C.P. No. 197
(17,287)
A.R.C. Technical Report

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MINISTRY OF SUPPLY

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Requirements for Uniformity of Flow in Supersonic Wind Tunnels

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LONDON: HER MAJESTY'S STATIONERY OFFICE

1955

Price 1s. 6d. net

C.P. No. 197

U.D.C. No. 533.6.071.4.011.5

Technical Note No. Aero.2340

September, 1954

ROYAL AIRCRAFT ESTABLISHMENT

Requirements for uniformity of flow in supersonic wind tunnels

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SUMMARY

An analysis is made of the effects of non-uniformity of flow on the pressure measurements on the surface of a model and also on the force and moment measurements and the following standards of flow uniformity are derived - variations in flow direction to be less than $\pm 0.1^\circ$ in the range M = 1.4 to 3; variation in Mach number to be less than ± 0.003 at M = 1.4 increasing to ± 0.01 at M = 3.

A brief analysis is made of the errors in model manufacture and their effects on force and pressure measurements. Using the same standards as were used in deducing the requirements for flow uniformity quoted above it is concluded that present standards of model manufacture are satisfactory overall, though for accurate pressure plotting tests at low supersonic Mach numbers ($M \approx 1.4$) a higher standard is desirable.

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1 Introduction

In considering the standard required of the flow in a supersonic tunnel it is important to bear in mind the nature of the flow distribution. Calibrations of supersonic tunnels show that after the nozzles have been corrected to remove obvious defects there is left a random variation of Mach number (or flow direction) superimposed on systematic variations. The order of the variations that occur is normally about ± 0.005 in Mach number overall. The problem is to determine the maximum variations of flow distribution consistent with the standards of accuracy required in test measurements.

In this note a brief analysis is made of both random and systematic variations of the flow. The accuracy of measurements of overall forces and moments on a model are considered and also the accuracy of pressure measurements on the surface of a model. It is difficult to interpret the significance of random variations in the flow with reference to overall forces and moments* but, on the other hand, the effects of systematic variations in flow on these quantities can be assessed and a standard of flow uniformity derived. The random variations and the errors they introduce can be considered in terms of pressure measurements on the surface of the model and thereby a standard of flow uniformity obtained.

Direct effects only are considered and no account is taken of, for example, the possible effects of flow irregularities on boundary layer conditions.

2 Errors in pressure measurements

Two standards are set depending on the nature of the work done in the tunnel. For most purposes it is sufficient to know the local pressure coefficient (C_p) to ± 0.005 but where comparisons with the more exact theories, especially on bodies, have to be made, it is desirable to know C_p to ± 0.002. This latter requirement is regarded as the limit and a standard of flow uniformity corresponding to it is necessary only in pure research work.

To derive from the accuracy required of C_p the allowable non-uniformity of flow distribution it must be remembered that each wing surface will only be affected by disturbances from the wall it faces (assuming for simplicity that the sidewalls are perfectly smooth). The disturbance is reflected from the surface of the wing which will double its effect as measured by the change in pressure coefficient when compared with uniform flow. When a pressure calibration of the flow is done in the empty tunnel the measured result is a combination of the disturbances from the opposite walls (again assuming smooth side walls). It can be assumed that the disturbances from the two walls are random and equal and thus the measured flow variation is, statistically, $\sqrt{2}$ times the flow variation from a single wall. Thus the limits of flow variation shown by a calibration correspond to $\sqrt{2}/2$ times the required accuracy of C_p.

Corresponding to a limiting error in C_p of ± 0.005, the following limits apply to the flow as shown by a normal calibration - at $M = 1.4$, $\Delta M = \pm 0.0035$, at $M = 2$, $\Delta M = \pm 0.006$ and at $M = 3$, $\Delta M = \pm 0.014$. The

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* In considering variations in flow direction over the tailplane the tail movement with change of incidence is comparatively small and in this case systematic variation tends to the same meaning as random variation.

corresponding increments in flow direction are $\pm 0.1^{\circ}$, $\pm 0.15^{\circ}$ and $\pm 0.28^{\circ}$ approximately. The increments in Mach number and flow direction corresponding to a limiting error in C_p of ± 0.002 are in direct proportion.

3 Errors in force and moment measurements

It is assumed from experience that the following accuracies are required in force and moment measurements.

(i) Errors in drag to be less than 1%.

(ii) Errors in pitching moment to correspond to less than 0.1° in tail setting to trim.

(iii) Errors in neutral point position to be less than 1% chord.

The analysis is simplified if a particular model is considered, but it must be noted that the results may vary slightly with the form of the model. For instance the lift and pitching moment on a body alone would be very sensitive to variations in pressure normal to its axis in the incidence plane.

The particular model chosen is in the form of an aircraft with rectangular wings; the dimensions in terms of the body diameter d are as follows:-

Wing	Span	= 7.5d	Tailplane	Span	= 3.75d
	Chord	= 2.5d		Chord	= 1.25d
Body length ℓ		= 15d	Tail volume coefficient		= 0.6

The wings and tailplane have 3% thick sections.

3.1 Drag

The drag of the model has been estimated on the following assumptions:-

(i) There is no interference between components.

(ii) The skin friction coefficient is 0.002.

(iii) The model has a bluff base and the estimated base pressure applies over the whole of the base.

(iv) The wave drag coefficient of the body is $(d/\ell)^2$.

The estimated value of C_D (based on wing area) varies from 0.023 at M = 1.4 to 0.013 at M = 3.

The error in drag is assumed to arise only from the buoyancy effects of a longitudinal pressure gradient and the error term is

$$C_{D_p} = - \frac{\text{model volume}}{q\,S} \cdot \frac{dp}{dx} \qquad \text{where } S = \text{wing area}$$

The pressure gradient corresponding to the buoyancy term equalling 1% of the drag varies from 0.002 at M = 1.4 to 0.0005 at M = 3.0, where the pressure gradient is expressed as the change in P/H per model length, H being the stagnation pressure. This corresponds to a Mach number gradient of approximately 0.4% of nominal Mach number per model length (0.025% per body diameter) over the range M = 1.4 to 3.0.

3.2 Pitching moment

There are two possible sources of error due to flow variations, firstly in the flow angle at the tailplane and secondly in the pitching moment on the wing body. It can be shown that the effect of the second is small; for convenience this is done by showing that the flow variation required to produce a pitching moment change equivalent to 0.1° change in tailplane setting is comparatively large.

The effects of the pressure gradient on the body can be ignored, leaving the only contribution to the pitching moment error that from the variation in flow direction along the wing chord, which gives an effective camber of the wing. This error can be evaluated from the relationship

$$C_{m_o} = \frac{1}{3\beta} \cdot \frac{d(\vartheta)}{d(\frac{x}{c})}$$

where $\beta = \sqrt{M^2 - 1}$ and ϑ is the flow direction.

The increment in C_{m_o} due to a 0.1° change in tail setting varies from 0.0035 at M = 1.4 to 0.0015 at M = 3, which corresponds to changes of flow angles along the chord of 0.6° in the range M = 1.4 to 3. This order of variation is much larger than the $\pm 0.1^\circ$ variation derived from the direct effect of flow direction at the tailplane.

In terms of Mach number gradient the variation deduced from the pitching moment on the wing becomes 0.5% of the nominal Mach number per body diameter. Compared with the requirement derived in Section 4.1 this is very large and therefore can be ignored in getting a standard of flow.

3.3 Neutral point

In the model design chosen the wings are large compared to the body and the effects of the body are neglected. The aerodynamic centre of the wings alone is fairly insensitive to Mach number changes and the effect of variation of tailplane lift only is calculated. During changes of incidence the model is assumed to rotate about a point in the region of the wing causing the tailplane to traverse the flow in an arc about this point.

The most important term to consider is that due to change in flow direction at the tailplane with displacement. For a shift in neutral point of 1% the corresponding variation in flow direction at the tail is 0.015α, α being the incidence change, which for a tail arm of 6d gives a gradient of 0.15° per body diameter. This is equivalent to a gradient of about 0.3% of nominal Mach number per body diameter.

4 Discussion

Consideration of possible errors in force and moment measurements due to non-uniformity of flow shows that the most important effects are the one affecting drag (buoyancy term) and that causing an error in pitching moment through a change in flow direction at the tailplane. The former gives a limitation in Mach number gradient of approximately

± 0.005 over the body length at $M = 1.4$, ± 0.008 at $M = 2$ and ± 0.012 at $M = 3$. The other limitation is that flow direction at the tailplane should be known to $\pm 0.1^{\circ}$. This requirement can be reasonably extended to the whole of the working section. The limitations on flow variations derived in this way are in good agreement with those derived from analysing the accuracy required of pressure measurements for general purpose tunnels, but a higher standard of flow variation is required of a tunnel intended for pure research work. It is worth noting in this connection that in some cases the accuracy of obtaining pressure measurements can be improved considerably by doing a comprehensive calibration and making corrections to the observed pressure distribution, provided the flow is fairly good.

In the analysis, changes in flow direction and changes in Mach number have been regarded as exact equivalents, which is correct for single disturbances or even disturbances from one wall. Where the disturbances from two opposite walls intersect, it is possible to have a large change in flow direction for no change in Mach number and vice versa. Therefore it is a necessary precaution in stating the requirements of flow uniformity as shown by a normal calibration of the flow to give the limitations in terms of both Mach number and flow direction.

In deriving the buoyancy error in drag a constant gradient in Mach number is assumed and no account is taken of the superimposed random variations. The wavelengths of the random variations are small compared to the length of the body and therefore an overall limitation in Mach number, somewhat less in magnitude than that allowed by the deduced limitation in gradient will give an approximately equivalent standard of accuracy, bearing in mind that the superimposed variations are random. This is done to simplify the statement of the requirements.

In the following table the requirements derived in various ways are combined to form a desirable standard of flow uniformity for general purpose tunnels:

M	Maximum variation	
	Flow direction	Mach number
1.4	$\pm 0.1^{\circ}$	± 0.003
2	$\pm 0.1^{\circ}$	± 0.005
3	$\pm 0.1^{\circ}$	± 0.010

Were it not for requiring to know the flow direction at the tailplane to $\pm 0.1^{\circ}$, the maximum variation allowable in flow direction would be $\pm 0.15^{\circ}$ at $M = 2$ and approximately $\pm 0.25^{\circ}$ at $M = 3$. It should be noted that the standard set by substituting these values in the appropriate places in the above table would be adequate for normal pressure plotting work. Where the higher standard of accuracy required for comparisons with the more exact theories are desirable the above standards of flow uniformity, with the flow direction limitations modified as discussed, should be approximately halved. This represents an extremely high standard of flow uniformity and is difficult to achieve over the test section, bearing in mind the order of surface finish it implies.

It is of interest to compare the errors that arise from non-uniformity of flow in the working section with those caused by manufacturing errors in the model. In considering the overall accuracy of a model the main concern is with the angular settings of the wing and the tailplane. The measurement of these quantities when the profiles are aerofoil shapes involves accurate location of the leading edge. Wing settings can be measured to about $\pm 0.025^{\circ}$ and tail settings to $\pm 0.05^{\circ}$ on typical aircraft models having wing spans of about 10-15 inches. The error will increase as the model size is reduced.

There are irregularities in the surfaces of models. These are random and can be considered in relation to the measurement of local surface pressures. As examples, the manufacturing errors on three typical steel wings are given in this report. All the models have aerofoil shape profiles and are made of high tensile steel. They are of approximately the same size having a mean chord of about 4 inches. Model A is a rectangular wing with the profile unchanged along the span and therefore comparatively simple to manufacture. It was made on a milling copying machine and afterwards hand-finished to template. Models B and C are delta wings made by the tangent plane grinding method, which generally results in a higher standard of accuracy. Model B is a plain delta wing but in Model C the chordline is twisted along the span giving a complex shape of wing surface. Both methods of manufacture ensure that there are no "steps" in the profile and the errors that occur are small local deviations of slope from the true shape. Measurements were made of the ordinates at stations about 0.1 inches apart along chord lines and errors in local slopes derived. The results are given in Fig.1 as points showing the percentage of the readings taken having errors in slope less than a given amount.

The standard of manufacture of models B and C is representative of the best than can be achieved, but it must be noted again that the complex surface shape of wing C presented an extremely difficult problem. The accuracy obtained on model A represents a good average. It is concluded that the curve drawn in Fig.1 is typical of what can be obtained by a good standard of manufacture.

The above discussion refers to wings with aerofoil profiles. A higher standard of accuracy can be expected, with care, on wings formed by a few planes.

To compare the errors in local surface pressure coefficients arising from faults in model manufacture with those caused by non-uniformity of flow, it is reasonable to take errors in slope of $\pm 0.15^{\circ}$, which covers 75% of the errors according to the curve of Fig.1. The errors in local pressure coefficient C_p corresponding to this error in slope are approximately ± 0.005 at $M = 1.4$, ± 0.003 at $M = 2$ and ± 0.002 at $M = 3$.

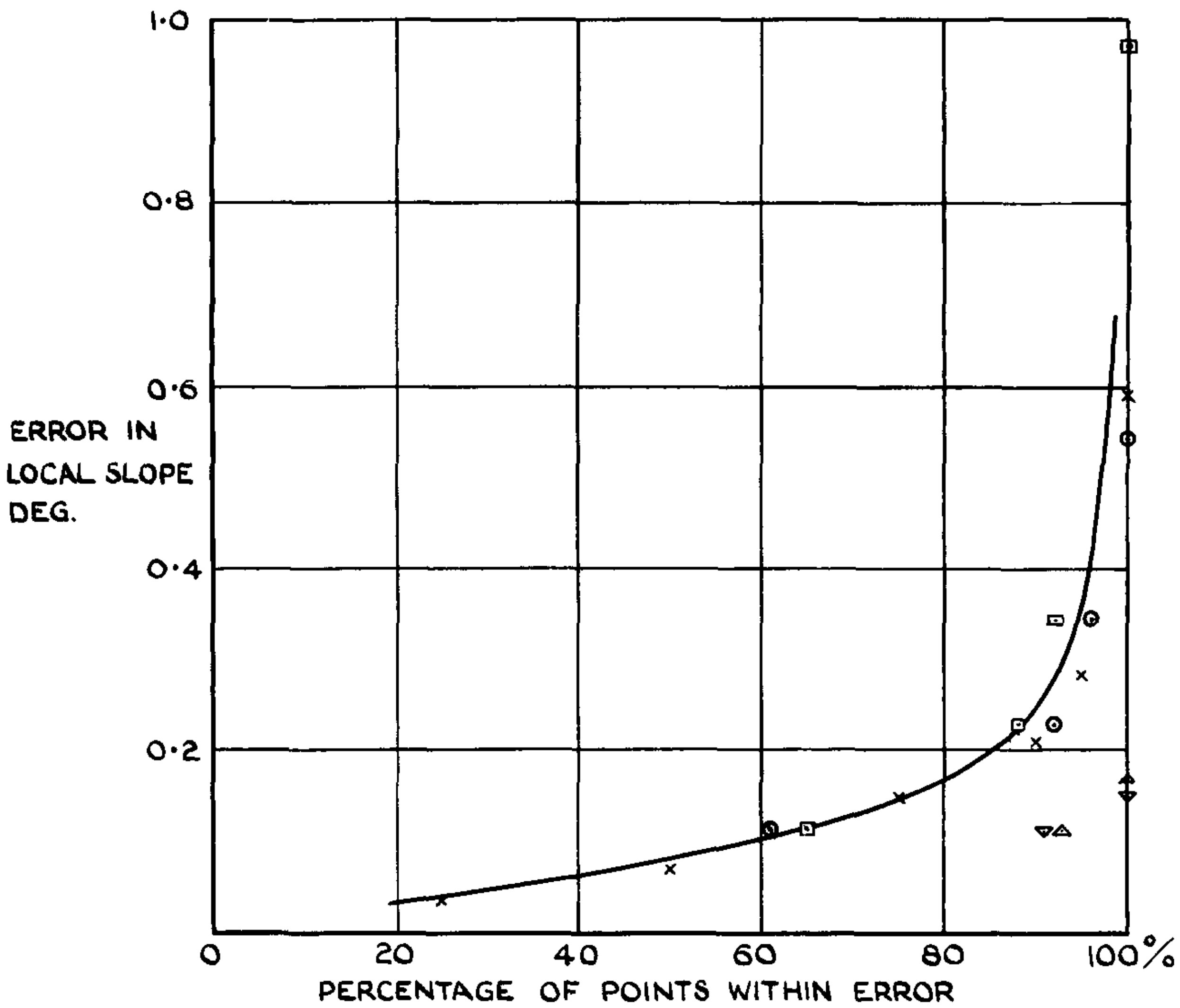
An error in the measured value of C_p of ± 0.005 was assumed to be reasonable in setting a standard of flow uniformity. Thus in order to retain this accuracy overall in the measurement of C_p at low supersonic Mach numbers ($M \approx 1.4$) with the standard of flow uniformity set in this report a higher standard of manufacture of models than is currently normal is necessary. At the higher Mach numbers, assuming that the probable combined error due to flow irregularity and errors of manufacture of the model, which are both random, can be determined by taking the square root of the sum of the squares of the separate errors, the errors due to the model itself are relatively small and can be neglected. Thus at about $M = 2$ and above the current standard of model manufacture is adequate. By an identical argument the overall standard of model manufacture, as shown by tail setting, is adequate for models having 10 inches span and above.

5 Conclusions

Based on an analysis of the effects of non-uniformity of flow on the pressure measurements on the surface of a model and also on the force and moment measurements the following standards of flow uniformity were derived - variations in flow direction to be less than $\pm 0.1^{\circ}$ and variation of Mach number to be within ± 0.003 at $M = 1.4$ increasing to ± 0.01 at $M = 3$.

Comparison of the effects of non-uniformity of flow with those of errors in model manufacture shows that present standards of model construction are satisfactory overall but that at low supersonic Mach numbers ($M \approx 1.4$) a higher standard of model accuracy is required for pressure measurements.

Wt.2078.CP.197.K3. Printed in Great Britain.



MODEL A	RECTANGULAR WING	UPPER SURFACE	×
MODEL B	PLAIN DELTA WING.	UPPER	△
		LOWER	▽
MODEL C	TWISTED DELTA WING	UPPER	□
		LOWER	⊙

MEAN CHORDS ≈ 4 INCHES
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RESULTS FOR MODEL A BASED ON 110 INTERVALS.
RESULTS FOR MODELS B & C BASED ON 75 INTERVALS PER SURFACE.

FIG. 1. MANUFACTURING ERRORS IN STEEL WINGS OF WIND TUNNEL MODELS.

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1955

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C.P. No. 197