C.P. No. 194
(16, 728)
A.R.C. Technical Report

C.P. No. 194

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A Note on the Sound from Weak Disturbances of a Normal Shock Wave

By

Alan Powell

of the Department of Aeronautical Engineering
University of Southampton

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1955

Price 1s 6d net

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A Note on the Sound from Weak Disturbances of a Normal Shock Wave

- by -
Alan Powell
of the
Department of Aeronautical Engineering,
University of Southampton

15th April, 1954

SUMMARY

The disturbances of a shock wave by sound waves or temperature fluctuations is studied in one dimension to a first order approximation. In general, both sound waves and temperature fluctuations arise behind the shock wave. Expressions are given for their amplitudes, and calculated for $\gamma = 1.4$. Sound waves colliding with the shock wave are amplified, but sound waves are almost annihilated by weak shock waves if originally travelling in the same direction as the shock wave. Small temperature fluctuations give rise to much sound, on an acoustical scale.

Introduction

A crucial link in the theory of the characteristic noise produced by supersonic jets (Powell 1952, 1953)[3,4] is that the disturbance of the 'standing' shock waves in the jet-stream by the passage of a train of eddies gives rise to sound energy. It was argued that velocity or pressure fluctuations swept into the shock would give rise to both velocity and pressure changes on the downstream side. These might be imagined to separate out into sound waves moving in opposite directions: but due to the relative flow velocities at the shock the upstream part can never leave the shock to either side and so will simply cause its strength to fluctuate further. Thus we immediately see, even on this crude basis, that the shock-wave position will be disturbed, its fluctuations in strength will result in entropy changes which will be swept away with the flow, and sound will be radiated away into the downstream flow.

Viewed loosely from the acoustical point of view the shock wave marks a discontinuity in acoustic impedance so that, in the absence of mean motion, transmitted and reflected waves would arise. But one of these two dependent variables, the upstream sound wave, cannot arise because of the supersonic upstream and subsonic downstream relative velocities, nor can the governing equations be satisfied by a single resultant sound wave, as will be indicated. The second dependent variable required is the entropy.

Possible/

Possible disturbances in one-dimensional flow are limited to temperature inhomogenities (assuming gas of a single type) and sound waves since pressure and velocity fluctuations are bound together in this form, but there is choice in their direction. Three cases apparently arise: sound waves in the upstream flow may collide head-on with the shock wave or be swept into it (i.e., with a stationary upstream fluid the shock would catch up with the sound waves) or finally sound waves in the subsonic downstream flow may catch up with the shock wave.

This paper which was partially completed some time ago was intended as a possible first step of a more general consideration bearing on the jet noise problem. However, in the meantime both H. S. Ribner[5] and M. J. Lighthill[2] have published papers on this topic, more directly from the turbulence point of view. Ribner (1953) works in two-dimensions and considers shear waves (being a constituent of turbulent motion), and Lighthill extends his theory of aerodynamic noise to the energy scattered by the interaction of sound waves, and finally weak shock waves, with turbulence.

Moreover, towards the end of this work Dr. Hans Liepmann kindly pointed out to the author an earlier paper by Burgers (1946)[1] who considered the interaction of sound waves with a shock wave on precisely the same assumptions as in this note, having shown that the entropy must be considered as the second dependent variable. However, this note is slightly more general and the results are considered in more detail; for the sake of completeness the whole analysis is given.

2. The Equations Concerned

The Steady State Equations

The usual assumptions associated with the treatment of a shock wave as a discontinuity of flow are made, i.e., the effects of viscosity, conduction and radiation of heat are ignored. If we take a frame of reference in which the undisturbed shock wave is stationary, and the regions just upstream and just downstream of the shock are respectively labelled 1 and 2, (Fig. 1) the equations of continuity of mass, momentum, energy and state are

$$\rho_1 u_1 = \rho_2 u_2 \qquad \ldots(1)$$

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2 \qquad \ldots(2)$$

$$\frac{u_1^2}{2} + C_p T_1 = \frac{u_2^2}{2} + C_p T_2 \qquad \ldots(3)$$

$$\frac{p_1}{\rho_1 T_1} = R = \frac{p_2}{\rho_2 T_2}. \qquad \ldots(4)$$

The following well known solutions may be found useful in considering the final results

$$M_2^2 = \left(\frac{\gamma + 1}{2\gamma}\right)^2 \frac{2\gamma}{2\gamma M_1^2 - (\gamma - 1)} + \frac{\gamma - 1}{2\gamma}$$

$$\frac{\rho_1}{\rho_2} = \frac{2}{\gamma + 1} \cdot \frac{1}{M_1^2} + \frac{\gamma - 1}{\gamma + 1}$$

where M is the Mach number u/c.

The/

The Equations with Small Disturbances

All the variables in the above equations will be increased by a small increment denoted by a single prime, e.g., ρ_1 becomes $\rho_1 + \rho_1'$. The shock wave will be assumed to move relative to the frame of reference with velocity ξ (Fig. 2). Then on dismissing second orders of small quantities, and taking out the original equations from each of the above,

$$\rho_1' u_1 + \rho_1(u_1' - \dot{\xi}) = \rho_2' u_2 + \rho_2(u_2' - \dot{\xi}) \qquad \ldots(1')$$

$$p_1' + \rho_1' u_1^2 + \rho_1 u_1(2\, u_1' - \dot{\xi}) = p_2' + \rho_2' u_2^2 + \rho_2 u_2(2\, u_2' - \dot{\xi}) \qquad \ldots(2')$$

$$u_1 u_1' + C_p T_1' = u_2 u_2' + C_p T_2' \qquad \ldots(3')$$

$$\frac{p_1'}{p} - \frac{\rho_1'}{\rho_1} - \frac{T_1'}{T_1} = 0 = \frac{p_2'}{p_2} - \frac{\rho_2'}{\rho_2} - \frac{T_2'}{T_2} \,. \qquad \ldots(4')$$

Specification of the Disturbances

Temperature fluctuations arise in the sound waves, but the process is isentropic: thus if we associate the temperature waves with entropy no confusion with temperature fluctuations of sound waves can arise. Also, if the sound waves are distinguished by pressure changes they are completely defined.

It will be useful to compare the amplitude of the temperature waves with the temperature elevation occurring within a sound wave. Since by the definition of entropy

$$\frac{S'}{C_p} = \frac{T'}{T} - \frac{R}{C_p} \cdot \frac{p'}{p} \qquad \ldots(5')$$

the amplitude of the temperature wave, which is a function of $(Mct - x)$, is

$$T' = \left(\frac{S'T}{C_p}\right)$$

and the temperature elevation of the sound wave is

$$T' = \frac{p'}{C_p} \cdot \frac{RT}{p} = \left(\frac{p'}{C_p \rho}\right).$$

There is a well known unique relationship between the particle velocity and pressure increment of a sound wave, provided the amplitude is small. This is

$$p' = \pm \rho c \,.\, u'$$

according to whether the wave is travelling in the positive or negative direction, and being functions of $\{(M + 1)\,ct - x\}$ and $\{(M - 1)\,ct - x\}$ respectively. The speed of sound c is of course given by $c^2 = p'/\rho' = \gamma p/\rho$. Both cases arise for incident waves in the upstream flow, but only the former can appear in the downstream flow. For the upstream waves the special symbol Σ will be used, to be taken as plus or minus (one) when appropriate. Thus the upstream sound waves are functions of $\{(M_1 + \Sigma 1)\,ct - x\}$ with

$$p_1' = \Sigma \rho_1 c_1 \,.\, u' \qquad \ldots(6'a)$$

and

$$p_2' = \rho_2 c_2 \,.\, u_2' \,, \qquad \ldots(6'b)$$

a/

a function of $\{(M_2 + 1)\,ct - x\}$, for the downstream transmitted wave. The latter notation for the resultant downstream wave will be retained when the incident wave approaches from the downstream direction, this incident wave being denoted by double primes:-

$$p_2'' = -\rho_2 c_2 \,.\, u_2''. \qquad \ldots(6'c)$$

3. Disturbances Originating from Upstream

The solution to equations (1') to (6') are required with p_1' and S_1' non-zero. Take equation (2'): eliminate ξ by use of equation (1), ρ' by equation (4'), T' by equation (5'), and u' by (6'a) or (6'b). The momentum equation (2') then becomes

$$p_1'\,(1 + \Sigma M_1)^2 - (\gamma - 1)\rho_1 M_1^2 S_1' T_1 = p_2'\,(1 + M_2)^2 - (\gamma - 1)\rho_2 M_2^2 S_2' T_2 \qquad \ldots(2'a)$$

where M is the local Mach number of the flow, u/c.

The energy equation (3'), by use of equation (5') becomes

$$\frac{p_1'}{\rho_1}\,(1 + \Sigma M_1) + S_1'\,T_1 = \frac{p_2'}{\rho_2}\,(1 + M_2) + S_2'\,T_2. \qquad \ldots(3'a)$$

It is easily seen that a sound wave alone, or a temperature wave alone cannot give rise to waves of only one type, i.e., sound or temperature waves, since for no value of Mach number can the resultant expressions be satisfied. An exception to this arises in the case of a shock wave overtaking a sound wave, when a unique value of $M \doteq 1.37$ results for zero transmitted sound wave. Thus in this case the interaction simply gives rise to temperature fluctuations convected with the flow, but only at that Mach number.

Eliminating S_2' and p_2' in turn from these equations gives the following results, on making first the initial condition S_1' and then p_1' zero.

Disturbance by Sound Waves in Upstream Region

The interaction of a sound wave of amplitude p_1' and temperature elevation $\left(\frac{p_1'}{C_p\rho_1}\right)$ with a shock wave produces behind the shock a sound wave of temperature elevation

$$\left(\frac{p_2'}{C_p\rho_2}\right) = \left(\frac{p_1'}{C_p\rho_1}\right) \underset{S\ \ S}{\overset{\Sigma}{\mathcal{T}}} \qquad \ldots(8)$$

together with a temperature wave of amplitude

$$\left(\frac{S_2'\,T_2}{C_p}\right) = \left(\frac{p_1'}{C_p\rho_1}\right) \underset{S\ \ T}{\overset{\Sigma}{\mathcal{T}}} \qquad \ldots(9)$$

where

$$\underset{S\ \ S}{\overset{\Sigma}{\mathcal{T}}} = \frac{(1 + \Sigma M_1)\,\{(1 + \Sigma M_1)\,\rho_1/\rho_2 + (\gamma - 1)\,M_2^2\}}{(1 + M_2)\,\{(1 + M_2) + (\gamma - 1)\,M_2^2\}} \qquad \ldots(8a)$$

and

$$\underset{S\ \ T}{\overset{\Sigma}{\mathcal{T}}} = \frac{(1 + \Sigma M_1)\,\{(1 + M_2) - (1 + \Sigma M_1)\,\rho_1/\rho_2\}}{\{(1 + M_2) + (\gamma - 1)\,M_2^2\}} \qquad \ldots(9a)$$

and/

are transmission coefficients depending only upon the strength of the shock wave. The first, ${}_S\tau_S^{\Sigma}$ concerns the transmitted sound wave, with Σ as +1 in the expression for the sound wave meeting the shock wave, and ${}_S\tau_S^{-}$ for the case when the sound wave and shock wave propagate in the same direction, Σ being then taken as -1. Similarly ${}_S\tau_T^{\Sigma}$ deals with the sound wave giving rise to a temperature wave.

As would be expected, for the sound wave colliding with a shock of vanishing strength, the sound pressure is unaffected, and no complementary temperature wave results (Fig. 3, γ = 1.4). As the shock strength increases, not only does a temperature wave of like sign appear with its amplitude finally approaching the temperature elevation of the original sound wave, but the sound wave undergoes amplification, its temperature elevation being increased by a factor of 1.4 at M_1 = 2. This corresponds to a sound intensity* amplification, $\left({}_S\tau_S^{+}\right)^2 \rho_2 c_1/\rho_1 c_2$, of four-fold. Sound energy is thus leaving the shock about twice as fast as it is being taken in, a 'power ratio' of 2.11. At M_1 = 4, the intensity amplification is 16.6 and the power ratio 4.77.

The shock wave perturbation velocity can be found by taking from equation (1') and then eliminating ρ', T', p_2', S_2' and finally p_1' by equations (4'), (5'), (8), (9) and (6'a) in turn. Then

$$\dot{\xi} = u_1' \, {}_S\tau_{\dot{\xi}}^{\Sigma} \qquad \ldots(10)$$

where

$${}_S\tau_{\dot{\xi}}^{\Sigma} = \frac{\rho_1}{\rho_2-\rho_1}\left\{\Sigma\frac{\rho_2 c_1}{\rho_1 c_2}\left[(1+M_2)\,{}_S\tau_S^{\Sigma} - (\gamma-1)M_2\,{}_S\tau_T^{\Sigma}\right] - (1+\Sigma M_1)\right\}. \qquad \ldots(10a)$$

The variation of this coefficient is also shown in Fig. 3; it is interesting to note that the shock front oscillates with an amplitude rather less than that of the 'particle', whereas at an ordinary discontinuity the amplitude is the same.

When the shock wave overtakes the sound wave (Fig. 4) the latter is almost annihilated at small Mach numbers, and exactly so at M_1 = 1.37, there being a relatively large complementary temperature wave. Both these waves tend to zero as the shock strength vanishes. The shock wave movement is smaller in this case, decreasing so that at M = 2.4 approx. it is undisturbed in position by the sound waves, and above that its movement is opposite to that of the 'particle' velocity.

Disturbance by Temperature Inhomogenities

With p_1' zero in equations (2'a) and (3'a), it is found that the temperature inhomogenities of amplitude $S_1' T_1/C_p$ interact with the shock wave to give rise to sound waves of temperature elevation

$$\left(\frac{p_2'}{C_p\rho_2}\right) = \left(\frac{S_1' T_1}{C_p}\right) {}_T\tau_S \qquad \ldots(11)$$

where/

---

*The rate at which sound energy traverses a reference plane moving with the fluid.

where $$\underset{T\ S}{\mathcal{T}} = -\frac{(\gamma - 1)}{(1 + M_2)} \cdot \frac{\{M_1^2\ \rho_1/\rho_2 - M_2^2\}}{\{(1 + M_2) + (\gamma - 1)\ M_2^2\}} \ . \qquad \ldots(11a)$$

This is the complementary wave, the transmitted temperature wave being

$$\left(\frac{S_2'\ T_2}{C_p}\right) = \left(\frac{S_1'\ T_1}{C_p}\right)_T \underset{T}{\mathcal{T}} \qquad \ldots(12)$$

where $$\underset{T\ T}{\mathcal{T}} = \frac{\{(1 + M_2) + (\gamma - 1)\ M_1^2\ \rho_1/\rho_2\}}{\{(1 + M_2) + (\gamma - 1)\ M_2^2\}} \qquad \ldots(12a)$$

while the shock wave perturbation velocity is given by

$$\dot{\xi} = c_1' \underset{T\ \dot{\xi}}{\mathcal{T}} \ . \qquad \ldots(13)$$

Here c_1' is the increment in the wave velocity of sound in the upstream flow due to the temperature fluctuation. It is connected to the entropy by the relation $c_1' = S_1'c_1/2C_p$, while the corresponding temperature elevation is $2c_1'T_1/c_1$.

The non-dimensional coefficient is

$$\underset{T\ \dot{\xi}}{\mathcal{T}} = \frac{2\rho_1}{\rho_2 - \rho_1}\left\{M_1 - \frac{1}{\gamma - 1} \cdot \frac{c_1\ \rho_2}{c_2\ \rho_1}\left[(\gamma - 1)\ M_2 \underset{T\ T}{\mathcal{T}} - (1 + M_2) \underset{T\ S}{\mathcal{T}}\right]\right\} \ldots(13a)$$

Perhaps the main interest here is the production of the complementary pressure wave by the interaction of the shock wave with temperature fluctuations in the upstream fluid (Fig. 5). The temperature elevation of the pressure wave, which is zero at $M = 1$, increases to a value of 0.6 at a Mach number of four. The pressure change is so as to reduce the pressure ratio of the shock, if the initial temperature disturbance is an increase, so that although the shock advances into it, it does so with a disturbance velocity less than M_1 times c'.

Let us see what is the amplitude of the pressure wave resulting from a small temperature fluctuation, in acoustical measure. For example if $M_1 = 2$, with the initial flow under normal atmospheric conditions a temperature fluctuation of 0.1°C will result in a pressure amplitude of 1 lb/ft^2. This is not a lot on an engineering scale, but corresponds to a sound intensity of 126 db* - a level bordering on the threshold of pain in the ear.

This case also serves to illustrate the order of approximation made in the analysis. While acoustic energy is being radiated away from the shock, it is a second order effect resulting from the energy input fluctuating about the initial mean.

4. Disturbances Originating from Downstream

The only disturbances from behind the shock wave which can disturb it are pressure waves, which will be identified by double primes, as in equation (6'c). Thus in equations (1') to (4') all quantities on the left hand side are zero, except those involving ξ and on the right hand side the increments will have to include the incident wave. Proceeding as before, it is found that a sound wave approaching the shock from the downstream region is 'reflected' according to

$$\left(\frac{p_2'}{C_p\rho_2}\right) = \left(\frac{p_2''}{C_p\rho_2}\right)_S \underset{S}{\mathcal{R}} \qquad \ldots(14)$$

where/

*Above 0.0002 dynes/cm^2.

where the 'reflection' coefficient is

$$\underset{S\ S}{R} = -\frac{(1 - M_2)\ \{(1 - M_2) + (\gamma - 1)\ M_2^2\}}{(1 + M_2)\ \{(1 + M_2) + (\gamma - 1)\ M_2^2\}} \qquad \ldots(14a)$$

while the complementary temperature wave is

$$\left(\frac{S_2'\ T_c}{C_p}\right) = \left(\frac{p_2''}{C_p \rho_2}\right) \underset{S\ T}{R}$$

where

$$\underset{S\ T}{R} = -\frac{2\ M_2\ (1 - M_2)}{\{(1 + M_2) + (\gamma - 1)\ M_2^2\}}\ . \qquad \ldots(15)$$

The shock wave disturbance velocity is given by

$$\dot{\xi} = u_2'' \underset{S\ \dot{\xi}}{R} \qquad \ldots(16)$$

where

$$\underset{S\ \dot{\xi}}{R} = \frac{\rho_2}{\rho_1 - \rho_2} \left\{(1 - M_2) + (1 + M_2) \underset{S\ S}{R} + (\gamma - 1)\ M_2 \underset{S\ T}{R}\right\}. \qquad \ldots(16a)$$

The outstanding result here is the smallness of the 'reflected' sound wave and the complementary temperature waves (Fig. 6), especially for weak shock waves. This may remove the necessity for including reflections off the shock in certain unsteady phenomena, a fact which has already been made use of by Lighthill (1953).

Concluding Remarks

The transmission and reflection of sound waves at a shock wave have been studied in one dimension by taking first order approximations in the relevant equations. The resultant sound wave is accompanied by a complementary temperature wave; similarly temperature fluctuations give rise to complementary sound waves as well as the transmitted temperature waves. The shock wave oscillations are generally of the same order as those of the 'particle' in the sound wave.

Thus sound energy may be generated or absorbed at the shock wave, this energy actually being of second order. It must of course be extracted from, or supplied to, the main flow. Generally the disturbance will not cease after these waves have left the shock wave, since the pressure wave will be at least partially reflected back towards the shock again, of the same sense if from a solid boundary, as when a piston generates the shock wave in an otherwise steady flow (it then doing additional work), or of the opposite sense when from a constant pressure region, as in wind tunnels. The arrival of this reflected wave will virtually complete the history since its own reflections from the shock wave are so small, at least at the lower Mach numbers.

A standing wave system in the upstream region, having a 'group velocity' equal to the shock wave velocity, (obtained simply by postulating that the wave propagated towards the shock must have wavelength $(M + 1)/(M - 1)$ times that of the wave propagated in the opposite direction) enables the study of pressure or velocity fluctuations alone at the shock wave. The 'transmission' coefficients are then simply the sum or difference of those for sound waves propagating in each direction. However, it would be unwise to place much reliance on this so far as magnitudes in three-dimensional phenomena are concerned since vorticity can then be transmitted across the shock wave, it being possible for kinetic energy to be convected with the flow.

The/

The possibility of obtaining experimental values of at least some of the coefficients is being investigated: the shock tube technique seems promising in conjunction with optical methods of sound measurement (Powell, 1953).

Finally, this work supports the general hypothesis of the theory of choked jet noise that disturbances of shock waves give rise to noise (as of course, do the other works referred to). Further, the fact that noise arises from shock-wave interaction with temperature fluctuations draws attention to the probable existence of a further source of noise in choked jets, of turbo-jets or rockets in particular where considerable temperature fluctuations may arise.

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Figs 1-4

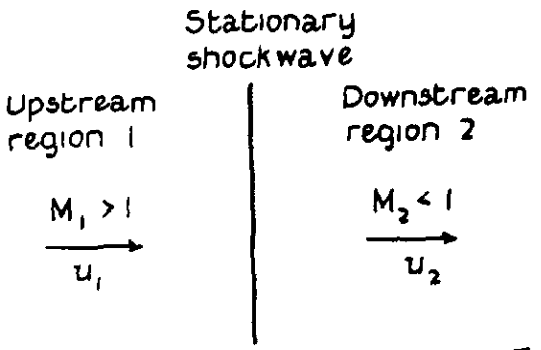
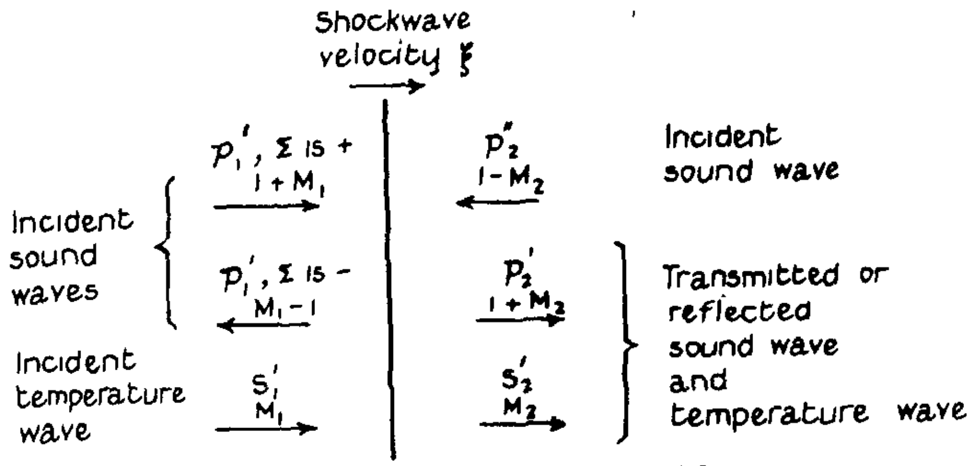


Fig 1.

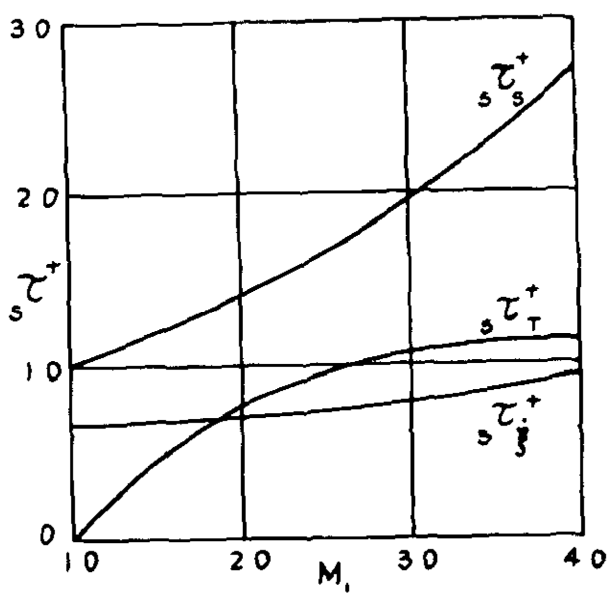
Steady state shockwave

Fig 2.



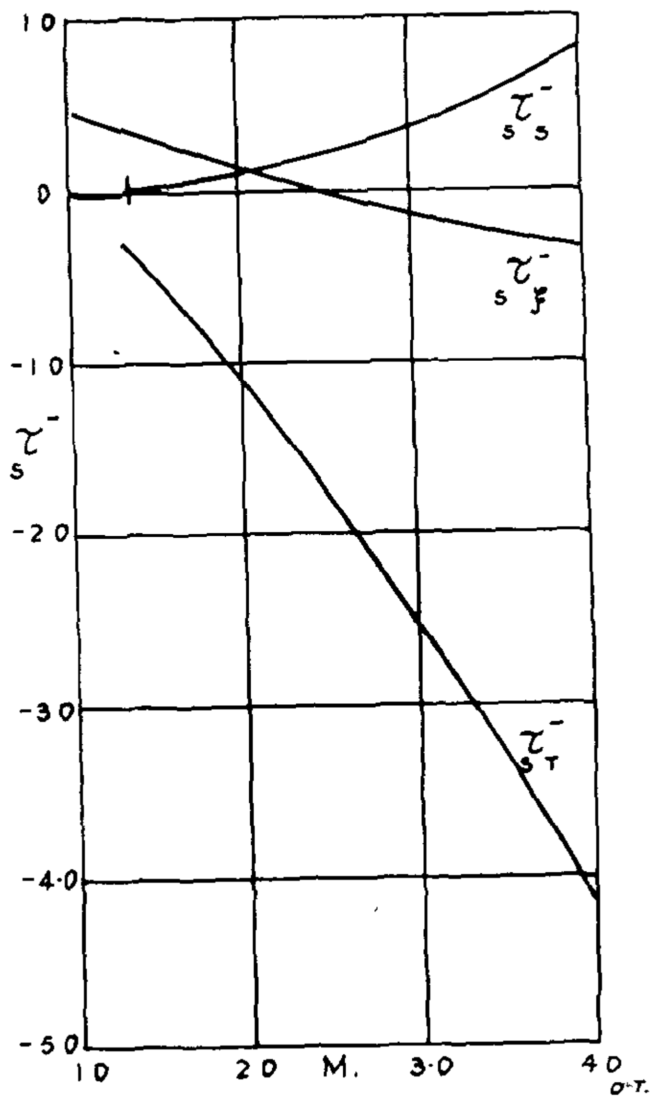
Perturbations at shockwave

Fig. 3



Coefficients for head-on collision of sound wave with shockwave.

Fig. 4

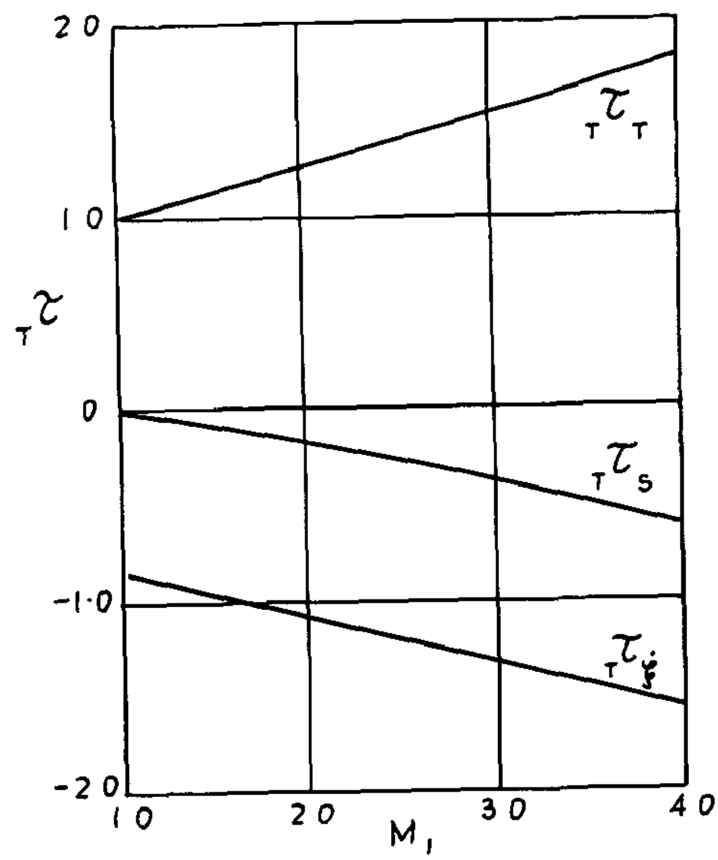


Coefficients for shock overtaking sound wave

O-T.

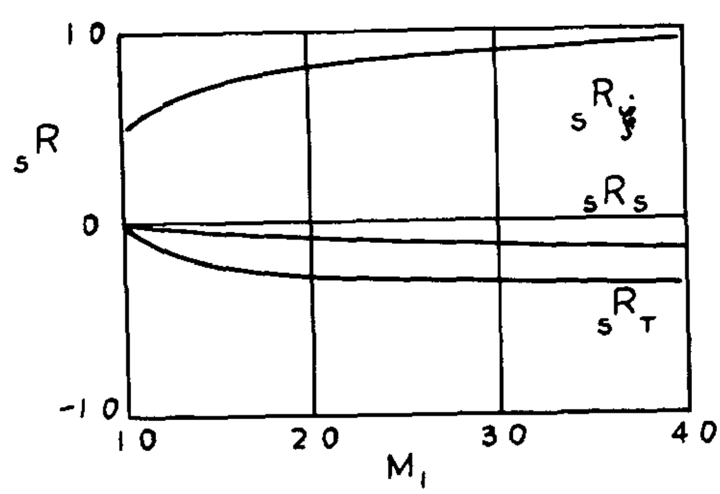
FIGS 5 & 6

FIG 5.



Coefficients for interaction of temperature wave and shockwave.

FIG 6.



Coefficients for reflection of soundwave off shockwave.

C.P. No. 194
(16, 728)

A.R.C. Technical Report

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