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The Theoretical Interference Velocity on the Axis of a Two-dimensional Wind Tunnel with Slotted Walls

By

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Summary

A calculation is made of the interference velocity on the axis in the flow of an incompressible fluid past a line doublet in a two-dimensional slotted tunnel i.e. a rectangular tunnel whose shorter sides are slotted in the direction of the flow. The method of analysis is basically the same as that used for cylindrical tunnels by Wright and Ward. Numerical solutions are obtained which show that if the width of (slot + slat) is one twelfth of the tunnel height (e.g. 6 slots in the shorter side of a 2 x 1 tunnel), the interference velocity is little different from the corresponding open-jet value - i.e. no slats - for all slot/slat ratios greater than 1/40. Further calculations are not proposed as the labour involved would be very great, and also it appears likely that all cases which give conditions uniform across the centre plane of the tunnel will also give an interference velocity which is close to the open Jet figure.

Introduction

The idea of reducing wind tunnel interference, or eliminating it, by use of a tunnel whose boundaries consist partly of solid walls and partly of free jet surfaces is not a new one. The problem has been treated by a number of writers in various countries[2,3,4,5,6], though until recently interest lay mainly with lifting surfaces. More recently Wright and Ward[1] applied the methods of Fourier analysis to obtain a numerical solution to the interference velocity on the axis of a cylindrical tunnel with slotted walls and experimental work was carried out which, in part, verified their conclusions. The present paper applies the same technique to the corresponding two-dimensional case.

Notation

$2d$	slat width
$2b$	(slat + slot) width
$2h$	tunnel height
u	longitudinal velocity component due to doublet
u^x	longitudinal interference velocity component
$x, y, z.$	running co-ordinates
ϕ	doublet potential

ϕ^x/

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Notation (contd.)

ϕ^x	interference potential
ξ , η	non-dimensional co-ordinates x/h , y/h
θ	non-dimensional co-ordinate $\pi z/b$
ω	$\pi d/b$

Theory of Slotted Tunnel in Incompressible Potential Flow

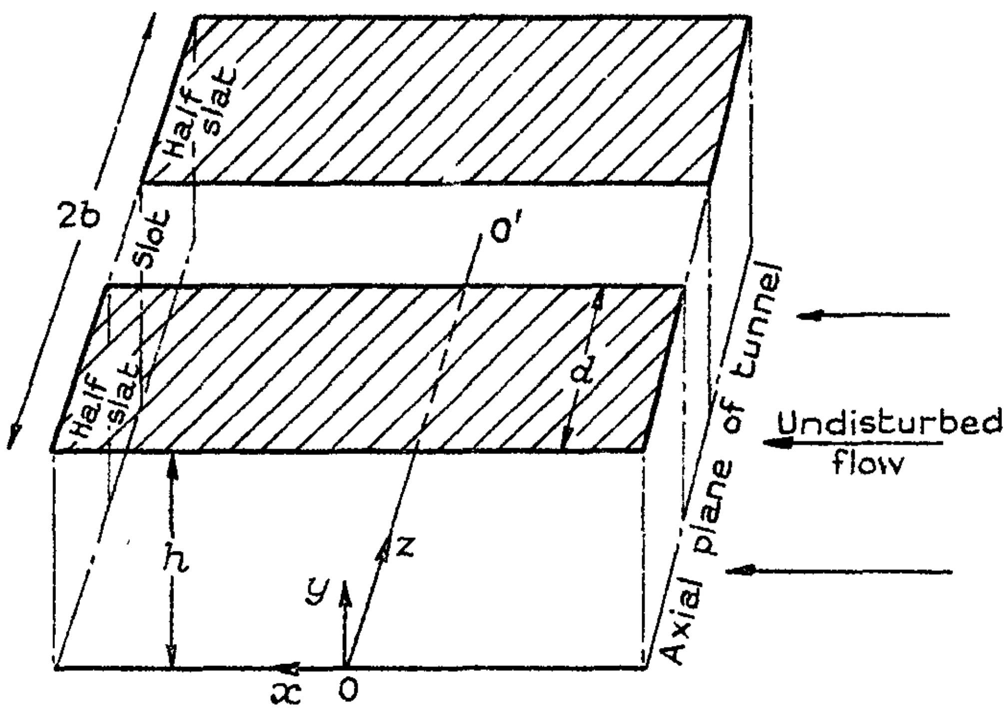

FIG. 1

Consider the flow when a line doublet OO' is placed on the centre line of the tunnel (Fig. I). From reasons of symmetry it is only necessary to consider the flow in the region between two planes parallel to the flow and normal to the line doublet, and which bisect adjacent slats. Take axes of reference so that the line doublet occupies the z-axis, the undisturbed flow is in the x-direction, and these two adjacent planes are $z = 0$ and $z = 2b$ respectively.

It will be convenient to work in non-dimensional quantities referred either to b or h, the tunnel semi-height, and we take the new co-ordinates (ξ, η, θ) given by

$$\xi = x/h, \quad \eta = y/h, \quad \theta = \pi z/b \ .$$

We/

We also write $\omega = \pi d/b < \pi$.

It is evident that the interference potential, with all its derivatives, must be periodic functions of θ with period 2π .

The equation of potential flow

$$\phi_{xx} + \phi_{yy} + \phi_{zz} = 0$$

can then be expressed in the form

$$\phi_{\xi\xi} + \phi_{\eta\eta} + \left(\frac{\pi h}{b}\right)^2 \phi_{\theta\theta} = 0 . \qquad \ldots (1)$$

We shall consider the interference on the flow past a doublet of strength $2\pi h$. The potential ϕ of the undisturbed flow is given by

$$\phi = \frac{h^2 x}{x^2 + y^2} = \frac{h\xi}{\xi^2 + \eta^2}$$

$$u = \phi_x = \phi_\xi / h = - \frac{(\xi^2 - \eta^2)}{(\xi^2 + \eta^2)^2} , \quad v = \phi_y = \phi_\eta / h = \frac{-2\xi\eta}{(\xi^2 + \eta^2)^2} \qquad \ldots (2)$$

let ϕ^x be the interference potential. Then me have for our boundary conditions

$$(\phi + \phi^x)_{\eta=1} = 0 \qquad 2\pi - \omega > \theta > \omega$$

$$(\phi_\eta + \phi^x_\eta)_{\eta=1} = 0 \qquad 0 < \theta < \omega \qquad \ldots (3)$$

ϕ^x must also satisfy the equation (I), so that

$$\phi^x_{\xi\xi} + \phi^x_{\eta\eta} + \left(\frac{\pi h}{b}\right)^2 \phi^x_{\theta\theta} = 0 . \qquad \ldots (4)$$

The periodic behaviour enforced on ϕ^x by the boundary conditions suggests the use of a Fourier cosine series in θ to represent ϕ^x , and we write

$$\phi^x (\xi,\eta,\theta) = \frac{1}{\pi} \psi_o (\xi,\eta) + \frac{2}{\pi} \sum_1^\infty \psi_s (\xi,\eta) \cos s\theta \qquad .** (5)$$

where ψ_s is given by

$$\psi_s (\xi,\eta) = \int_0^\pi \phi^x (\xi,\eta,\theta) \cos s\theta \, d\theta . \qquad \ldots (6)$$

Equation/

Equation (4) may then be written in the form

$$(\psi_s)_{\xi\xi} + (\psi_s)_{\eta\eta} - \left(\frac{s\pi h}{b}\right)^2 \psi_s = 0 \qquad \ldots(7)$$

with boundary conditions

$$\psi_s(\xi,1) = -\phi(\xi,1)\,\frac{\sin s\pi - \sin s\omega}{s} + \int_0^{\omega} \phi^x(\xi,1)\cos s\theta\, d\theta$$

$$\left[\frac{\partial\psi_s}{\partial\eta}\right]_{\eta=1} = -\left(\frac{\partial\phi}{\partial\eta}\right)_{\eta=1}\frac{\sin s\omega}{s} + \int_{\omega}^{\pi}\left(\frac{\partial\phi^x}{\partial\eta}\right)_{\eta=1}\cos s\theta\, d\theta \qquad \ldots(8)$$

Now, from (5),

$$\int_0^{\omega} \phi^x \cos s\theta\, d\theta = \frac{1}{\pi}\psi_0\frac{\sin s\omega}{s} + \frac{1}{\pi}\sum_j \psi_j\left[\frac{\sin(j+s)\omega}{j+s} + \frac{\sin(j-s)}{j-s}\right]$$

$$\int_{\omega}^{\pi} \frac{\partial\phi^x}{\partial\eta}\cos s\theta\, d\theta = \frac{1}{\pi}\frac{\partial\psi_0}{\partial\eta}\,\frac{\sin s\pi - \sin s\omega}{s} - \frac{1}{\pi}\sum_j \frac{\partial\psi_j}{\partial\eta}\left[\frac{\sin(j+s)\omega}{j+s} + \frac{\sin(j-s)\omega}{j-s} - \frac{\sin(j-s)\pi}{j-s}\right] \qquad \ldots(9)$$

and where

$$\frac{\sin(j-s)\pi}{j-s} = \begin{cases}\pi, & j = s\\ 0, & j \neq s\end{cases}$$

Hence the boundary conditions (8) become, on $\eta = 1$,

$$\psi_s = -\phi\,\frac{\sin s\pi - \sin s\omega}{s} + \frac{1}{\pi}\psi_0\frac{\sin s\omega}{s} + \frac{1}{\pi}\sum_{j=1}^{\infty}\psi_j\left[\frac{\sin(j+s)\omega}{j+s} + \frac{\sin(j-s)\omega}{j-s}\right]$$

$$\frac{\partial\psi_s}{\partial\eta} = -\frac{\partial\phi}{\partial\eta}\frac{\sin s\omega}{s} + \frac{1}{\pi}\psi_0\,\frac{\sin s\pi - \sin s\omega}{s} - \frac{1}{\pi}\sum_{j=1}^{\infty}\frac{\partial\psi_j}{\partial\eta}\left[\frac{\sin(j+s)\omega}{j+s} + \frac{\sin(j-s)\omega}{j-s} - \frac{\sin(j-s)\pi}{j-s}\right].$$

or

$$\psi_s = -\phi \frac{\sin s\pi - \sin s\omega}{s} + \frac{1}{\pi}\psi_o \frac{\sin s\omega}{s} + \frac{1}{\pi}\sum_{j=1}^{\infty} a_{js}\,\psi_j$$

$$\frac{\partial\psi_s}{\partial\eta} = -\frac{\partial\phi}{\partial\eta}\frac{\sin s\omega}{s} + \frac{1}{\pi}\frac{\partial\psi_o}{\partial\eta} + \frac{1}{\pi}\sum_{j=1}^{\infty} a_{js}\frac{\partial\psi_j}{\partial\eta} + \frac{\partial\psi_s}{\partial\eta} \qquad \text{. . . (10)}$$

where $$a_{js} = \frac{\sin (j+s)\omega}{j+s} + \frac{\sin (j-s)\omega}{j-s}$$

Assuming the variables in ψ_s to be separable, so that

$$\psi_s = X_s(\xi)\,Y_s(\eta).$$

then from (7) we get

$$\frac{1}{X_s}\frac{\partial^2 X_s}{\partial\xi^2} + \frac{1}{Y_s}\frac{\partial^2 Y_s}{\partial\eta^2} - \left(\frac{s\pi h}{b}\right)^2 = 0$$

whence

$$\frac{\partial^2 X_s}{\partial\xi^2} = -\lambda^2 X \quad \text{i.e.} \quad X_s = A_s \sin\lambda\xi$$

and

$$\frac{\partial^2 Y_s}{\partial\eta^2} = \left\{\lambda^2 + \left(\frac{s\pi h}{b}\right)^2\right\} Y_s, \quad \text{i.e.} \quad Y_s = B_s \cosh\eta\sqrt{\lambda^2 + \left(\frac{s\pi h}{b}\right)^2}.$$

The particular solutions have been chosen to give the correct behaviour for ψ with respect to ξ and η.

We can thus write

$$\psi_s = \int_0^{\infty} hK_{\lambda s}\sin\lambda\xi.\cosh\eta\sqrt{\lambda^2 + \left(\frac{s\pi h}{b}\right)^2}\,.\,d\lambda \qquad \text{...(11)}$$

Substituting (11) in (10) we get, at $\eta = 1$,

$$\int_0^{\infty} hK_{\lambda s}\sin\lambda\xi\,.\cosh\sqrt{\lambda^2 + \left(\frac{s\pi h}{b}\right)^2}\,.\,d\lambda = -\phi\,\frac{\sin s\pi - \sin s\omega}{s} +$$

$$+ \frac{\sin s\omega}{s\pi}\int_0^{\infty} hK_{\lambda o}\sin\lambda\xi\,.\cosh\lambda.d\lambda$$

$$+ \frac{1}{\pi}\sum_j a_{js}\int_0^{\infty} hK_{\lambda j}\sin\lambda\xi.\cosh\sqrt{\lambda^2 + \left(\frac{s\pi h}{b}\right)^2}\,.\,d\lambda$$

and/

$$\int_0^\infty hK_{\lambda s}\sqrt{\lambda^2+\left(\frac{s\pi h}{b}\right)^2}\cdot\sin\lambda\xi\cdot\sinh\sqrt{\lambda^2+\left(\frac{s\pi h}{b}\right)^2}\cdot d\lambda = -\frac{\partial\phi}{\partial n}\,\frac{\sin s\omega}{s}$$

$$+\int_0^\infty hK_{\lambda s}\sqrt{\lambda^2+\left(\frac{s\pi h}{b}\right)^2}\cdot\sin\lambda\xi\cdot\sinh\sqrt{\lambda^2+\left(\frac{s\pi h}{b}\right)^2}\cdot d\lambda$$

$$+\frac{\sin s\pi-\sin s\omega}{s\pi}\int_0^\infty \lambda hK_{\lambda o}\sin\lambda\xi\cdot\sinh\lambda\cdot d\lambda$$

$$-\frac{1}{\pi}\Sigma\, a_{js}\int_0^\infty hK_{\lambda j}\sqrt{\lambda^2+\left(\frac{s\pi h}{b}\right)^2}\cdot\sin\lambda\xi\cdot\sinh\sqrt{\lambda^2+\left(\frac{s\pi h}{b}\right)^2}\cdot d\lambda \quad \ldots(12)$$

Furthermore we can write ϕ and $\frac{\partial\phi}{\partial y}$ in a similar form

$$\phi = \frac{h\xi}{\xi^2+\eta^2} = h\int_0^\infty e^{-\lambda\eta}\sin\lambda\xi\cdot d\lambda$$

$$\frac{\partial\phi}{\partial\eta} = -\frac{2\xi\eta h}{\xi^2+\eta^2} = -h\int_0^\infty \lambda e^{-\lambda\eta}\sin\lambda\xi\cdot d\lambda\,. \quad \ldots(13)$$

Substituting from (13) in (12) and equating terms in $\sin\lambda\xi$, we get (putting $\mu_s = \sqrt{\lambda^2+\left(\frac{s\pi h}{b}\right)^2}$).

For s = 0

$$K_{\lambda s}\cosh\mu_s = e^{-\lambda}\cdot\frac{\sin s\omega}{s}+\frac{\sin s\omega}{s\pi}K_{\lambda o}\cosh\lambda+\frac{1}{\pi}\sum_j a_{js}K_{\lambda j}\cosh\mu_j \quad \ldots(14a)$$

$$\lambda K_{\lambda s}\sinh\mu_s = \lambda e^{-\lambda}\cdot\omega+\frac{\pi-\omega}{\pi}\lambda K_{\lambda o}\sinh\lambda-\frac{1}{\pi}\sum_j a_{jo}K_{\lambda j}\sinh\mu_j \quad \ldots(14b)$$

For s > 0

$$K_{\lambda s}\cosh\mu_s = e^{-\lambda}\frac{\sin s\omega}{s}+\frac{\sin s\omega}{s\pi}K_{\lambda o}\cosh\lambda+\frac{1}{\pi}\sum_j a_{js}K_{\lambda j}\cosh\mu_j \quad \ldots(15a)$$

$$[illegible]\ \lambda e^{-\lambda}\frac{\sin s\omega}{s}-\lambda K_{\lambda o}\sinh\lambda\ \frac{\sin s\omega}{s\pi}-\frac{1}{\pi}\sum_j \mu_j a_{js}K_{\lambda j}\sinh\mu_j \quad \ldots(15b)$$

We/

We thus have two infinite sets of simultaneous equations for the singly infinite set of variables $K_{\lambda s}$. It is however apparent from the derivation of these equations, that neither set of equations is sufficient to determine the $K_{\lambda s}$ uniquely, This can most easily be understood by considering a similar case of two sets of equations for m variables. Then the first set would only contain r independent equations, and the second (m - r). A linear combination of the two would in general, contain m independent equations. We shall assume that the same is true of these infinite sets of equations, and shall combine the two sets.

Dividing (14b) and (15b) by μ_s, and adding to (14a) and (15a) respectively, we get

$$\left(\frac{\pi - \omega}{\pi} + \frac{w}{\pi} \tanh A\right) K_{\lambda o} \cosh \lambda = e^{-\lambda} (2\omega - \pi) + \frac{1}{\pi} \sum_j a_{jo} K_{\lambda j} \cosh \mu_j \left(1 - \frac{\mu_j \tanh \lambda}{\lambda}\right) \quad \ldots(16)$$

and

$$K_{\lambda s} \text{ oosh } \mu_s = e^{-\lambda}\left(1 + \frac{\lambda}{\mu_s}\right)\frac{\sin s\omega}{s} + \left(1 - \frac{\lambda \tanh \lambda}{\mu_s}\right) K_{\lambda o} \cosh \mu_s \frac{\sin sw}{s\pi} + \frac{1}{\pi} \sum_j a_{js} K_{\lambda j} \cosh \mu_j \left(1 - \frac{\mu_j \tanh \lambda}{\mu_s}\right) \quad \ldots(17)$$

It is apparent that tho inclusion of oceffioients like $\cosh \mu_j$ which become large for quite small values of μ_j is going to complicate the numerical work and we substitute

$$\ell_{\lambda j} = K_{\lambda j} \cosh \mu_j \quad \ldots(18)$$

whence

$$\left\{1 - \frac{\omega}{\pi}(1 - \tanh \lambda)\right\} \ell_{\lambda o} = e^{-\lambda} (2\omega - \pi) + \frac{2}{\pi} \sum_{j=1}^{\infty} \frac{\sin j\omega}{j} \ell_{\lambda j} \left(1 - \frac{\mu_j \tanh \lambda}{\lambda}\right) \quad \ldots(19)$$

and

$$\ell_{\lambda s} = e^{-\lambda}\left(1 + \frac{\lambda}{\mu_s}\right)\frac{\sin s\omega}{3} + \ell_{\lambda o}\left(1 - \frac{\lambda \tanh \lambda}{\mu_s}\right)\frac{\sin s\omega}{s\pi} + \frac{1}{\pi}\sum_{j=1}^{\infty} a_{js} \ell_{\lambda j}\left(1 - \frac{\mu_j \tanh \mu_j}{\mu_s}\right), \quad s \geqslant 1 \quad \ldots(20)$$

where

$$a_{js} = \frac{\sin (j + s)\omega}{(j + s)} + \frac{\sin (j - s)\omega}{(j - s)}$$

and

$$\mu_j = \sqrt{\lambda^2 + \left(\frac{j\pi h}{b}\right)^2}$$

Once/

Once the $K_{\lambda j}$ have been found from (18), (19) and (20) we can obtain ϕ^x using (5) and (11)

$$\phi^x(\xi,\eta,\theta) = \frac{h}{\pi}\int_0^\infty K_{\lambda o}\sin\lambda\xi \,.\, \cosh\lambda\eta \,.\, d\eta + \frac{2h}{\pi}\sum_{s=1}^{\infty}\cos s\theta \,.\int_0^\infty K_{\lambda s}\sin\lambda\xi \,.\, \cosh\mu_s\eta \,.\, d\lambda \qquad \ldots(21)$$

from which we obtain the axial interferenoe velocity u^x

$$u^x = \left(\frac{1}{h}\frac{\partial\phi^x}{\partial\xi}\right)_{\eta=0} = \frac{1}{\pi}\int_0^\infty \lambda K_{\lambda o}\cos\lambda\xi \,.\, d\lambda + \frac{2}{\pi}\sum_{s=1}^{\infty}\cos s\theta \,.\int_0^\infty \lambda K_{\lambda s}\cos\lambda\xi \,.\, d\lambda \qquad \ldots(22)$$

<u>Numerical Solution of the equations</u>

No general method of solution of an infinite set of equations, such as (19) and (20), is known, and it remained to be seen if it was possible to obtain a good approximation by solving a finite number of them for the corresponding number of variables. This was done by the Mathematics Division, N.P.L. The equations were found to be highly convergent, the solutions obtained from 7, 11 and 13 sets-of equations not being significantly different. The final calculations were done with 13 sets of equations. The values of $K_{\lambda o}$ obtained from (18), (19) and (20) are given in Table 1, for the case where $b/h = 1/12$.

<u>Table 1</u>

A	$-K_{\lambda o}$ $(\omega = 3\pi/4)$	$-K_{\lambda o}$ $(\omega = 7\pi/8)$
0	π	π
	2.328	2.297
.5	1.616	1.566
	1.064	1.007
1.0	0.6376	0.6228
	.4148	.3744
1.5	.2511	.2210
	.1503	.1290
2.0	.0893	.0749
	.0528	.0434
2.5	.0312	.0251
	.0185	.0144
3.0	.0109	.00825
	.0064	.00474
3.5	.00375	.00272
	.0022	.00155
4.0	.0013	.00088
	.0006	.00050
4.5	.0003	.00015
	.0001	.00008
5.0	.0000	.00004

The/

The $K_{\lambda j}$ were all found to be negligible for $j \geqslant 1$, and so we get, from (22)

$$u^x = \frac{1}{\pi} \int_0^\infty K_{\lambda o} \cos \lambda \xi \, . \, d\lambda \quad \ldots(23)$$

The numerical values of this integral for $\xi = {}^1/_2, 1, {}^3/_2$ are given in Tnble 2 (on page 11).

Open Jet Tunnel

If the upper and lower surfaces of the tunnel are free jet surfaoesso that, in the above notation, $\omega = 0$, it is apparent from (14) and (15) that

$$K_{\lambda o} = - \pi h e^{-\lambda} \operatorname{sech} \lambda = \frac{-2\pi h}{e^{2\lambda} + 1}$$

$$K_{\lambda s} = 0 \qquad s \geqslant 1$$

$$u^x = -2 \int_0^\infty \frac{\lambda \cos \lambda \xi}{e^{2\lambda} + 1} d\lambda \quad \ldots(74)$$

To evaluate the above Integral, consider the integral of $\dfrac{e^{i\xi z}}{e^{2z} + 1}$ taken round the contour C of Fig. 2.

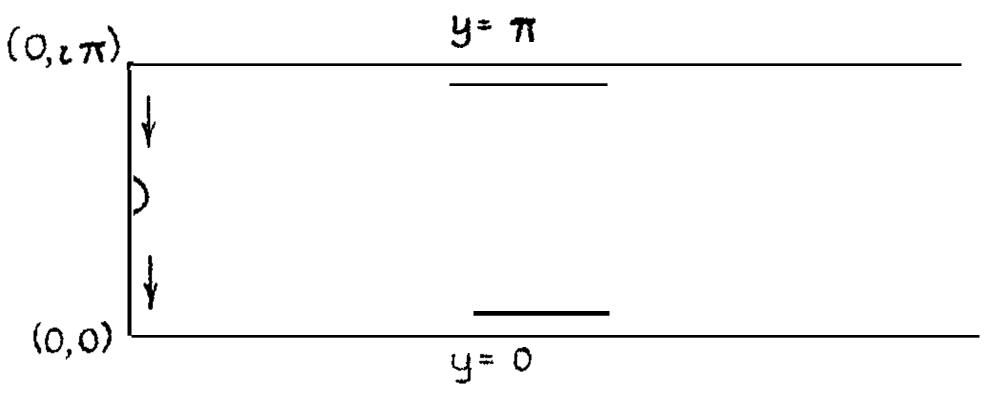

FIG 2

Then/

Then, provided the contour is suitably indented round the pole at (0, in/Z), we have

$$\int_C \frac{e^{i\xi z}}{e^{2z}+1}\,dz = 0.$$

i.e.

$$\int_0^\infty \frac{e^{i\xi x}}{e^{2x}+1}\,dx - e^{-\pi\xi}\int_0^\infty \frac{e^{i\xi x}}{e^{2x}+1}\,dx - i\int_0^\pi \frac{e^{-\xi y}}{e^{2iy}+1}\,dy - i\pi R\,(0,\ i\pi/2) = 0 \qquad \ldots(25)$$

where R (0, iπ/2) is the residue of $\frac{e^{\xi z}}{e^{2z}+1}$ at (0, iπ/2), whence

$$R\,(0,\ i\pi/2) = -\tfrac{1}{2}e^{-\frac{1}{2}\pi\xi}.$$

Also $\qquad 1 + e^{2iy} = 1 + \cos 2y + i\sin 2y$

$$= 2\cos y . e^{iy},$$

Substituting these in (25), we get

$$\left(1 - e^{-\pi\xi}\right)\int_0^\infty \frac{e^{i\xi x}}{e^{2x}+1}\,dx = -\frac{i\pi}{2}e^{-\frac{1}{2}\pi\xi} + i\int_0^\pi \frac{e^{-\xi\eta}(\cos y - i\sin y)}{2\cos y}\,dy.$$

Taking the imaginary part

$$\left(1 - e^{i\pi\xi}\right)\int_0^\infty \frac{\sin x\xi}{e^{2x}+1}\,dx = -\frac{\pi}{2}e^{-\frac{1}{2}\pi\xi} + \frac{1}{2}\int_0^\pi e^{-\xi y}\,dy$$

$$= -\frac{\pi}{2}e^{-\frac{1}{2}\pi\xi} + \frac{1 - e^{-\pi\xi}}{2\xi}$$

Whence

$$\int_0^\infty \frac{\sin x\,\xi}{e^{2x}+1}\,dx = \frac{1}{2\xi} - \frac{\pi}{4}\operatorname{cosech}\tfrac{1}{2}\pi\xi \qquad \ldots(26)$$

Finally, differentiating (26) w.r.t. ξ

$$\int_0^\infty \frac{x\cos x\xi}{e^{2x}+1}\,dx = -\frac{1}{2\xi^2} + \frac{\pi^2}{8}\operatorname{cosech}\tfrac{1}{2}\pi\xi \cdot \coth\tfrac{1}{2}\pi\xi. \qquad \ldots(27)$$

so that from (24)

$$u^x = \frac{1}{\xi^2} - \frac{\pi^2}{4}\operatorname{cosech}\tfrac{1}{2}\pi\xi \cdot \coth\tfrac{1}{2}\pi\xi. \qquad \ldots(28)$$

Closed/

Closed Tunnel

For a closed tunnel $\omega = \pi$,

$$\left.\begin{aligned} K_{\lambda o} &= \pi e^{-\lambda} \operatorname{cosech} \lambda = \frac{2\pi}{e^{2\lambda} - 1} \\ K_{\lambda j} &= 0 \qquad j \geqslant 1 \end{aligned}\right\} \quad \ldots(29)$$

whenoe

$$u^x = 2\int_0^\infty \frac{\lambda \cos \lambda\xi}{e^{2\lambda} - 1} d\lambda \, . \quad \ldots(30)$$

The integral in equation (30) can be evaluated by the same method as in the previous section to obtain the result

$$u^x = \frac{1}{\xi^2} - \frac{\pi^2}{4} \operatorname{cosech}^2 \tfrac{1}{2}\pi\xi \quad \ldots(31)$$

The results of equations (28) and (31) are not new, having been obtained before by the method of images[7], but the analysis is given here since it does give an indication of the correctness of the method.

Results

The numerical values of u^x , the axial interference velocity, for the oases discussed are given in Table 2, and plotted in Figs. 3 and 4.

Table 2 Values of u and u^x

x	1/2 h	h	3/2 h
u (for doublet)	4.00	1.00	0.444
u^x , w = 0 (free jet)	-0.33	-0.16	-0.036
w = $3\pi/4$	-0.30	-0.17	-0.049
ω = $7\pi/8$	-0.28	-0.16	-0.055
w = π (closed tunnel)	+0.73	+0.54	+0.35

It can be seen from Fig. 4 that there are insufficient values calculated to give any reliable values for ω at which the interference is zero, but this value is certainly greater than 0.975π. The inference from this is that the slot/slat ratio for zero interference must bo less than 1 : 40 and might easily be 1 : 100 or even less. The small slot widths arising from such a ratio would hardly be praotioable, and in such oases viscous effects would probably play such a large part near the wall that the potential flow calculations would not be valid. For slot/slat ratios greater than 1 : 40 the flow near the axis is, to all intents and purposes, the same as would be obtained in a free jet. It is also worth

noting/

noting that the slight reduction in interference that is evident near the body due to the slotted walls, is lost further from the body $\left(\text{at } x = \frac{3h}{2} \right)$. At this point, however, the interferenoe is 80 small as to be insignificant.

Another important point is that the results are independent Of z ; i.e. there is no 'ripple', or variations in the interference potentials on the axis due to the mixed boundary* This arises from the fact that the $K_{\lambda s}$ $(s > 0)$ are all zero. It would appear to be at least likely that whenever ω is small enough to produce smooth flow on the axis (independent of z), then under suoh conditions the flow on the axis will always behave in a free jet,

This result is in acoordnnoe with the results already obtained by Pistolesi[5,6] in the consideration of the interferenoe on a lifting surface i.e. due to the dipole trailing vortex system. (This paper is not available in this country, but the results are discussed in Ref. 6.) It was found that if the value of the slot/slat ratio was kept unaltered, the interference correction beoame more end more like the free jet correction as the number of slots was increased. Similar results are obtained with the flow of fluids through a grating, and also the behaviour of sound waves under similnr conditions[8], it being found in both these cases that the effect of such gratings is far less than their solidity would suggest. The crude approximation that the interference is directly proportional to the slot/slat ratio, is quite useless and misleading,

It is worth noting that the effect of oomprcssibility, to the first order as oaloulated from Prandtl-Glauert theory, is to increase tho effective magnitude of the doublet, and hence of the interference velocity. To this first order, the solution is altered in magnitude but not in form, and the general conclusions reached in this paper should still apply.

It would be useful and interesting to calculate the axial inturfcrenoe velocity for different slot/slat ratios, with fewer slots - but the labour involved is so great as to be out of proportion to the results. Thero is every reason to believe from experiments that freo jet behaviour persists even when there are just two or three slots. It would appear, therefore, that the value of a slotted wall tunnel so far as interference effeots are concerned lies not in produoing a tunnel with no interference, but rather in producing one with free Jet interferenoe oharaoteristios, though with improved behaviour[9].

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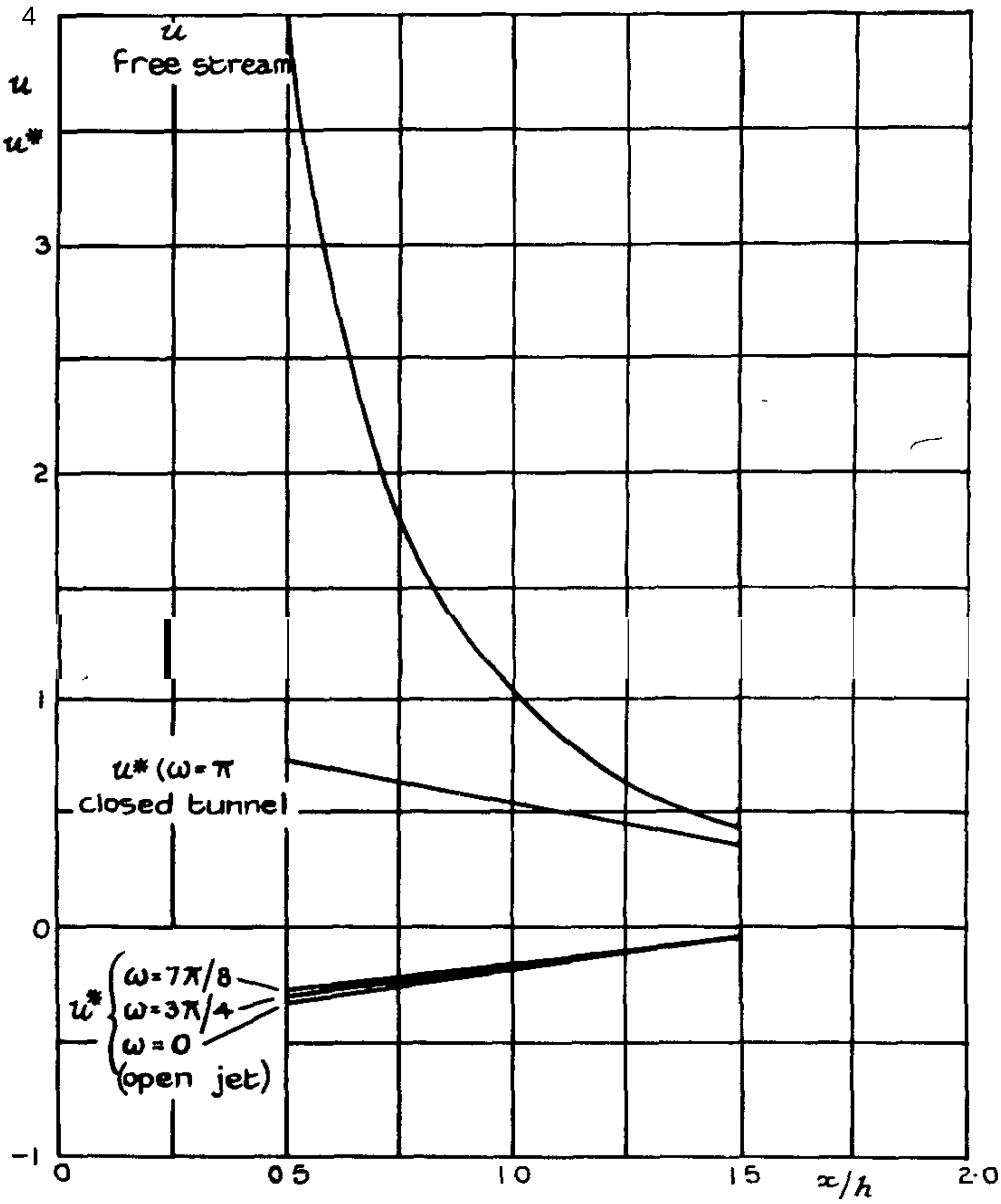
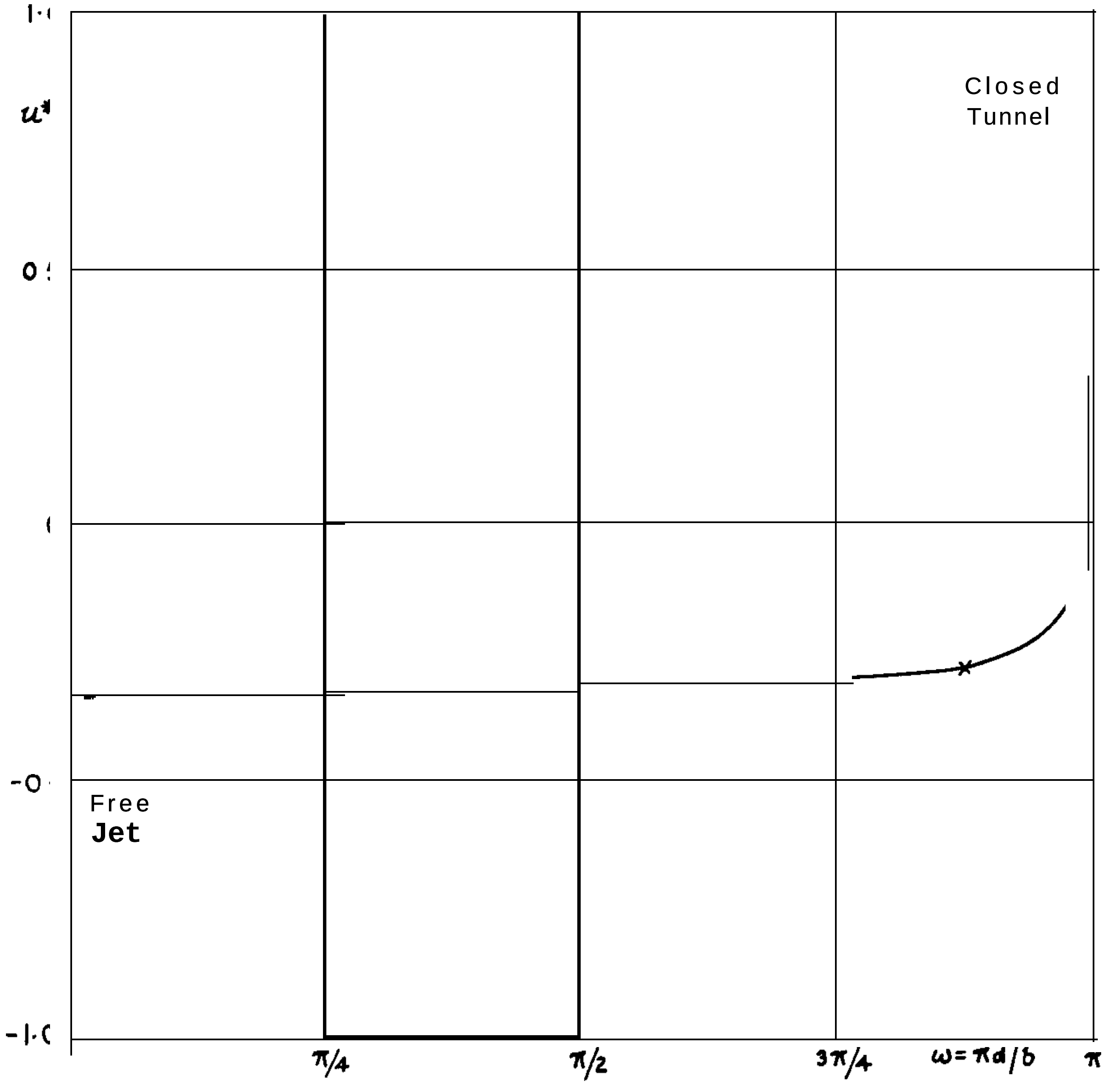


Doublet and Interference velocities vs Distance Upstream

N.B Velocity in infinite stream = 4.



Interference velocity For difference thickness of slats.

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