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Optimum Designs for Reinforced Circular Holes

By

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E. H. Mansfield, M.A.

SUMMARY

The design of reinforced circular holes in an infinite sheet is considered theoretically. The stress system in the main body of the sheet is assumed to be one in which the principal stresses are in the ratio 1 : -1 (i.e. shear), 1 : 0 (i.e. tension), 1 : 1 or 1 : $\frac{1}{2}$.

The reinforcement may vary round the hole and families of such reinforcements with constant total weight are considered: the peak stresses in the sheet are evaluated so that optimum weight-strength designs are determined.

LIST OF CONTENTS

LIST OF ILLUSTRATIONS

1 Introduction

When a sheet has a reinforced hole there will usually be a perturbation of the general stress pattern in the vicinity of the hole and a consequent weakening of the structure due to stress concentrations. It has been shown in a previous paper[1] by the writer that the general stress pattern will remain unaltered in the region of the reinforced hole if the shape of the hole as well as the distribution of reinforcing material is suitably chosen. Such holes are called Neutral Holes. Frequently, however, engineering considerations pre-determine a circular shape for the hole, in which event there will generally be stress concentrations. The problem now is to determine what distribution of reinforcing material will produce the least stress concentrations, and if this optimum distribution is too heavy, what then is the optimum distribution for reinforcements of a given smaller total weight.

1.1 Assumptions

The following assumptions are made:

(i) stress-strain relations are linear,

(ii) buckling does not take place,

(iii) rivet flexibility is negligible,

(iv) the bending stiffness of the reinforcing member is negligible compared with its tensile stiffness.

Assumptions (i) to (iii) are standard practice and assumption (iv) has been shown to be justifiable by the author[1] and by Reissner[2].

2 List of Symbols

r, θ	=	polar co-ordinates
ϕ	=	Airy stress function
R	=	radius of hole
t	=	thickness of sheet
A	=	cross-sectional area of reinforcement
σ_r, σ_θ	=	direct stresses in the sheet
$\tau_{r\theta}$	=	shear stress in the sheet
S	=	octahedral stress concentration factor
U	=	$\frac{6G}{t}$ × elastic energy of distortion per unit area
W	=	weight of reinforcement/weight of sheet removed
ν	=	Poissons ratio (taken as $\frac{1}{3}$ in the numerical calculations)
$\alpha, \beta, \gamma, \delta, \varepsilon$	=	perturbation stress coefficients

$\lambda_1, \lambda_2, \lambda_3$ = coefficients introduced in equation (34)

c, C = constants

3 The reinforced circular hole

If the stress function in the sheet is ϕ it can be shown[1] that at the boundary of the reinforced hole, where $r = R$,

$$\phi = 0 \tag{1}$$

and the section area of the reinforcement is given by

$$\frac{A}{Rt} = \frac{\left(\frac{\partial\phi}{\partial r}\right)_R}{R\left(\frac{\partial^2\phi}{\partial r^2}\right)_R - \nu\left(\frac{\partial\phi}{\partial r}\right)_R} . \tag{2}$$

The method of solution is an inverse one. A stress function will be chosen which satisfies equation (1), the stress conditions at infinity and the equation of compatibility ($\nabla^4\phi = 0$), and which gives rise to perturbation stresses round the hole. The number of independent perturbation stress systems will usually be restricted to two and their magnitudes will be determined by arbitrary coefficients. Substitution of the stress function in equation (2) gives the corresponding section area of the reinforcement as a function of θ and the two arbitrary coefficients. The weight-strength characteristics of different reinforcements may now be considered by varying the arbitrary coefficients. In view of the two-dimensional character of the stresses it is more realistic to base the stress concentration factor on the octahedral shear stress[3] rather than the greater of the two principal stresses. This "Octahedral stress concentration factor" is then given by

$$S = \sqrt{\frac{U_{maximum}}{U_{reference}}} \tag{3}$$

where

$$U = \sigma_r^2 + \sigma_\theta^2 - \sigma_r\,\sigma_\theta + 3\,\tau_{r\theta}^2 . \tag{4}$$

The weight of the reinforcement will be expressed in terms of the weight of sheet removed, i.e.

$$\left.\begin{aligned} W &= \frac{1}{\pi Rt}\oint A\,d\theta \\ &= 2\left(\frac{A}{Rt}\right)_{mean} \end{aligned}\right\} \tag{5}$$

3.1 Uniform stress in all directions

This is a trivial case since, from considerations of symmetry, the optimum reinforcement will not vary with θ and accordingly only one perturbation stress system is possible. The appropriate stress function satisfying equation (1) and the conditions at infinity is given by

$$\phi = r^2 - R^2 + 2\alpha R^2 \log (r/R) \qquad (6)$$

where the log term gives rise to perturbation stresses and α is an arbitrary coefficient.

Substituting equation (6) in equation (2) gives

$$\frac{A}{Rt} = \frac{1+\alpha}{1-\nu-\alpha(1+\nu)} \qquad (7)$$

so that from equation (5)

$$W = \frac{2(1+\alpha)}{1-\nu-\alpha(1+\nu)} . \qquad (8)$$

The octahedral stress concentration factor is found from equations (3) and (4):

$$S = (1+3\alpha^2)^{\frac{1}{2}} . \qquad (9)$$

The minimum value of S occurs when α is zero giving a value of W of $\left(\frac{2}{1-\nu}\right)$ which corresponds to the special case of a neutral hole. For other values of W it is found that

$$S = \left[1 + 3\left\{\frac{W(1-\nu)-2}{W(1+\nu)+2}\right\}^2\right]^{\frac{1}{2}} \qquad (10)$$

which has been plotted in Figure 2.

It will be noted for example that if W is to be limited to unity the value of S is 1.2.

3.2 Principal stresses in the ratio 2 : 1

Such a stress system occurs in a thin-walled cylinder under internal pressure. The appropriate stress function is given by

$$\phi = r^2 (3+\cos 2\theta) - 3R^2 - \alpha R^2 \cos 2\theta - \frac{\beta R^4 \cos 2\theta}{r^2} + 2\gamma R^2 \log (r/R) \qquad \ldots\ldots(11)$$

The first term in equation (11) satisfies the conditions at infinity and the last three terms give rise to perturbation stresses. In order to satisfy equation (1) it will be seen that

$$\alpha + \beta = 1 \tag{12}$$

so that these perturbation stress systems are not independent.

Substituting equations (11) and (12) in equation (2) and taking ν equal to $\frac{1}{3}$ gives

$$\frac{A}{Rt} = \frac{3\{3 + \gamma + (1+\beta)\cos 2\theta\}}{2\{3 - 2\gamma + (1-5\beta)\cos 2\theta\}} \ . \tag{13}$$

Now A is essentially positive everywhere and this fact limits the possible values of β and γ to a region in the (β,γ) plane bounded by the lines

$$\left.\begin{aligned} 3 + \gamma \pm (1+\beta) &= 0 \\ \text{and} \qquad 3 - 2\gamma \pm (1-5\beta) &= 0 \end{aligned}\right\} \tag{14}$$

3.21 <u>The stress concentration factor S</u>

From equation (4)

$$\tfrac{1}{4} U_R = 3(3+\gamma^2) + 12(1+\beta)^2 + 6(1+2\beta\gamma-\beta)\cos 2\theta - (11+26\beta-\beta^2)\cos^2 2\theta \qquad \ldots\ldots(15)$$

so that

$$\tfrac{1}{4} U_{max} = \left\{\begin{aligned} & 3\gamma^2 + 13\beta^2 + 4(4+3\beta\gamma-2\beta) \\ \text{or} \quad & 3(3+\gamma^2) + 12(1+\beta)^2 + \frac{9(1+2\beta\gamma-\beta)^2}{11+26\beta-\beta^2} \end{aligned}\right\} \tag{16}$$

whichever is the greater.

Furthermore,

$$U_\infty = 48,$$

so that

$$S = \sqrt{\frac{U_{max}}{48}} \ . \tag{17}$$

Contours of S as a function of β and γ have been drawn in Figure 3.

3.22 <u>The weight of reinforcement W</u>

Substituting equation (13) in equation (5) gives

$$W = \frac{3(1+\beta)}{1-5\beta} - \frac{9(6\beta+\beta\gamma-\gamma)}{(1-5\beta)\sqrt{\{(2-2\gamma+5\beta)(4-2\gamma-5\beta)\}}} \ . \tag{18}$$

Contours of W as a function of β and γ have been drawn in Figure 3.

3.23 The optimum W-S relationship

The optimum W-S relationship can be regarded as occurring when W is least for a given S or when S is least for a given W. It occurs in the (β,γ) plane where the W-contours and the S-contours are tangential, i.e. along the broken line in Figure 3. This optimum W-S relationship has been plotted in Figure 4, and the corresponding values of β and γ in Figure 5.

3.3 Pure shear

The appropriate stress function is given by

$$\phi = r^2 \cos 2\theta - \alpha R^2 \cos 2\theta - \frac{\beta R^4 \cos 2\theta}{r^2} + \varepsilon R^6 \cos 6\theta \left(\frac{1}{r^4} - \frac{R^2}{r^6}\right) \quad (19)$$

where

$$\alpha + \beta = 1 . \quad (20)$$

The first term in equation (19) satisfies the conditions at infinity and the other terms give rise to perturbation stresses. Perturbation stress systems varying as $\cos 3\theta$, $\cos 4\theta$ and $\cos 5\theta$ will not occur because of considerations of symmetry.

Substituting equations (19) and (20) in equation (2) and simplifying gives

$$\frac{A}{Rt} = \frac{3(1 + \beta - \varepsilon + 2\varepsilon \cos 4\theta)}{2(1 - 5\beta + 17\varepsilon - 34\varepsilon \cos 4\theta)} . \quad (21)$$

The fact that A is essentially positive everywhere limits the possible values of β and ε to a region in the (β,ε) plane bounded by the lines

$$\left.\begin{aligned} 1 + \beta - \varepsilon \pm 2\varepsilon &= 0 \\ \text{and} \qquad 1 - 5\beta + 17\varepsilon \pm 34\varepsilon &= 0 \end{aligned}\right\} \quad (22)$$

3.31 The stress concentration factor S

From equation (4)

$$\left.\begin{aligned} \tfrac{1}{4} U_R &= c_o + c_1 \cos^2 2\theta + c_2 \cos^4 2\theta + c_3 \cos^6 2\theta \\ \text{where} \quad c_o &= 12(1 + \beta - 3\varepsilon)^2 \\ c_1 &= -11 - 26\beta + \beta^2 + 390\varepsilon + 114\beta\varepsilon + 225\varepsilon^2 \\ c_2 &= -328\varepsilon + 40\beta\varepsilon - 600\varepsilon^2 \\ c_3 &= 400\varepsilon^2 \end{aligned}\right\} \quad (23)$$

so that

$$\left.\begin{aligned} \tfrac{1}{4}U_{max} &= c_o \\ \text{or} \quad & c_o + c_1 + c_2 + c_3 \\ \text{or} \quad & c_o - \frac{c_1 c_2}{3c_3} + \frac{2c_2^3}{27c_3^2} \pm \left(\frac{2c_1}{9c_3} - \frac{2c_2^2}{27c_3^2}\right)\sqrt{(c_2^2 - 3c_1 c_3)} \end{aligned}\right\} \qquad (24)$$

whichever is the greatest.

Furthermore,

$$U_\infty = 12$$

so that

$$S = \sqrt{\frac{U_{max}}{12}} \; . \qquad (25)$$

Contours of S as a function of β and ε have been drawn in Figure 6.

3.32 The weight of reinforcement W

Substituting equation (21) in equation (5) gives

$$W = -\frac{3}{17} + \frac{18(3+2\beta)}{17\sqrt{\{(1-5\beta+51\varepsilon)(1-5\beta-17\varepsilon)\}}} \qquad (26)$$

Contours of W as a function of β and ε have been drawn in Figure 6.

3.33 The optimum W-S relationship

The optimum W-S relationship occurs in the (β,ε) plane where the W-contours and the S-contours are tangential, i.e. along the broken line in Figure 6. This optimum W-S relationship has been plotted in Figure 7, and the corresponding values of β and ε in Figure 8.

3.4 Uniform stress in one direction

The analysis for this problem falls naturally into two sections. The first section treats the case when the stress in the reinforcement does not change sign with θ. The second section treats the more important case when the stress in the reinforcement changes sign with θ.

The first case

The first case is shown to be an inefficient way of reinforcing the sheet, and only the bare results of the analysis are given:

The appropriate stress function is given by

$$\phi = r^2(1+\cos 2\theta) - R^2 - \alpha R^2 \cos 2\theta - \frac{\beta R^4 \cos 2\theta}{r^2} + 2\gamma R^2 \log (r/R) \qquad (27)$$

where

$$\alpha + \beta = 1 . \qquad (28)$$

Substitution into equation (2) gives

$$\frac{A}{Rt} = \frac{3\{1+\gamma+(1+\beta)\cos 2\theta\}}{2\{1-2\gamma+(1-5\beta)\cos 2\theta\}} . \qquad (29)$$

The possible values of β and γ are limited to a region in the (β,γ) plane bounded by the lines

$$\left.\begin{aligned} \beta - \gamma &= 0 , \\ 5\beta - 2\gamma &= 0 , \\ 2 - 2\gamma - 5\beta &= 0 . \end{aligned}\right\} \qquad (30)$$

The second case

The appropriate stress function is now given by

$$\phi = r^2 (1+\cos 2\theta) - R^2 - \alpha R^2 \cos 2\theta - \frac{\beta R^4 \cos 2\theta}{r^2} + 2\gamma R^2 \log (r/R) + \delta R^4 \cos 4\theta \left(\frac{1}{r^2} - \frac{R^2}{r^4}\right) \qquad \ldots\ldots(31)$$

where

$$\alpha + \beta = 1 . \qquad (32)$$

Now adjacent to the reinforcement

$$\left.\begin{aligned} \tfrac{1}{2}\sigma_\theta &= 1 - \gamma + 7\delta + (1-3\beta)\cos 2\theta - 14\delta \cos^2 2\theta \\ \text{and} \quad \tfrac{1}{2}\sigma_r &= 1 + \gamma - \delta + (1+\beta)\cos 2\theta + 2\delta\cos^2 2\theta \end{aligned}\right\} \qquad (33)$$

and as we have stipulated a change of sign in the stress in the reinforcement as θ varies the expressions above must also change sign at the same value of θ. The expressions in equation (33) must therefore have a common factor and we can write

$$\left.\begin{aligned} &1 - \gamma + 7\delta + (1-3\beta)\cos 2\theta - 14\delta\cos^2 2\theta \equiv -14\delta\,(\lambda_1+\cos 2\theta)(\lambda_2+\cos 2\theta), \text{ say} \\ &\text{and} \\ &1 + \gamma - \delta + (1+\beta)\cos 2\theta + 2\delta\cos^2 2\theta \equiv 2\delta\,(\lambda_1+\cos 2\theta)(\lambda_3+\cos 2\theta), \text{ say.} \end{aligned}\right\} \quad (34)$$

By comparing the coefficients of the different powers of $\cos 2\theta$ it is possible to solve these equations to give δ, λ_1, λ_2 and λ_3 in terms of β and γ:

$$\left.\begin{aligned} \delta &= \frac{(2+\beta)(2\beta+\beta\gamma-\gamma)}{(4+3\gamma)^2 - 2(2+\beta)^2}, \\ \lambda_1 &= \left(\frac{4+3\gamma}{4+2\beta}\right), \\ \lambda_2 &= \frac{\gamma(1+3\gamma) - (1+\beta)(2+3\beta)}{7(2\beta+\beta\gamma-\gamma)}, \\ \lambda_3 &= \frac{\gamma(7+3\gamma) + 2 - 3\beta - \beta^2}{2\beta+\beta\gamma-\gamma}. \end{aligned}\right\} \quad (35)$$

Substituting equations (31), (32) and (35) in equation (2) gives

$$\frac{A}{Rt} = \frac{-3(\lambda_3 + \cos 2\theta)}{21\lambda_2 + \lambda_3 + 22\cos 2\theta}. \quad (36)$$

The possible values of β and γ are limited to regions in the (β,γ) plane bounded by the curves:

$$\left.\begin{aligned} \lambda_3 \pm 1 &= 0, \\ 21\lambda_2 + \lambda_3 \pm 22 &= 0, \\ \lambda_1 - 1 &= 0. \end{aligned}\right\} \quad (37)$$

3.41 The stress concentration factor S

From equation (4)

$$
\left.
\begin{aligned}
\tfrac{1}{4} U_R &= C_o + C_1 \cos 2\theta + C_2 \cos^2 2\theta + C_3 \cos^3 2\theta + C_4 \cos^4 2\theta \\
\text{where} & \\
C_o &= 13 + 24\beta + 12\beta^2 + 3\gamma^2 + 6\delta - 24\gamma\delta \cdot 57\delta^2 \\
C_1 &= 2 - 2\beta + 102\delta + 12\beta\gamma + 42\beta\delta \\
C_2 &= -11 - 26\beta + \beta^2 - 12\delta + 48\gamma\delta - 36\delta^2 \\
C_3 &= -108\delta + 12\beta\delta \\
C_4 &= 36\delta^2
\end{aligned}
\right\} \qquad (38)
$$

so that

$$
\tfrac{1}{4} U_{max} = C_o + C_2 + C_4 + |C_1 + C_3|
$$

or

$$
\tfrac{1}{4} U(\theta_o)
$$

where

$$
C_1 + 2C_2 \cos 2\theta + 3C_3 \cos^2 2\theta + 4C_4 \cos^3 2\theta = 0, \qquad (39)
$$

whichever is the greater.

Furthermore,

$$
U_\infty = 16
$$

so that

$$
S = \tfrac{1}{4} (U_{max})^{\frac{1}{2}} . \qquad (40)
$$

Contours of S as a function of β and γ have been drawn in Figure 9.

3.42 <u>The weight of reinforcement W</u>

Substituting equation (36) in equation (5) gives

$$
W = -\frac{3}{11} + \frac{63 \, |\lambda_3 - \lambda_2|}{11 \sqrt{\{(21 \lambda_2 + \lambda_3 - 22)(21 \lambda_2 + \lambda_3 + 22)\}}} \qquad (41)
$$

Contours of W as a function of β and γ have been drawn in Figure 9.

3.43 The optimum W-S relationship

The optimum W-S relationship occurs in the (β,γ) plane where the W-contours and the S-contours are tangential, i.e. along the broken line in Figure 9. This optimum W-S relationship has been plotted in Figure 10, and the corresponding values of β and γ in Figure 11.

4 Examples and Discussion

4.1 Principal stresses in the ratio 2 : 1

Suppose that W is to be limited to a value of 0.5; what distribution of reinforcing material then gives the lowest value of S?

From Figures 4 and 5 the lowest value of S is 1.6 and it occurs when

$$\beta = -0.1$$

$$\gamma = -1.8$$

whence from equation (13):

$$\frac{A}{Rt} = \frac{1.2 + 0.9 \cos 2\theta}{4.4 + \cos 2\theta}$$

which has been plotted in Figure 12.

It is interesting to note that if $W = 1.1$ $(S = 1.42)$ the optimum distribution of reinforcing material is independent of θ. For values of W greater than 1.1 the optimum distribution of material is such that the maximum reinforcement occurs at the points where the reinforcement line is normal to the direction of the greater applied stress. This, at first sight anomalous result, is due primarily to the effect of Poisson's ratio on the strains in the sheet; a similar result occurs in the optimum distribution for the elliptical neutral hole[1]. Increasing the value of W above 1.1 does not however lower the value of S significantly, as can be seen from Figure 4.

4.2 Pure shear

(i) What reinforcement gives the lowest possible value of S?

From Figures 7 and 8 the lowest possible value of S is 1.17 and it occurs when

$$W = 0.566$$

$$\varepsilon = 0$$

$$\beta = -0.416$$

whence

$$\frac{A}{Rt} = 0.283$$

and it will be noted that this optimum distribution is independent of θ .

With this optimum reinforcement the peak octahedral shear stress has a constant value around the edge of the reinforcement.

(ii) If S is to be limited to a value of 1.5, how should the reinforcing material be distributed for least weight?

From Figures 7 and 8 the least value of W is 0.20 which occurs when

$$\varepsilon = 0.031$$

$$\beta = -0.66$$

and the variation of reinforcing material is found from equation (21):

$$\frac{A}{Rt} = 0.0876\left(\frac{5.00 + \cos 4\theta}{4.55 - \cos 4\theta}\right)$$

which has been plotted in Figure 13.

4.3 Simple tension

From Figure 9 it will be seen that the permissible region in the (β,γ) plane has a "waist" which vanishes at the point $\beta = -0.414$, $\gamma = -0.586$, corresponding to a constant reinforcement with $W = 0.57$. In this waist, say from $W = 0.4$ to 0.65, the range of possible distributions of reinforcement is therefore somewhat limited. This is a fault of the method of analysis; by taking another term in the stress function of equation (31) it would be possible to arrive at more efficient reinforcements in this region.

If $W = 0.76$ ($S = 1.54$) the optimum distribution of reinforcing material is independent of θ. For values of W greater than 0.76 the optimum distribution is such that the maximum reinforcement occurs at the points where the reinforcement line is normal to the direction of the applied stress. The reason for this is identical with that given in para. 4.1.

(i) *As a first example consider the optimum distribution for* $W = 0.3$. Note that from Figure 10 we should have to take $W = 0.5$ if the reinforcement did not vary with θ to get as low a value of S. From Figure 11 we find

$$\beta = -0.49$$

$$\gamma = -0.65$$

whence, from equation (35)

$$\lambda_2 = -4.31$$

$$\lambda_3 = +4.57$$

so that from equation (36)

$$\frac{A}{Rt} = 0.136\left(\frac{4.57 + \cos 2\theta}{3.90 - \cos 2\theta}\right)$$

which has been plotted in Figure 14.

(ii) If it is necessary to reduce S to a value of 1.4, say, we should have to take W = 2.0 and, from Figure 11,

$$\beta = -0.24$$

$$\gamma = -0.34$$

so that

$$\lambda_2 = +2.40$$

$$\lambda_3 = -10.8$$

whence, from equation (36),

$$\frac{A}{Rt} = 0.136 \left(\frac{10.8 - \cos 2\theta}{1.8 + \cos 2\theta}\right)$$

which has been plotted in Figure 15.

5 Conclusions

The optimum weight-strength design of reinforced circular holes in an infinite sheet is considered theoretically. The stress system in the main body of the sheet is assumed to be one in which the principal stresses are in the ratio 1 : -1, 1 : 0, 1 : 1 or 1 : $\frac{1}{2}$. The method of solution is an inverse one; a stress function is chosen which satisfies the equation of compatibility and the stress conditions at infinity, and which gives rise to a number of arbitrary perturbation stress systems round the hole. The section area of the reinforcement is then determined in terms of these arbitrary perturbation stress systems. The weight-strength characteristics of different reinforcements is then investigated by varying the perturbation stress systems.

It is shown that (apart from the trivial case of the 1 : 1 stress field) for a given total weight of reinforcement the stress concentration in the sheet may be reduced by suitably varying the reinforcement around the hole. The reduction in stress concentration is greatest when the weight of reinforcement is small. For the 1 : 0 and 1 : $\frac{1}{2}$ stress fields there is a reinforcement which gives least stress concentrations when it does not vary around the hole; for reinforcements with a greater total weight the optimum variation is then such that the maximum reinforcement occurs at the points where the reinforcement line is normal to the direction of the greater applied stress.

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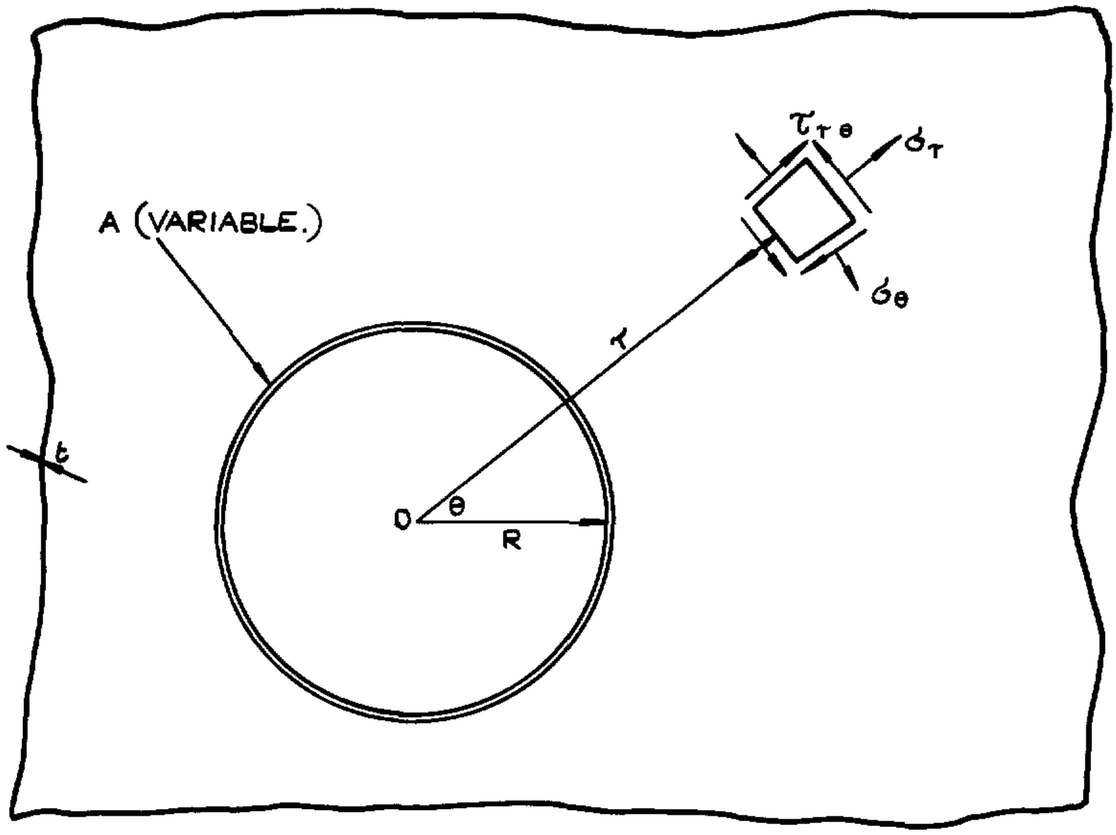

FIG. 1. FIGURE SHOWING NOTATION.

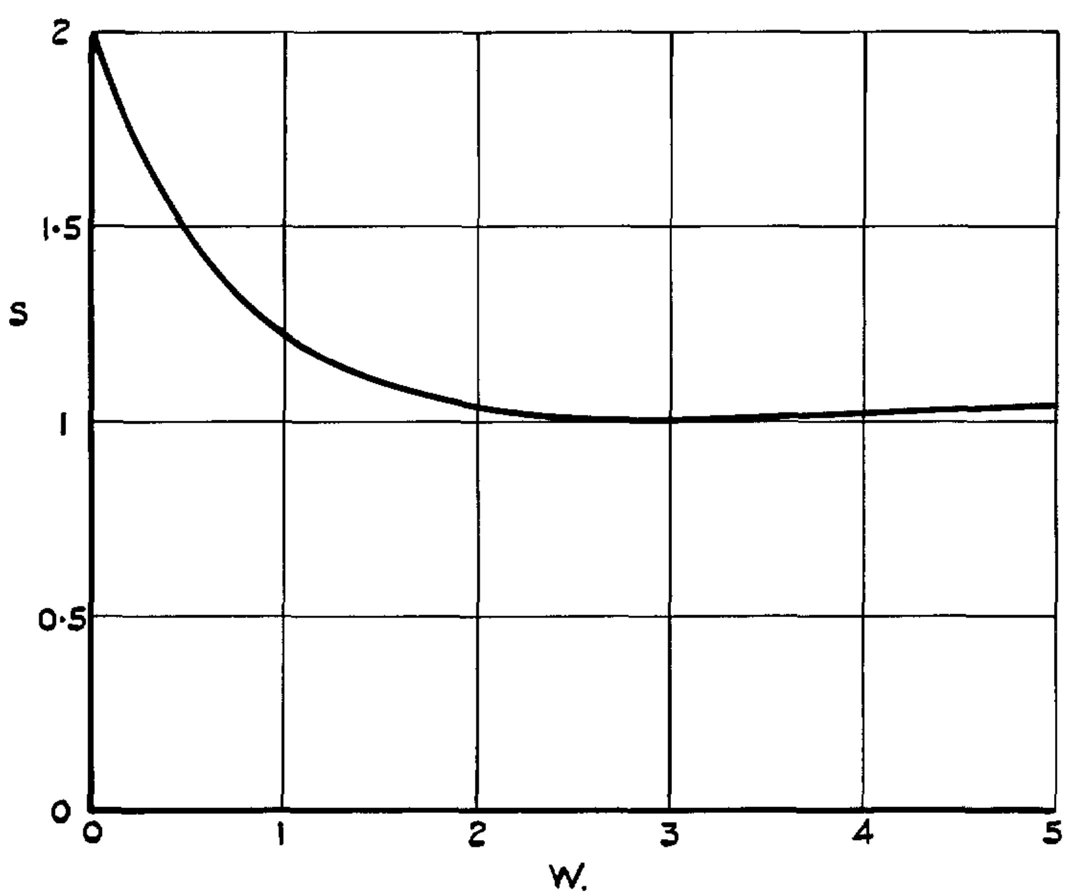

FIG. 2. OPTIMUM W-S RELATION FOR UNIFORM STRESS IN ALL DIRECTIONS.

FIG. 3.

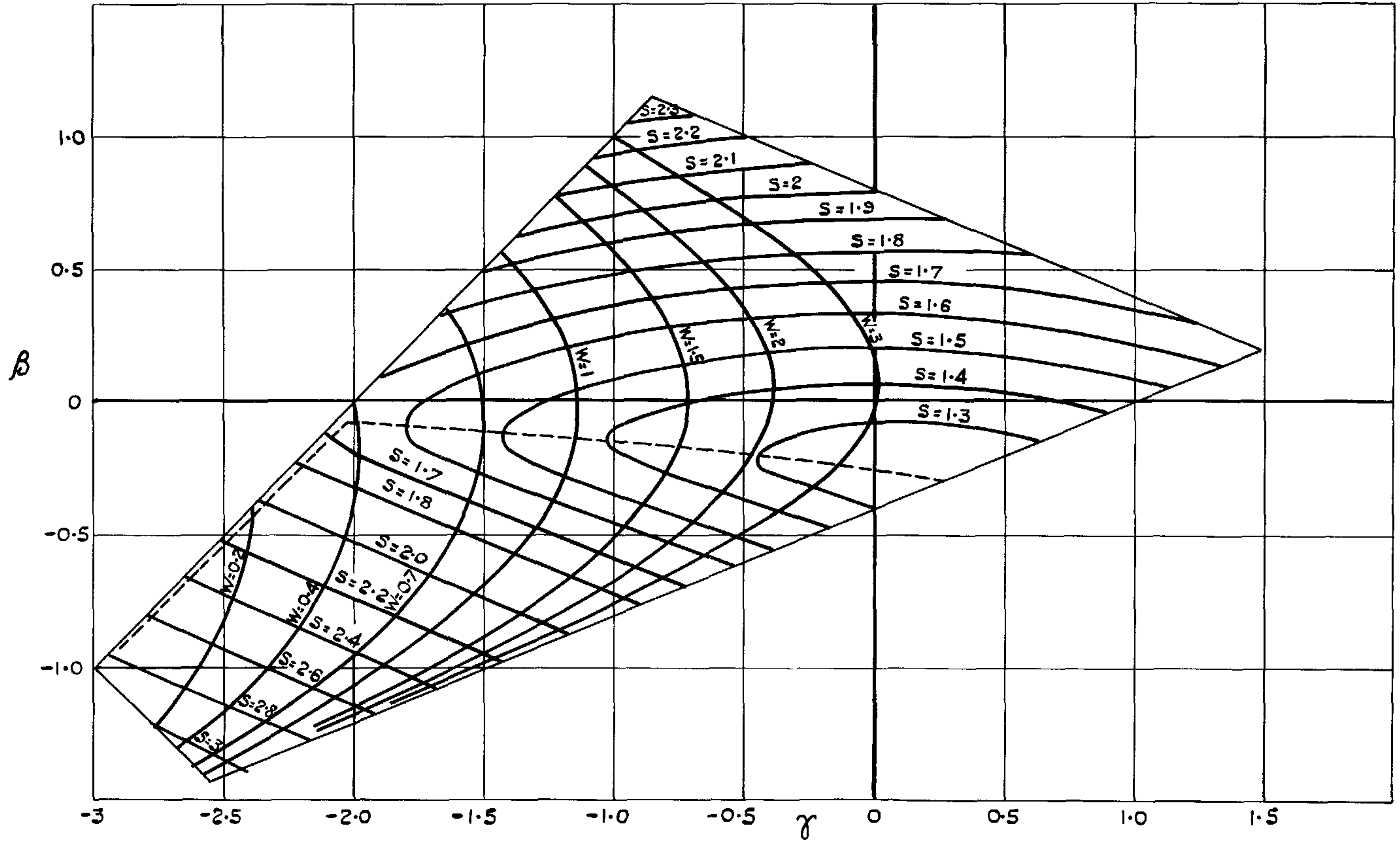

FIG. 3. W AND S CONTOURS FOR PRINCIPAL STRESSES IN THE RATIO 2:1.

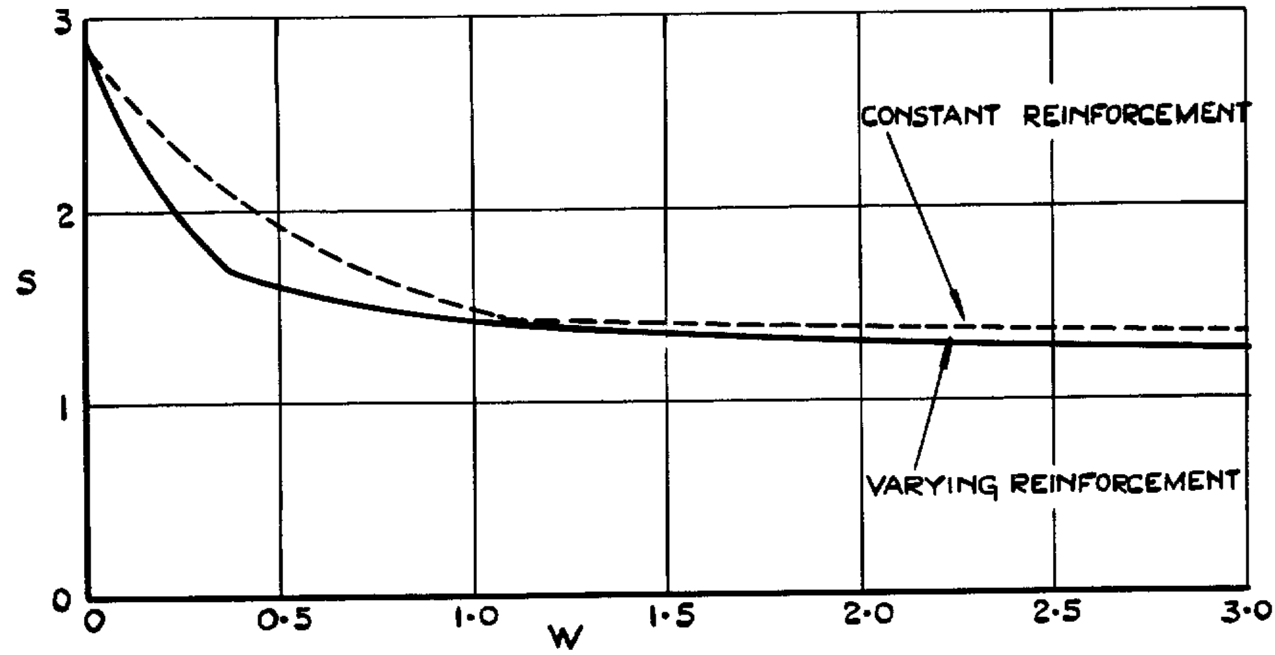

FIG. 4. OPTIMUM W-S RELATION FOR PRINCIPAL STRESSES IN THE RATIO 2:1.

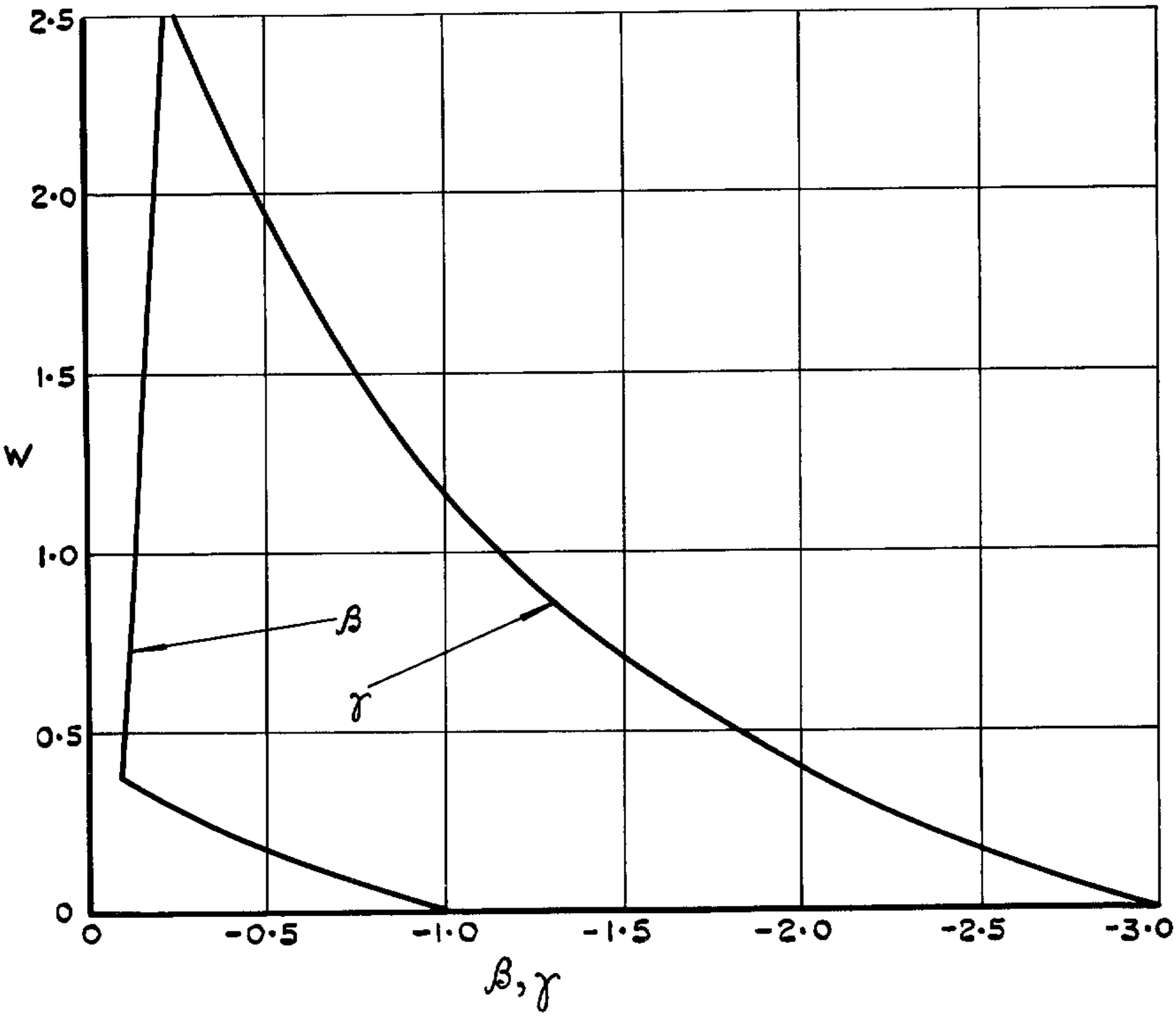

FIG. 5. VALUES OF β AND γ CORRESPONDING TO OPTIMUM W-S RELATION FOR PRINCIPAL STRESSES IN THE RATIO 2:1.

FIG. 6.

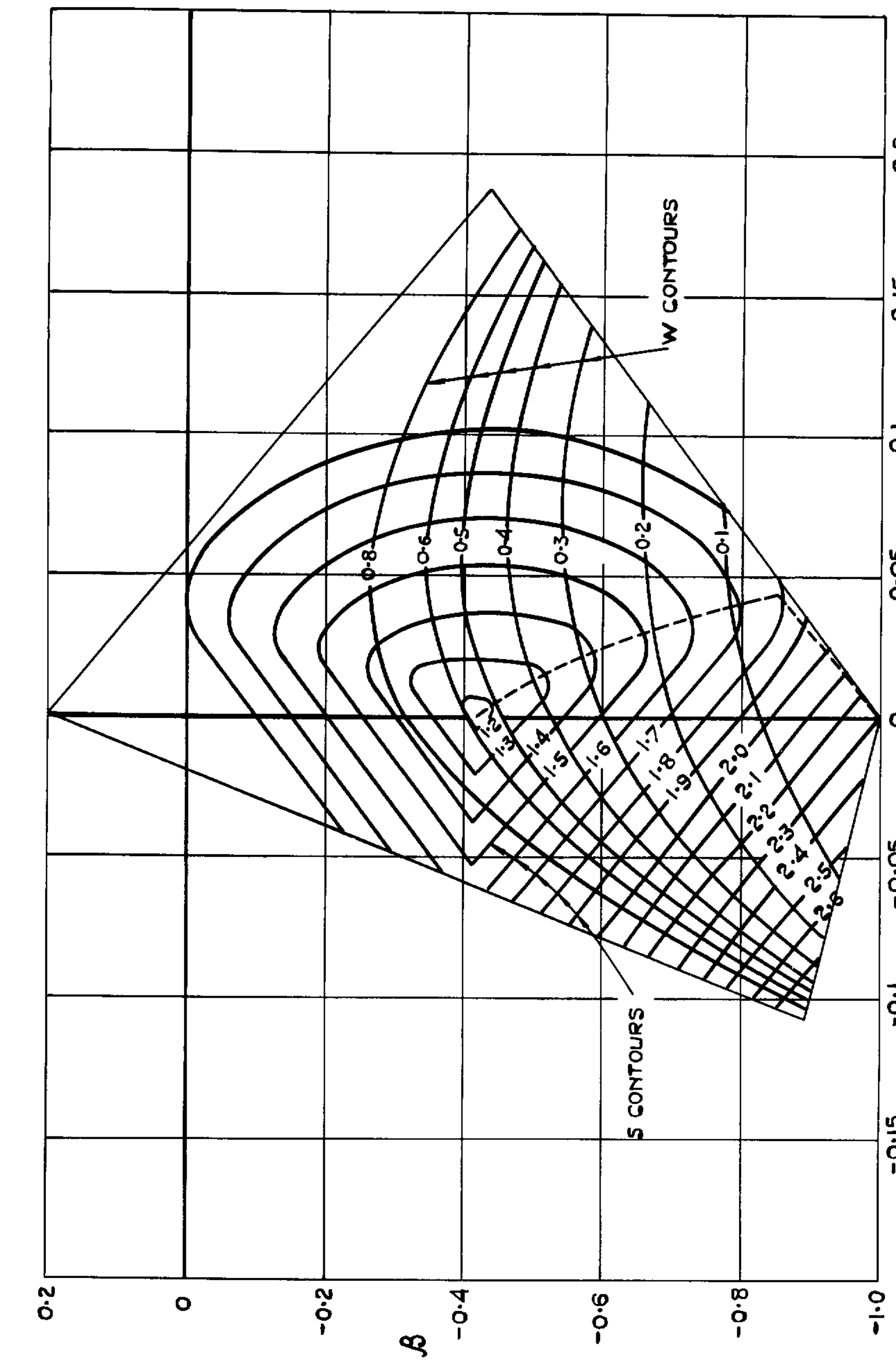

FIG. 6. W AND S CONTOURS FOR PURE SHEAR.

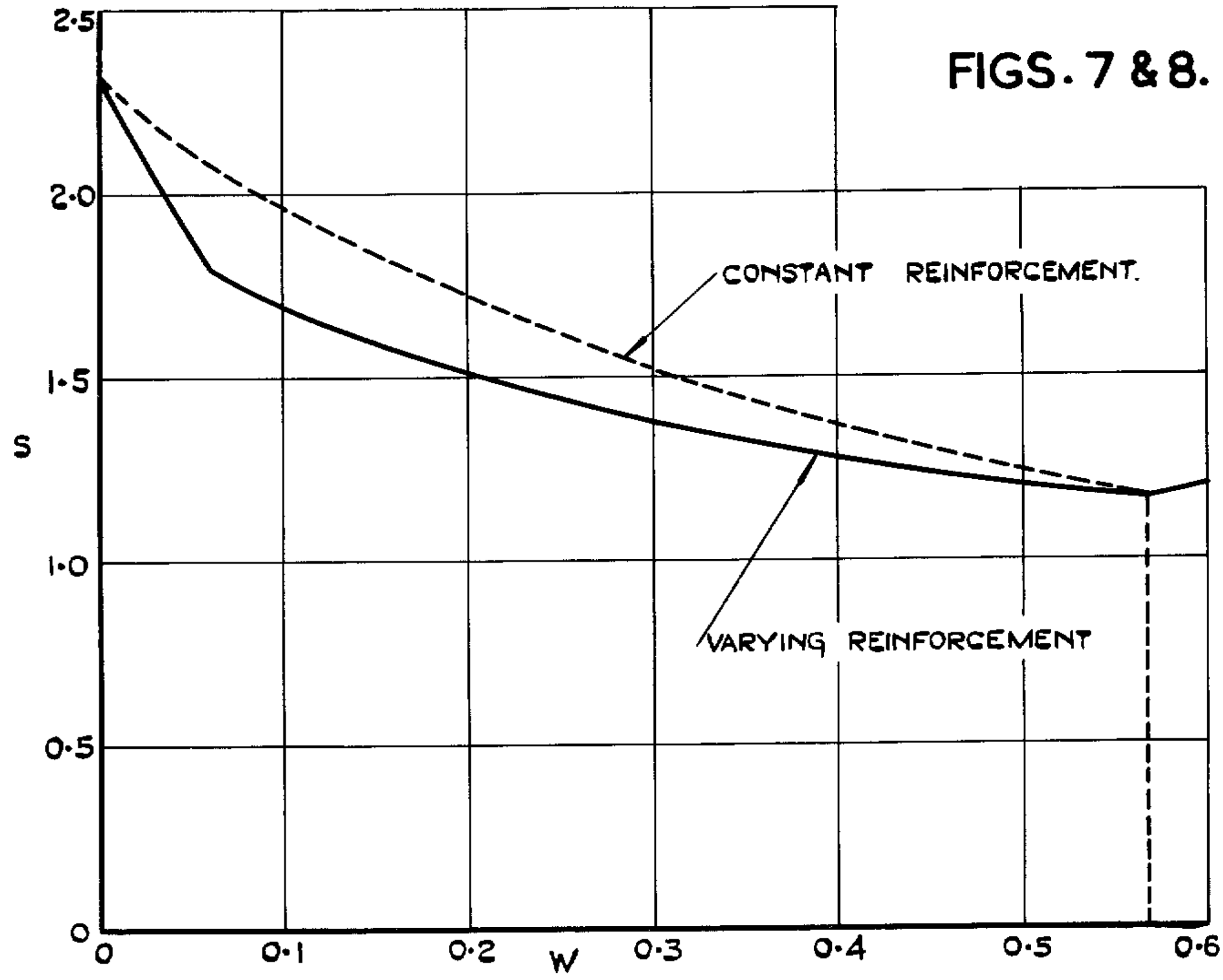

FIG. 7. OPTIMUM W-S RELATION FOR PURE SHEAR.

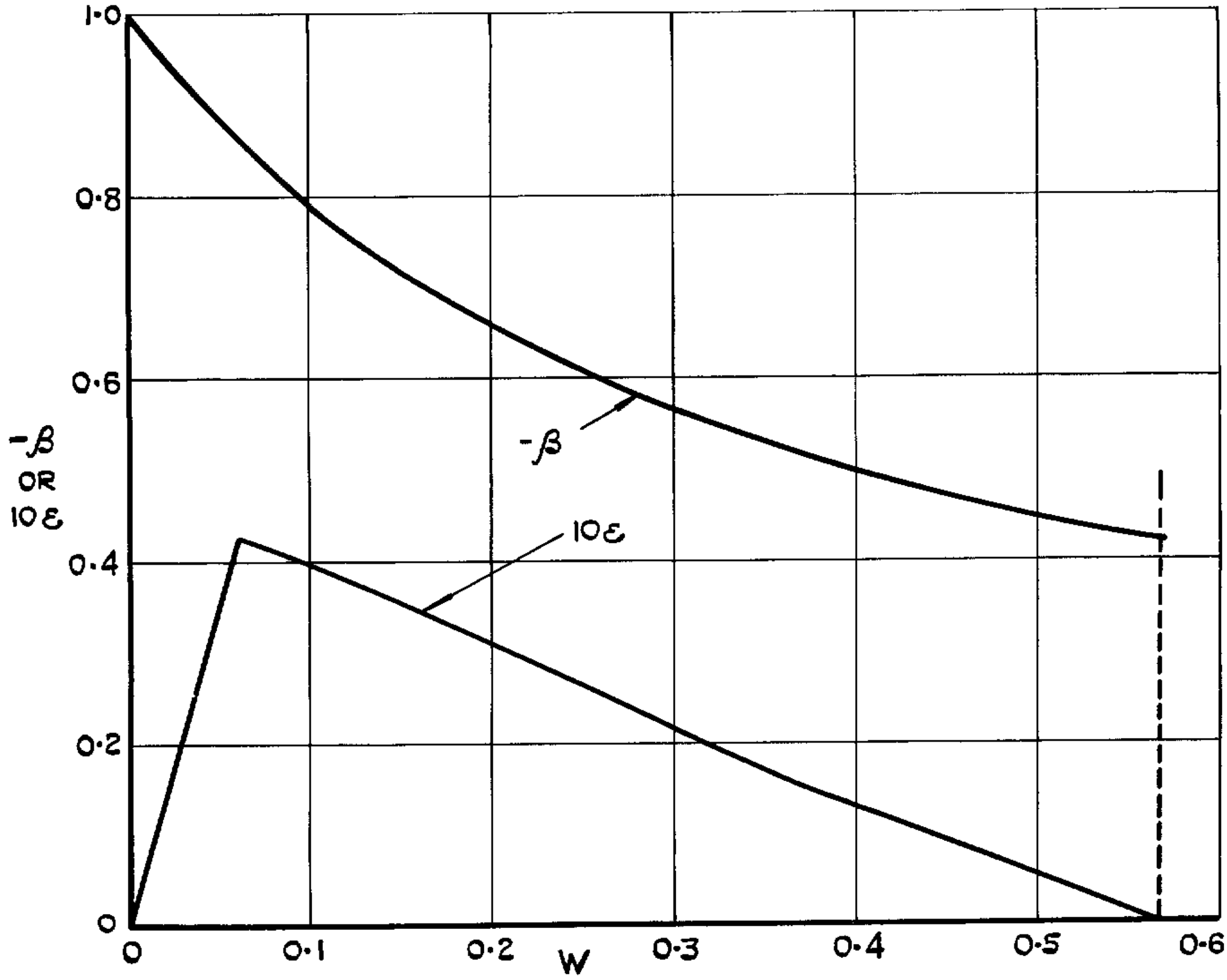

FIG. 8. VALUES OF β AND ε CORRESPONDING TO OPTIMUM W-S RELATION FOR PURE SHEAR.

FIG. 9.

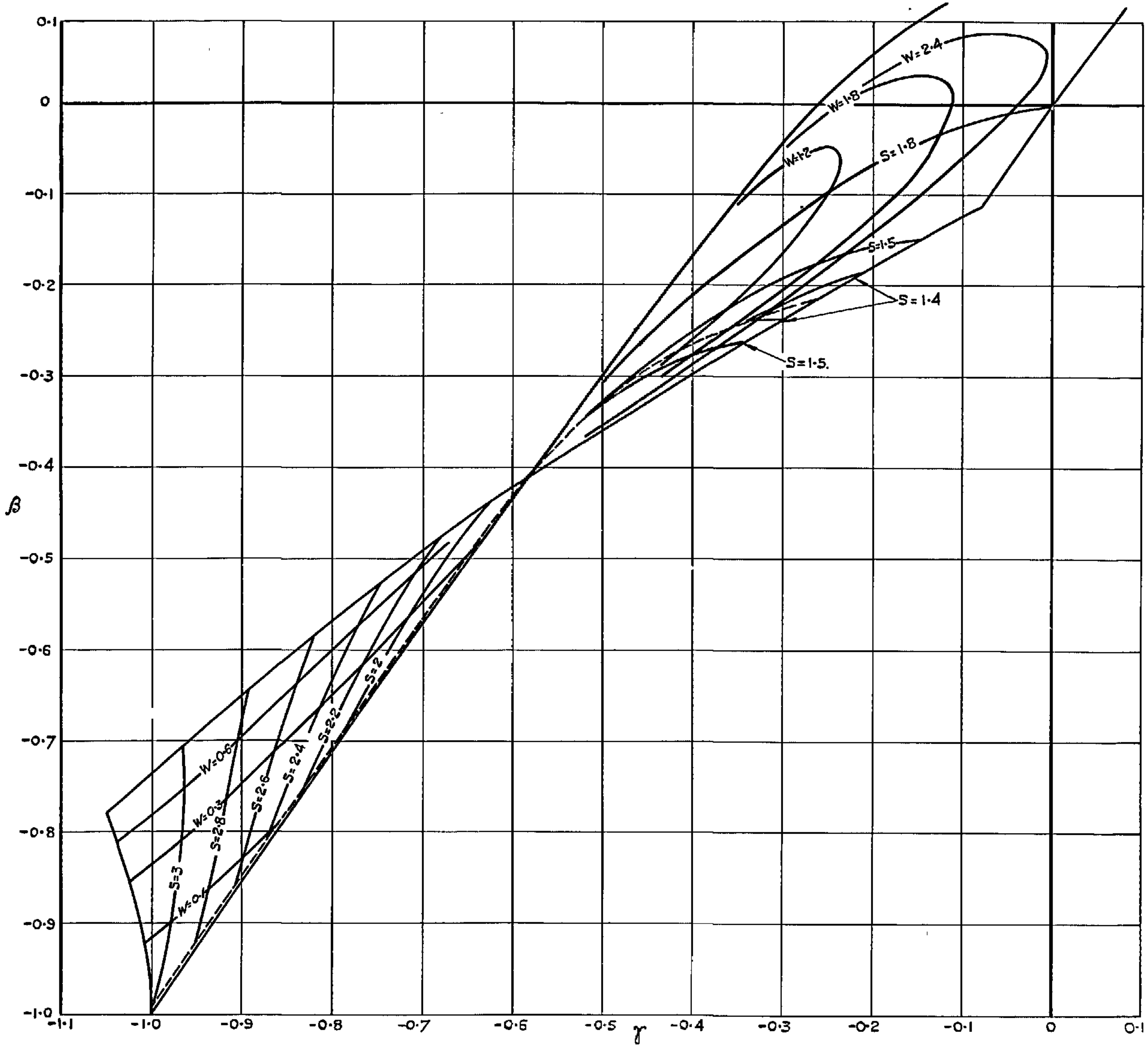

FIG. 9. W AND S CONTOURS FOR PURE TENSION.

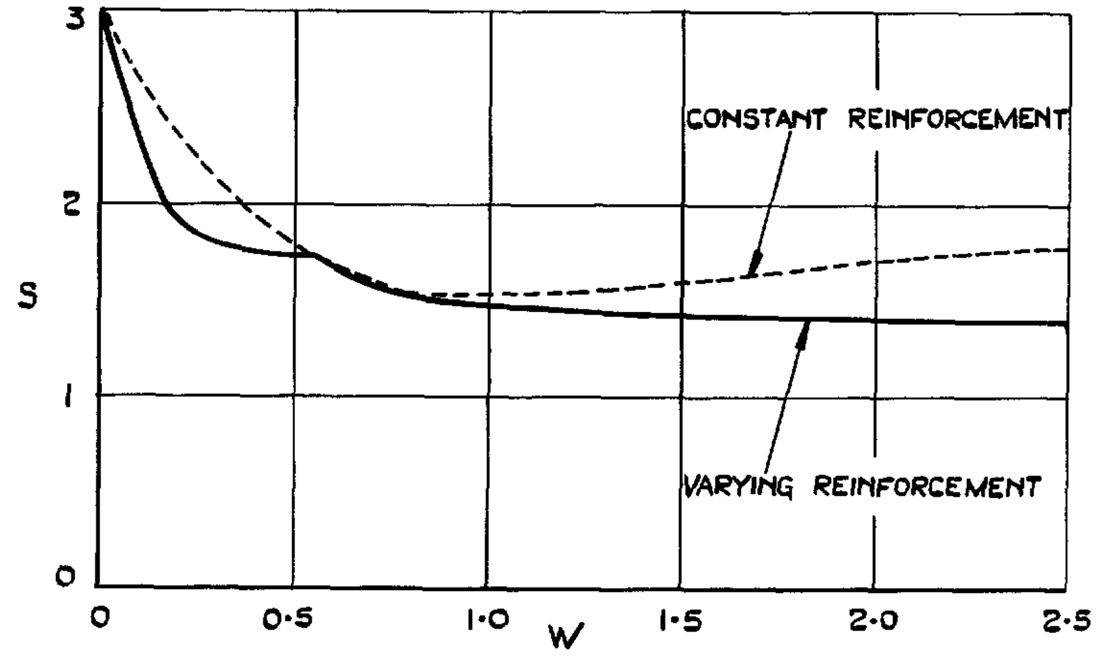

FIG. 10 OPTIMUM W-S RELATION FOR PURE TENSION.

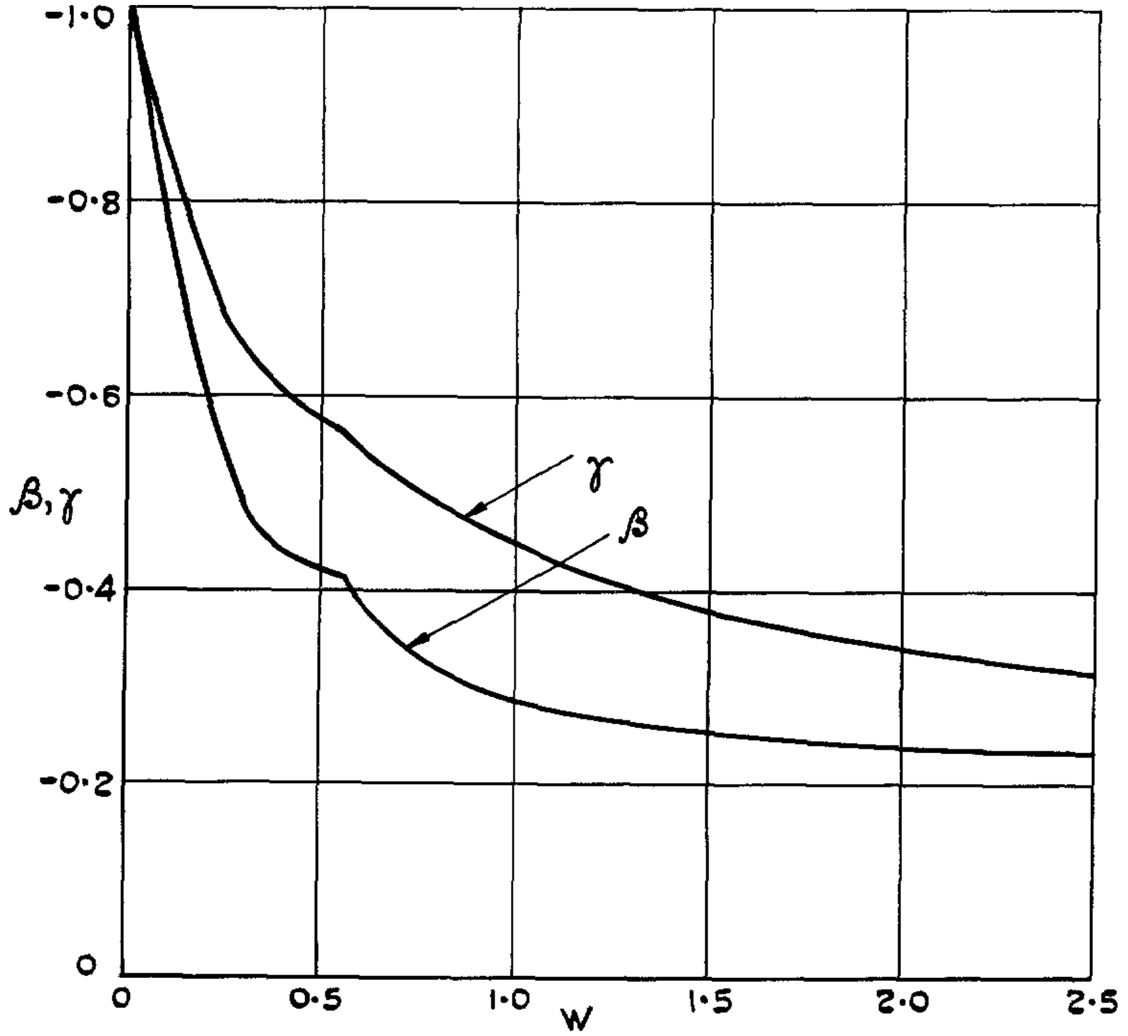

FIG. 11 VALUES OF β AND γ CORRESPONDING TO OPTIMUM W-S RELATION FOR PURE TENSION.

FIG. 12.

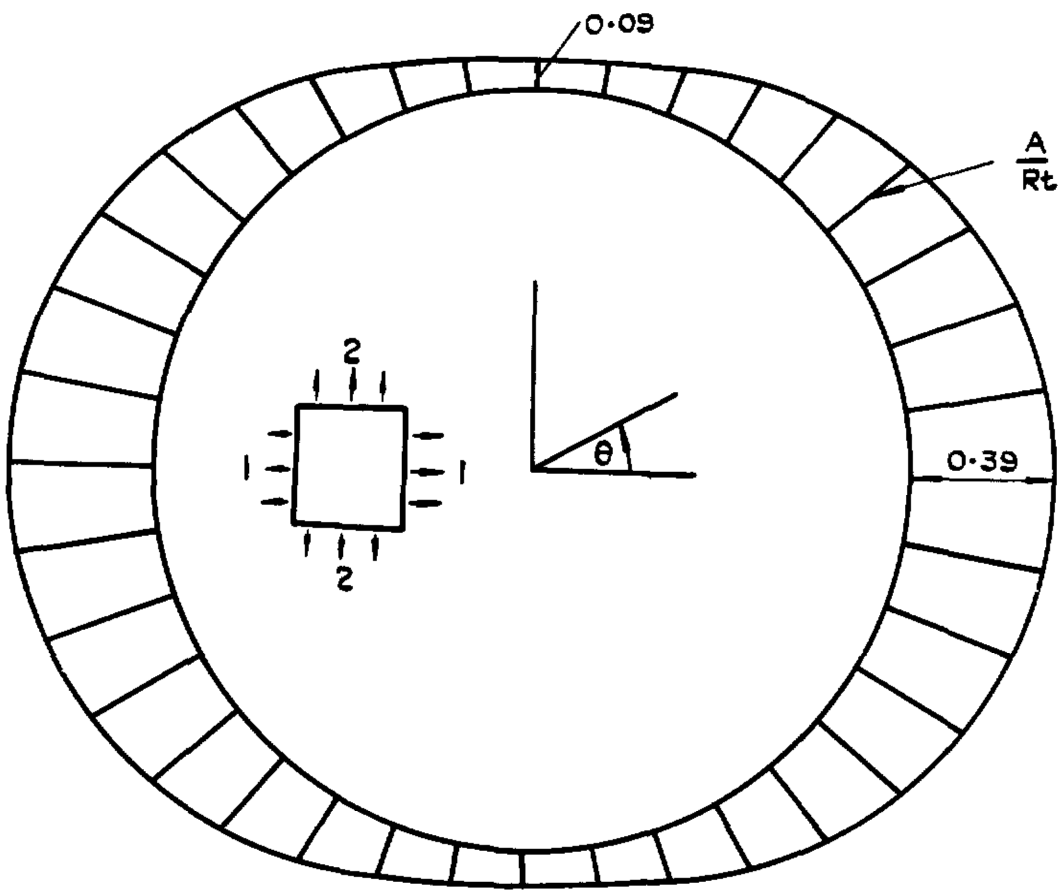

FIG. 12. OPTIMUM DISTRIBUTION OF A/Rt FOR 2:1 STRESS FIELD WITH W=0·5, S=1·6.

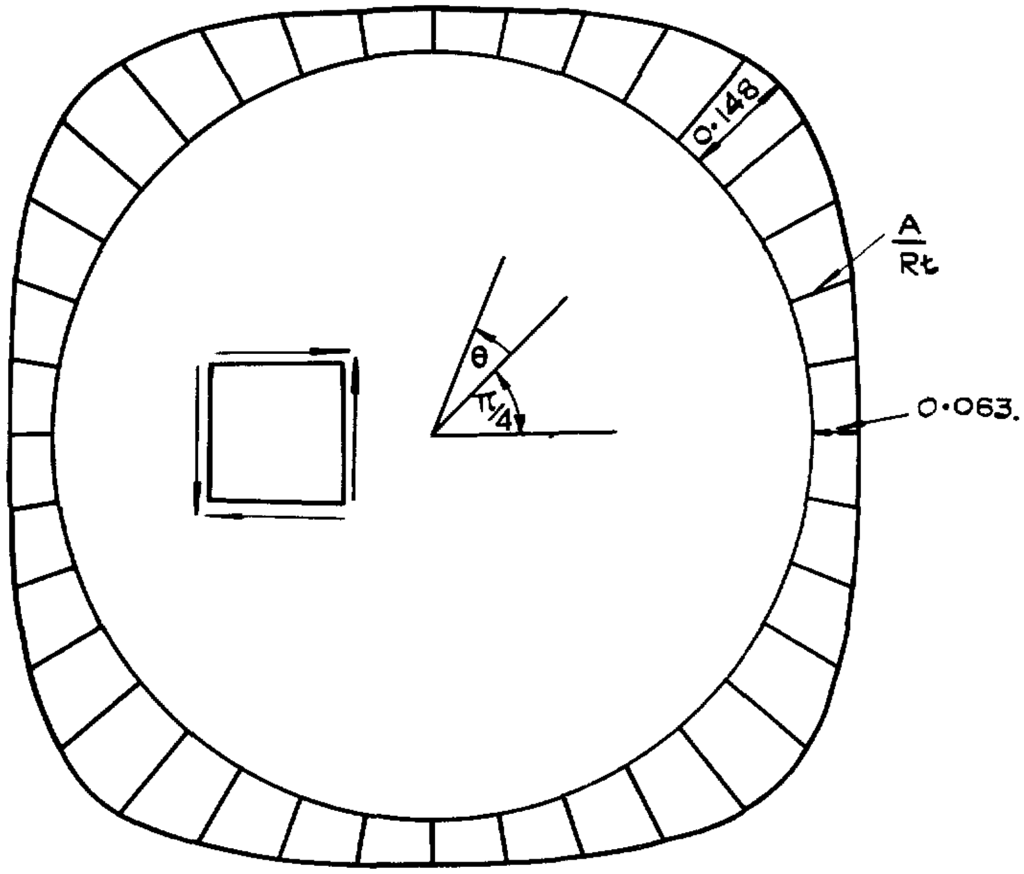

FIG. 13. OPTIMUM DISTRIBUTION OF A/Rt FOR PURE SHEAR WITH W=0·2, S=1·5.

FIG. 14.

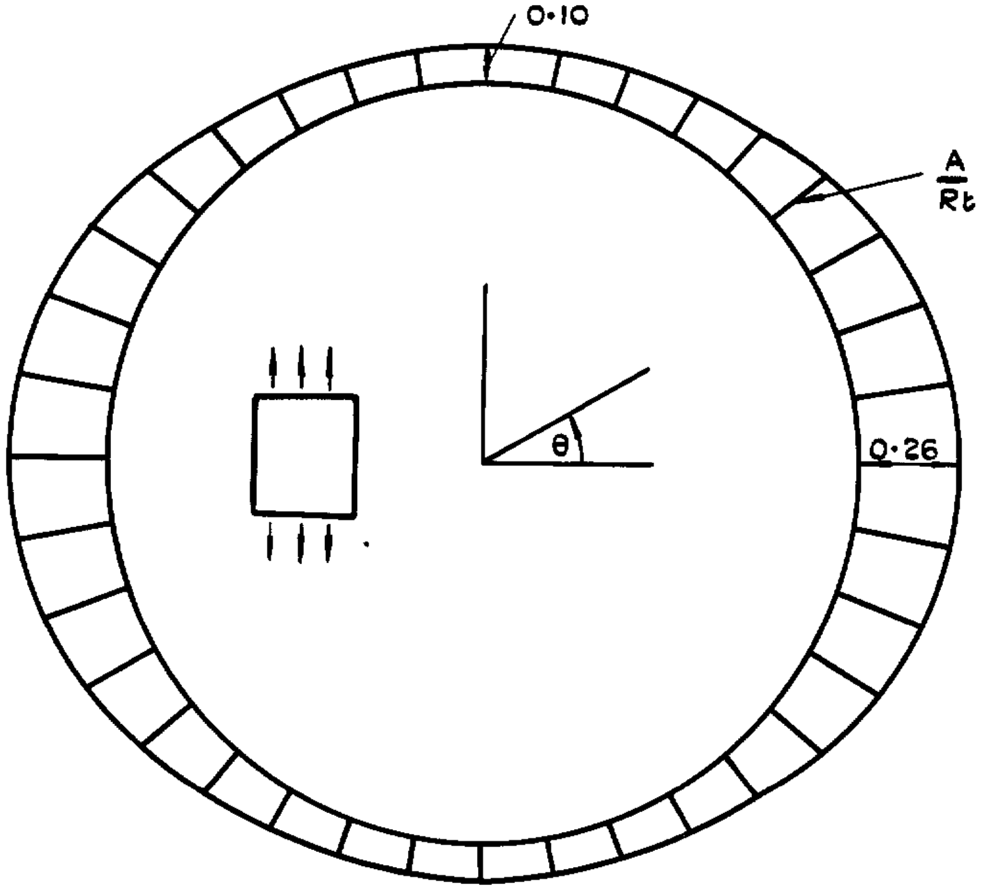

FIG. 14. OPTIMUM DISTRIBUTION OF A/Rt FOR TENSION WITH W=0·3, S=1·8.

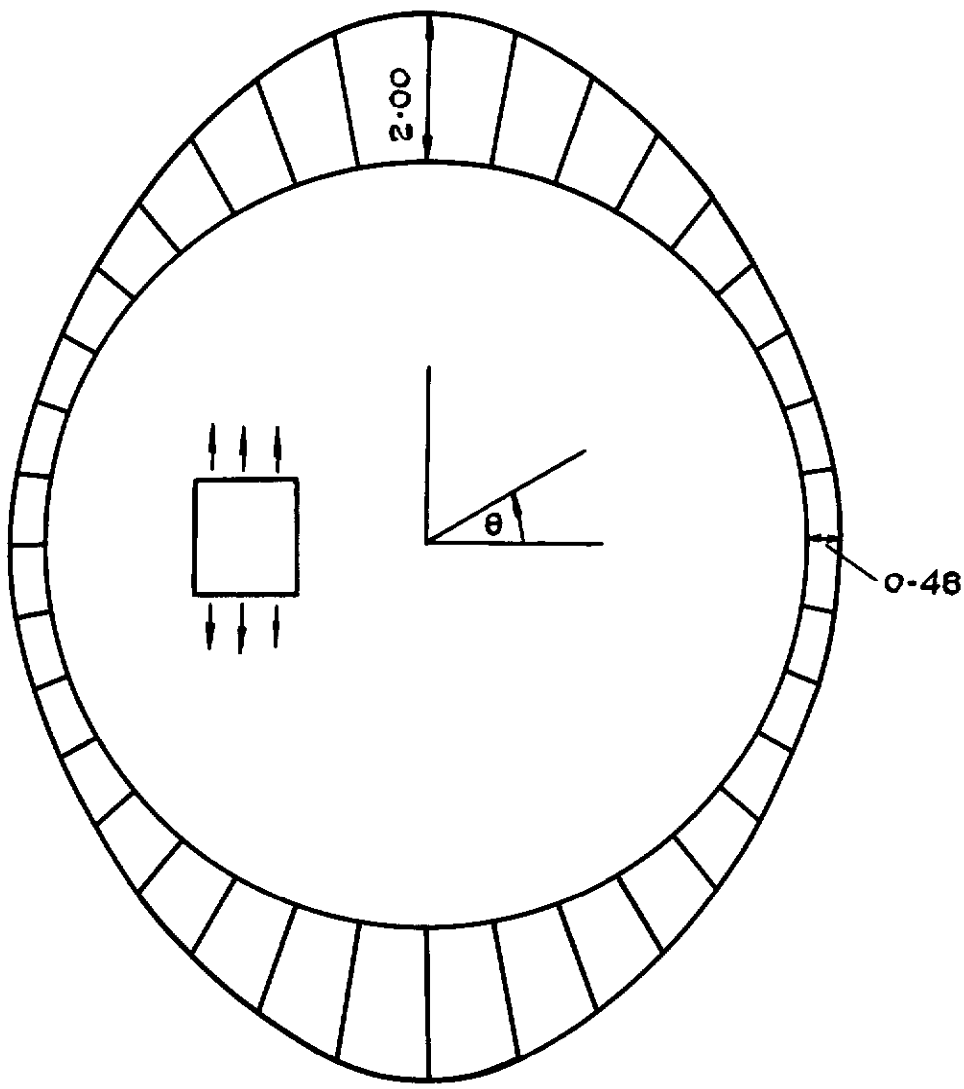

FIG. 15. OPTIMUM DISTRIBUTION OF A/Rt FOR TENSION WITH W = 2·0, S = 1·4.

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