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Factors Influencing the Optimum Aerodynamic Design of Cooled Turbines

By

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SUMMARY

Owing to the losses in performance which increase with the rate of heat extraction required for turbine cooling, it is desirable to know what range of aerodynamic designs is associated with low values of this quantity. Different aerodynamic designs of turbine, all passing the same mass flow and having approximately the same disc and blade stresses, have been compared on the basis of the ratio of heat extraction rate to work output. It is found that high values of flow coefficient (V_a/U_m) are beneficial in this respect, and that impulse turbines have a slight advantage over reaction turbines.

Unfortunately, turbines which have low heat extraction rates tend to have low, uncooled, aerodynamic efficiencies. Calculations for one specific gas turbine plant indicate that no net gain of thermal efficiency is to be obtained by using low reaction turbines, but that the use of high flow coefficients may result in a slight gain. This improvement may be appreciable in cases where the blade speed is low and the difference between the gas and blade temperature is large.

Although attention is directed mainly towards industrial turbines, some of the conclusions apply equally to aircraft turbines.

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ILLUSTRATIONS

1.0 Introduction

It is common knowledge that certain losses in turbine performance are incurred when an attempt is made to cool the turbine, (see Ref. (1)). These losses increase with the amount of heat which must be extracted by the cooling medium to reduce the blade temperature to the desired value. For any given work output and gas temperature, the rate at which this heat must be extracted depends largely upon the aerodynamic design of the turbine. It is clearly desirable to know what range of aerodynamic designs is associated with a low rate of heat extraction, and the primary purpose of this note is to define this range with somewhat more precision than has been attempted hitherto. A secondary aim is to decide whether sufficient precision can now be achieved with the heat transfer data already obtained, and if not what further experimental work might be usefully undertaken for this purpose.

The first attempts to find the optimum aerodynamic design of turbine for a minimum rate of heat extraction are reported in Ref. (8) and Ref. (1). In Ref. (1) a range of designs was covered by considering three classes of turbine, each designed with a range of flow coefficient, V_a/U_m. This method has also been adopted here, the three types of turbine being.

(1) Impulse, with no swirl at exit from the stage, i.e. the nozzle blades have zero inlet angle.

(2) 50 per cent reaction, with the inlet angle of the blades equal to one-half the outlet angle.

(3) 50 per cent reaction, with inlet angle of the blades equal to 0^o.

Such a classification means that for each type, the flow coefficient is a unique function of rotor blade outlet angle α_2. It also implies a constant temperature drop coefficient, $2C_p\, g\, J\, \Delta T/U_m^2$, of -4 and -2 for turbines (1) and (3) respectively, and a variation of this coefficient from approximately -3 to -5 in the case of turbines (2). Ref. (2), which gives estimates of design point stage efficiency from the results of cascade tests, indicates that turbines (3) are operating with a temperature drop coefficient giving approximately maximum efficiency for any given value of flow coefficient, V_a/U_m, while turbines (1) and (2) operate with a range of temperature drop coefficients which are numerically rather larger than the optimum value of about -2. A summary of the main features of the three types of turbine is given in Fig. 1.

The present investigation differs from the previous ones in so far as the assumptions on which the rate of heat extraction is calculated are more realistic. Firstly, instead of assuming a constant relationship between Nusselt number and Reynolds number for each type of turbine blade regardless of outlet angle, a suitable variation is assumed. Secondly, instead of assuming that all the turbines have blades of the same chord and surface area, these quantities have been determined by satisfying the requirement that the blade centrifugal tensile and gas bending stresses should be the same in all turbines. The comparison has been made, therefore, between turbines of different aerodynamic design but all designed to pass the same mass flow, with the same rim speed (i.e., approximately the same disc or rotor stresses) and blade stresses. The work output per stage, tip diameter, or rotational speed, will not, however, be the same for all cases.

In addition, this note includes an estimate of the rate of heat extraction required to cool the annulus walls. For lack of any experimental information about the heat transfer coefficients involved, these were assumed to be equal to the blade coefficients.

2.0 Method of presenting the results

As in Ref. (1), it has been found convenient to express the rate of heat extraction in terms of stage heat extraction coefficient, f, defined by:

$$f = \frac{\Delta H}{W} \cdot \frac{U_m^2}{gJCp\ (T_g - T_b)} \quad \ldots \quad (1)$$

ΔH is the total rate of heat extraction per stage, and W is the work output per stage. $(T_g - T_b)$ is the difference between the mean effective gas temperature and the blade or annulus wall temperature, and is assumed to be constant throughout the stage. The ΔH term comprises the heat extracted from stator and rotor blades, and annulus walls. Consequently, the stage heat extraction coefficient can be regarded as the sum of four coefficients, i.e.,

$$f = f_S + f_R + f_{AS} + f_{AR}$$

where f_{AS} and f_{AR} refer to the annulus walls adjacent to the stator and rotor blades respectively.

As shown in Appendix I, equation (1) when applied to a row of blades can be expressed as:

$$f_S \text{ or } f_R = \frac{S}{c} \cdot k\,Re_2^{\,n} \cdot \frac{\lambda}{C_p} \cdot \frac{(-\tan\alpha_o - \tan\alpha_1)}{(\tan\alpha_1 - \tan\alpha_2)} \cdot \frac{1}{\ell V_a s} \quad \ldots \quad (2)$$

This assumes that the mean Nusselt number for the blade is given by $Nu = k\,Re_2^{\,n}$, where the Reynolds number is based on gas conditions at exit from the blade row, relative velocity at exit, and blade chord. Both V_a and U_m are assumed to be constant throughout the stage. Equation (2) may be applied to the stator and rotor in turn to give the following expressions for f_S and f_R,

$$f_S = k\left(Re_u\right)^{n-1} \cdot \frac{S/c}{s/c} \cdot \frac{1}{P_r} \cdot \frac{(-\tan\alpha_o - \tan\alpha_1)^{2-n}}{(\tan\alpha_o - \tan\alpha_1)} \cdot \frac{1}{(-\cos\alpha_o)^n} \quad (3)$$

$$f_R = k\left(Re_u\right)^{n-1} \cdot \frac{S/c}{s/c} \cdot \frac{1}{P_r} \cdot \frac{(-\tan\alpha_o - \tan\alpha_1)^{2-n}}{(\tan\alpha_o - \tan\alpha_1)} \cdot \frac{1}{(-\cos\alpha_2)^n} \quad (4)$$

Re_u is the Reynolds number based on mean blade speed, but otherwise unchanged from that used in Equation (2).

When applying these equations to the range of turbines under consideration, a constant Prandtl number of 0.71 is assumed. It is found that the gas angles are all unique functions of α_2 for each class of turbine, and consequently f_S and f_R are functions of k, Re_u, n, S/c, s/c, α_2 and the class of turbine. The determination of the first five variables in terms of the remaining two is discussed in Section 3.0.

An approximate value of f_A has been estimated assuming the heat transfer coefficient between gas and wall to be the same as that between gas and adjacent blade. On this basis, the ratio of f_{AS} to f_S, or f_{AR} to f_R, will be in the ratio of the respective surface areas. If it is assumed that the axial length of annulus wall associated with each blade row is $1.2\ c \cos\xi$ (where ξ is the stagger) then, as shown in Appendix II,

$$\frac{f_{AS}}{f_S} \text{ or } \frac{f_{AR}}{f_R} = \frac{2.4 \cos\xi \cdot s/c}{L/c \ . \ S/c} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (5)$$

Thus the annulus heat extraction coefficients are functions of the blade coefficients, S/c, s/c, L/c and ξ. ξ is a function of α_2 and the class of turbine if the incidence is assumed to be zero and a camber line is chosen. The only quantity additional to those already required to determine the blade coefficients is the aspect ratio, L/c, and this is also discussed in the next section.

3.0 Assumptions

So far we have assumed:

(a) Constant U_m and V_a through stage

(b) Constant temperature difference $(T_g - T_b)$ through stage

(c) Prandtl number constant at 0.71 for all turbines

(d) Annulus length equal to 1.2x projected chord of blade

(e) $Nu = k\ Re_2^{\ n}$. This relationship implies the assumption that the temperature ratio, T_g/T_b, is constant for all turbines.

It remains to find k, Re_u, n, S/c, s/c and L/c as functions of α_2 and the class of turbine.

3.1 Map of heat transfer coefficients ($Nu = k\ Re_2^{\ n}$)

All the turbines considered use one or two of the following three types of blade; nozzle blade $\alpha_1 = 0$, impulse blade $\alpha_1 = -\alpha_2$, reaction blades $\alpha_1 = -\alpha_2/2$. The results of hot cascade tests on each type of blade have been reported by Andrews and Bradley[3], Pohlmann[4], and Bammert and Hahnemann[5]; they are summarised in Ref. (6). Only one value of outlet angle was used in each case. Ref. (9) provides a result for a second nozzle blade of smaller outlet angle (25° as compared with the 70° of Andrews and Bradley). As expected, the Nusselt number and Reynolds number exponent are both appreciably increased by the reduction of outlet angle.

For this investigation, it was decided to assume a map of heat transfer coefficients in the form of a curve of Nusselt number versus outlet angle for each type of blade, plotted for a Reynolds number, Re_2, of 2×10^5, and annotated with values of the Reynolds number exponent (see Fig. 2). It is assumed that the three curves would meet at $\alpha_2 = 0$, giving the Nusselt number relationship for a streamline body which was taken from Ref. (3). The map refers to blades operating with zero incidence.

Although this map is only tentative, it is thought that a comparison of the results obtained from its use, with those obtained by assuming a constant Nusselt number relationship for each blade, would show the magnitude of the error likely to arise from an imperfect knowledge of the variation of heat transfer coefficient with outlet angle. It should also indicate whether an extended programme of cascade tests would be useful or not.

3.2 Pitch/chord ratio ($^s/_c$)

Results of pressure loss measurements on cascades of turbine blades have been reported in Ref. (7). Curves of optimum $^s/_c$ ratio versus outlet angle are given therein for nozzle and impulse blades, together with a method of interpolation for deducing the corresponding curves for blades intermediate between these two extremes. The interpolation has been carried out for blades having $\alpha_1 = -\alpha_2/2$, and the result is given in Fig. 3 with the reproduced curves for nozzle and impulse blades.

3.3 Reynolds number (Re_u)

Assuming that the mean blade speed and the gas conditions are the same for all turbines, Re_u becomes proportional to the blade chord. The rotor blade chords may be determined by postulating constant blade stresses, in the following way.

The centrifugal tensile stress is a function of tip/root radius ratio, r, and mean blade speed, U_m. Hence, for a given stress and blade speed, r is constant. An approximate expression for the gas bending stress may be found by ignoring the axial bending moment, and is given by:

$$\text{Gas bending stress} \propto \frac{s}{c} \cdot \frac{1}{Z\,c^2} \cdot \frac{Q\Delta T}{U_m} \cdot \frac{(r - 1)}{(r + 1)}$$

where Z is the section modulus of a blade having one-inch chord. Thus, when comparing turbines having the same mass flow, blade speed, and blade stresses, the chord is a function of $^s/c$, Z and ΔT.

An unpublished approximate rule by D.G. Ainley gives Z in terms of camber angle, for any given thickness/chord ratio which for this purpose is assumed to be 20 per cent. If the blade angles are assumed to equal the gas angles, Z becomes a function of outlet angle and class of blade. Also, from the reference to the temperature drop coefficient in the Introduction, it may be inferred that ΔT is a function of α_2 and class of turbine when U_m is constant. Consequently, the choice of a datum turbine having certain values for the air angles and a rotor blade chord of one inch, determines the chords of all other turbine rotor blades having the same stresses. The datum chosen in this note is an impulse turbine with $\alpha_1 = -\alpha_2 = 45^\circ$, $\alpha_0 = -70^\circ$, and s/c = 0.7, and the chords for the three classes of blade are plotted against outlet angle in Fig.4. An assumption that the datum turbine operates with $Re_u = 2 \times 10^5$, enables a scale of Re_u to be added to the figure.

The chords of the stator blades are made equal to those of the rotor blades in the case of the reaction turbines, and are fixed at an arbitrary value of 1.5 inches for the impulse turbines. When finding Re_u for the stator row, the change in gas density between outlet of stator and outlet of rotor is neglected.

3.4 Perimeter/chord ratio $(^S/_c)$

An approximate rule is used for the perimeter of a blade section, viz. perimeter = 2.23 x length of camber line, and circular arc camber lines are assumed. Thus the $^S/_c$ ratio is a function of camber angle, i.e., a function of outlet angle and class of blade, and the relationship is given in Fig. 5.

3.5 Aspect ratio $(^L/_c)$

Assuming constant centrifugal tensile stress, blade speed, gas density and mass flow, the rotor blade aspect ratio may be regarded as a function of the flow coefficient and chord (see Appendix II). Both these variables are functions of α_2 and the class of turbine. For this investigation, a value of 3 is taken for the aspect ratio of the rotor blades in the datum turbine, and the corresponding values are then determined for all the turbines considered.

The aspect ratio of the stator blades is fixed by making the assumption that the heights of corresponding rotor and stator blades are equal.

4.0 Results

In Fig. 6(a), values of the heat extraction coefficient for the blades alone are plotted against flow coefficient for the three classes of turbine. It is seen that high values of V_a/U_m are accompanied by low heat extraction coefficients, and that impulse turbines are slightly better than reaction turbines over the useful range of V_a/U_m. Under the conditions assumed here, the decrease of heat extraction coefficient with increase of V_a/U_m is mainly due to the accompanying decrease of total surface area of the blade row. Fig. 6(b) shows that the general picture is unchanged when the annulus walls are taken into account, although from Fig. 7 it is clear that at high values of V_a/U_m the rate of heat extraction required to cool the annulus walls may become as much as thirty per cent of the total.

It will be appreciated that the values of the heat extraction coefficients are only relative, since their magnitude depends upon the chord and Reynolds number assumed for the blades of the datum turbine. Inspection of Equations (3) and (4), remembering that $n < 1$, will show that a reduction of this Reynolds number, Re_u, would lead to an increase in the values for the heat extraction coefficient. A reduction of centrifugal tensile stress would be accompanied by a reduction of the blade chord, if it was obtained by reducing the tip/root radius ratio without changing the gas bending stress. The blade chord would also be reduced if the permissible gas bending stress was increased. In either case, if the reduction of blade chord was accompanied by a reduction of Re_u, the value of the heat extraction coefficient would be increased. Thus, either a reduction of centrifugal tensile stress or an increase of gas bending stress might increase the values of f. Although the ordinate scale of Fig. 6 would be altered, the general shape would, however, remain unchanged.

To compare the relative effects of V_a/U_m upon efficiency and heat extraction rates, the efficiency curves of Fig. 7(d) were deduced from those in Ref. (2)*. The order of merit in which the turbines appear from the point of view of low heat extraction coefficient is reversed; clearly a compromise must be made between high aerodynamic efficiency and low heat extraction rate. It is not possible to say what reduction of heat extraction coefficient will compensate for a given drop of turbine aerodynamic efficiency; only detailed performance estimates of particular gas turbine plant can show this in specific cases. The question arises as to whether the heat transfer data available is sufficiently extensive to make such estimates worthwhile, and Fig. 8 is an attempt to answer this question.

Fig. 8 illustrates the effect of using constant values of k and n for each type of blade instead of values taken from the map of Nusselt numbers. The greatest effect is noticed with the fifty per cent reaction turbines having $\alpha_1 = 0$, because these use nozzle type blades for both stator and rotor, and such blades have the greatest change of Nusselt number with outlet angle. It may be concluded that only major errors in the map would make a material difference to the relative values of the heat extraction coefficient.

5.0 Effect of choice of flow coefficient and type of turbine on overall thermal efficiency of a possible gas turbine plant

It is not possible to make a generalisation about the relative effect of heat extraction rate and stage aerodynamic efficiency upon the overall performance of gas turbines. Nevertheless it might be of value to illustrate the effect of the choice of flow coefficient and type of turbine on the overall thermal efficiency of one possible gas turbine plant. The specification of the selected plant is detailed in Appendix III; briefly, it is a simple gas turbine with heat-exchange, operating with a pressure ratio of 6:1 and a maximum cycle temperature of 1,200°C. The temperature difference $(T_g - T_b)$ is assumed to be 600°C. and the mean blade speed of the turbine 850 ft. per sec.

The calculations take account of the loss due to "negative reheat", and the loss due to the "dilution effect", i.e., reduction of temperature rise in the heat exchanger incurred by lower exhaust temperatures. These losses have been estimated using the equations given in Appendix III Ref. (8). The equations are based on the assumption that $\Delta H/W$ is constant throughout the turbine, and that the number of stages is large. (If the number of stages is small the loss due to negative reheat will be overestimated). Although in practice $(T_g - T_b)$ will fall through the turbine, f will probably be made to increase by designing the later stages for a lower value of V_a/U_m to reduce the exit volute loss. Thus the assumption of constant $\Delta H/W$ will be approximately correct.

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*This efficiency is the isentropic stage efficiency of an uncooled turbine of the given aerodynamic design, it may be termed the "aerodynamic efficiency".

In Fig. 9, the overall thermal efficiency of the plant is plotted against heat extraction coefficient for various turbine polytropic efficiencies. The results should be regarded as applying to a water cooled turbine, although the thermal value of the steam formed has not been included. If the turbine is air-cooled other effects will be present, such as the work done by the cooling air in expanding through the turbine and the additional work required to provide the cooling air.

If the stage efficiencies of Fig. 7(b) are used as polytropic efficiencies, the following table can be compiled from Figs. 7 and 9.

Class of Turbine	Flow coeff. $-V_a/U_m$	Polytropic turbine effy. %	f	Overall thermal effy. %	Net gain of thermal effy. %
50% reaction (α_1 = 0)	from 0.5 to 1.3	93 88	0.031 0.016	36.3 38.8	2.5
Impulse (α_3 = 0)	from 0.5 to 1.3	87 85	0.033 0.013	33.7 38.5	4.8
From 50% reaction To impulse	0.5 0.5	93 87	0.031 0.033	36.3 33.7	-2.6
From 50% reaction To impulse	1.3 1.3	88 85	0.016 0.013	38.8 38.5	-0.3

The estimated turbine efficiencies are, according to Ref. (2), on the optimistic side for low reaction turbines, so it is clear that no advantage is to be gained by choosing impulse rather than 50 per cent reaction designs even with high flow coefficients. There does appear to be a net gain to be obtained by using a high flow coefficient, although this is unlikely to be as large as would appear from the table. This is because a change from low to high flow coefficient will almost certainly increase the exit volute loss (see Ref. (1)) which has not been taken into account in the calculations.

It must be emphasised that these figures only refer to one specific gas turbine. Had the value chosen for the mean blade speed been lower, or the temperature difference (T_g - T_b) been higher, the relative importance of designing for a low heat extraction coefficient rather than a high turbine aerodynamic efficiency would have been increased. So, too, if the values of f in Fig. 7 had been increased, by altering the blade chord or Reynolds number of the datum turbine for example.

6.0 Conclusions

(1) High flow coefficients imply low heat extraction rates for all types of turbine; impulse turbines showing a slight advantage over reaction turbines.

(2) The heat extraction rate for the annulus walls may be as much as thirty per cent of the total at high flow coefficients, and consequently it may be important to determine the heat transfer coefficients involved experimentally.

(3) Small errors in the assumed map of Nusselt numbers have little effect on the blade heat extraction coefficients, and consequently extensive cascade tests to find the variation of mean Nusselt number with outlet angle for different classes of blade may not pay large dividends.

(4) Turbines which have low heat extraction rates tend to have low aerodynamic efficiencies. It is not possible to make a generalisation about the relative effect of these two quantities on the overall performance of gas turbine plant. Calculations for one specific gas turbine indicate that no net gain of thermal efficiency is to be obtained by using impulse designs in preference to reaction designs, but that the use of high flow coefficients in preference to low flow coefficients may result in a slight gain. This gain may become appreciable for plant using turbines with low blade speeds and high values of $(T_g - T_b)$.

Acknowledgments

The author is indebted to the earlier work of R. Staniforth, and to A.G. Smith for his guidance and suggestions.

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NOTATION

α_0	stator outlet gas angle	
α_1	rotor inlet gas angle	
α_2	rotor outlet gas angle	
α_3	stator inlet gas angle	
ξ	stagger angle	
V_a	axial velocity	(ft./sec.)
V_2	gas velocity relative to blade at outlet	(ft./sec.)
U_m	mean blade speed	(ft./sec.)
ϱ	density at exit from blade row	(lb./ft.^{3})
S	perimeter of blade	(ft.)
s	pitch of blade	(ft.)
c	chord of blade	(ft.)
L	height of blade	(ft.)
Z	section modulus of blade having 1 inch chord	(in.^{3})
n_b	number of blades per row	
N	rotational speed	(r.p.s.)
r	tip/root radius ratio of annulus	
q	blade centrifugal tensile stress	(tons/sq.in.)
Q	mass flow	(lb./sec.)
h	heat transfer coefficient	(CHU/sq.ft./sec./^{o}C)
λ	thermal conductivity of gas	(CHU/ft./sec./^{o}C.)
μ	viscosity	(lb./ft./sec.)
C_p	specific heat at constant pressure	(CHU/lb./^{o}C.)
ΔH	rate of heat extraction	(CHU/sec.)
ΔT	stage temperature drop	(^{o}C.)
W	stage work done	(CHU/sec.)
f	heat extraction coefficient	
Nu	Nusselt number (hc/λ)	

NOTATION (Cont'd)

Pr	Prandtl number $(\mu C_p/\lambda)$
Re_2	Reynolds number $(\ell V_c/\mu)$ based on V_2
Re_u	Reynolds number based on U_m
k, n	Constant and exponent in the relation, $Nu = k\, Re_2^{\,n}$

Suffices

A	refers to annulus
R	refers to rotor
S	refers to stator

APPENDIX I

Expressions for the Blade Heat Extraction Coefficients

The stage heat extraction coefficient is defined by,

$$f = \frac{\Delta H}{W} \cdot \frac{U_m^2}{gJC_p\,(T_g - T_b)} \quad \ldots \ldots \ldots \ldots \quad (1)$$

The work output per stage is given by,

$$W = \frac{U_m^2}{gJ} \cdot \frac{(\tan\alpha_1 - \tan\alpha_2)}{(-\tan\alpha_o - \tan\alpha_1)} \cdot \ell V_a L s n_b$$

(The sign convention described in Ref. (2) is used for the gas angles and velocities. As this could result in a negative value of W and consequently f, the usual equation has been multiplied throughout by -1 thus giving $(-\tan\alpha_o - \tan\alpha_1)$ for the denominator).

The heat extraction for a blade row is,

$$\Delta H_b = n_b L S h \;(T_g - T_b)$$

Substituting for W and ΔH_b in (1), and writing $\frac{Nu\lambda}{c}$ for h, we have

$$f_b = \frac{S}{c}\,\frac{Nu\lambda}{C_p} \cdot \frac{(-\tan\alpha - \tan\alpha_1)}{(\tan\alpha_1 - \tan\alpha_2)} \cdot \frac{1}{\ell V_a s}$$

And, if $Nu = k\,Re_2^{\;n}$

$$f_b = \frac{s}{c}\,k\,Re_2^{\;n}\,\frac{\lambda}{C_p}\,\frac{(-\tan\alpha_o - \tan\alpha_1)}{(\tan\alpha_1 - \tan\alpha_2)} \cdot \frac{1}{\ell V_a s} \quad \ldots \ldots \quad (2)$$

Applying Equation (2) to a row of rotor blades, a substitution may be made for V_a and V_2 as follows:

$$V_a = \frac{-U_m}{(-\tan\alpha_o - \tan\alpha_1)}\;,\quad U_m \text{ being negative.}$$

Hence,

$$f_R = \frac{S}{c}\,k\left(\frac{-\ell U_m c}{\mu}\right)^n \left(\frac{V_2}{U_m}\right)^n \frac{\lambda}{C_p}\,\frac{(-\tan\alpha_o - \tan\alpha_1)^2}{(\tan\alpha_1 - \tan\alpha_2)}\;\frac{1}{\left(\frac{-\ell U_m c}{\mu}\right)\mu \cdot \frac{s}{c}}$$

And, since

$$V_2 = \frac{V_a}{\cos \alpha_2} = \frac{-U_m}{\cos \alpha_2 \ (-\tan \alpha_o - \tan \alpha_1)} , \quad \text{we have finally,}$$

$$f_R = k \ (Re_u)^{n-1} \cdot \frac{S/_c}{s/_c} \cdot \frac{1}{Pr} \cdot \frac{(-\tan \alpha_o - \tan \alpha_1)^{2-n}}{(\tan \alpha_1 - \tan \alpha_2)} \cdot \frac{1}{(-\cos \alpha_2)^n} \quad .. \quad (3)$$

A similar expression may be obtained for the stator, but with the $(-\cos \alpha_2)^n$ term replaced by $(-\cos \alpha_o)^n$.

APPENDIX II

Method of Determining the Annulus Heat Extraction Coefficients

Assuming equal wall and blade heat transfer coefficients we have,

$$\frac{f_{AR}}{f_R} \text{ or } \frac{f_{AS}}{f_S} = \frac{\text{Surface area of annulus walls}}{\text{Surface area of blades}}$$

If r_m is the mean radius of the annulus, then for one blade row,

$$\text{surface area of walls} = 2 \times 2\pi r_m \; 1{\cdot}2 \; c \; \text{Cos} \; \xi$$

$$\text{surface area of blades} = n_b LS = \frac{2\pi r_m LS}{s}$$

$$\therefore \quad \frac{f_{AR}}{f_R} \text{ or } \frac{f_{AS}}{f_S} = \frac{2{\cdot}4 \; \text{Cos} \; \xi \; . \; s/c}{L/c \; . \; S/c}$$

L/c may be shown to be a function of the flow coefficient and rotor blade chord as follows:

$$L = \frac{Q}{\ell V_a s \; n_b} \qquad \dots \quad \dots \quad \dots \quad \dots \quad (1)$$

Centrifugal tensile stress, $q = K \, n_b s L N^2$, where K is a constant and N is the rotational speed.

$$\therefore \qquad N = \sqrt{\frac{q}{K n_b s L}} \qquad \dots \quad \dots \quad \dots \quad \dots \quad (2)$$

Also,

$$n_b = \frac{U_m}{sN} \qquad \dots \quad \dots \quad \dots \quad \dots \quad (3)$$

Substituting for N in Equation (3), we have

$$n_b = \frac{U_m^{\,2} \; . \; L \; K}{s \; . \; q}$$

And substituting this in Equation (1),

$$\left(\frac{L}{c}\right)^2 = \frac{Q \cdot q}{\ell V_a U_m^{\,2} \; K} \cdot \frac{1}{c^2}$$

Thus for given values of q, U_m, ℓ and Q,

$$\left(\frac{L}{c}\right)^2 \propto \frac{U_m}{V_a} \cdot \frac{1}{c^2}$$

Both U_m/V_a and c are known as functions of outlet angle and class of turbine, so that if a value of L/c is attributed to the rotor blade of the datum turbine, for which the values of U_m/V_a and c are known, the rotor blade aspect ratio of all the turbines is determined.

APPENDIX III

Specification of cooled Gas Turbine Plant

Simple gas turbine with heat exchange.

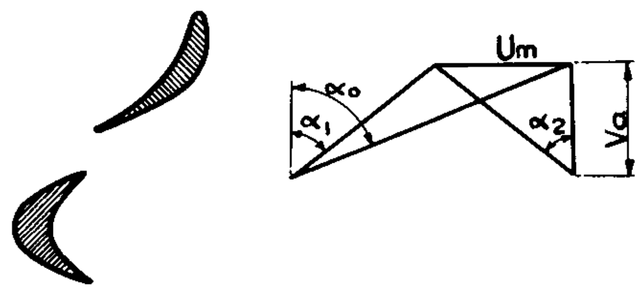
Compressor pressure ratio	6:1
Compressor polytropic efficiency	85 per cent
Combustion efficiency	100 per cent
Combustion pressure loss	1 lb./in.^{2}
Thermal ratio of heat exchanger	0.8
Pressure loss on air-side	2 lb./in.^{2}
Pressure loss on gas-side	1 lb./in.^{2}
Mechanical efficiency	99 per cent
Maximum cycle temperature	1200°C.
Temperature difference, $T_g - T_b$	600°C.
Mean blade speed of turbine	850 ft./sec.
Ambient conditions	15°C. and 14.7 lb./in.^{2}

For gas $\gamma = 1.333$ and $C_p = 0.274$

For air $\gamma = 1.4$ and $C_p = 0.24$

/

(1) IMPULSE WITH $\alpha_3 = 0$

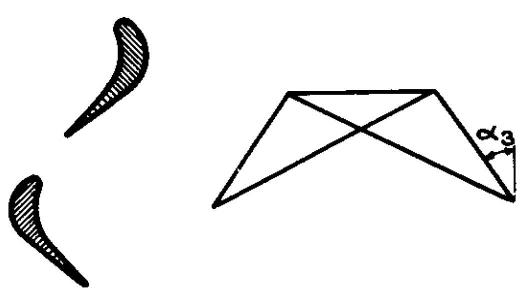


$\alpha_1 = \alpha_2$

$$\frac{V_a}{U_m} = \frac{1}{\tan\alpha_2}$$

$$2gJC_p\frac{\Delta T}{U_m^2} = -4$$

(2) 50% REACTION WITH $\alpha_1 = -\alpha_2/2$

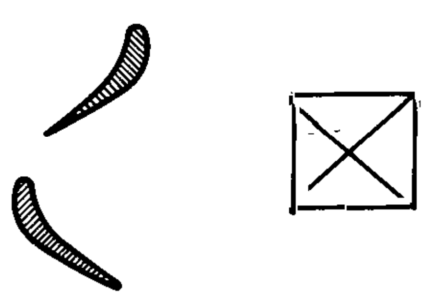


$\alpha_0 = \alpha_2$ AND $\alpha_1 = \alpha_3$

$$\frac{V_a}{U_m} = \frac{1}{\tan\alpha_2 - \tan\left(\frac{\alpha_2}{2}\right)}$$

$$2gJC_p\frac{\Delta T}{U_m^2} = 2\left[\frac{\tan\alpha_1 - \tan\alpha_2}{\tan\alpha_0 + \tan\alpha_1}\right]$$ AND VARIES BETWEEN −3 AND −5 OVER THE RANGE OF Va/Um CONSIDERED

(3) 50% REACTION WITH $\alpha_1 = 0$



$\alpha_0 = \alpha_2$ AND $\alpha_1 = \alpha_3$

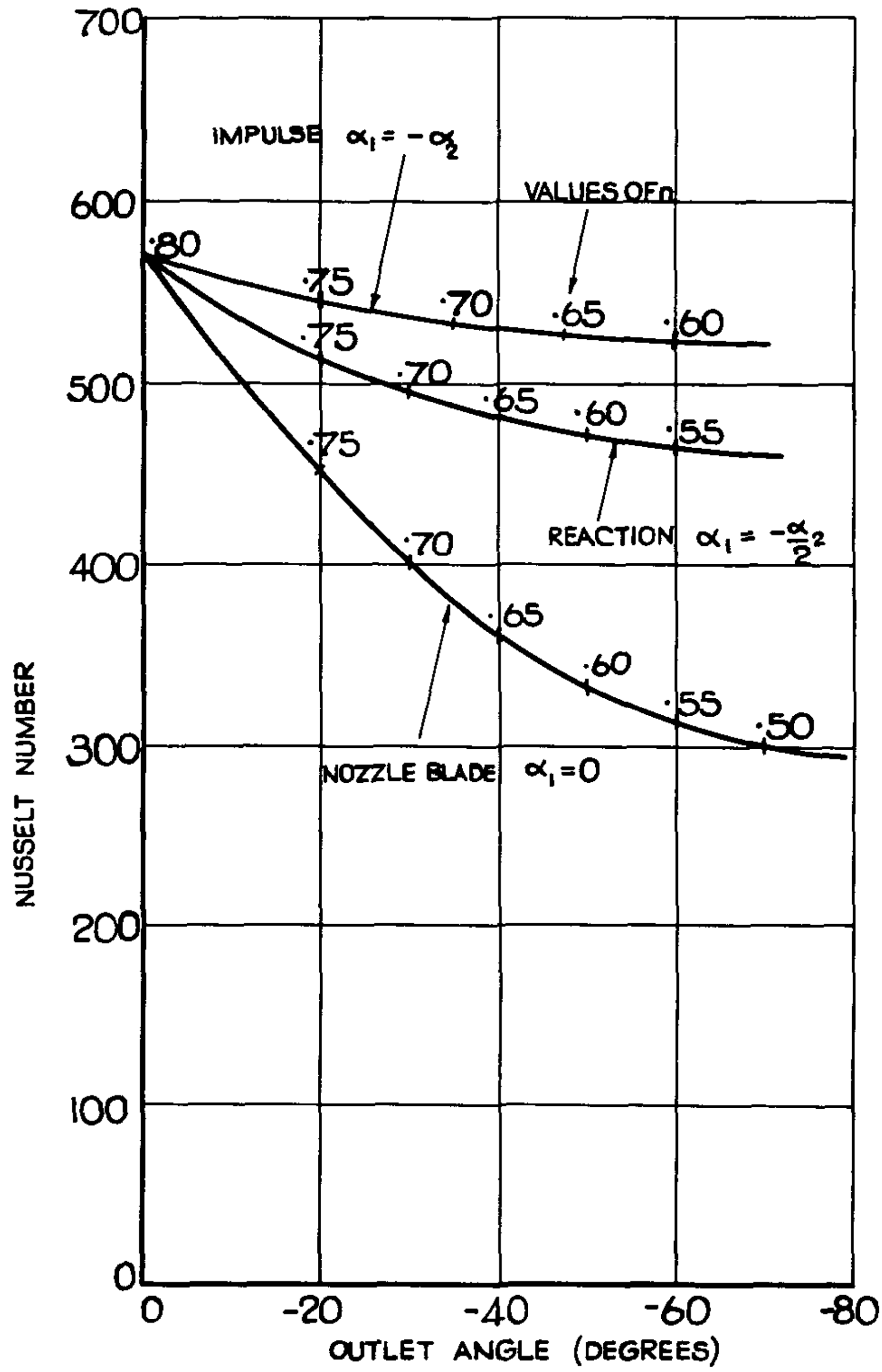
$$\frac{V_a}{U_m} = \frac{1}{\tan\alpha_2}$$

$$2gJC_p\frac{\Delta T}{U_m^2} = -2$$

TYPES OF TURBINE CONSIDERED.

Va IS CONSTANT THROUGH THE STAGE IN ALL CASES

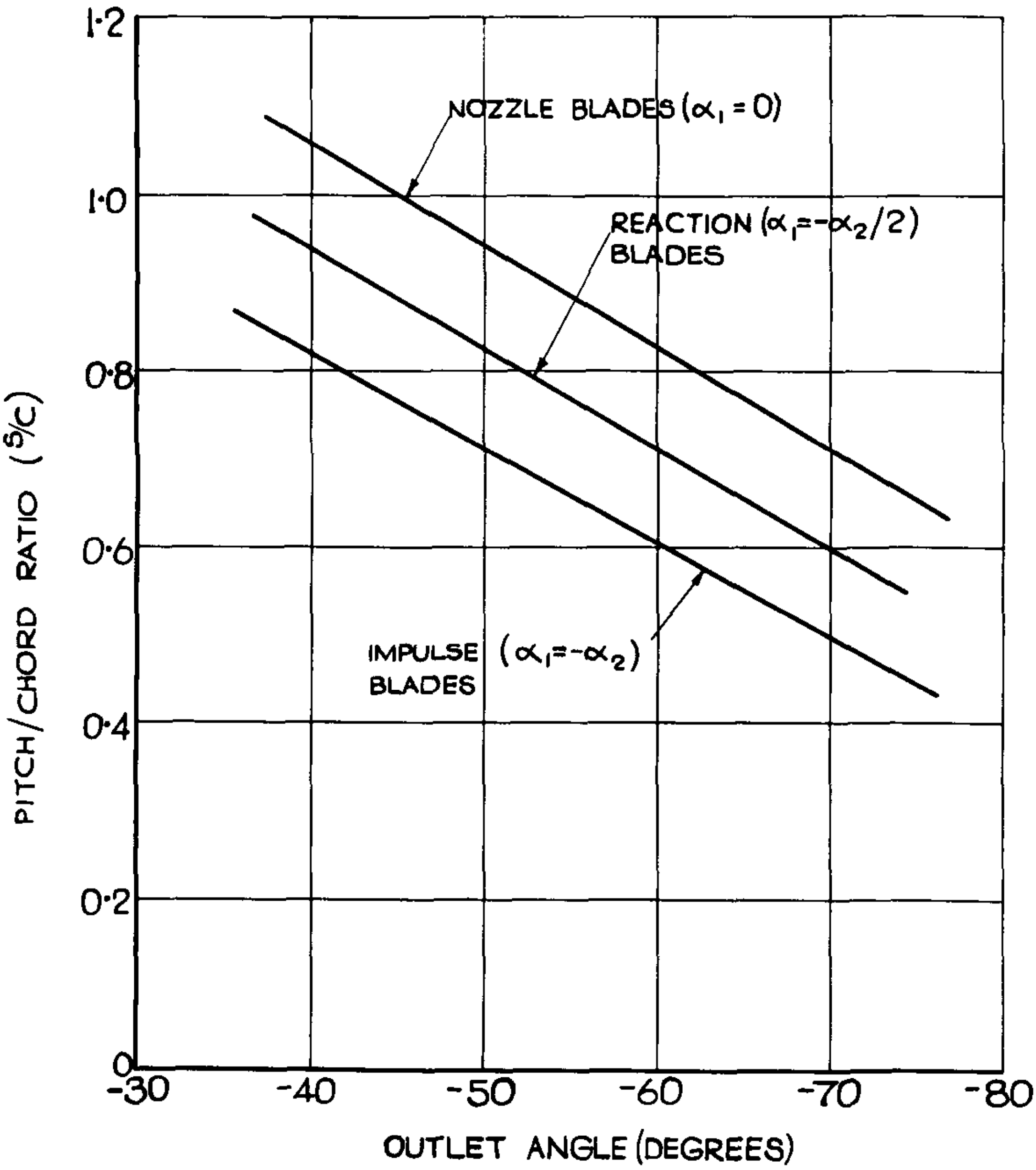
FIG. 2.



ATLAS OF BLADE HEAT TRANSFER COEFFICIENTS.

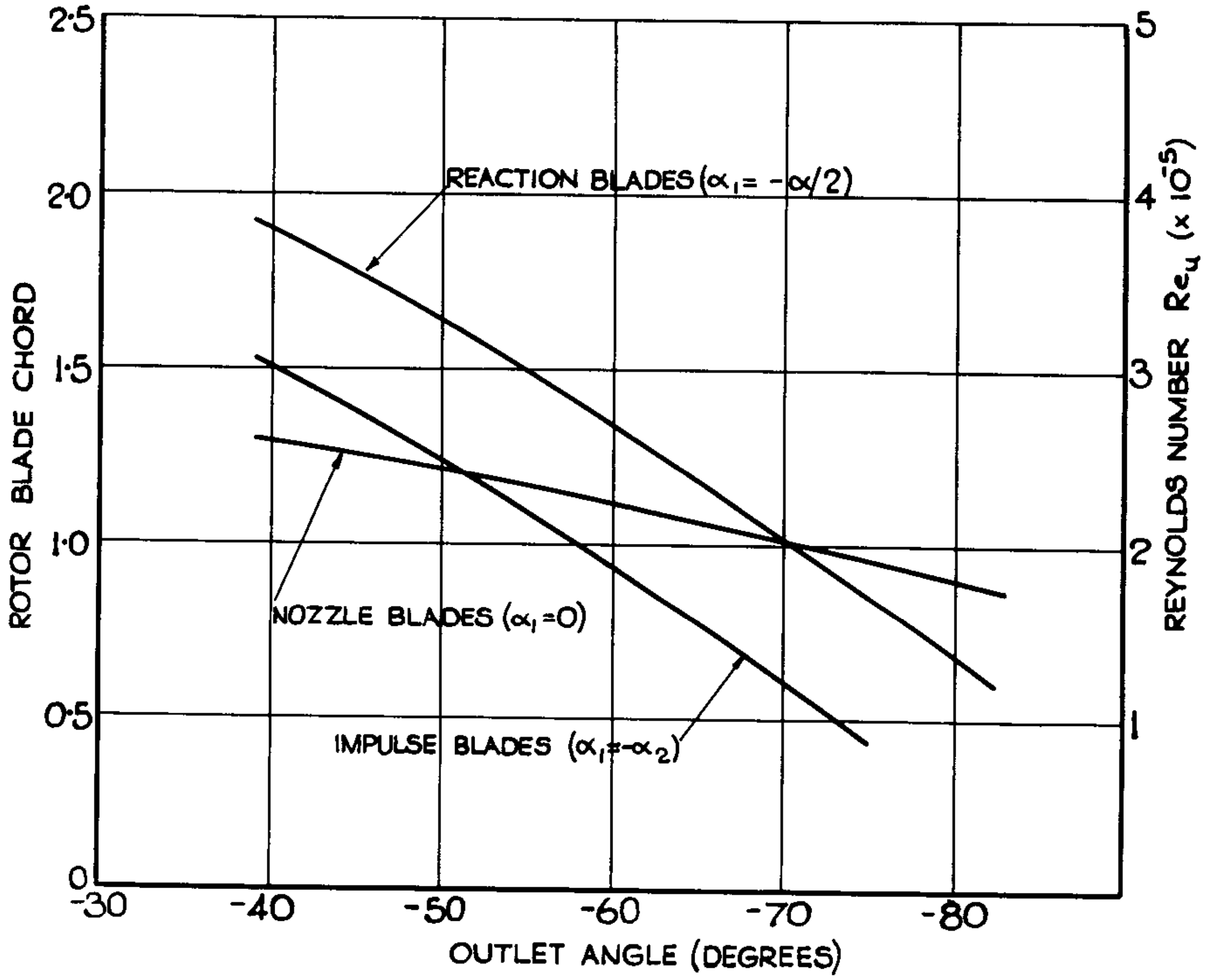
NUSSELT NUMBER V OUTLET ANGLE FOR $Re_2 = 2 \times 10^5$ WHERE $Nu = k\, Re_2^n$

FIG. 3.



PITCH/CHORD CURVES FOR TURBINE BLADES.

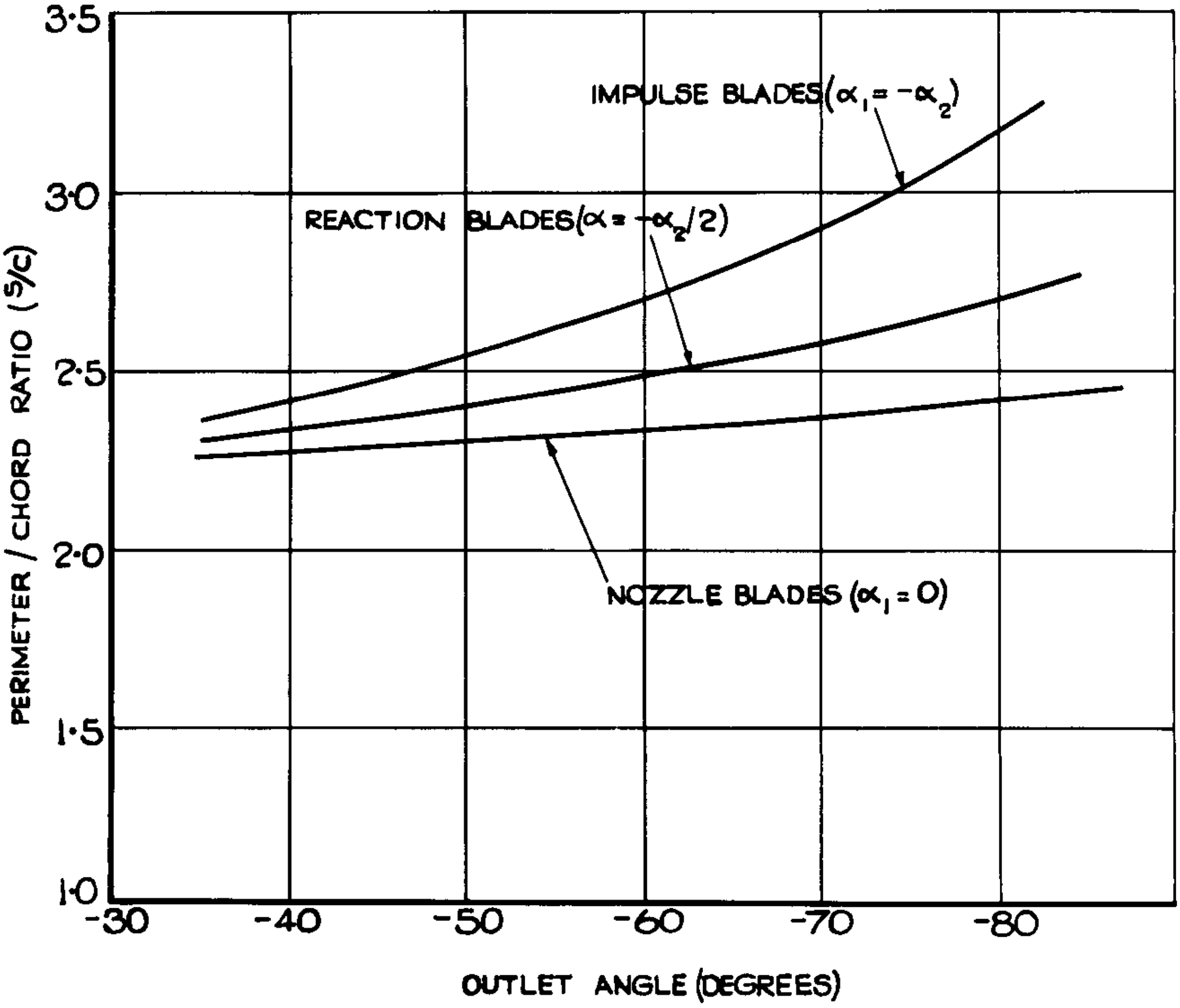
FIG. 4.



CHORD AND REYNOLDS NUMBER RELATIVE TO THE VALUES ASSUMED FOR THE DATUM TURBINE.

FOR DATUM TURBINE HAVING $\alpha_1 = -\alpha_2 = 45^\circ$, $\alpha_0 = -70^\circ$, $s/c = 0{\cdot}7$,
THE ROTOR BLADE CHORD IS 1 INCH AND $Re_u = 2 \times 10^5$

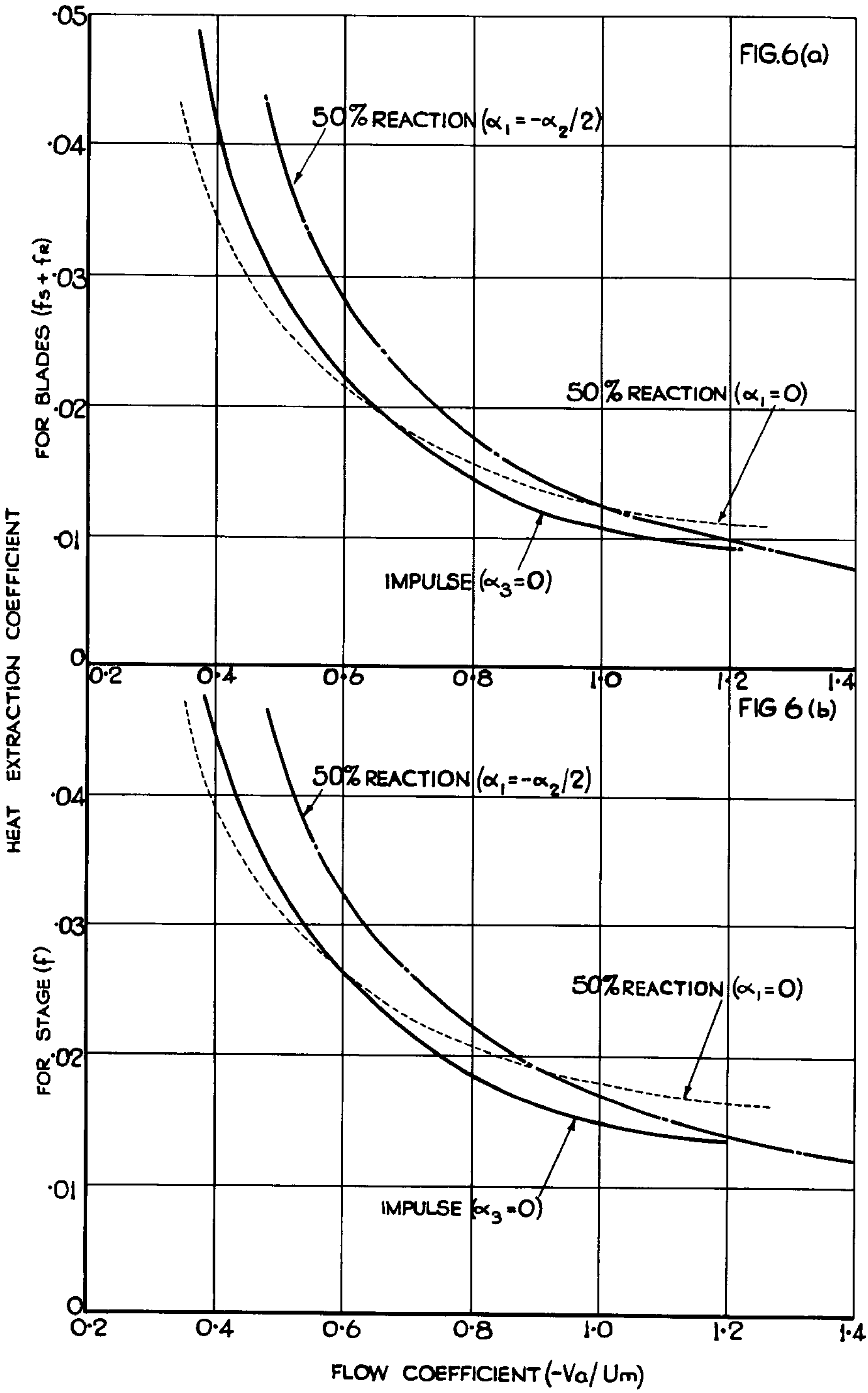
FIG. 5.



S/C CURVES FOR TURBINE BLADES.

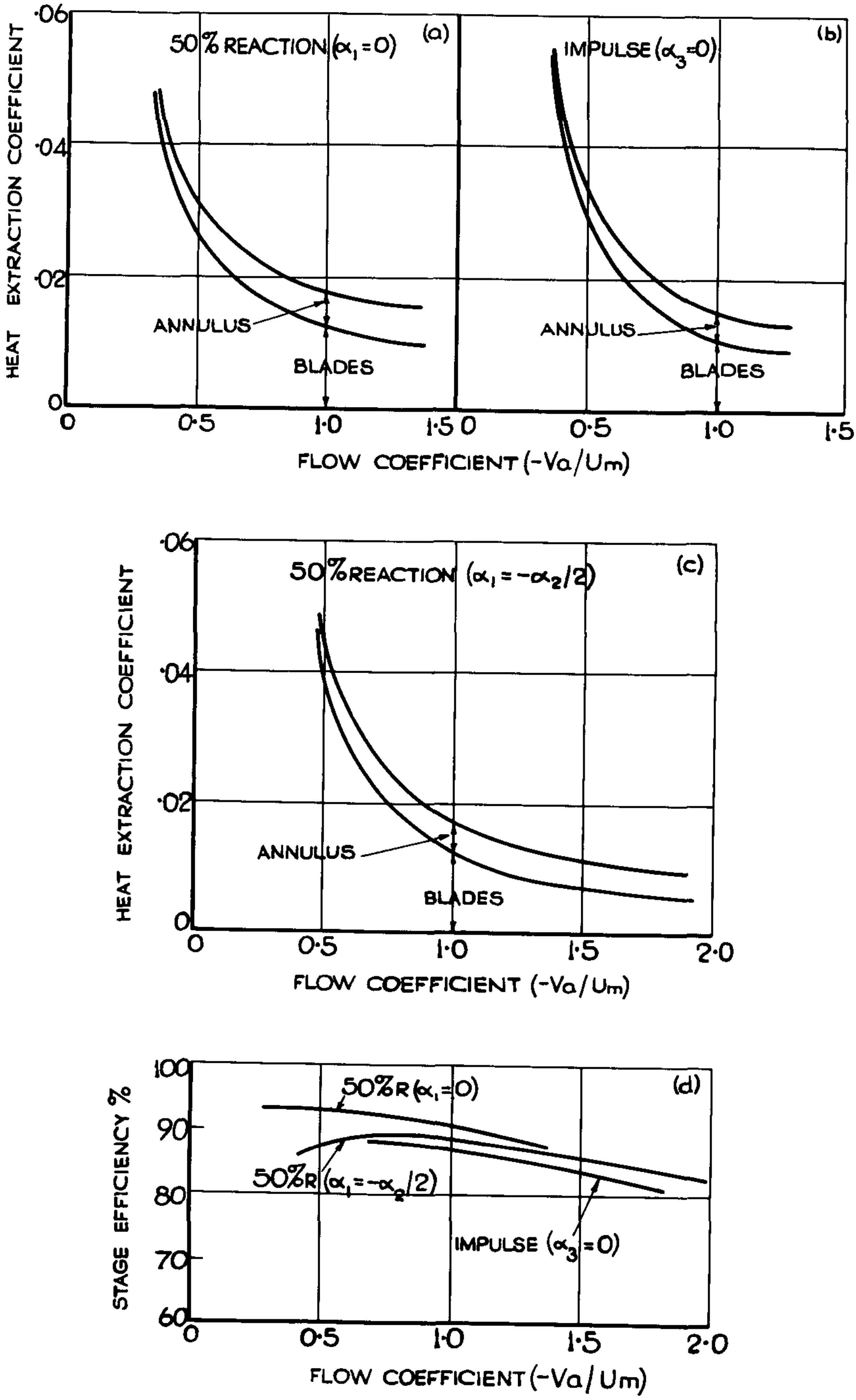
(ASSUMING CIRCULAR ARC CAMBER LINES AND PERIMETER = 2·23 x CAMBER LENGTH)

FIG. 6.



HEAT EXTRACTION COEFFICIENTS.

FIG. 7.



RELATIVE EFFECTS OF FLOW COEFFICIENT UPON BLADE AND ANNULUS COEFFICIENTS, AND STAGE EFFICIENCY.

FIG. 8.

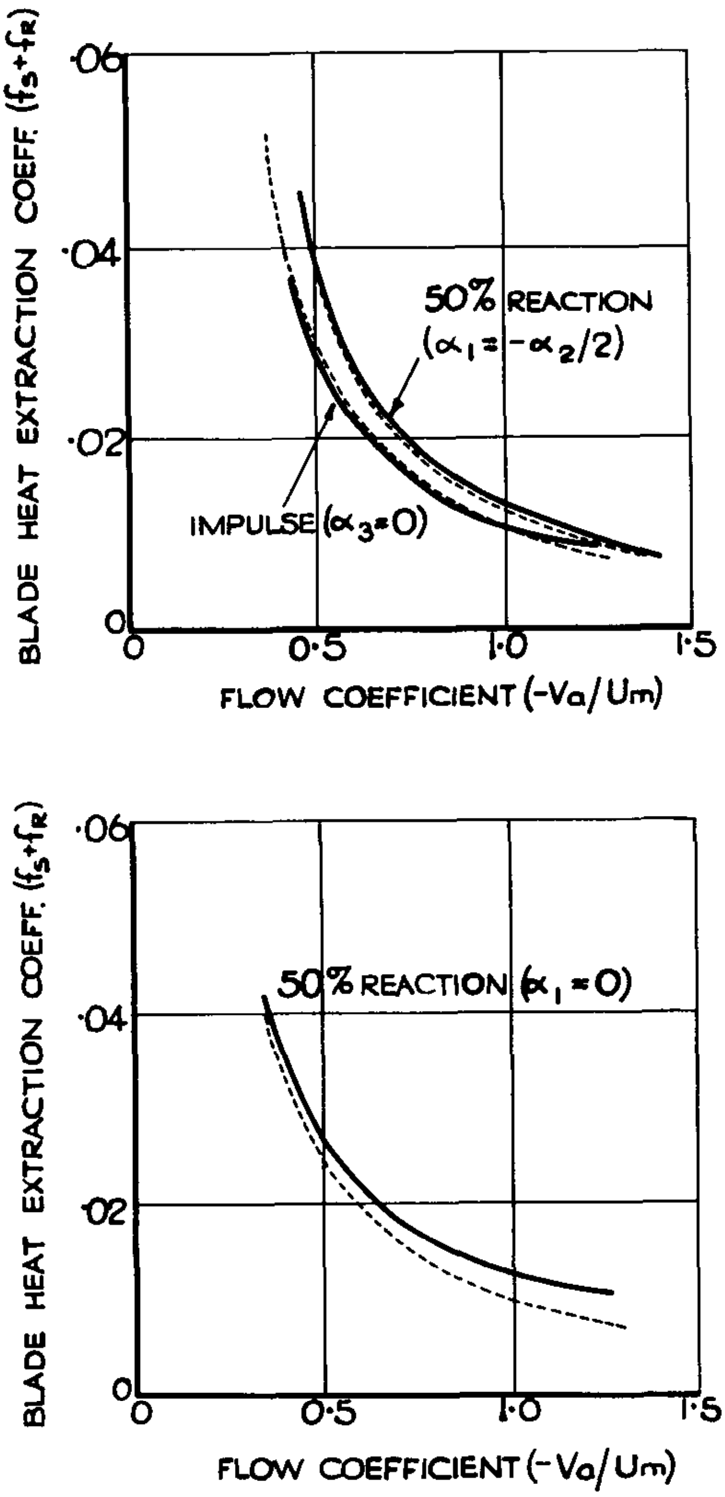
——— USING THE ATLAS OF HEAT TRANSFER COEFFICIENTS

------- USING THE FOLLOWING RELATIONSHIPS :-

NOZZLE BLADES ($\alpha_1 = 0$), $Nu = 0{\cdot}65\ Re_2^{0{\cdot}5}$

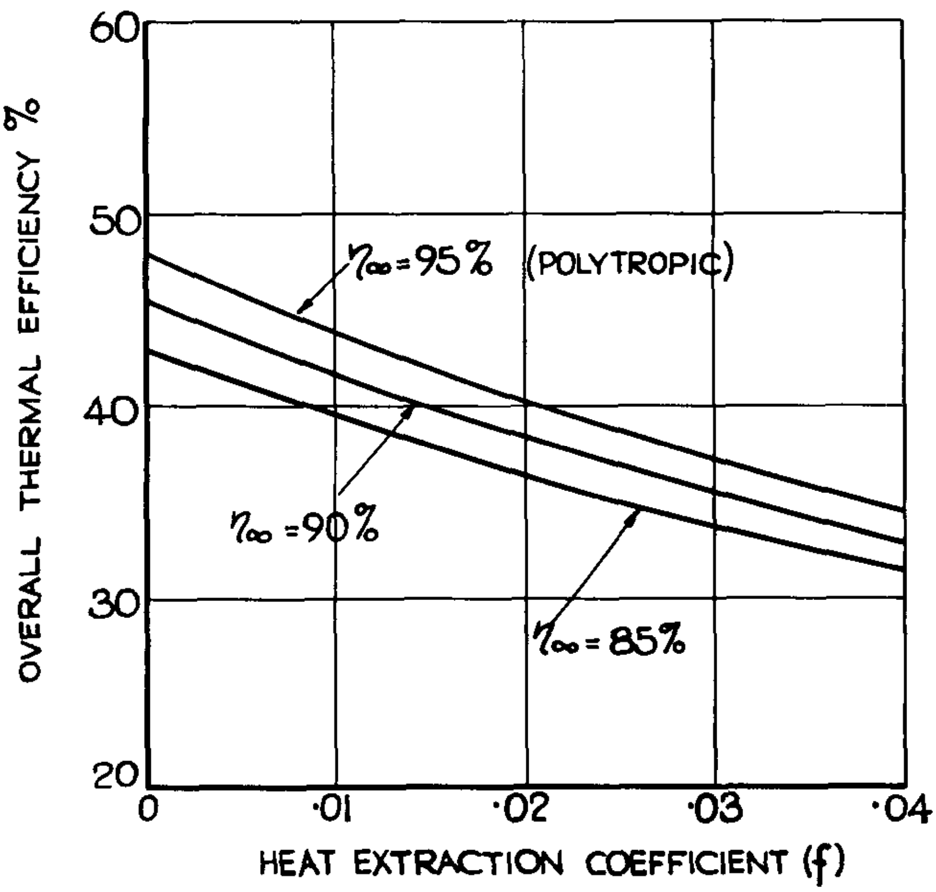
IMPULSE BLADES ($\alpha_1 = -\alpha_2$), $Nu = 0{\cdot}169\ Re_2^{0{\cdot}66}$

REACTION BLADES ($\alpha_1 = -\alpha_2/2$), $Nu = 0{\cdot}55\ Re_2^{0{\cdot}55}$



EFFECT OF ASSUMING CONSTANT NUSSELT NUMBER RELATIONSHIPS.

FIG. 9.



OVERALL THERMAL EFFICIENCY v. HEAT EXTRACTION COEFFICIENT FOR VARIOUS TURBINE EFFICIENCIES.

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