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by

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CORRIGENDA

p. 1, first paragraph, last line: for "disturbed" read "distorted".

p. 1, second paragraph, sub-paragraph (iii), first line: for "band" read "based".

p. 8, Ref. 4: for "VIIIth" read "VIIth".

p. 8, Ref. 8: for "Proc. Roy. Soc., etc." read "A.R.C., R. & M. 1010 (1926)".

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WATER TUNNEL BOUNDARY EFFECTS ON AXIALLY SYMMETRIC FULLY DEVELOPED CAVITIES

by I. J. Campbell and G. E. Thomas

ABSTRACT

The limitation on cavitation number attainable when axially symmetric cavities are formed in a fixed wall tunnel (Simmons' blockage effect1 and related effects are discussed in some detail. Estimates of the way in which the dimensions of cavities formed behind a circular disc depend on the cavitation number and on the ratio of model diameter to tunnel diameter are provided.

INTRODUCTION

1. The effects of the finite extent of the stream on the formation of cavities in a jet bounded by fixed or free walls have been thoroughly explored in the two-dimensional case by Birkhoff, Plesset and Simmons (Ref.1). Knowledge of tunnel boundary effects on cavities in the three-dimensional axially symmetric case is very much more limited. Armstrong and Tadman (Ref.2) have given an approximate theory from which they derive the boundary effects in some detail for cavities in a free jet and give first order corrections for cavities in a fixed wall tunnel. Specifically they enquire what will be the effect of the stream boundaries on the dimensions of the cavity for a given value of the cavitation number. For the case of the fixed wall tunnel this approach does not give the whole story since, owing to blockage effects, well-developed cavities may be formed behind obstacles, which are quite small compared with the tunnel diameter, at cavitation numbers quite different from those at which they would be formed in an unbounded stream: indeed the range of cavitation numbers of interest in an unbounded stream may be quite unattainable in the tunnel. The present report discusses in some detail the limitations on cavitation number attainable when axially symmetric cavities are formed in a fixed wall tunnel (the blockage effect found by N. Simmons) and related effects. Theoretical estimates are provided of the dimensions of cavities formed behind a circular disc at various cavitation numbers in a fixed wall tunnel and the extent to which a cavity of given fineness ratio may be distorted by the influence of fixed boundaries is illustrated.

CAVITIES IN UNBOUNDED FLOW

2. Axially symmetric cavities formed in an unbounded stream will be discussed first. For the present, the radius R in Figure 1 is assumed to be infinite. Reichardt (Ref.3) carried out experiments in a specially designed tunnel and gave good reasons for supposing that his results were little affected by boundary effects. He showed that:

(i) The fineness ratio (l/a) of the cavity depends exclusively on the cavitation number. The dependence of l/a on Q is given in Figure 2.

(ii) If $C_D(Q)$ is the drag coefficient based on the frontal area of the wetted portion of the obstacle, when the cavitation number is Q, then for obstacles with a fixed separation point

$$C_D(Q) = \frac{D}{\frac{1}{2}\rho U^2 \pi b^2} = C_D(0)(1+Q) \quad \ldots\ldots (1)$$

(iii) The drag coefficient based on the maximum frontal area of the cavity depends exclusively on the cavitation number and a good approximation to the experimental data is given by

$$K_D = \frac{D}{\frac{1}{2}\rho U^2 \pi a^2} = 0.9\,Q \qquad \ldots\ldots (2)$$

The value of $C_D(0)$ is now all that is required to give the absolute dimensions, l and a, for a cavity behind an obstacle with a fixed separation point for any given value of Q. a/b is given from (1) and (2) by

$$\frac{a}{b} = \sqrt{\frac{C_D(0)(1+Q)}{0.9\,Q}} \qquad \ldots\ldots (3)$$

and

$$\frac{l}{b} = \frac{l}{a}\cdot\frac{a}{b},$$

where a/b is known from (3) and l/a from (1) above (Figure 2).

A reasonably comprehensive theoretical account of axially symmetric cavities can be considered to have been achieved if the dependence of fineness ratio on Q, equations (1) and (2) and the value of $C_D(0)$ can be derived theoretically. Such an account will be sketched below as a preliminary to developing a similar account for cavities formed in a bounded stream. No exact theory for three dimensional cavities exists and the arguments are plausible rather than undeniable.

To relate l/a to Q. Simmons (Ref.4) made the simplifying assumption that the flow round a cavity can be represented by a source-sink distribution; the strength of which varies linearly from a positive value at $x = -l$ (Figure 1) to an equal negative value at $x = +l$. By varying the strength of the distribution, the values of a and of u_c/U, which is equal to $(1 + Q)^{\frac{1}{2}}$, can be varied together. Simmons' relation between l/a and Q is as follows:

$$(1 + Q)^{\frac{1}{2}} = 1 + c\left[\ln\left\{\frac{l}{a} + \left(\frac{l^2}{a^2} + 1\right)^{\frac{1}{2}}\right\} - \frac{\frac{l}{a}}{\left(\frac{l^2}{a^2} + 1\right)^{\frac{1}{2}}}\right] \tag{4}$$

where

$$\frac{1}{c} = \frac{l}{a}\left(\frac{l^2}{a^2} + 1\right)^{\frac{1}{2}} - \ln\left\{\frac{l}{a} + \left(\frac{l^2}{a^2} + 1\right)^{\frac{1}{2}}\right\} . \tag{5}$$

The agreement between this relation and the experimental values is not unreasonable (Figure 2). (Better agreement was obtained by Reichardt by using more elaborate source-sink distributions (Ref.5): the simpler model is preferred here because it is used later in connection with cavities in a bounded stream).

Reichardt made equation (1) at least plausible by making simple assumptions about the pressure distribution over the wetted portion of the obstacle. Reichardt's considerations are reproduced in a slightly different form in Appendix I and discussed in relation to a "hydraulic principle" of Birkhoff (Ref.6).

To obtain (2) theoretically, Reichardt showed from considerations of momentum that

$$K_D = Q - \frac{2}{a^2}\int_a^\infty \Delta^2\, r dr, \qquad \ldots\ldots(6)$$

where $u(0,r) = U(1 + \Delta)$. This relation is derived in Appendix II. If the integral is evaluated using Simmons' representation of the external flow, close agreement with (2) is obtained, as is illustrated in Table I,

TABLE I

l/a	3	5	9
K_D (from (6) and Simmons' model)	0.216	0.033	0.045
0.9 Q (Q from (4) and (5))	0.220	0.081	0.045

The value of $C_D(0)$ depends, of course, on the shape of the obstacle. By assuming the pressure distribution to be the same as that on a flat plate in two-dimensional cavity flow at zero cavitation number, Reichardt obtained $C_D(0)$ for a circular disc. (Plesset and Shaffer (Ref.7) have made similar estimates for a family of cones). The value of $C_D(0)$ obtained in this way for a circular disc is 0.81 and the experimentally determined value is 0.79. It hardly matters which of these values is used but in fact the value 0.79 has been used subsequently in this report. For a circular disc normal to the flow the relation between a/b and Q can now be obtained from equation (3) and the comparison with directly measured experimental values is shown in Figure 3. The relation between l/b and Q is likewise plotted and compared with experimental results in Figure 4

CAVITIES IN A FIXED WALL TUNNEL

3. Some idea of the general effects to be expected on axially symmetric cavities from constraining the stream inside a fixed wall tunnel may be gained from elementary considerations.

Noting that the cavitation number is given by

$$Q = \frac{p_0 - p_c}{\frac{1}{2}\rho U^2} = \left(\frac{u_c}{U}\right)^2 - 1, \qquad \ldots\ldots\ (7)$$

we consider a small cavity In a tunnel. The effect of the presence of fixed walls is to speed up the flow at the model so that the model is effectively in a stream Of velocity U_M, where $U_M > V$. Assuming that the cavity pressure Is fixed, e.g. at the vapour pressure, the effective cavitation number, $(u_c/U_M)^2 - 1$, is less than the (nominal) cavitation number, $(u_c/U)^2 - 1$. So It Is to be expected that the presence of the walls will encourage the production at a given cavitation number of a more fully developed cavity, i.e. a cavity of larger fineness ratio (Figure 2).

Again, since the minimum pressure occurs at the cavity wall,

$$\frac{u_c}{U} > \frac{\bar{u}}{U} \left\{\text{where } \bar{u} \text{ denotes the mean value } \frac{1}{(R^2 - a^2)}\int_a^R u(0,r)\, r dr\right\}$$

$$= \frac{R^2}{R^2 - a^2}$$

Hence

$$Q > Q_B = \left[\frac{1}{1 - \frac{a^2}{R^2}}\right]^2 - 1 > 2\frac{a^2}{R^2} \qquad \ldots\ldots\ (8)$$

In the limiting case in which the cavity becomes infintely long the cavity radius will tend asymptotically to a constant value and the transverse velocity distribution will tend to become uniform: the corresponding cavitation number is then the blockage cavitation number, Q_B. This Is the tunnel blockage phenomenon, discovered by Simmons. In any case the cavitation number certainly exceeds twice the cavity blockage (a^2/R^2). According to Reichardt's results a cavity of fineness ratio 10 will be formed In unbounded flow at a Q of 0.033: thus the speed at which such a cavity would be found at a given free stream pressure In a fixed wall tunnel would be 35% lower if the cavity blockage were 4% and more than 55% lower if the cavity blockage were 9%.

Likewise, If we assume that C_D^* Is Independent of changes in Q and the presence Of boundaries (see Appendix I), we can make some assessment of the Influence of tunnel wall constraint on C_D. Thus

$$C_D = C_D^*(1 + Q) > C_D^*(1 + Q_B) > C_D^*\left[1 + 2\frac{a^2}{R^2}\right]$$

For a cavity of fineness ratio 10 in an unbounded stream, the value of C_D Is roughly 1.03 C_D^* (where C_D^* depends on the shape of the obstacle): In a fixed wall tunnel C_D must be at least 1.08 C_D^* If the cavity blockage Is 4% and at least 1.18 C_D^* If the cavity blockage is 9%.

Conclusions concerning K_D can be derived from Reichardt's relation

$$K_D = Q - \frac{2}{a^2}\int_a^R \Delta^2\, r dr \qquad \ldots\ldots\ (9)$$

where $u_c = U(1 + \Delta)$ (see Appendix II). Assuming, as seems reasonable, that Δ is always positive,

$$\frac{2}{a^2}\int_a^R \Delta^2\, r dr \leq \Delta_{max}\frac{2}{a^2}\int_a^R \Delta\, r dr .$$

By continuity requirements

$$2\pi\int_a^R \{u(0,r) - U\}\, r dr = U\pi a^2$$

i.e.

$$\frac{2}{a^2}\int_a^R \Delta\, r dr = 1 \quad .$$

The maximum value of Δ must occur at the cavity wall where

$$U(1 + \Delta_{max}) = u_c$$

$$= U(1 + Q)^{\frac{1}{2}} .$$

Hence

$$\frac{2}{a^2}\int_a^R \Delta^2\, r dr \leq (1 + Q)^{\frac{1}{2}} - 1$$

and so

$$K_D > (1 + Q)^{\frac{1}{2}}[(1 + Q)^{\frac{1}{2}} - 1] > \tfrac{1}{2}Q > \tfrac{1}{2}Q_R > \frac{a^2}{R^2} \qquad \ldots\ldots (10)$$

In the limiting case when the cavity becomes infinitely long

$$K_{DB} = (1 + Q_R)^{\frac{1}{2}}[(1 + Q_R)^{\frac{1}{2}} - 1] = \left(\frac{\frac{a}{R}}{1 - \frac{a^2}{R^2}}\right)^2 \qquad \ldots\ldots (11)$$

K_D certainly exceeds the cavity blockage. According to Reichardt's results the value of K_D for a cavity of fineness ratio 10 is about 0.03 in unbounded flow: thus a fixed wall tunnel measurement for a cavity of fineness ratio 10 will overestimate K_D by more than 30% if the cavity blockage is 4% and by more than 200% if the cavity blockage is 9%

It is also possible to draw some general conclusions about the effect of the tunnel wall constraint on the cavity diameter. From the definitions of C_D and K_D

$$\frac{a^2}{b^2} = \frac{C_D}{K_D} \leq \frac{C_D^*(1 + Q)}{(1 + Q)^{\frac{1}{2}}[(1 + Q)^{\frac{1}{2}} - 1]} = \frac{C_D^*}{1 - \frac{1}{(1 + Q)^{\frac{1}{2}}}}$$

$$\leq \frac{C_D^*}{1 - \frac{1}{(1 + Q_R)^{\frac{1}{2}}}} = C_D^*\frac{R^2}{a^2}$$

So

$$\frac{a}{b} \leq (C_D^*)^{\frac{1}{4}}\left(\frac{R}{b}\right)^{\frac{1}{2}} , \qquad \ldots\ldots (12)$$

where the equality sign holds in the limiting case of an infinitely long cavity. Figure 5 shows how a/b depends on b/R in the case of a circular disc placed symmetrically in a tunnel under blockage conditions. Thus if b/R is 0.10, i.e. If the model blockage (b^2/R^2) is 1% the value of a/b can never exceed about 3: for cavities In an unbounded stream the value of a/b Is greater than 3 for cavities of quite moderate fineness ratios $(l/a > 5)$ and tends to Infinity when the cavity length tends to infinity.

It will be seen also that, under blockage conditions,

$$\frac{a^2}{R^2} = (C_D^*)^{\frac{1}{2}} \frac{b}{R} \qquad \ldots\ldots (13)$$

so that (8) may be written

$$Q_B = \left\{ \frac{1}{1 - (C_D^*)^{\frac{1}{2}} \frac{b}{R}} \right\} - 1 . \qquad \ldots\ldots (14)$$

in the case of a circular disc in a tunnel, this relation Is plotted in Figure 6, from which it will be seen that, even when the model blockage is only 1%, the blockage cavitation number already exceeds 0.20: at this cavitation number in a virtually unbounded stream only a shot-t cavity with a fineness ratio of about 2 could be expected.

4. To reach more detailed conclusions, it will be assumed here, as in the paper by Armstrong and Tadman (Ref. 2), that the flow round a cavity in a fixed wall tunnel can be represented by a distribution of sources, between the points $X = \pm l$ on the axis of the tunnel, such that the source strength at $X = \eta$ Is $-2\pi c U\eta$ per unit length. As has been seen, this representation leads to conclusions in reasonable agreement wlth experiment when the boundary effects are negligible.

The velocity potential due to a point source of strength S at the origin on the axis of a circular tube of radius R was given by Lamb lRef.81, namely

$$\phi = \frac{S}{2\pi R} \left[-\frac{|x|}{R} + \sum \frac{J_0\left(\lambda \frac{r}{R}\right)}{\lambda J_0^2(\lambda)} e^{-\lambda \frac{|x|}{R}} \right] ,$$

where the summation Is over all the positive zeros of $J_1(\lambda)$. It follows that the velocity In the plane half-way along the cavity due to the singularitiy distribution assumed here is

$$u(0,r) = U + c\left[\left(\frac{l}{R}\right)^2 + 2\sum \frac{J_0\left(\lambda \frac{r}{R}\right)}{\lambda J_0^2(\lambda)} \cdot \left\{ \frac{1}{\lambda}\left(1 - e^{-\lambda \frac{l}{R}}\right) - \frac{l}{R} e^{-\lambda \frac{l}{R}} \right\} \right] \qquad \ldots\ldots (15)$$

Since the velocity at the wall is $u(0,a)$, it follows that

$$(1 + Q)^{\frac{1}{2}} = 1 + c\left\{ \left(\frac{l}{R}\right)^2 + 2 S_1 \right\} \qquad \ldots\ldots (16)$$

where

$$S_1 = \sum \frac{J_0\left(\lambda \frac{a}{R}\right)}{\lambda^2 J_0(\lambda)} \left\{ \frac{1}{\lambda}\left(1 - e^{-\lambda \frac{l}{R}}\right) - \frac{l}{R} e^{-\lambda \frac{l}{R}} \right\} . \qquad \ldots\ldots (17)$$

C Is determined from the condition that the boundary of the cavity passes through the point $(0,a)$; continuity requires that

$$\int_a^R \{u(0,r) - U\}\, 2\pi r dr = U\pi a^2 ,$$

whence
$$\frac{1}{c} = \left[\frac{l}{a}\right]^2 \left\{1 - \left[\frac{a}{R}\right]^2\right\} - 4\frac{R}{a} S_2 \qquad \ldots\ldots (18)$$

where

$$S_2 = \sum \frac{J_1\left[\lambda \frac{a}{R}\right]}{\lambda^2 J_0^2(\lambda)} \left\{\frac{1}{\lambda}\left[1 - e^{-\lambda \frac{l}{R}}\right] - \frac{l}{R} e^{-\lambda \frac{l}{R}}\right\} \qquad \ldots\ldots (19)$$

The summations In (15), (17) and (19) are overall the positive zeros of $J_1(\lambda)$. (16) and (18) correspond to (4) and (5) in the unbounded case and relate l/a to Q for various values of a/R. The method used to sum S_1 and S_2 is briefly described in Appendix II. The results are presented in Figure 7.

For various values of l/a and a/R, Δ was also evaluated from (15) and from these results value: of

$$\frac{2}{a^2}\int_a^R \Delta^2\, r dr$$

were obtained by numerical integration. Hence values of K_D could be obtained from (9).

Assuming that C_D^* Is independent of changes In Q or the presence of boundaries, a/b CM be evaluated from the relation

$$\frac{a^2}{b^2} = \frac{C_D^*\,(1 + Q)}{K_D} .$$

The numerical values obtained are summarised in Table II: here the values of a/b apply to cavities formed behind a circular disc $(C_D^* = 0.79)$. These values were used in the preparation of Figure 8: the "blockage barrier" In Figure 8 represents the relation between Q_B and a/R given In (8) and the curves of constant a/b Intersect with the blockage barrier In points given by (13): the intersections of the curves Of constant a/b with the abscissa are given in Figure 3. Figures 7 and 8 were used to construct Figures 9 and 10, which show how l/b and a/b vary with Q for different values of b/R in the case when the obstacle is a symmetrically placed circular disc. Finally Figure 11, in which a/b Is plotted against l/a for various values of b/R, indicates, again for the case of the circular disc, how the shape of a cavity of given fineness ratio is distorted by the presence of the tunnel walls. Similar results could, of course, be obtained for cavities formed behind any obstacle on which the separation point is fixed simply by taking the appropriate value of C_D^*.

TABLE II

(The values of a/b given in this table apply to cavities formed behind a circular disc, for which C_D^* is 0.791

$\frac{l}{a}$	$\frac{a}{R} = 0.10$			$\frac{a}{R} = 0.15$			$\frac{a}{R} = 0.20$			$\frac{a}{R} = 0.25$			$\frac{a}{R} = 0.30$		
	Q	K_D	$\frac{a}{b}$	Q	K_D	$\frac{a}{b}$	Q	K_D	$\frac{a}{b}$	Q	K_D	$\frac{a}{b}$	Q	K_D	$\frac{a}{b}$
3	0.244	0.215	2.14	0.255	0.218	2.13	0.276	0.225	2.12	0.311	0.238	2.08	0.363	0.260	2.04
4	0.161	0.146	2.52	0.193	0.150	2.49	0.204	0.158	2.45	0.242	0.173	2.38	0.299	0.199	2.27
5	0.097	0.083	3.23	0.113	0.088	3.15	0.142	0.099	3.01	0.186	0.119	2.81	0.249	0.150	2.56
8	0.067	0.055	3.92	0.086	0.062	3.73	0.117	0.075	3.43	0.165	0.098	3.06	0.230	0.131	2.73
9	0.058	0.047	4.22	0.078	0.054	3.98	0.111	0.069	3.56	0.159	0.092	3.16	0.226	0.127	2.75
10	0.051	0.040	4.57	0.072	0.049	4.14	0.106	0.064	3.70	0.155	0.088	3.23	0.222	0.123	2.80

SUMMARY AND CONCLUSIONS

5. The I imitations on cavitation number attainable and related effects when axial ly symmetric cavities are formed in a fixed wall tunnel have been discussed In some detail. In the case of cavltles formed behind a circular disc symmetrically placed In the tunnel It Is estimated that the lowest cavitation number attainable, which corresponds to a cavity of infinite length, exceeds 0.2 even when the model blockage Is only 1%: at a cavitation number of 0.2 in a virtually unbounded stream only a short cavity with a fineness rat lo of about 2 would be expected. For the saw case of the disc with a model blockage of 1% in the tunnel It is estimated that the ratio of maximum cavity diameter to disc diameter can never exceed about 3: for cavities In an unbounded stream thls ratio is greater than 3 for cavities of quite moderate flneness ratios ard tends to Infinity when the cavity length tends to infinity. Estimates of the way in which the dimensions of cavltles formed behind a circular disc depend on the cavitation number and on the ratio of model diameter to tunnel diameter are presented graphically. A further diagram shows the extent to which cavities of various fineness ratios may be distorted by the Inf luence of fixed boundaries.

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APPENDIX I

THE RELATION BETWEEN $C_D(Q)$ AND $C_D(0)$ FOR CAVITIES IN UNBOUNDED FLOW AND BIRKHOFF'S "PRINCIPLE OF STABILITY OF THE PRESSURE COEFFICIENT"

Let C_D^* denote the drag coefficient based on the frontal area of the wetted portion of the obstacle and on the velocity on the cavity wall. Then, If p_N Is the pressure at any point on the wetted portion and p_C the pressure In the cavity,

$$\tfrac{1}{2} \rho u_C^2 S C_D^* = D = \int_S (p_N - p_C) \, dS$$

i.e.

$$C_D^* = \frac{1}{S} \int_S \left[\frac{p_N - p_C}{\tfrac{1}{2} \rho u_C^2}\right] dS .$$

S denotes the frontal area of the wetted portion. Clearly

$$\left[\frac{p_N - p_C}{\tfrac{1}{2} \rho u_C^2}\right]$$

Is always 1 at the front stagnation point and is always zero at the separation point. If changes in the shape of the pressure distribution at Intermediate points can be neglected, then C_D^* will be independent of Q. Since

$$C_D(Q) = C_D^* \left[\frac{u_C}{U}\right]^2 = C_D^* (1 + Q)$$

and, In particular, when $Q = 0$, $C_D(0) = C_D^*$, the assumption that C_D^* Is independent of Q leads to Riechardt's relation

$$C_D(Q) = C_D(0) \, (1 + Q).$$

Birkhoff (Ref.6) has made the additional suggestion that

$$\frac{p_N - p_C}{\tfrac{1}{2} \rho u_C^2}$$

and so also C_D^* are Independent not only of changes in Q but also of the presence of boundaries. Certainly in the two-dimensional case of blockage cavities formed behind a flat strip symmetrically placed In a closed channel exact theory show that C_D^* is almost Independent of the model blockage in the tunnel (Ref.1). This conclusion about C_D^* could not, of course, be expected to hold in cases where the separation position might change with Q.

APPENDIX II

THE RELATION $K_D = Q - \frac{2}{a^2}\int_a^R \Delta^2 \, r\,dr$

Consider the fluid inside a control surface consisting (in Figure 1) of two planes normal to the stream, one far upstream and one at the maximum diameter of the cavity, the walls of the tunnel, the wetted portion of the nose and part of the cavity wall.

The flux of momentum out of the control surface to the right

$$= -\rho U^2 \int_0^R 2\pi r\,dr + \rho \int_a^R u^2(0,r)\, 2\pi r\,dr$$

$$= -\rho U^2 \pi a^2 + 2\pi\rho \int_a^R (u^2 - U^2)\, r\,dr$$

$$= -\rho U^2 \pi a^2 + 2\pi\rho U^2 \int_a^R (\Delta^2 + 2\Delta)\, r\,dr$$

where $u(0,r) = U(1+\Delta)$.

The integral of pressures, parallel to the axis of symmetry, over the control surfaces

$$= p_0 \int_0^R 2\pi r\,dr - \int_0^b p_N \cdot 2\pi r\,dr - p_C \int_b^a 2\pi r\,dr - \int_a^R p_e\, 2\pi r\,dr$$

$$= p_0\, \pi a^2 - \int_0^b (p_N - p_C)\, 2\pi r\,dr - p_C \int_0^a 2\pi r\,dr + 2\pi \int_a^R (p_0 - p_e)\, r\,dr$$

$$= (p_0 - p_C)\, \pi a^2 - D + \pi\rho \int_a^R (u^2 - U^2)\, r\,dr$$

$$= (p_0 - p_C)\, \pi a^2 - D + \pi\rho U^2 \int_a^R (\Delta^2 + 2\Delta)\, r\,dr$$

Equating these two expressions, we have

$$(p_0 - p_C)\,\pi a^2 - D + \pi\rho U^2 \int_a^R (\Delta^2 + 2\Delta)\, r\,dr = -\rho U^2 \pi a^2 + 2\pi\rho U^2 \int_a^R (\Delta^2 + 2\Delta)\, r\,dr$$

i.e. $$D = (p_0 - p_C)\,\pi a^2 + \rho U^2 \pi a^2 - \pi\rho U^2 \int_a^R (\Delta^2 + 2\Delta)\, r\,dr.$$

From continuity considerations

$$U \pi a^2 = \int_a^R (u - U)\, 2 \pi r dr$$

i.e.

$$U \pi a^2 = \pi U \int_a^R 2 \Delta r dr$$

Hence

$$D = (p_O - p_C) \pi a^2 - \pi \rho U^2 \int_a^R \Delta^2 r dr$$

and so

$$K_D \equiv \frac{D}{\frac{1}{2} \rho U^2 \pi a^2} = Q - \frac{2}{a^2} \int_a^R \Delta^2 r dr .$$

APPENDIX III

SUMMATION OF S_1 AND S_2

Since S_1 is a slowly convergent series, it was found convenient to determine an asymptotic expression, $a_{(1)m}$, for the successive terms, $S_{(1)m}$, of S_1, to sum the difference between $S_{(1)m}$ and $a_{(1)m}$ (requiring only a few terms) and to convert $\Sigma a_{(1)m}$ into a rapidly convergent series which can also be summed with only a few times. The same procedure was followed in summing S_2.

S_1 was written as follows:

$$S_1 = \sum_{m=0}^{\infty} a_{(1)m} + \sum_{m=0}^{\infty} (S_{(1)m} - a_{(1)m}),$$

where

$$a_{(1)m} = \alpha \frac{\cos\left\{\left(m\frac{a}{R} + \beta\right)\pi\right\}}{\left(m + \frac{5}{4}\right)^{3/2}} + \frac{\alpha}{8\pi}\left(\frac{R}{a} + 3\frac{a}{R}\right)\frac{\sin\left\{\left(m\frac{a}{R} + \beta\right)\pi\right\}}{\left(m + \frac{5}{4}\right)^{5/2}},$$

$$\alpha = \frac{1}{\pi\left(2\frac{a}{R}\right)^{\frac{1}{2}}} \quad \text{and} \quad \beta = \frac{1}{4}\left(5\frac{a}{R} - 1\right)$$

Similarly

$$S_2 = \sum_{m=0}^{\infty} a_{(2)m} + \sum_{m=0}^{\infty} (S_{(2)m} - a_{(2)m})$$

where

$$a_{(2)m} = \frac{\alpha}{\pi} \frac{\sin\left\{\left(m\frac{a}{R} + \beta\right)\pi\right\}}{\left(m + \frac{5}{4}\right)^{5/2}} + \frac{3\alpha}{8\pi^2}\left(\frac{R}{a} - \frac{a}{R}\right)\frac{\cos\left\{\left(m\frac{a}{R} + \beta\right)\pi\right\}}{\left(m + \frac{5}{4}\right)^{7/2}}$$

$a_{(1)m}$ and $a_{(2)m}$ were derived by employing the asymptotic expansions of the Bessel Functions in $S_{(1)m}$ and $S_{(1)m}$.

$$\sum_{m=0}^{\infty} a_{(1)m} \quad \text{and} \quad \sum_{m=0}^{\infty} a_{(2)m}$$

were summing by using the following results, which were derived by MacFarlane's method (Ref.91 of summing slowly convergent series:

$$\sum_{m=0}^{\infty} \frac{\cos\left\{\left(m\frac{a}{R} + \beta\right)\pi\right\}}{\left(m + \frac{5}{4}\right)^{3/2}} = \sum_{n=0}^{\infty} \frac{\sin\left\{(2n+1)\frac{\pi}{4}\right\}\zeta\left(-n + \frac{3}{2}, \frac{5}{4}\right)}{n!}\left(\pi\frac{a}{R}\right)^n$$

$$\sum_{m=0}^{\infty} \frac{\sin\left\{\left(m \frac{a}{R} + \beta\right) \pi\right\}}{\left(m + \frac{5}{4}\right)^{5/2}} = \sum_{n=0}^{\infty} \frac{\sin\left\{(2n-1)\frac{\pi}{4}\right\} \zeta\left(-n + \frac{5}{2}, \frac{5}{4}\right)}{n!} \left(\pi \frac{a}{R}\right)^n$$

$$\sum_{m=0}^{\infty} \frac{\cos\left\{\left(m \frac{a}{R} + \beta\right) \pi\right\}}{\left(m + \frac{5}{4}\right)^{7/2}} = \sum_{n=0}^{\infty} \frac{\sin\left\{(2n+1)\frac{\pi}{4}\right\} \zeta\left(-n + \frac{7}{2}, \frac{5}{4}\right)}{n!} \left(\pi \frac{a}{R}\right)^n$$

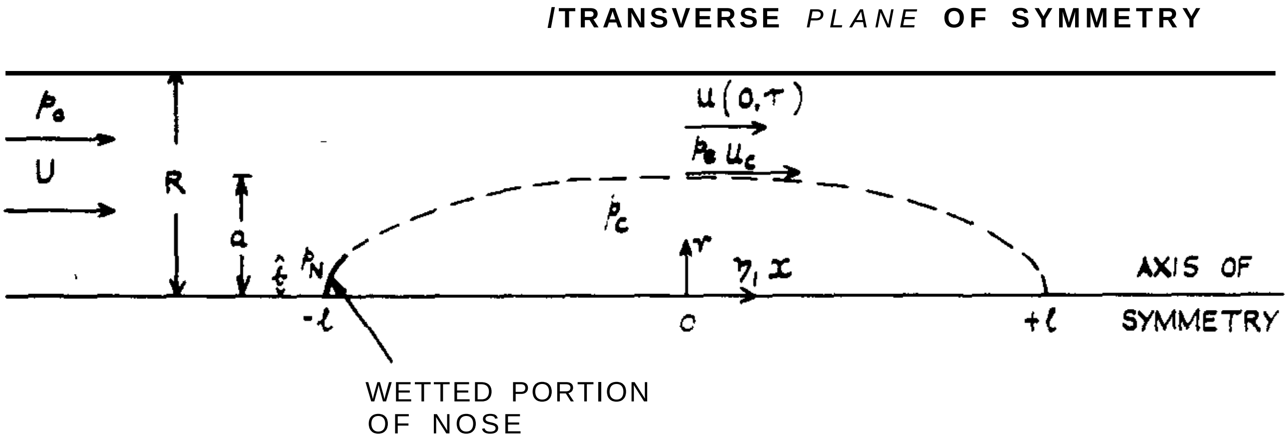
$\zeta(\nu,h)$ denotes the generalised Riemann ~-function, which is defined for $\nu > 1$ by

$$\zeta(\nu,h) = \sum_{q=0}^{\infty} (q+h)^{-\nu} \qquad (\nu > 1)$$

and for $\nu < 1$ by analytic continuation. Tables of $\zeta(\nu,h)$ are given in Ref.10. These series of generalised Riemann ~-functions are very rapidly convergent.

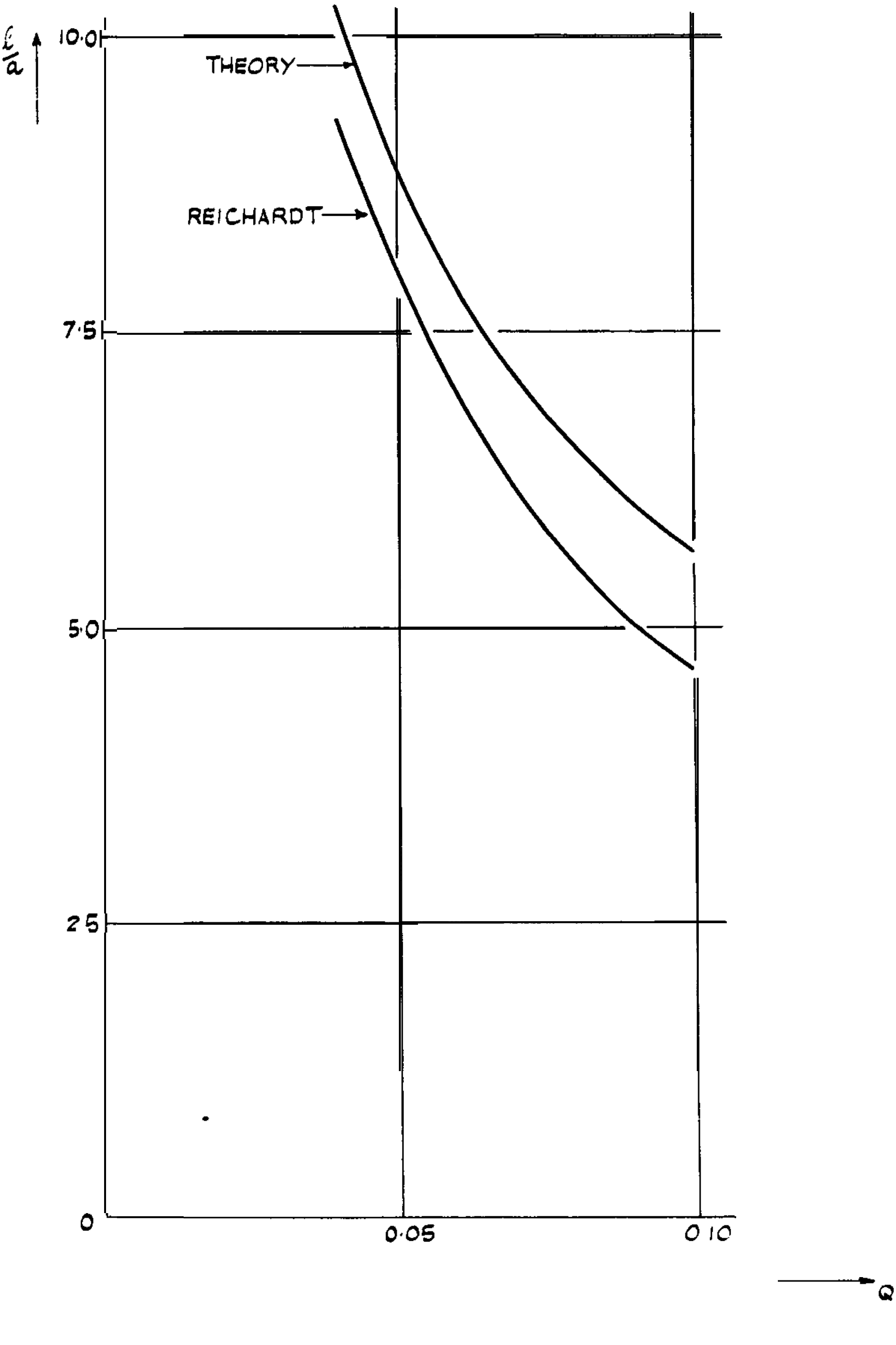
LIST OF SYMBOLS

a	maximum radius of cavity
b	maximum radius of wetted portion of obstacle
C	a parameter governing strength of singularity distributions
C_D	drag coefficient based on frontal area of wetted portion of obstacle and upstream velocity $(D = \frac{1}{2} \rho U^2 \pi b^2 C_D)$
C_D^*	drag coefficient based on frontal area of wetted portion of obstacle and velocity on cavity wall $(D = \frac{1}{2} \rho u_c^2 \pi b^2 C_D^*)$
D	drag
J_0, J_1	Bessel functions
K_D	drag coefficient based on frontal area of cavity and upstream velocity $(D = 3 \rho U^2 \pi a^2 K_D)$
l	half-length of cavity
p_c	pressure in cavity
p_e	pressure un water at points In plane normal to axis of symmetry and situated half-way along cavity
p_N	pressure at points on wetted portion of obstacle
4	free stream pressure
Q	cavitation number $\left\{Q = \frac{p_0 - p_c}{\frac{1}{2} \rho U^2}\right\}$
Q_B	blockage cavitation number (lower limit)
R	radius of tunnel
r	radial distance from axis of symmetry
U	velocity far upstream
$u(0,r)$	velocity at points In plans normal to axis of symmetry and situated half-way along cavity
u_c	velocity on cavity wall
X	axial distance
A	defined by $u(0,r) = U(1 + 4$
η	axial distance
ρ	density of fluid
ϕ	velocity potential
$\frac{a^2}{R^2}$	Is taken as the measure of "cavity blockage"
$\frac{b^2}{R^2}$	Is taken as the measure of "model blockage".



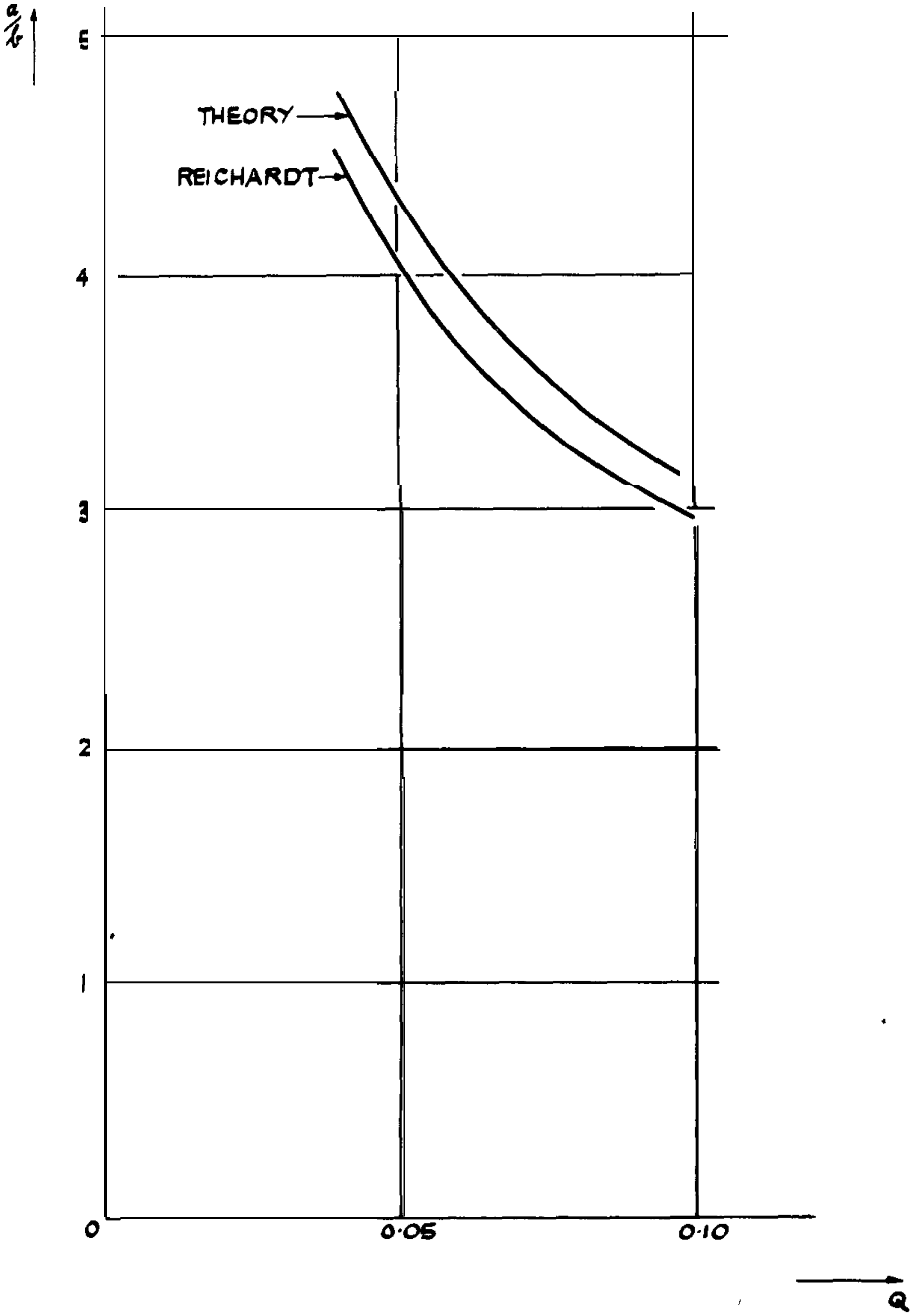
NOTATION

FIG. I.



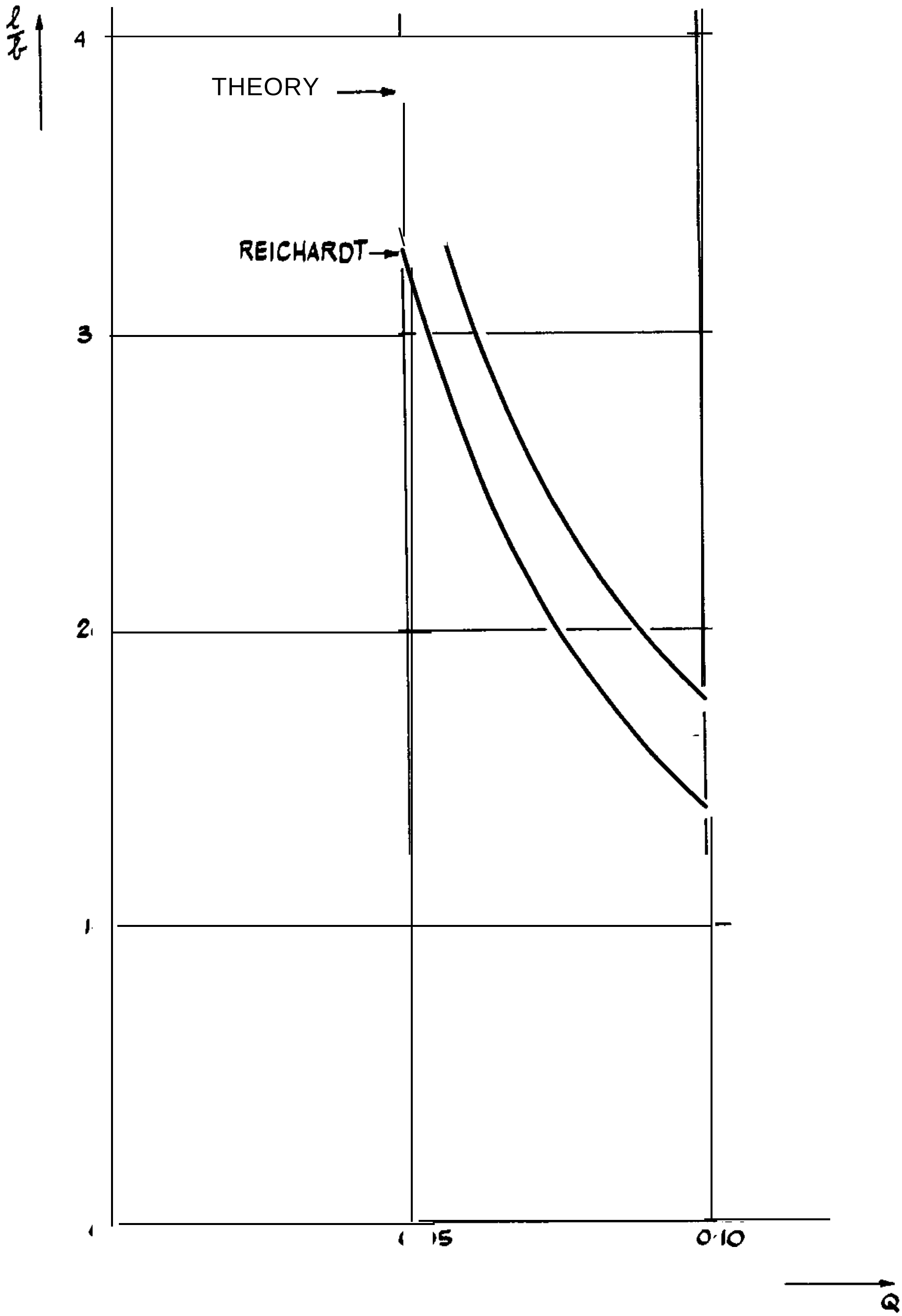
FINENESS RATI0 : CAVITIES IN UNBOUNDED FLOW.

FIG 2.



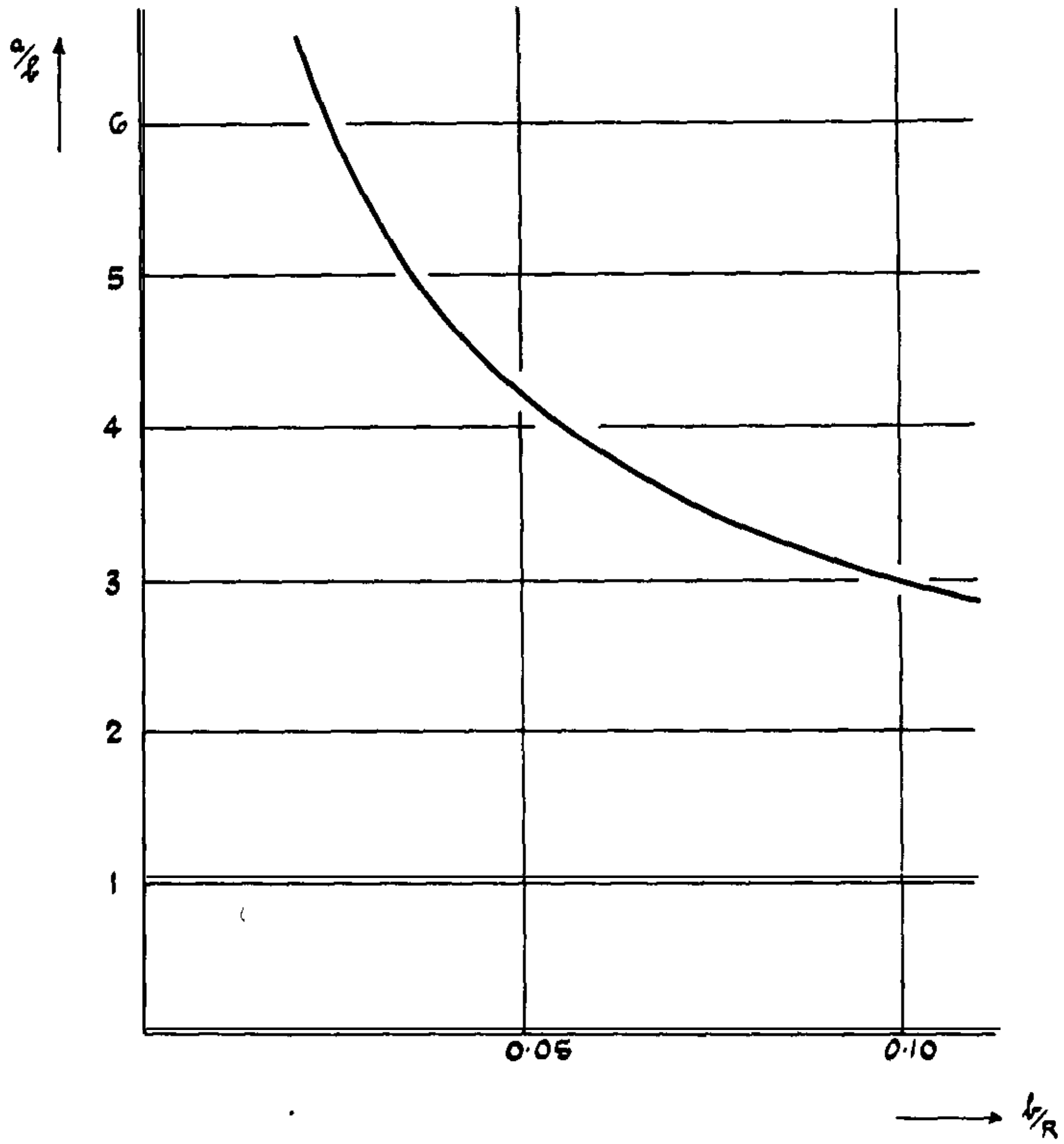
CAVITY DIAMETER: CAVITIES BEHIND CIRCULAR DISC, HELD NORMAL TO STREAM, IN UNBOUNDED FLOW.

FIG.3.



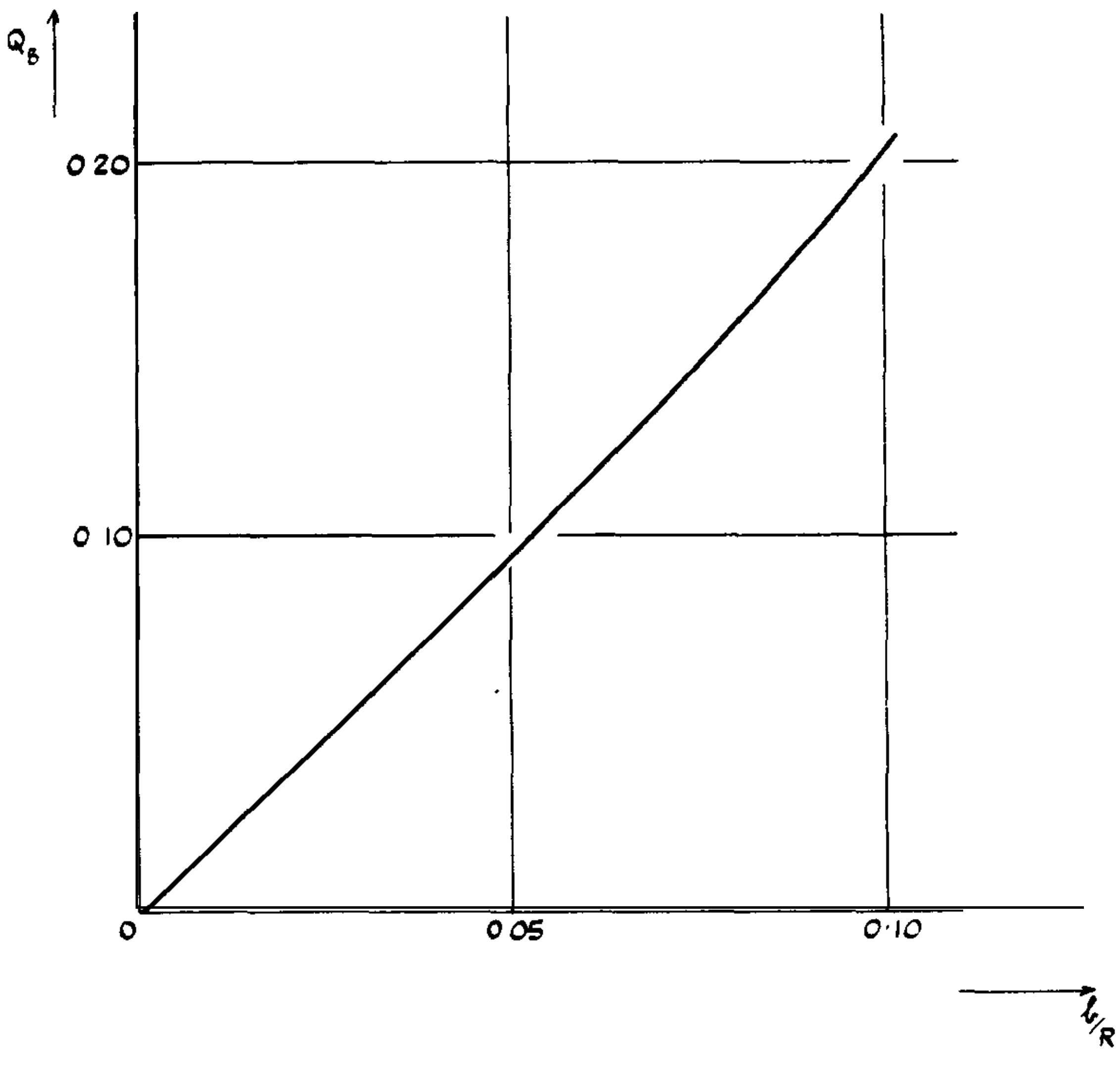
CAVITY LENGTH: CAVITIES BEHIND CIRCULAR DISC, HELD NORMAL TO STREAM, IN UNBOUNDED FLOW.

FIG.4.



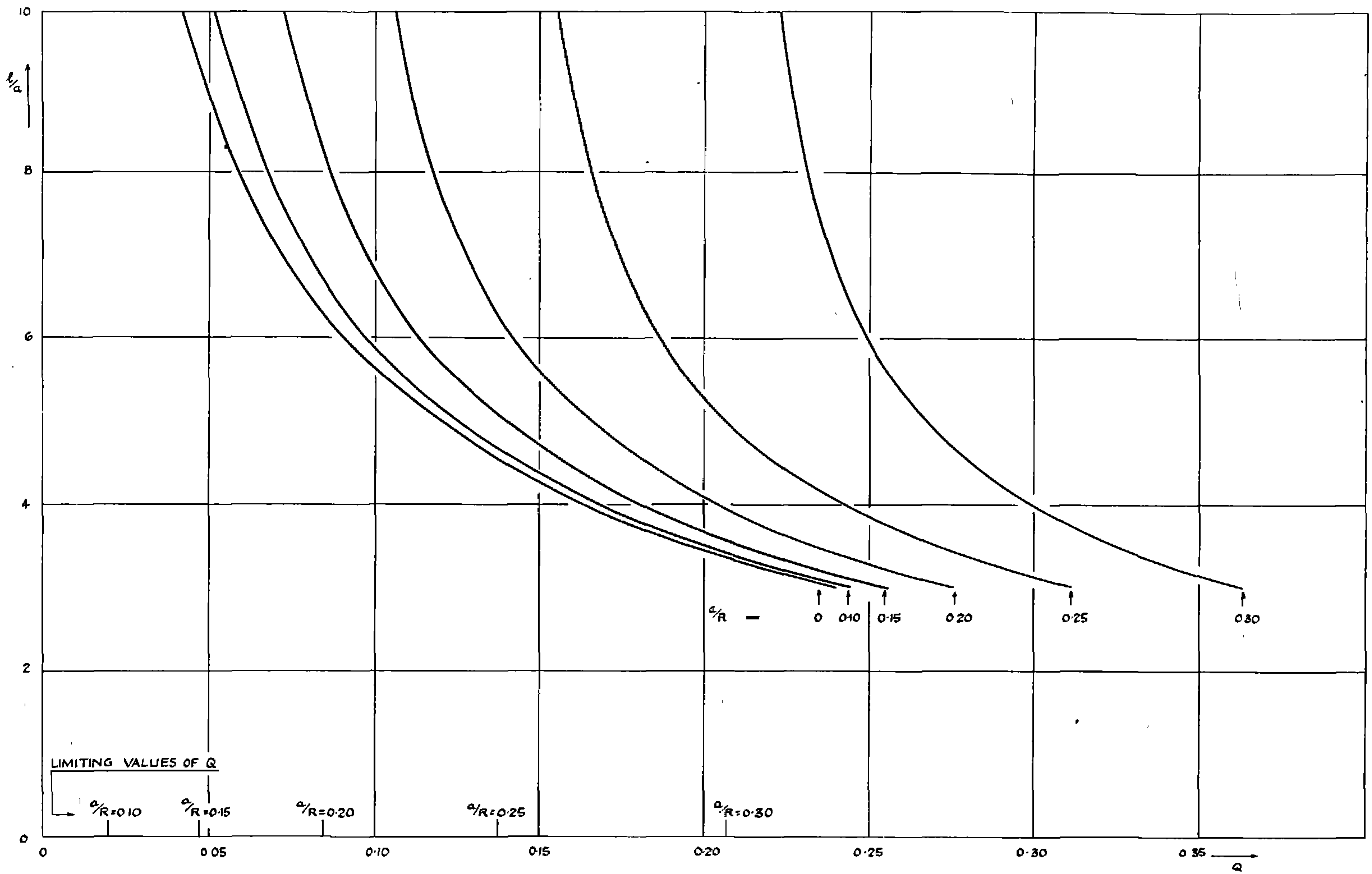
BLOCKAGE VALUES OF CAVITY DIAMETER: CAVITIES BEHIND CIRCULAR DISC SYMMETRICALLY PLACED IN A FIXED WALL TUNNEL.

FIG. 5.



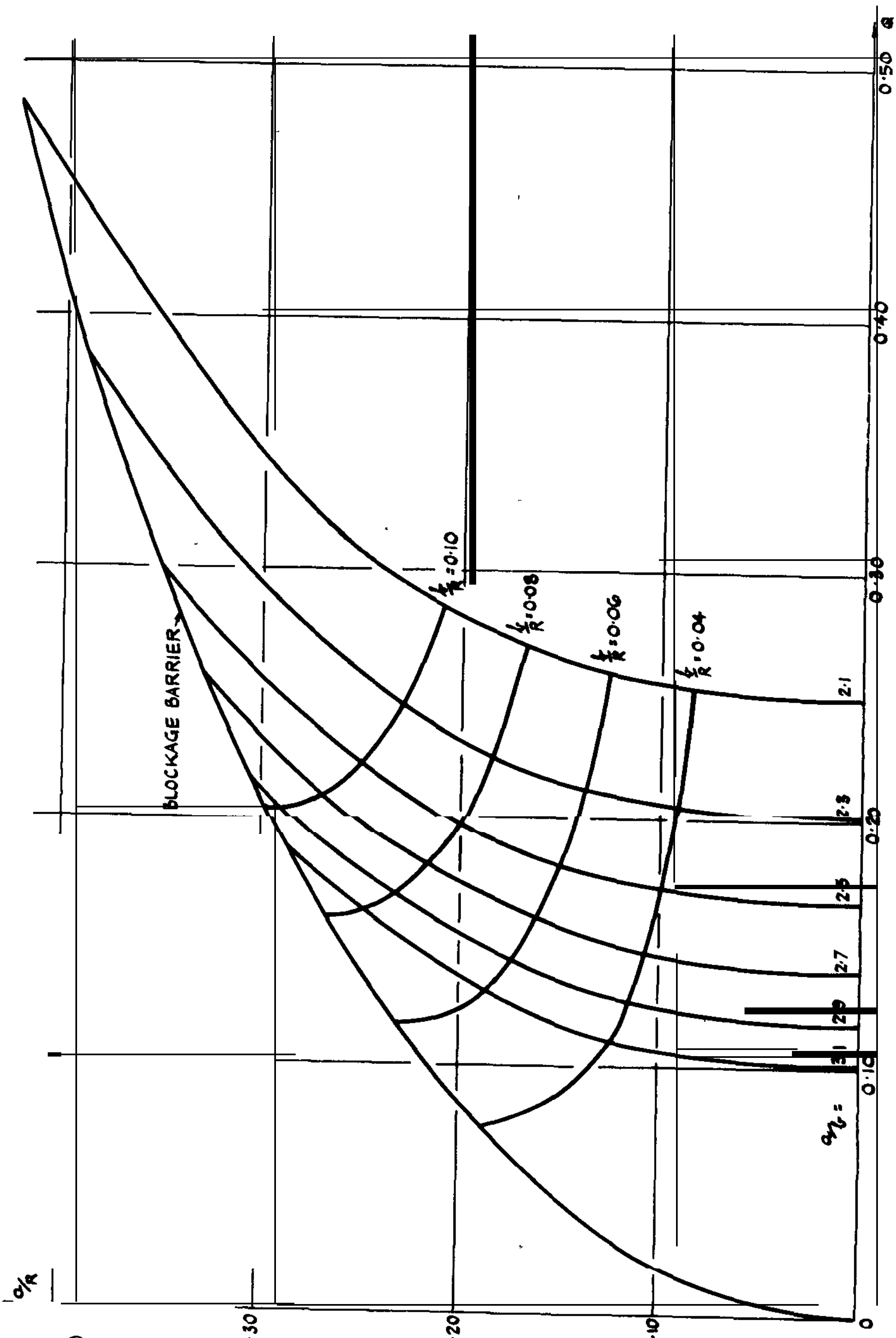
BLOCKAGE CAVITATION NUMBERS : CAVITIES BEHIND CIRCULAR DISC SYMMETRICALLY PLACED IN A FIXED WALL TUNNEL.

FIG. 6.



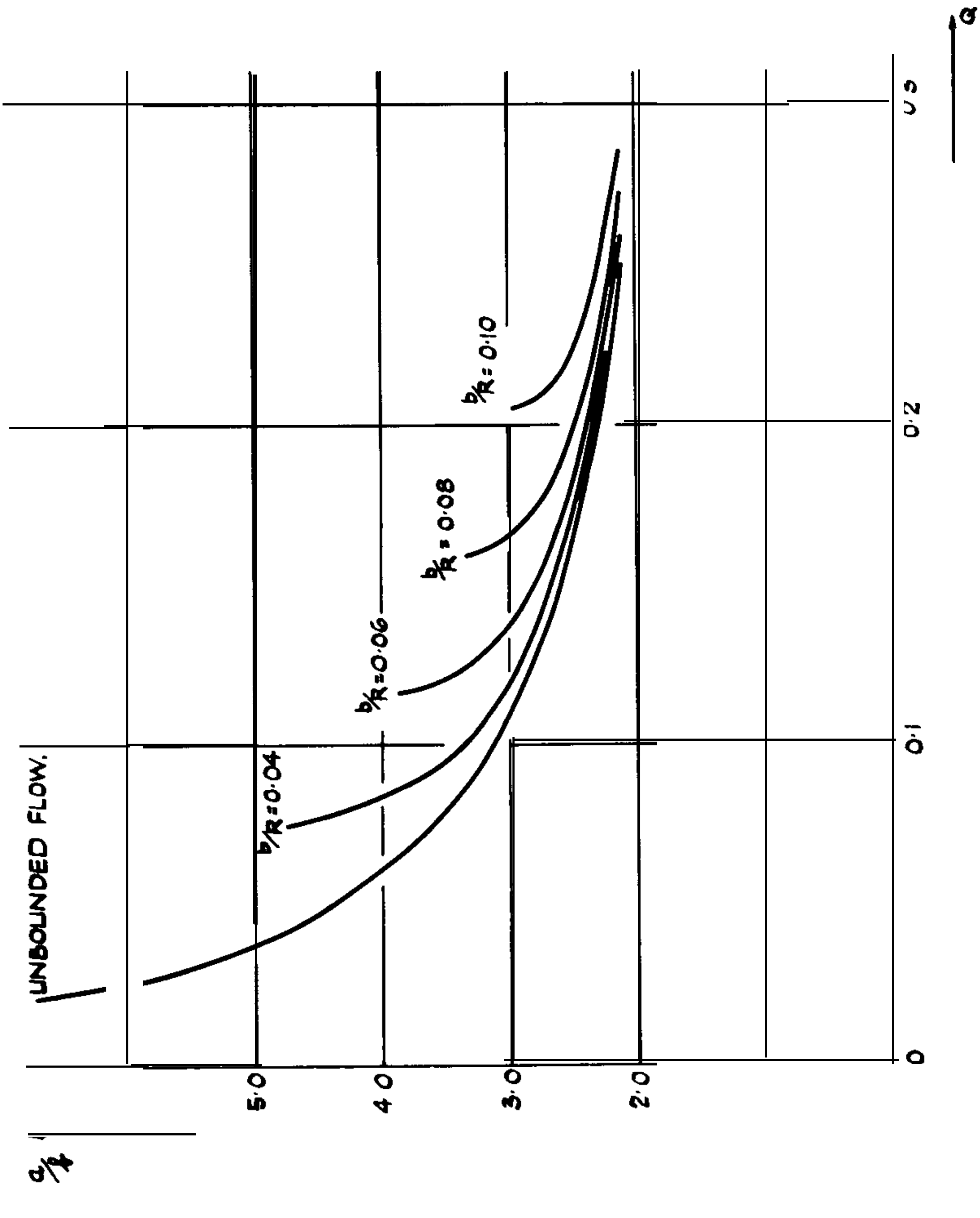
FINENESS RATIO: CAVITIES IN A FIXED WALL TUNNEL.

FIG. 7.



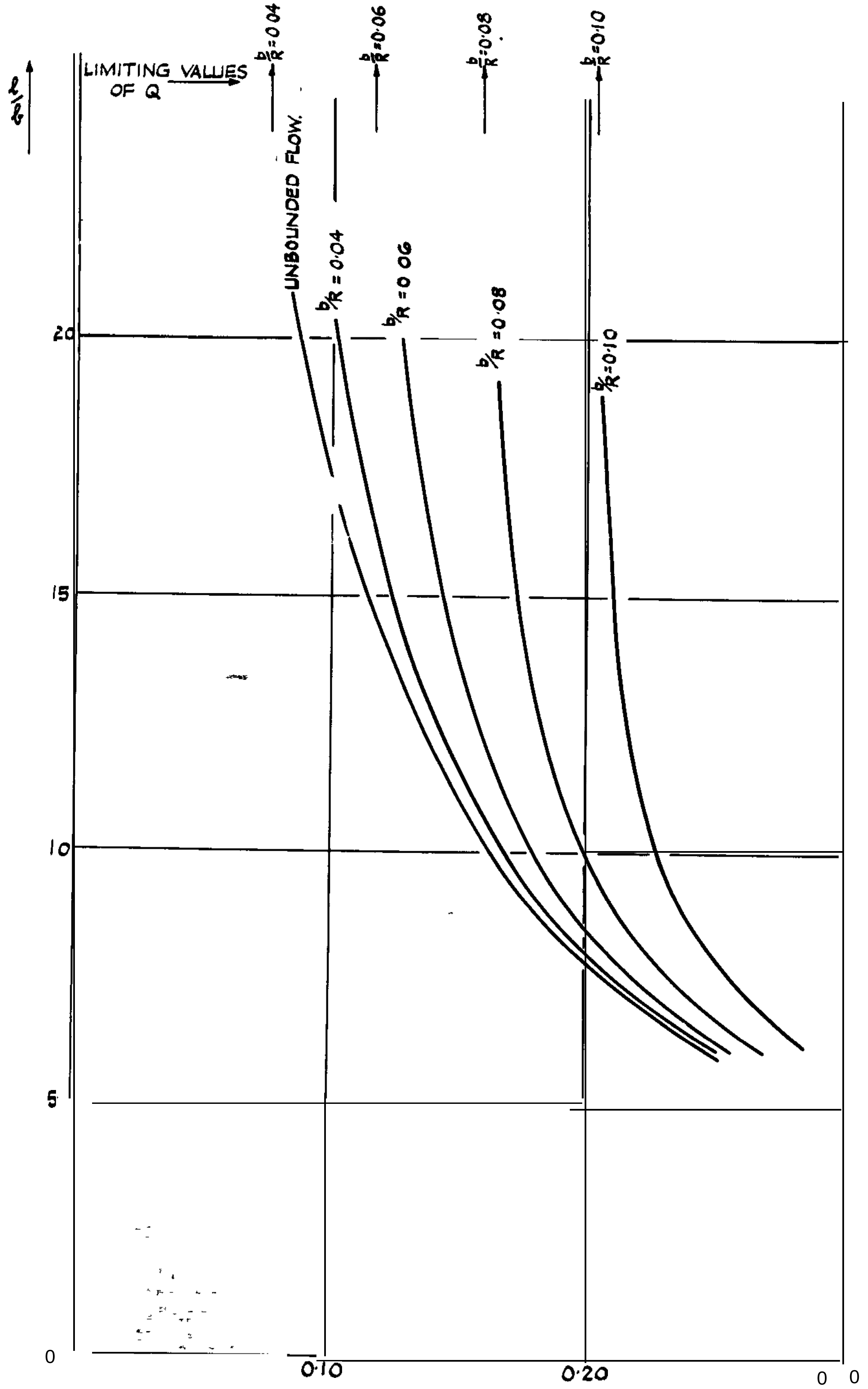
THIS DIAGRAM APPLIES TO CAVITIES FORMED BEHIND A CIRCULAR DISC SYMMETRICALLY PLACED IN A FIXED WALL TUNNEL.

FIG. 8.



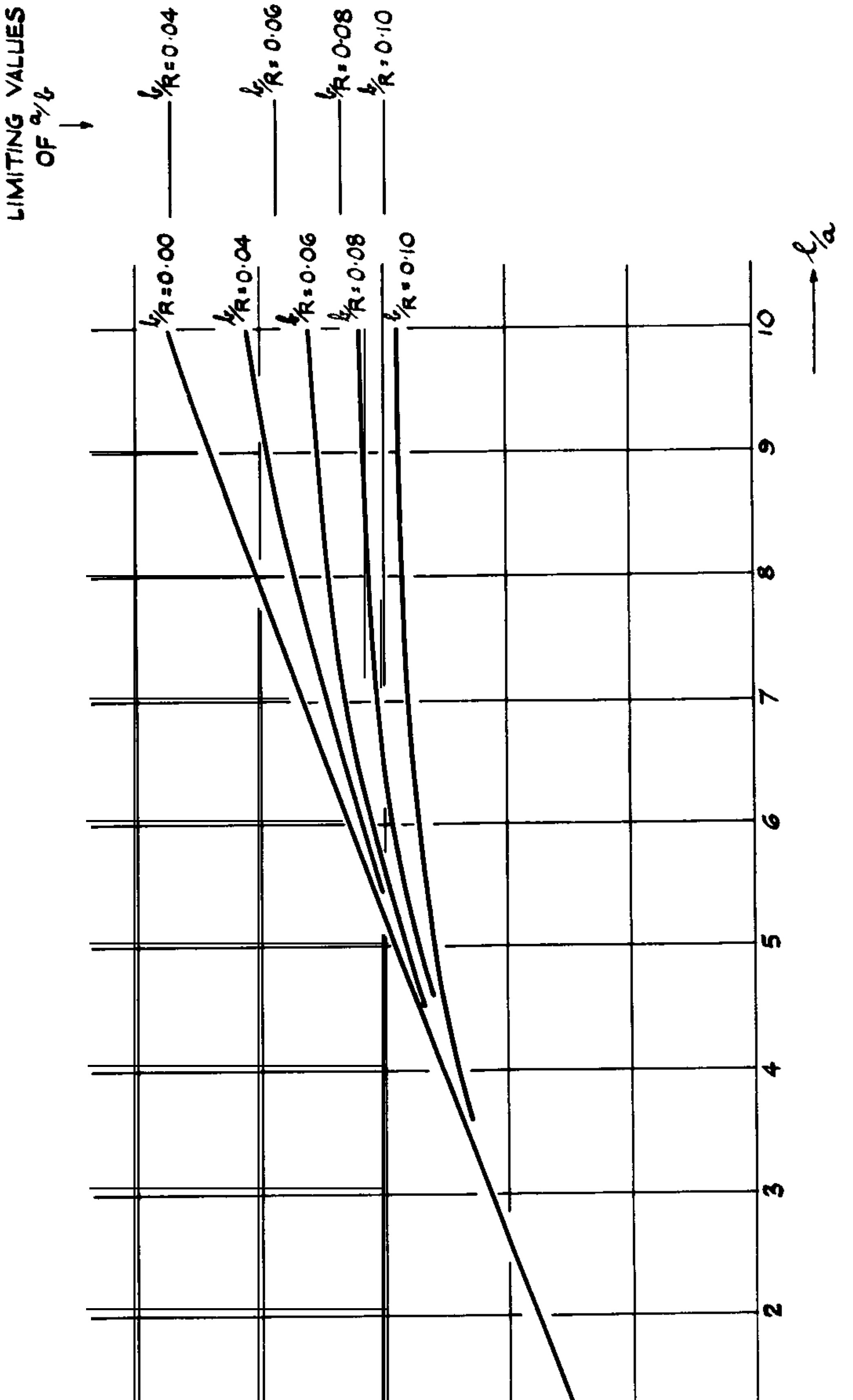
MAXIMUM CAVITY RADIUS : CAVITIES BEHIND CIRCULAR DISC SYMMETRICALLY PLACED IN A FIXED WALL TUNNEL.

FIG. 9.



CAVITY LENGTH : CAVITIES BEHIND CIRCULAR DISC SYMMETRICALLY PLACED IN A FIXED WALL TUNNEL.

FIG.10.



GEOMETRY OF CAVITIES FORMED BEHIND CIRCULAR DISC SYMMETRICALLY PLACED IN A FIXED WALL TUNNEL.

. FIG. II.

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