As the numerical values taken in the above example can be regarded as highly typical, one may safely generalise so far as to say that the increase in hoop stress brought about by inter-stringer swelling does not constitute a significant factor, and in no way invalidates the main thesis.

APPENDIX II

"Window arrangement appropriate to the multiple-band scheme of construction advocated in this note"

In the text of this Note nothing is said regarding the effect of the multiple-band method of pressure-cabin construction on the design of cabin windows. If, as advocated, the bands are pitched about 10 in. apart it is clear that, either they must cut across the windows, or the windows must be accommodated in between the bands. A third possibility is to break off near the edge of the window the band that, if carried across the window, would divide it into two halves. This would undoubtedly create difficulties both of design and construction and is to be avoided if possible. The second of the first two alternatives seems equally unacceptable, as it would entail windows of a size altogether too small. One is driven to the conclusion therefore that the best course is to carry the interfering band right across the window.

When examined, this proposition has many solid advantages. These are:-

(i) By not breaking the continuity of the bands, the ratio between the hoop tension carried by skin and bands remains constant over the circumference of the cabin.

(ii) If this ratio is about one-to-one (as it should be), the hoop-tension load carried by the skin, both near and far from a window, (assuming the window to cover a <u>neutral</u> hole) is half the total hoop-tension load. And, since the total hoop load per unit width of wall is twice the longitudinal load, the load carried by the skin (per unit width of wall) in the circumferential direction is approximately equal to the longitudinal load (per unit width) carried by skin and stringers.

Any <u>neutral</u> hole, or cut-out, in the cabin wall, such as that made to accommodate a window, will therefore have its periphery under an outward pull per unit length of periphery, that is normal to the edge and constant all along the edge. The shape of the neutral hole, and hence the correct shape for the window, is therefore a circle. This is satisfactory both from the manufacturing and the aesthetic points of view.

(iii) The appearance of a window cut across by one of the bands will be something like that shown in Fig.1.

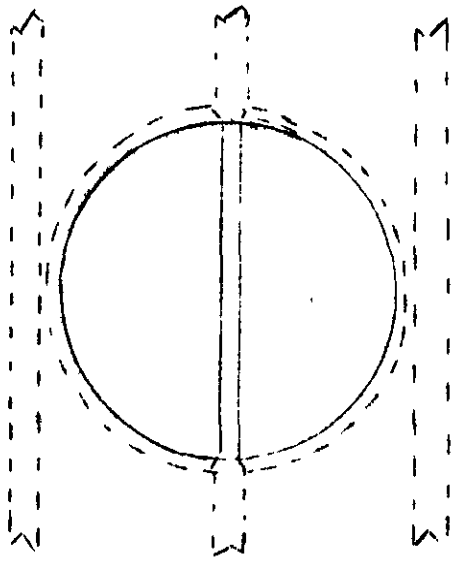

Fig.1

Here the middle band has its width cut down, where it traverses the window, from the normal width of $1\frac{1}{2}$ or 2 in. to about $\frac{3}{4}$ in, without of course reducing its cross-sectional area. Thus, while causing a minimum of interference with the passenger's view, the central strip not only greatly strengthens the window but ensures that only small objects can be blown out of the cabin in the event of the window breaking - distinctly contributing to the peace of mind of the nearby passenger.

It hardly needs emphasising that the load in the middle band of Fig.1, bypassing the window as it does, has no effect on the shape of the neutral hole. For clearly, only those forces that have to be transmitted via the reinforcing member round the edge of the hole need be considered in determining the correct shape of the neutral hole.

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