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A Theoretical Investigation of the Factors Affecting Stalling Flutter of Compressor Blades

By

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A theoretical investigation of the factors affecting stalling flutter of compressor blades

- by -

A. D. S. Carter

SUMMARY

Recent work has drawn attention to the importance of the critical flutter velocity in applying cascade data to axial compressors. The hope has been expressed that it was primarily an aerodynamic phenomenon, though the possibility of mechanical damping being an important factor was not excluded. In this Report the mechanism of stalling flutter has been re-examined to establish, as far as is at present possible, the major parameters involved.

On the basis of the hypothesis advanced, the factors governing the critical flutter velocity have been isolated. The variation of this velocity with respect to blade thickness has been deduced, and compared with test results. Agreement is good, but further work is necessary before the hypothesis can be accepted "in toto". Mechanical damping appears to be a major parameter, and may severely limit the practical application of the theoretical solution.

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1.0 Introduction

In a recent report (Reference 1) attention has been drawn to the importance of the critical flutter velocity, or Mach number, in applying cascade data to axial compressors. The hope was expressed in that report that it was primarily an aerodynamic phenomenon, though the possibility of mechanical damping being an important factor was not excluded. In this Report the mechanism of stalling flutter has been re-examined in order to establish, as far as is at present possible, the major parameters involved. The "critical flutter velocity" has been defined as the velocity at which stalling flutter becomes appreciable. This term is used even though stalling flutter appears to be a "soft" vibration.

Throughout the Report standard notation (Reference 2) has been adopted. Additional symbols have been defined in the text.

2.0 Theoretical investigation

2.1 Review of previous work

The general equation of motion of a vibrating blade can be written

$$m\ddot{x} + \frac{\omega}{\pi} \cdot \delta \cdot m\dot{x} + \omega^2 m x = F(x\,\dot{x}\,\ddot{x}) \quad .. \quad .. \quad (1)$$

where x = displacement

ω = frequency of vibration

δ = logarithmic decrement in vacuo (strictly $\delta = f(x)$)

m = mass of blade per unit length

and $F(x\,\dot{x}\,\ddot{x})$ = the deviation of the total aerodynamic force per unit length from its equilibrium value in steady flow.

The function $F(x\,\dot{x}\,\ddot{x})$ is extremely complicated. This is particularly so in the stalling region. So far as the author is aware only two attempts have been made to represent the function mathematically and so solve the equation. Both solutions are based on the assumption that the aerodynamic force on a blade during vibration is the same as that on a static blade at the instantaneous inlet angle. This is not a strict representation of the physical phenomenon, of course. It neglects aerodynamic hysteresis, which is known to be present (Reference 3 and Figure 1). Nevertheless it forms a useful starting point.

The first solution to equation (1) was given by Den Hartog in connection with the vibration of electric transmission lines (Reference 4). He put

$$F(x\,\dot{x}\,\ddot{x}) = k\,{}^{\dot{x}}/_{V} = c_1\,\dot{x} \quad .. \quad .. \quad .. \quad .. \quad .. \quad (2)$$

$$\delta = c_2\,x^2 \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (3)$$

where k, c_1 and c_2 are constants. Equation (1) then reduces to Van der Pols equation for which a solution is known. Sisto (Reference 5) carries the solution a step further by making the aerodynamic force a function of the inlet angle. He puts

$$F\,(x\ \dot{x}\ \ddot{x}) = f(\alpha_i) \text{ where } \alpha_i = \alpha_m + \dot{x}/V_m \quad \ldots \quad (4)$$

$$\delta = \text{constant} \quad \ldots \quad (5)$$

$F\,(x\ \dot{x}\ \ddot{x})$ is then expressed as a power series in α_1. By choosing the constants in the power series appropriately, Sisto's approach can be made a closer representation of physical case than Den Hartog's original treatment.

The extent to which Sisto's solution represents the actual physical phenomenon may be gauged from Figures 2 and 3, which compare some calculated and experimental results. The agreement is not very good though a certain similarity is present. In particular it will be noted that in the stalled region where the slope of the force coefficient-incidence curve is positive theory would predict no flutter. The reason for the discrepancy is, most probably, the neglect of aerodynamic hysteresis.

Both Pearson (References 6 and 7), Kilpatrick (References 8 and 9) and the author , have assumed that the damping, δ, can, in the limiting case, be taken as zero. If it is assumed as before that

$$F\,(x\ \dot{x}\ \ddot{x}) = c_1\,\dot{x} \quad \ldots \quad (2)$$

it can be shown (see References quoted) that

$$-\,c_1 = \frac{\partial F}{\partial \alpha_1} \cdot \frac{\text{Cos}\,(\alpha_1 - \zeta)}{M_n\,V_c} + \frac{\partial F}{\partial M_n}\,\frac{\text{Sin}\,(\alpha_1 - \zeta)}{V_c} \qquad (6)$$

where $\partial F/\partial \alpha_1$ and $\partial F/\partial M_n$ are the derivitives of the static force curve with respect to the fluid inlet angle and Mach number respectively. Flutter will occur when the aerodynamic damping $(-\,c_1\,\dot{x})$ becomes negative. Various attempts have been made to correlate flutter with equation (6) or parts of it. No attempt has been wholly satisfactory, again probably due to neglecting aerodynamic hysterisis.

2.2 Estimation of critical flutter velocity with phase lag

A closer approximation to the actual flow conditions can be obtained by assuming that the aerodynamic force on the blade lags behind the motion of the blade by an unknown angle, φ. Assuming sinusoidal motion the function $F\,(x\ \dot{x}\ \ddot{x})$ is then given by

$$F\,(x\ \dot{x}\ \ddot{x}) = F_{\dot{x}}\,x_o\,\omega\,\text{Cos}\,(\omega t - \varphi) \quad \ldots \quad (7)$$

where $F_{\dot{x}}$ is an aerodynamic constant and x_o is the maximum amplitude of the blade. It can then be shown (see Appendix I) that the work done per cycle by the aerodynamic force is

$$\Delta W = \pi/2\rho \; v_1 \; c \; \partial C_F/\partial \alpha \; \text{Cos} \; (\alpha_1 - \zeta) \; x_0^2 \; \omega \; \text{Cos} \; \varphi \quad .. \quad (8)$$

Flutter will occur if this is greater than the work done by the damping force, but no flutter will occur if it is less than that. The critical flutter velocity will be given by equating these quantities. From which, as shown in Appendix I

$$V_f = K_f \; t \; f \; \delta_0 \; \sigma/\rho \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (9)$$

where K_f is some constant dependent only on the aerodynamics, and δ_0 is the logarithmic decrement in the neutral position.

It is rather interesting to note that using Sisto's solution to the general equation (1) the critical flutter velocity will be given by the same equation (see Appendix II), though the constant K_f will have a different physical significance.

From a practical point of view, therefore, equation (9) can be used with some confidence to predict the critical flutter velocity irrespective of the fundamental theory used in deriving it, so long as the constant K_f is determined directly from the experimental results. One might expect that, for any blade, K_f will be a function of incidence and, on phase lag theory, a function of the frequency parameter.

2.3 Application of theory to experimental investigation

Of the quantities in equation (9), the constant K_f and the damping, as expressed by the decrement δ_0, are the most difficult to measure. The best test of equation (9) is therefore one in which both of these quantities are kept constant. This is most easily done by concentrating the tests on a given blade. The series of tests described in the next section were carried out by thinning down one typical compressor blade, so meeting the conditions outlined above. The blade natural frequency will be given by

$$f_b = k' \; t/h^2 \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (10)$$

We then have, since the blade material density and air density are constant, and also the blade height in these tests.

$$V_f = k'' \left(\frac{t}{h}\right)^2 = k \; t^2 \quad .. \quad .. \quad .. \quad .. \quad .. \quad (11)$$

where k is some constant.

3.0 Experimental investigation

3.1 Equipment

The tests were carried out in the very simple cascade tunnel shown in Figure 4. Air was supplied under pressure to the 2 in. square working section via a 20 : 1 contraction. It was then discharged to atmosphere through the cascade. The cascade was mounted in a turntable carried in the floor of the tunnel so allowing the incidence to be varied. The top of the tunnel was made of perspex so that any vibration could be observed directly.

No traversing gear was used. The only instrumentation consisted of a pitot situated in the inlet section and wall static tapping situated just ahead of the cascade.

The cascade was made up of blades having the following specification:-

Base Profile	C.1
Camber Line	Circular Arc
Camber Angle	45°
Stagger Angle	-17°
Chord	0.667 in.
Height	2.0 in.
Aspect Ratio	3.0
Pitch/Chord Ratio	0.80

The blades of the cascade had a maximum thickness of 13 per cent chord. However the central blade, which was the only one under test was thinned down successively from 9.7 per cent to 6.6 per cent chord. Vibration was recorded on the central blade only, and for the majority of the testing the remaining blades were not vibrated. No measurement of blade material and root damping was taken at the time of the tests.

3.2 Test technique

It should be pointed out that these tests were carried out some years ago, before the more refined techniques now being used (see References 9 and 10 for example) were developed. Nevertheless the data obtained is reliable within the limits of the test procedure, and is considered adequate for the analysis in hand.

Testing consisted of setting the cascade at a fixed incidence. The airspeed was then very gradually increased until flutter was observed visually. The pitot and static pressures were noted and the test repeated at other incidences, usually in 1° steps. The critical flutter velocity could then be calculated. Having witnessed both these tests and those carried out under modern conditions the author would estimate that the critical flutter velocity obtained in this manner corresponds to an alternating flutter stress of about $\pm$ 10 tons/in^2.

Tests were first carried out on a blade 0.064 in. maximum thickness. The blade thickness was then reduced by hand in steps of about 0.003 in. down to 0.045 in. This blade broke under test. All the tests reported in this note were carried out on the same blade so that the root damping may be supposed to be constant.

3.3 Test results

The test results have been presented graphically in Figures 5 and 6, where the critical flutter Mach number and the critical flutter velocity, respectively, have been plotted against incidence. Both torsional and flexural vibration was encountered on these blades, though the test notes indicate that the flexural form usually appeared first with increasing Mach number. The flutter appeared to be more uniform than present records indicate, but this may well have been an optical illusion. Some choking flutter was observed at high Mach number and negative incidences, but has not been recorded in this note. The curves plotted represent the stalling flutter boundary zone, and define the incidence and velocity limitations.

4.0 Discussion of results and further work

The critical flutter velocity has been plotted against section thickness on a logarithmic basis for a number of representative incidences in Figures 7 and 8. The best straight line through the test points has been drawn in and also, for comparison, the curve given by the expression

$$V_f = k\ t^2 \qquad \cdots \qquad (11)$$

Assuming that the test results can be represented by the equation

$$V_f = k\ t^n \qquad \cdots \qquad (12)$$

values of "n" have been determined and plotted against incidence in Figure 9. It will be seen that agreement between the experimental and theoretical values is good, though the test results would show a variation of "n" with incidence. Referring back to equation (9) it is difficult to see which of the factors is dependent on "t" in a manner which varies with incidence. All can be eliminated except K_f. Examination of isolated aerofoil data suggests that the blade maximum thickness has little effect on the aerodynamic behaviour above stall (see Reference 11 for example), i.e. one would expect K_f to be constant at any given incidence. However the departure of "n" from its theoretical value is so small that the theoretical curve could be used in most cases with sufficient practical accuracy.

On the evidence of the theoretical and experimental results quoted in this Report it would appear that estimation of the critical flutter velocity is amenable to very simple theoretical treatment. It is true that only two of the parameters have been varied. The general equation for the critical flutter velocity deduced in Section 2.2 is

$$V_f = K_f\ t\ f\ \delta_0\ \sigma/\rho \qquad \cdots \qquad (9)$$

and before any reliance can be put in this equation further work using refined techniques is necessary. An investigation into the effect of air density is already well in hand, and will be reported later. An additional examination of the frequency effect, by change of blade height, is also in hand. In all this work a major unknown factor arises from changes in the damping. Although damping can be measured it is doubtful if sufficient accuracy is being obtained by present techniques. More generally, the hope expressed in the introduction that the damping would only occur in the equation (9) as a second order term has not been fulfilled by the theoretical investigation. The inability to predict the damping of blades without resort to test seems an insuperable obstacle in the way of a general practical application of equation (9).

5.0 Conclusions

A theoretical investigation has been made into the factors governing the critical flutter velocity. On the basis of the hypothesis advanced, the variation of the critical flutter velocity with respect to blade thickness has been deduced and compared with test results. Agreement is good, but further work is necessary before the hypothesis can be accepted "in toto". It would appear that the blade damping is a major parameter in determining the critical flutter velocity, and this entails a severe practical handicap in generally applying the theoretical formula.

6.0 Acknowledgments

The author is indebted to Mrs. M. Mettam for carrying out the test work, and to Miss H. P. Hughes for the numerical work etc. used in this Report.

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APPENDIX I

Derivation of Equation (8)

It is known from the solution of the general equation (1) given by Den Hartog that, with typical values of the constants in equations (2) and (3) the motion of a blade in stalled flutter will be approximately sinusoidal. This is, of course, confirmed by experimental evidence. We can thus write

$$x = x_o \text{ Sin } \omega t \qquad \ldots \qquad (13)$$

$$\text{and} \quad \dot{x} = x_o \, \omega \text{ Cos } \omega t \qquad \ldots \qquad (14)$$

where x_o is the maximum amplitude, and may be a slowly varying function of time.

The aerodynamic force function $F\,(x\,\dot{x}\,\ddot{x})$ can then be written

$$F\,(x\,\dot{x}\,\ddot{x}) = \dot{x}\,F_{\dot{x}} = x_o\,\omega \text{ Cos } \omega t \cdot F_{\dot{x}} \qquad \ldots \qquad (15)$$

where $F_{\dot{x}}$ is a constant depending only on the aerodynamic conditions. Assuming that the aerodynamic force on a blade during vibration is the same as that on a static blade at the instantaneous angle of attack, it can be shown (References 7 and 10) that

$$F_{\dot{x}} = \frac{\partial F}{\partial \alpha_1} \; \frac{\text{Cos}(\alpha_1 - \zeta)}{V_1} \qquad \ldots \qquad (16)$$

where the term containing $\partial F / \partial Mn$ has been neglected when considering stalling flutter

$$\therefore \; F_{\dot{x}} = \tfrac{1}{2}\,\rho\,V_1\,c\;{}^{\partial C_F}/_{\partial\alpha}\,\text{Cos}(\alpha_1 - \zeta) \qquad \ldots \qquad (17)$$

A value for $F\,(x\,\dot{x}\,\ddot{x})$ may be obtained by substituting for $F_{\dot{x}}$ into equation (15), but a closer approximation to the physical phenomenon may be obtained by introducing an arbitrary phase lag, φ, of the aerodynamic forces behind the motion of the blade. Hence we get

$$F\,(x\,\dot{x}\,\ddot{x}) = x_o\,\omega\,\text{Cos}(\omega t - \varphi) \cdot \tfrac{1}{2}\,\rho\,V_1\,c\;{}^{\partial C_F}/_{\partial\alpha}\,\text{Cos}(\alpha_1 - \zeta) \qquad (18)$$

the work done per cycle by this force will be

$$\Delta W = \int_0^{2\pi/\omega} F\,(x\,\dot{x}\,\ddot{x})\,\dot{x}\,dt \qquad \ldots \qquad (19)$$

or $$\Delta W = \frac{\pi}{2} \rho V_1 \; c \, x_0^2 \; \omega \frac{\partial C_F}{\partial \alpha} \operatorname{Cos}(\alpha_1 - \zeta) \operatorname{Cos} \varphi \quad .. \quad .. \qquad (20)$$

Now the kinetic energy absorbed in damping per cycle is given by

$$\Delta W = 2 W \delta \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \qquad (21)$$

where $$W = \tfrac{1}{2} m \, x_0^2 \, \omega^2 \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \qquad (22)$$

m being the mass of the blade per unit length.

Hence $$\Delta W = m \, x_0^2 \; \omega^2 \; \delta \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \qquad (23)$$

Hence flutter will occur when

$$\frac{\pi}{2} \rho V_1 \; c \, x_0^2 \; \omega \frac{\partial C_F}{\partial \alpha} \operatorname{Cos}(\alpha_1 - \zeta) \operatorname{Cos} \varphi > m \; x_0^2 \; \omega^2 \; \delta \quad .. \qquad (24)$$

or the critical flutter velocity will be given by

$$V_f = \frac{m \, \omega \, \delta_0}{\pi/2 \; \rho \; c \; {\partial C_F}/{\partial \alpha} \operatorname{Cos}(\alpha_1 - \zeta) \operatorname{Cos} \varphi} \quad .. \quad .. \qquad (25)$$

writing $$m = k \, c \, t \, \sigma \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \qquad (26)$$

and $$\omega = 2 \pi f \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \qquad (27)$$

and $$K_f = \frac{4 k}{{\partial C_F}/{\partial \alpha} \operatorname{Cos}(\alpha_1 - \zeta) \operatorname{Cos} \varphi} \quad .. \quad .. \quad .. \quad .. \qquad (28)$$

we get $$V_f = K_f \, t \, f \, \delta_0 \, \sigma/\rho \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \qquad (29)$$

APPENDIX II

Derivation of Equation (8) from Sisto's solution of Equation I

In the notation adopted in this Report, Sisto gives the following expression for the equilibrium flutter amplitudes

$$\left(\frac{x_o}{c}\right)^2 = 2\,\frac{\left\{\partial C_F/\partial\alpha + \left(4m/\pi\,\rho\,c^2\right)\left(\omega c/2V\right)\left(\delta\right)\right\}\left\{2V/\omega c\right\}^2}{\partial^3 C_F/\partial\alpha^3} \qquad (30)$$

The critical flutter velocity will then be given by $x_o = o$

$$\therefore \quad -\frac{\partial C_F}{\partial\alpha} = \frac{4\,m}{\pi\,\rho\,c^2}\cdot\frac{\omega c}{2\,V_f}\cdot\delta \quad \ldots\ldots\ldots \quad (31)$$

Or writing as before

$$m = k\,c\,t\,\sigma \quad \ldots\ldots\ldots\ldots \quad (26)$$

and $$\omega = 2\,\pi\,f \quad \ldots\ldots\ldots\ldots \quad (27)$$

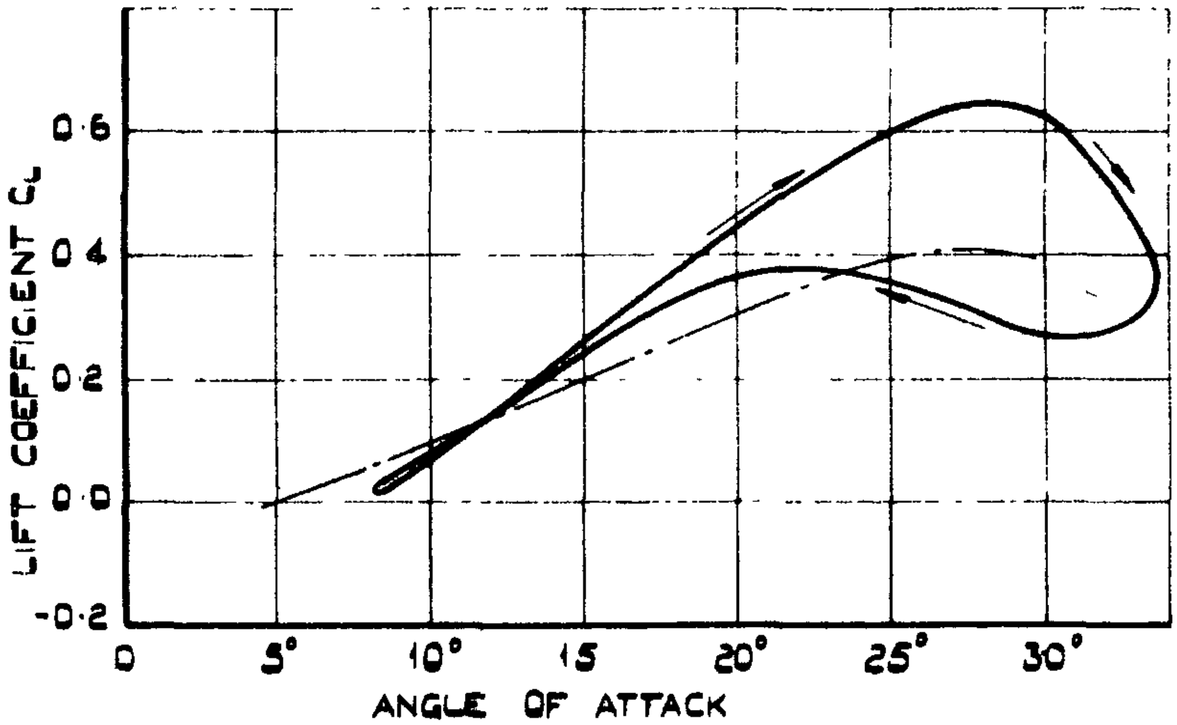
we get

$$V_f = \frac{8\,k\,t\,f\,\delta\,\sigma/\rho}{\partial C_F/\partial\alpha} \quad \ldots\ldots\ldots\ldots \quad (32)$$

or $$V_f = K_f\,t\,f\,\delta\,\sigma/\rho \quad \ldots\ldots\ldots\ldots \quad (33)$$

where $$K_f = 8\,k/\partial C_F/\partial\alpha. \quad \ldots\ldots\ldots\ldots \quad (34)$$

FIG. I.



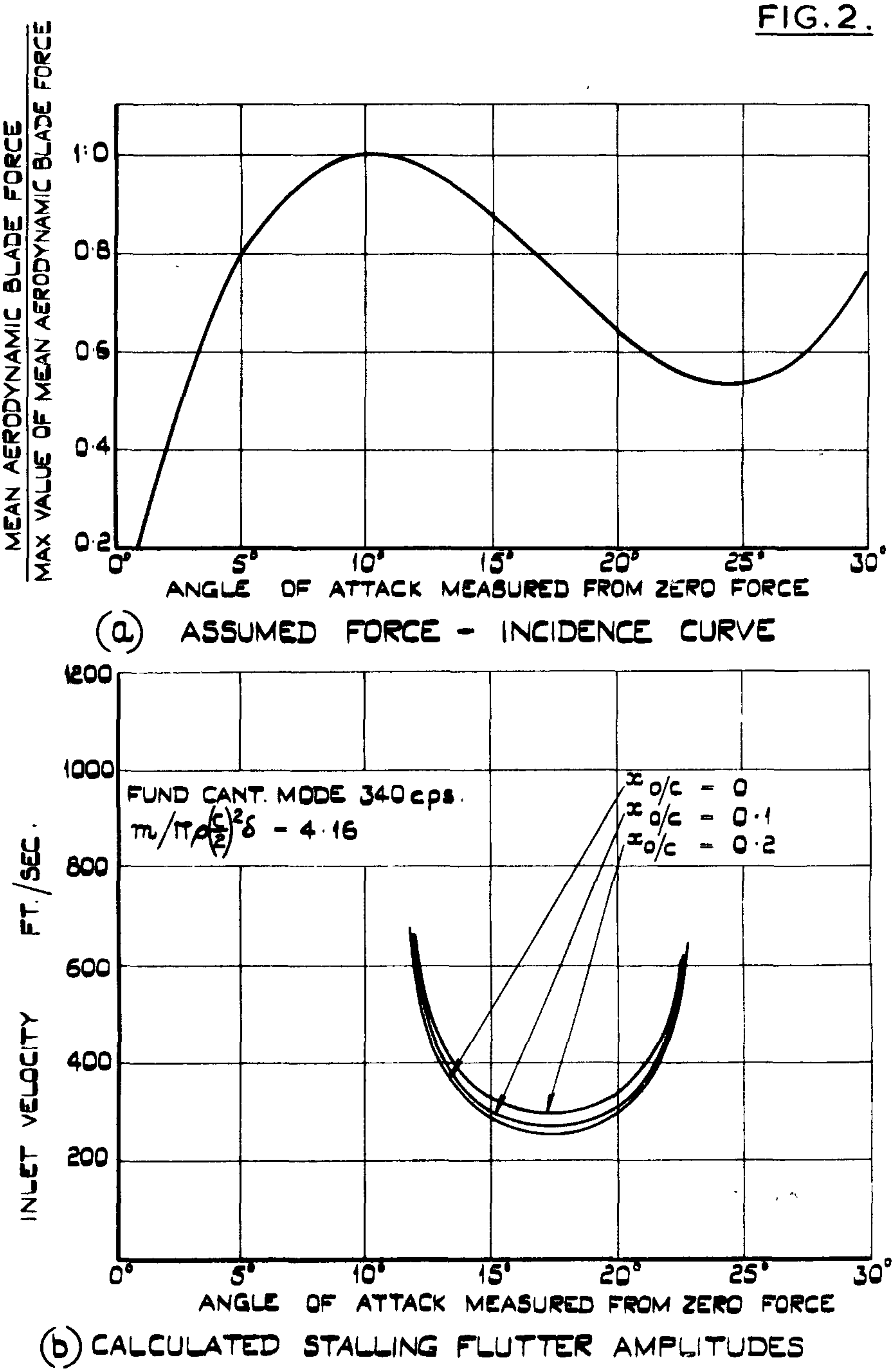
STATIC LIFT - INCIDENCE CURVE.

DYNAMIC LIFT - INCIDENCE CURVE.

AEROFOIL	NACA 6·5409
STAGGER	-52°
PITCH/CHORD	1·0

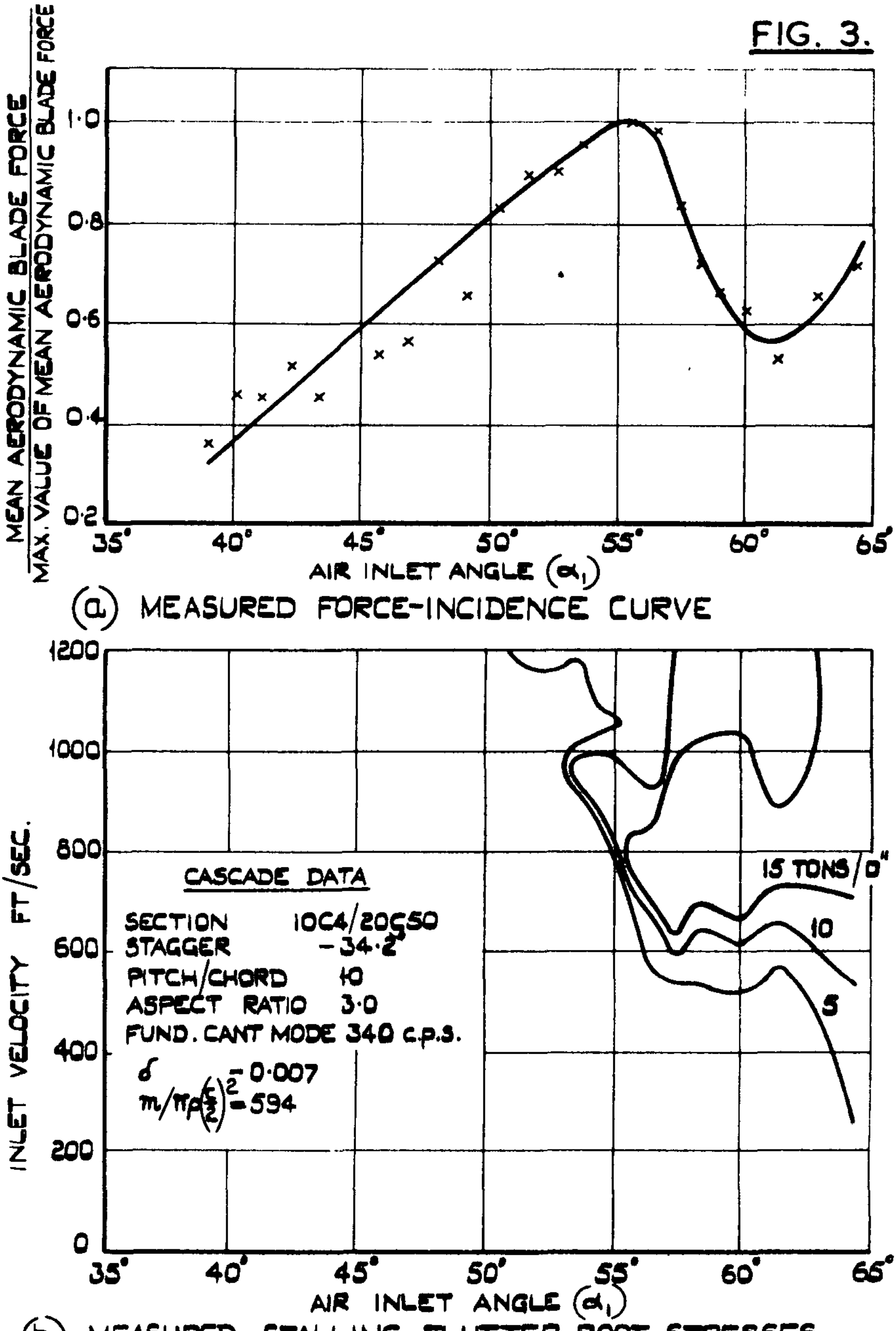
RESULTS GIVEN BY JAN R. SCHNITTGER REF 3

COMPARISON OF STATIC AND DYNAMIC LIFT INCIDENCE CURVES.



CALCULATED STALLING FLUTTER IN A REPRESENTATIVE CASCADE.

FIG. 3.

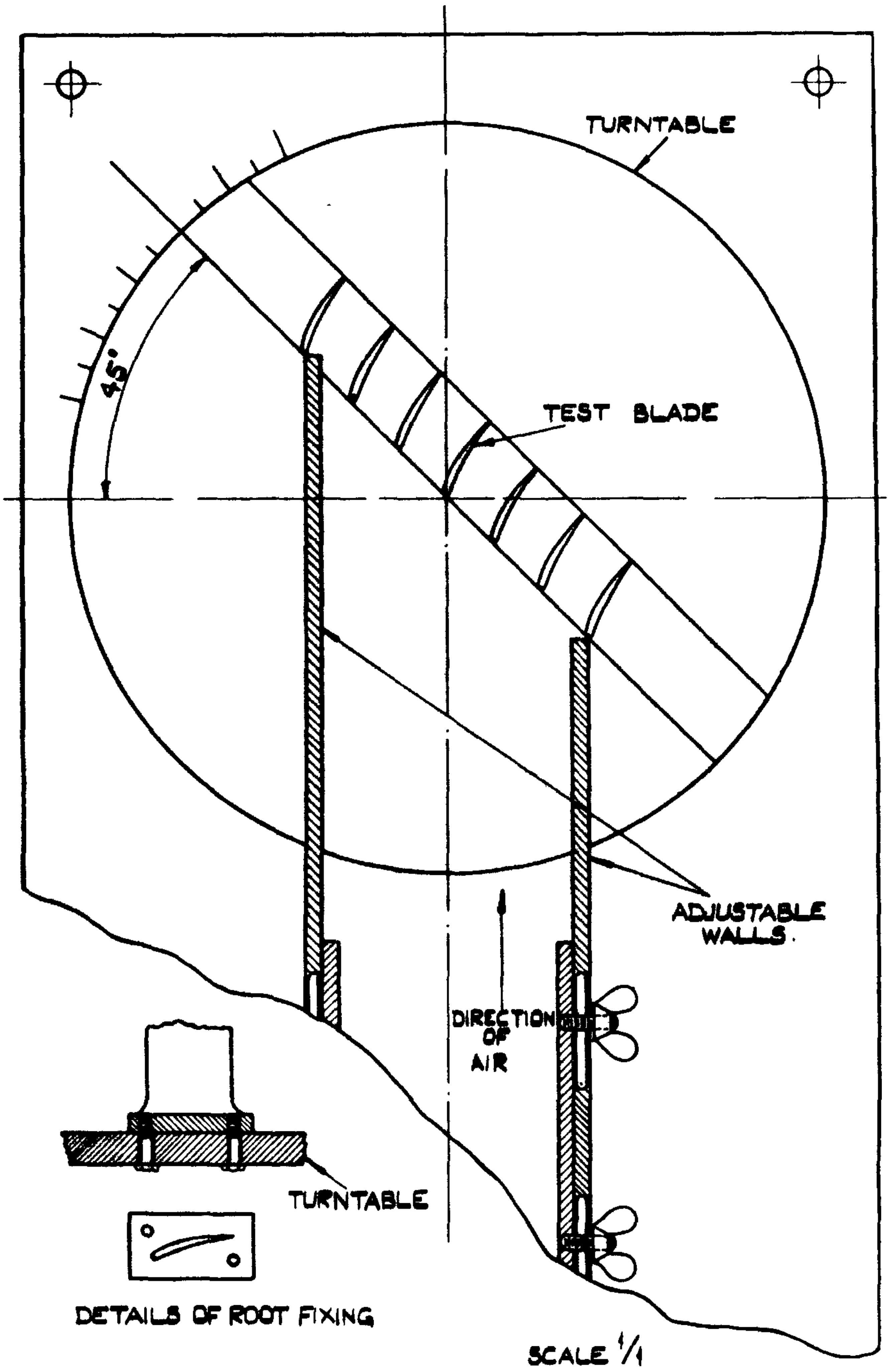


(a) MEASURED FORCE-INCIDENCE CURVE

(b) MEASURED STALLING FLUTTER ROOT STRESSES

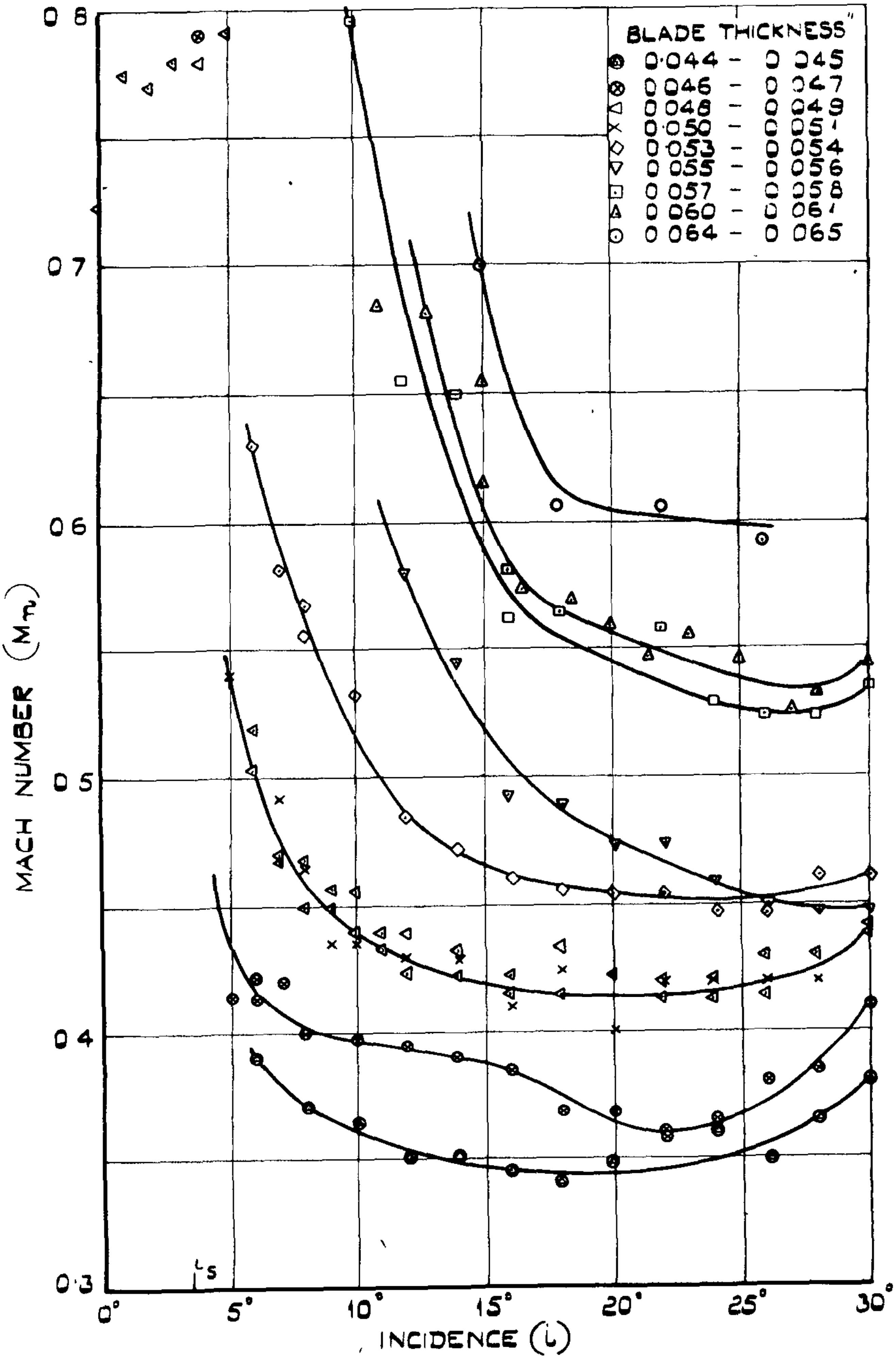
MEASURED STALLING FLUTTER IN A REPRESENTATIVE CASCADE.

FIG. 4.



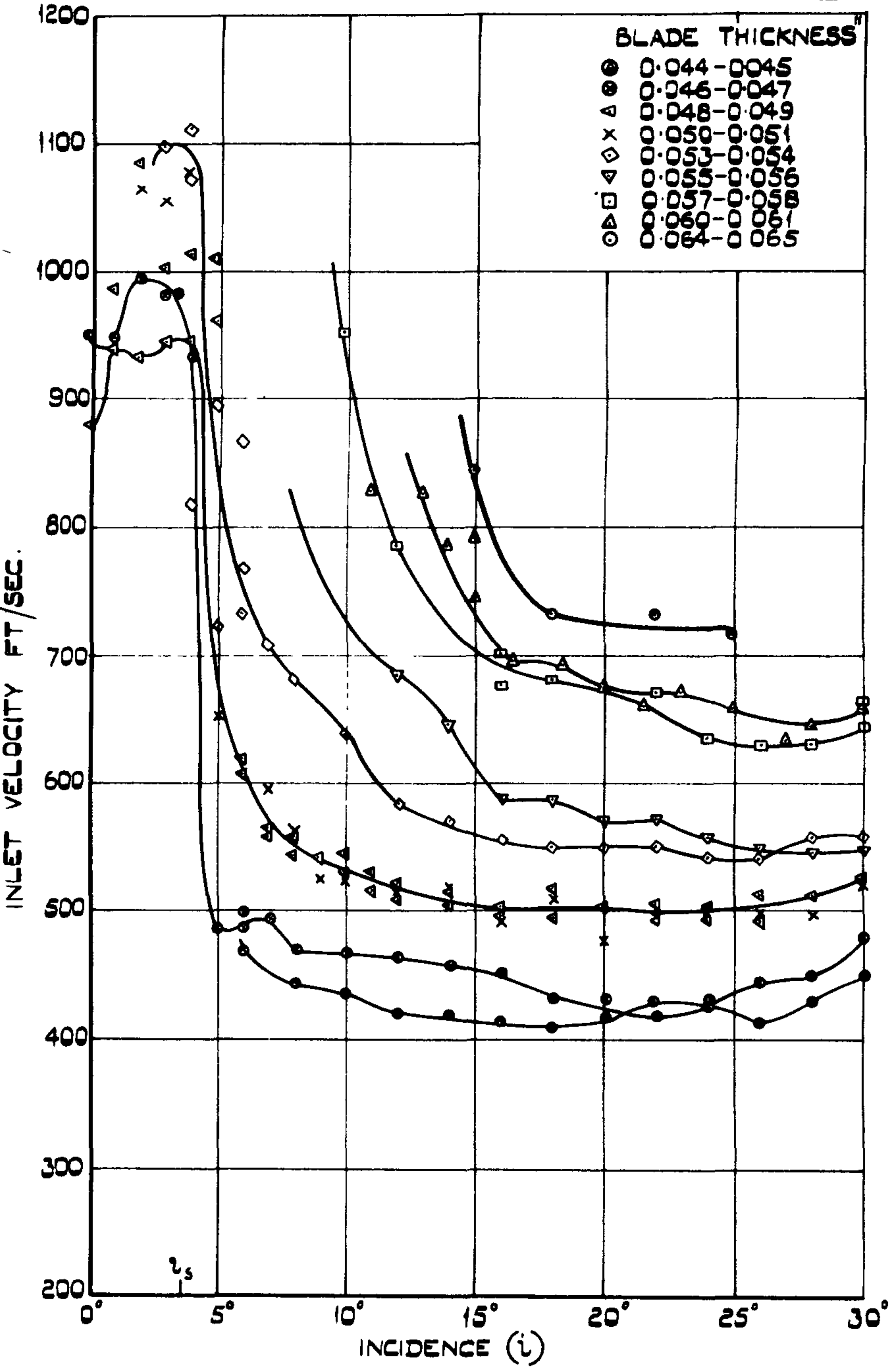
VIBRATION CASCADE TUNNEL

FIG. 5.



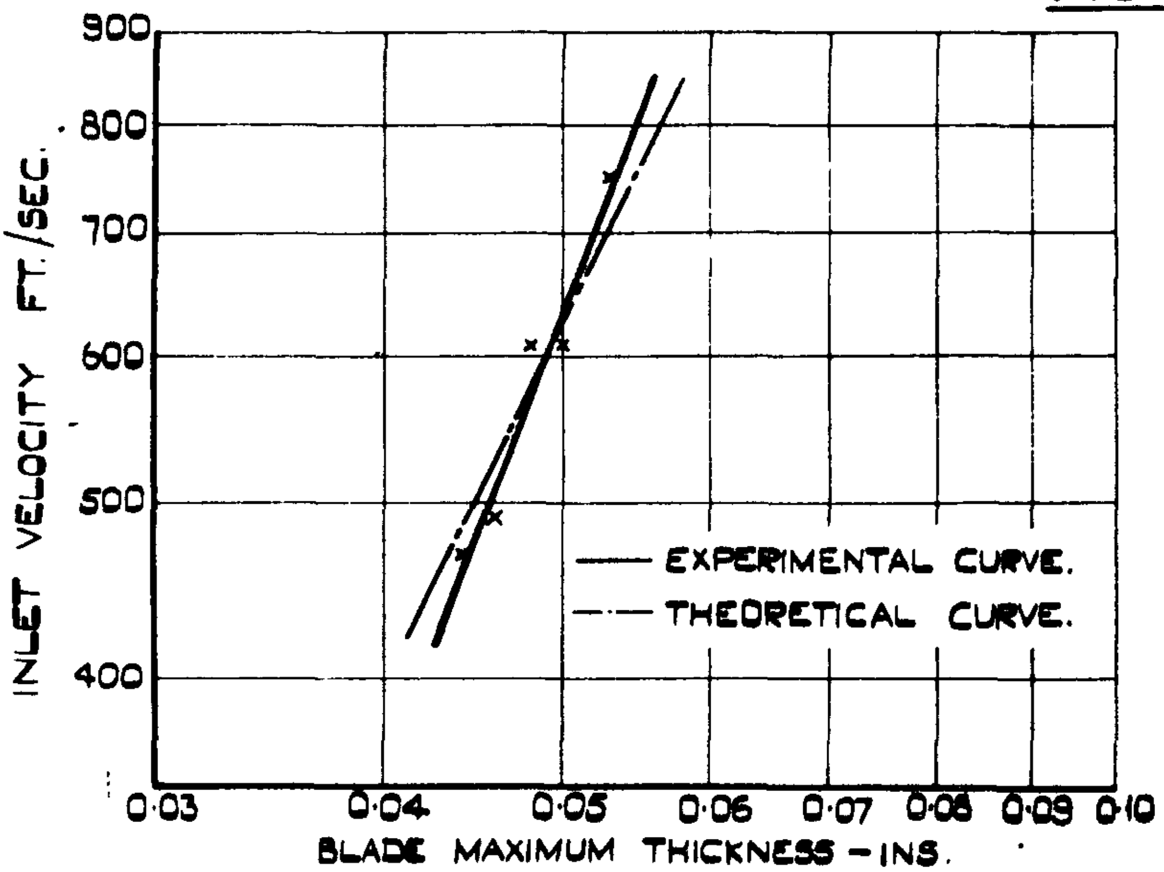
CRITICAL FLUTTER MACH NUMBER.

FIG. 6.

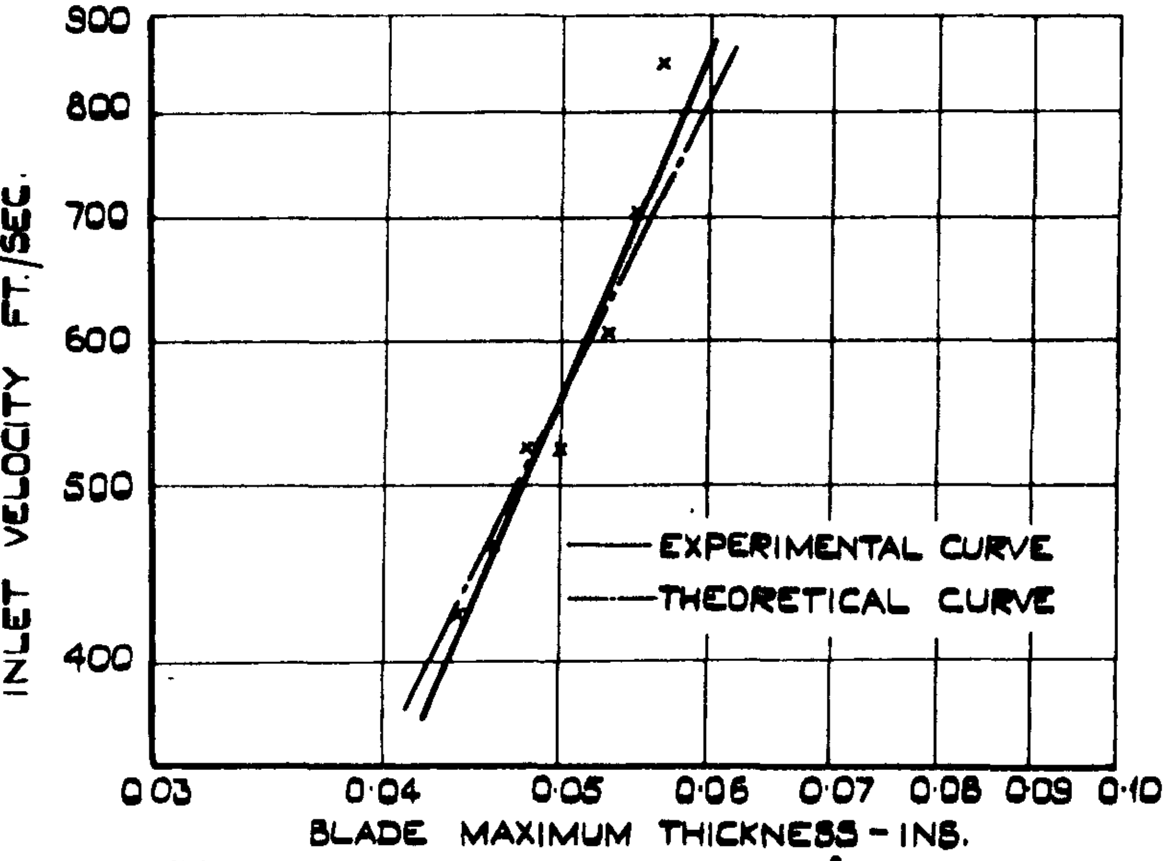


CRITICAL FLUTTER VELOCITY.

FIG. 7.



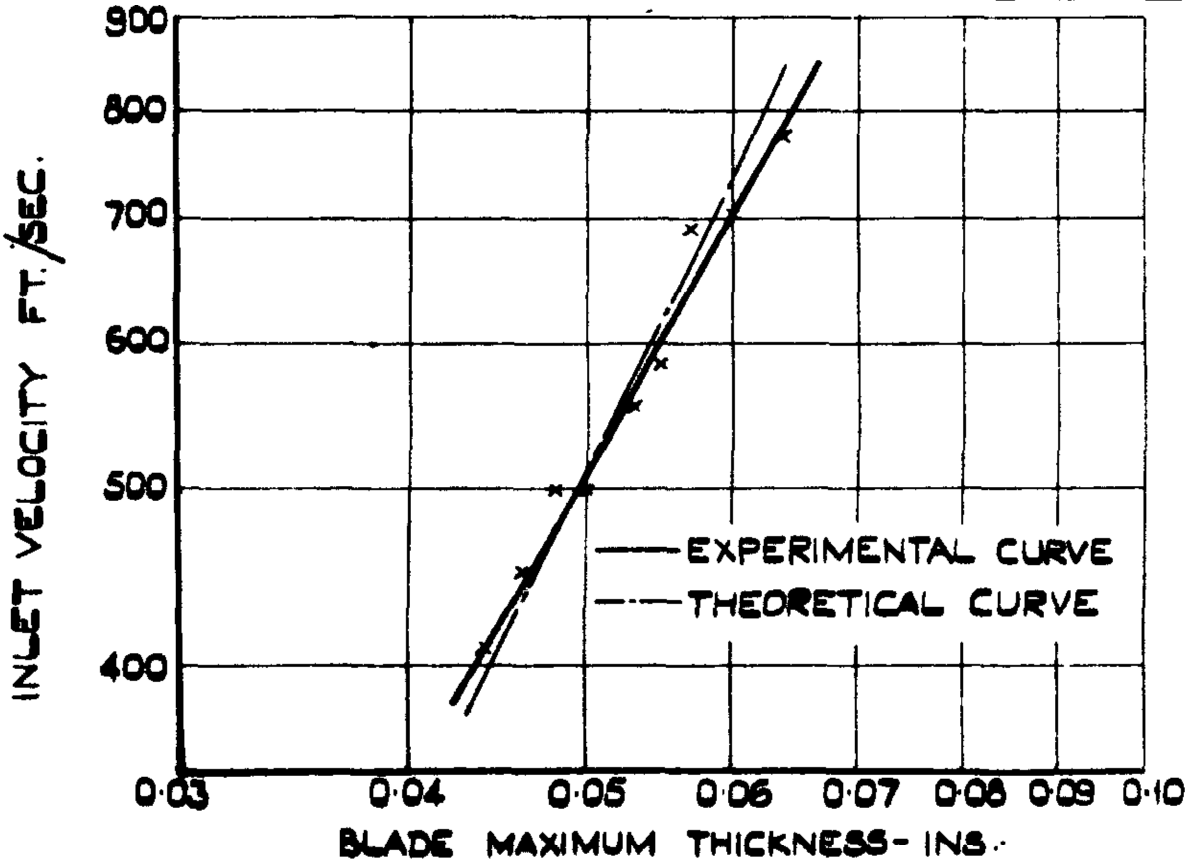
(a) INCIDENCE = 6·0°



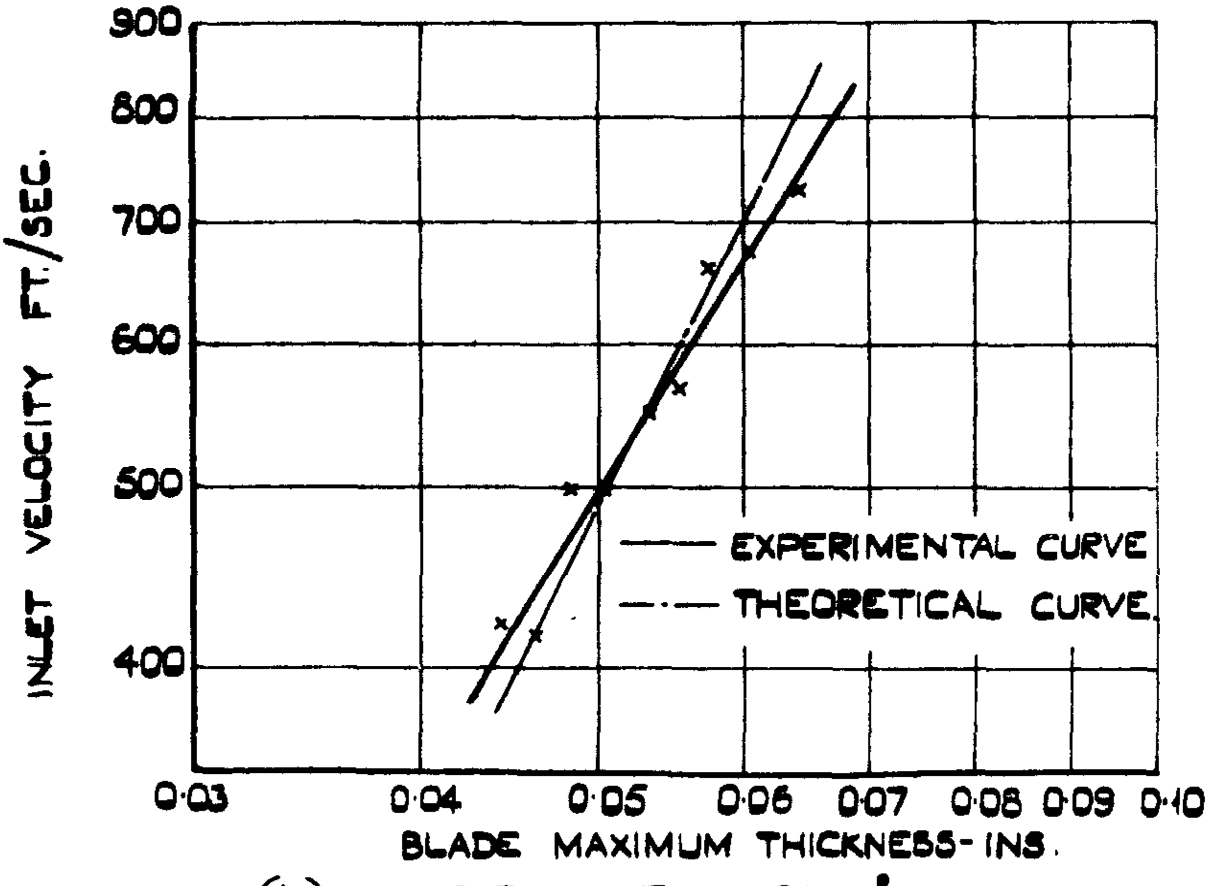
(b) INCIDENCE = 11·0°

CRITICAL FLUTTER VELOCITY v THICKNESS

FIG. 8.

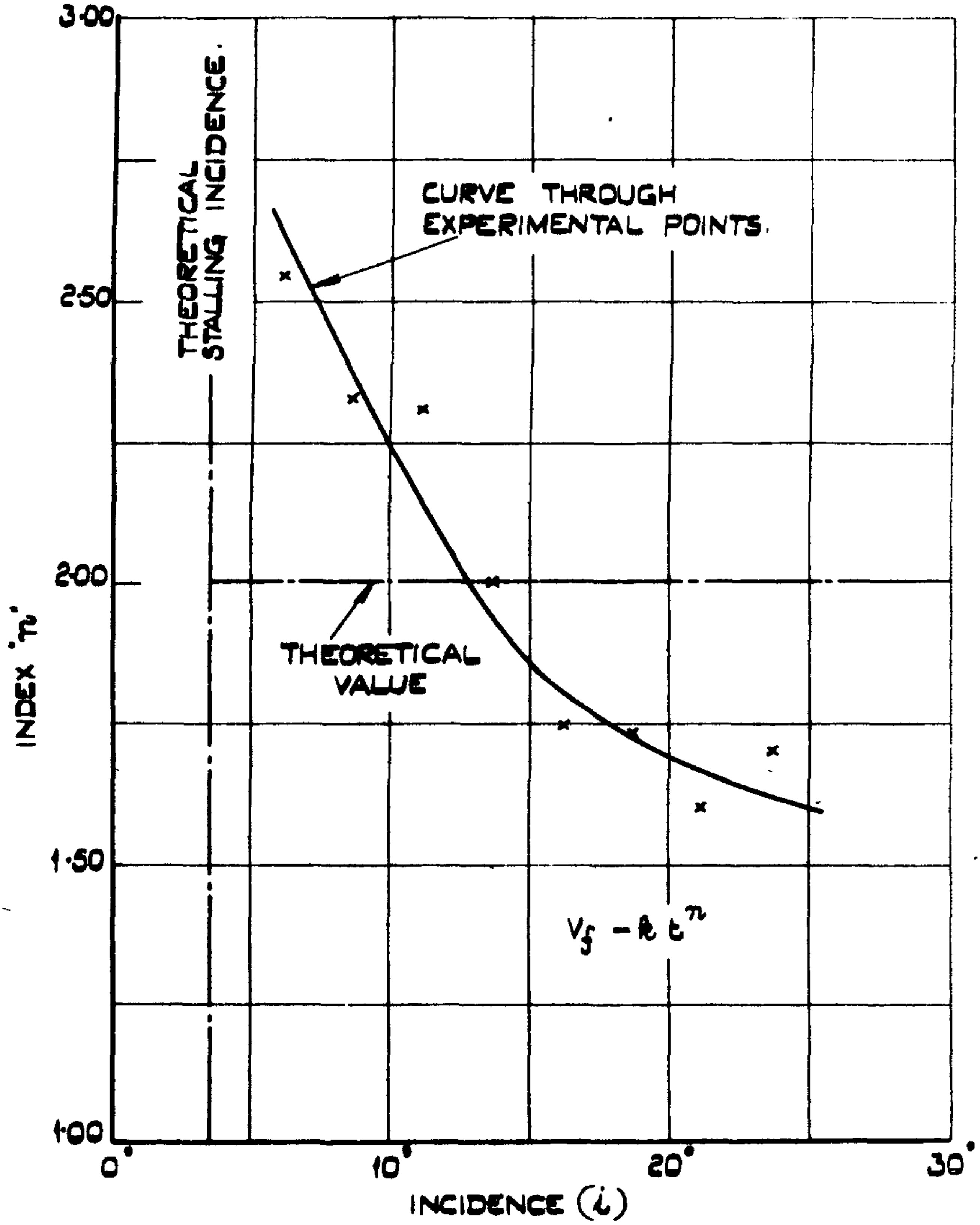


(a) INCIDENCE = 16·0°



(b) INCIDENCE = 21·0°

CRITICAL FLUTTER VELOCITY v THICKNESS.



VARIATION OF "t" INDEX WITH INCIDENCE.

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