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Note on the Effect of Thickness and Aspect Ratio on the Damping of Pitching Oscillations of Rectangular Wings Moving at Supersonic Speeds

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5th May, 1953

1. Summary

An estimate is made of the effect of variation of aspect ratio and thickness parameter on the damping derivative of a rectangular wing performing pitching oscillations in a supersonic stream. After a discussion of various theories of unsteady two-dimensional supersonic flow round aerofoils, that due to Van Dyke[7] is combined with the linearized supersonic theory for a rectangular flat plate to obtain the effect on the damping in pitch. The values given apply to wings of symmetrical biconvex section oscillating at low frequency, in a stream of Mach number such that the shock at the leading edge is attached.

2. Notation

c	chord of wing
k	thickness ratio = maximum thickness/c
V_o	free stream velocity
a	speed of sound in free stream
$M_o = V_o/a$	free stream Mach number
β_o	$\sqrt{M_o^2 - 1}$
b	distance of pitching axis downstream of leading edge
h	= b/c
f	frequency of oscillation
ω	$2\pi f$
$\lambda = \omega c/V_o$	frequency parameter
ρ_o	density of air in free stream
A	aspect ratio
$\alpha = \alpha' \exp(i\omega t)$	angle of pitch (positive if it tends to raise the leading edge and depress the trailing edge)
$\mathcal{M}$	pitching moment (positive in the same sense as α)
t	time

γ/

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γ ratio of specific heats (taken as 1.4)

$$N = \frac{(\gamma + 1)}{2} \cdot \frac{M_o^2}{\beta_o^2}$$

p pressure on surface of aerofoil

p_o free stream pressure

C_1, C_2, C_3, D Busemann coefficients (see Ref.2)

x distance measured downstream of leading edge

y distance perpendicular to free stream, positive upwards

$y = Y(x)$ equation of upper surface of aerofoil

$\mu_o = \sin^{-1} \frac{1}{M_o}$ Mach angle in free stream

$C_p = \frac{p - p_o}{\frac{1}{2} \rho_o V_o^2}$ pressure coefficient

Suffix 1 refers to conditions downstream of the shock at the apex of a wedge

m_α stiffness coefficient in pitching moment derivative

$m_{\dot{\alpha}}$ damping coefficient in pitching moment derivative.

3. Introduction

This note draws attention to the need for more information about the forces acting on oscillating wings at transonic and supersonic speeds. Linearized flat plate theory, which is available for two-dimensional motion and certain three-dimensional plan forms (see, e.g., Refs. 12, 20, 21), predicts negative damping in pitch at supersonic speeds under some circumstances, e.g., in two-dimensions when the pitching axis is forward of the two-thirds chord position and the Mach number is sufficiently near to unity. This prediction is supported by certain incidents which have occurred during tests of guided missiles and which appear to be due to one-degree-of-freedom instability of the all-moving control surfaces. However a knowledge of the effect of finite thickness is still required. Several two-dimensional theories dealing with the effect of thickness have been produced (Refs. 2 to 10), and are discussed in §4, but unfortunately they disagree, and no transonic or three-dimensional supersonic theory including thickness effects appears to be available. No systematic experimental investigation has been carried out, although a few results are available in particular cases, e.g., those of Ref.17. Tests at the N.P.L. on double and single wedge profiles which it is proposed to carry out in the near future may throw some light on the situation.

4. Consideration of Unsteady Two-dimensional Supersonic Flow Theories

(i) Discussion of Theories for General Profiles

Several authors (Refs. 2-10) have produced theories giving the effect of finite thickness on the forces acting on a two-dimensional aerofoil performing small amplitude oscillations in a supersonic stream. It is assumed that the shock at the leading edge is attached and, usually, that the frequency is small. Unfortunately the results differ widely from theory to theory, and in this section some

consideration/

consideration is given to the methods by which they were obtained. The only experimental work which seems to be available as a check is that described in Ref.17, and the results given there are not sufficient to decide which, if any, of the theories is correct.

Jones' theory of Ref.2 is an attempt to obtain the unsteady pressures for low frequency oscillations by taking the steady Busemann theory for two-dimensional aerofoils in the form

$$\frac{p - p_o}{\frac{1}{2}\rho_o V_o^2} = C_1\left(\frac{W}{V_o}\right) + C_2\left(\frac{W}{V_o}\right)^2 + \left(C_3 - \frac{C_1}{3}\right)\left(\frac{W}{V_o}\right)^3 - D\left(\frac{W_e}{V_o}\right)^3 \quad \ldots\ldots (1)$$

where C_1, C_2, C_3 and D are the Busemann coefficients, W/V_o is the tangent of the local angle of upward deflection of the flow, and W_e/V_o is the corresponding value at the leading edge, and replacing W/V_o by an appropriate equivalent unsteady value in order to obtain the unsteady pressures. For small frequencies, two-dimensional flat plate theory[1] suggests that W/V_o be replaced in (1) by an equivalent unsteady local deflection given by

$$\bar{W}/V_o = W/V_o - \frac{\tan^2 \mu_o}{V_o} \int_o^x \frac{\partial}{\partial t}\left(\frac{W}{V_o}\right) dx \quad \ldots\ldots (2)$$

where W is now the instantaneous upwash due to the shape and oscillating motion of the aerofoil. The justification is that when $\bar{W}$ is substituted for W in equation (1), and only the first term on the right hand side is retained, the value of $p - p_o$ is that given by oscillating flat plate theory. The integral in (2) thus allows for the effect of the motion of these parts of the aerofoil lying between the leading edge and the point corresponding to x. Since W/V_o is known from the prescribed oscillatory motion of the aerofoil, the pressure may be determined by (1) and (2). If it is assumed that the amplitude of oscillation is sufficiently small for its square to be neglected the pressure is given to first order in thickness by the first two terms on the right hand side of (1), so that the assumption involved in results to this accuracy is that the modification implied by (2) to the first term of (1) may also be used in the second. For a $7\frac{1}{2}\%$ biconvex wing pitching about the half-chord axis this theory gives better agreement with the experimental values of Ref.17 than does linearized flat plate theory, insofar as it predicts positive damping for pitching about the half-chord axis, whereas flat plate theory predicts negative damping at the Mach numbers considered.

The method of Ref.3 consists of a small perturbation procedure applied to the known steady flow round a two-dimensional profile, resulting in a linear differential equation for the velocity potential of the small unsteady part of the velocity field. For the case of an attached shock this equation is hyperbolic in type and is solved by a numerical step-by-step method starting at the leading edge shock wave and proceeding downstream. Near the leading edge (assumed pointed) the velocity potential is taken to be the solution for a flat plate oscillating in a stream of Mach number equal to that immediately behind the attached shock. This method is not restricted to low frequency, and provided the starting solution is correct it is difficult to see any objection to it, apart from the practical one that in general it involves a considerable amount of computation for each combination of frequency parameter and Mach number. In the case of a single-wedge profile the method becomes particularly simple since the step-by-step solution can be replaced by an analytical one; this profile will be considered later.

Wylly's theory (Refs. 4, 5 and 6) is based on an expansion of the velocity potential in a series of powers of the thickness parameter and frequency parameter, the resulting linear differential equations being solved by a complicated analytical method. The pressure coefficient on the upper surface of the aerofoil is given in the notation of the present report by

$$C_p = \frac{2}{\beta_o}\left\{(Y' - \alpha) + \frac{\dot{\alpha}}{V_o}\left[b + \frac{x}{\beta_o^2}(2 - M_o^2)\right] + (Y' - \alpha)^2\frac{A}{2\beta_o} + \frac{\dot{\alpha}}{V_o}(Y' - \alpha)\left[\frac{x}{\beta_o^3}D + \frac{b}{2\beta_o}B\right]\right\} \quad \ldots\ldots (3)$$

where $A = \dfrac{(\gamma + 1)M_o^4 - 4\beta_o^2}{2\beta_o^2}$, $B = M_o^2(\gamma + 1) - 4$

and $D = \dfrac{\gamma(4M_o^4 - M_o^6) + 3M_o^6 - 10M_o^4 + 10M_o^2}{2\beta_o^2}$.

This result is open to objection for two reasons. One, due to Van Dyke[7*], is based on the fact that at the leading edge the pressure is entirely determined by the local upwash, i.e., by $Y' - \alpha + \dot{\alpha}b/V_o$ and this condition is not satisfied by (3). The other rests on the fact that the limiting form of (3) for very large M_o does not agree with Lighthill's high Mach number theory[10] whereas the theories of Van Dyke[7] and Jones[2] do so agree. Wylly's theory for closed profiles predicts a large damping effect due to thickness for all axis positions, in disagreement with the other theories which predict destabilizing effects for forward axis positions.

The method of obtaining Van Dyke's result[7]

$$C_p = \frac{2}{\beta_o}(Y' - \alpha) + \frac{2\dot{\alpha}}{\beta_o V_o}\left(\frac{2 - M_o^2}{\beta_o^2}x + b\right) + \frac{(M_o^2N - 2)}{\beta_o^2}(Y'^2 - 2Y'\alpha) + \frac{2\dot{\alpha}}{V_o}\left[\frac{2M_o^2(N - 1)Y}{\beta_o^4} + \left\{\frac{(2 - M_o^2)(M_o^2N - 1)}{\beta_o^4}x + \frac{(M_o^2N - 2)}{\beta_o^2}\cdot b\right\}Y'\right] \quad (4)$$

has not been published so that no comment is possible. Equation (4) agrees with Lighthill's theory when M_o is very large, and moreover it agrees to second order in thickness[8] with an extension to any axis position of Carrier's theory[9] for a single wedge oscillating about its apex.

Lighthill's theory[10] is applicable to very high Mach numbers and uses the fact that when the Mach number is very large the variation of the flow with y is very large compared to the variation with x so that the problem reduces to one in which y is the only independent variable for each particular value of x. The range of Mach numbers considered is outside that for which negative damping in pitch may be expected so that the theory does not apply so far as this note is concerned, but, as has been stated, it gives a check on the limiting forms of the others.

Fig.1 shows the stability diagrams corresponding to these theories for a 5% thick biconvex aerofoil oscillating in pitch about an axis distance hc behind the leading edge. No curve corresponding to Wylly's theory appears since this predicts positive damping for all combinations of h and M_o . All the theories predict an increase in damping for axes behind the half-chord but vary for axes forward of this position.

(ii)/

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* Van Dyke's theory has now been published as "Supersonic flow past oscillating airfoils including non-linear thickness effects", N.A.C.A. Tech. Note 2982, July, 1953.

(ii) The Single Wedge Profile

A particularly simple case arises when the profile is a single wedge, since the stream behind the shock (assumed to be attached) is then uniform. It seems plausible to assume that the forces on the surface are those on a flat plate oscillating in a stream of Mach number, density, etc., equal to those behind the shock. If free stream values are denoted by the suffix 0 and those behind the shock by suffix 1, then the forces on a wedge in a free stream of Mach number M_0 are assumed to be those on a flat plate in a free stream of Mach number M_1. Since the relation between M_0 and M_1 is easily found from shock tables it is possible to plot a stability diagram for any particular wedge angle. An objection to this procedure is that since the Mach lines behind the shock are inclined at an angle μ_1 to the surface and cut the shock, the flat plate corresponding to the surface of the wedge is not oscillating in an unlimited region of Mach number M_1 but one limited by the shock-wave, so that the shock may have some effect on the aerofoil. Sewell[19] has shown that the small variations in shock position due to the oscillation will have no effect. A stability diagram plotted in this way for a wedge of 5° semi-apex angle is included in Fig.2; the procedure is equivalent to that of Ref.3 and the curve is so marked.

If this assumption is made the pressure on the upper surface is given, for low frequency, by flat plate theory as

$$p - p_1 = -\rho_1 V_1^2 \left(\frac{\alpha}{\beta_1} - \frac{x\dot{\alpha}}{V_1} \frac{(2 - M_1^2)}{\beta_1^3} - \frac{b\dot{\alpha}}{V_1 \beta_1} \right) . \qquad \ldots\ldots (5)$$

If this is transformed by using the approximate relations between corresponding quantities behind and ahead of the shock (Ref.18), i.e.,

$$\frac{M_1}{M_0} = 1 - \omega\beta_0(N - 1) \quad , \quad \frac{\beta_1}{\beta_0} = 1 - \omega \frac{M_0^2}{\beta_0} (N - 1)$$

$$\frac{p_1}{p_0} = 1 + \frac{\gamma M_0^2}{\beta_0} \omega \quad , \quad \frac{\rho_1}{\rho_0} = 1 + \frac{M_0^2}{\beta_0} \omega$$

$$\frac{V_1}{V_0} = 1 - \frac{\omega}{\beta_0}$$

where ω is the wedge semi-apex angle and the expressions are correct to first order in ω, then

$$p - p_0 = \tfrac{1}{2}\rho_0 V_0^2 \left\{ \frac{2}{\beta_0} (\omega - \alpha) + \frac{2\dot{\alpha}x}{V_0} \cdot \frac{2 - M_0^2}{\beta_0^3} - 2\alpha\omega \frac{(M_0^2 N - 2)}{\beta_0^2} + \frac{x\dot{\alpha}\omega}{V_0} \cdot \frac{2}{\beta_0^4} \right.$$

$$\times \left[(2 - M_0^2)(M_0^2 N - 1) + 2M_0^2(N - 1) \right]$$

$$\left. + \frac{b\dot{\alpha}}{V_0} \left[\frac{2}{\beta_0} + 2\omega \frac{(M_0^2 N - 1)}{\beta_0^2} \right] \right\} \qquad \ldots\ldots (6)$$

which corresponds to Van Dyke's result except for the last term where $M_0^2 N - 1$ replaces $M_0^2 N - 2$. This difference is presumably due to the difference in boundary conditions at the shock mentioned above.

Equation/

Equation (5) may also be obtained by taking a result proved for steady flow in Ref.3, i.e.,

$$p - p_1 = - \rho_1 V_1^2 \tan \mu_1 . \alpha , \qquad \ldots\ldots (7)$$

and substituting for α an equivalent unsteady angle of incidence as described in connection with Ref.2. Here suffix 1 refers to quantities in the undisturbed position of the aerofoil, and p is the pressure corresponding to a small angle of incidence α.

Fig.2 shows stability diagrams corresponding to the various theories for a 5° semi-apex angle single wedge profile. All agree in the general trend but differ as to its magnitude.

In this note Van Dyke's theory has been used though it cannot be regarded as certain that it is correct. Use of this theory involves restriction to low frequency parameter and an attached shock at the leading edge. As far as the subsequent work is concerned it would have been equally simple to use any of the others.

5. Formulae for Rectangular Wings

For a wing pitching about any given axis the pitching moment may be expressed in the form

$$\mathcal{M} = \tfrac{1}{2} \rho_0 V_0^2 c^3 e^{i\omega t} A \{m_\alpha + i\lambda m_{\dot\alpha}\} \qquad \ldots\ldots (8)$$

where m_α and $m_{\dot\alpha}$ are functions of M_0 and h. Here $m_{\dot\alpha}$ is the coefficient determining the damping in pitch, damping being positive if $m_{\dot\alpha} < 0$.

Several authors (Refs. 11 to 16) have investigated by linearized theory the problem of determining $m_{\dot\alpha}$ for a plane rectangular wing of zero thickness; the value used here is taken from the work of Watkins[10] where it is given in a convenient form. In the notation of §2,

$$-m_{\dot\alpha} = \frac{4}{\beta_0}\left\{h^2 - h + \frac{1}{3} + \frac{1}{\beta_0^2}\left(\frac{h}{2} - \frac{1}{3}\right)\right\} - \frac{2}{A\beta_0^2}\left\{h^2 - \frac{2}{3}h + \frac{1}{\beta_0^2}\left(\frac{2}{3}h - \frac{1}{2}\right)\right\} \qquad (9)$$

for small frequencies.

The first term corresponds to the two-dimensional flat-plate theory, the second gives the effect of finite aspect ratio.

Now according to Van Dyke's theory[7], for a symmetrical biconvex aerofoil,

$$-m_{\dot\alpha} = \frac{4}{\beta_0}\left\{h^2 - h + \frac{1}{3} + \frac{1}{\beta_0^2}\left(\frac{h}{2} - \frac{1}{3}\right)\right\} + k . \frac{4}{3\beta_0^2}\left\{h\,\frac{M_0^2(N-1)}{\beta_0^2} - (M_0^2 N - 2)(1 - 2h)\right\}$$

$$\ldots\ldots (10)$$

the first term again being the two-dimensional flat-plate result, and the second giving the thickness effect.

It seems reasonable to combine (9) and (10) and take

$$- m_{\dot{\alpha}} = \frac{4}{\beta_o}\left\{h^2 - h + \frac{1}{3} + \frac{1}{\beta_o^2}\left(\frac{h}{2} - \frac{1}{3}\right)\right\} - \frac{2}{A\beta_o^2}\left\{h^2 - \frac{2}{3}h + \frac{1}{\beta_o^2}\left(\frac{2}{3}h - \frac{1}{2}\right)\right\}$$

$$+ k\frac{4}{3\beta_o^2}\left\{h\frac{M_o^2(N-1)}{\beta_o^2} - (M_o^2N - 2)(1 - 2h)\right\} \qquad \ldots\ldots (11)$$

as the equation giving $-m_{\dot{\alpha}}$ for a rectangular wing of aspect ratio A, and biconvex section of thickness ratio k, oscillating at low frequency; that is to say it is assumed that the tip effect can be superimposed on the thickness effect.

This equation was used to compute $-m_{\dot{\alpha}}$ for various combinations of A, k, M_o and h, and the results are given in Figs. 3 to 7. Fig.3 shows the effect on the stability diagram of varying A alone with $k = 0$, and shows the stabilizing effect of reducing aspect ratio. Since equation (9) applies only to wings with $A \geqslant \frac{1}{\beta_o}$ it is not certain whether a sufficiently low aspect ratio will eliminate the instability altogether. Fig.4 illustrates the result of varying k alone, with $A = \infty$, showing the stabilizing effect for axes downstream of about 0.39 of the chord and destabilizing forward. Figs. 5 and 6 show the effect of combining these two variations. If the infinite flat plate $(A = \infty, k = 0)$ is taken as the basis of comparison, and only axes lying on the wing $(0 < h < 1)$ are considered, it appears that except for axes very near the leading edge, a reduction of aspect ratio to 6 is sufficient to overcome the destabilizing effect of an increase of k from 0 to 0.05, i.e., for axes on the wing, a wing with $k = 0.05$ and $A = 6$ is at least as stable as an infinite flat plate, except for axes very near the leading edge. A similar result is true for $A = 3$ and $k = 0.10$. Fig.7 shows some values of $-m_{\dot{\alpha}}$ corresponding to the values of A and k used in Fig.5 and leads to the same conclusion.

Since the leading edge shock becomes detached for $M_o < 1.27$ for $k = 0.05$, and for $M_o < 1.47$ if $k = 0.10$, curves corresponding to these thickness parameters become meaningless for Mach numbers below these values.

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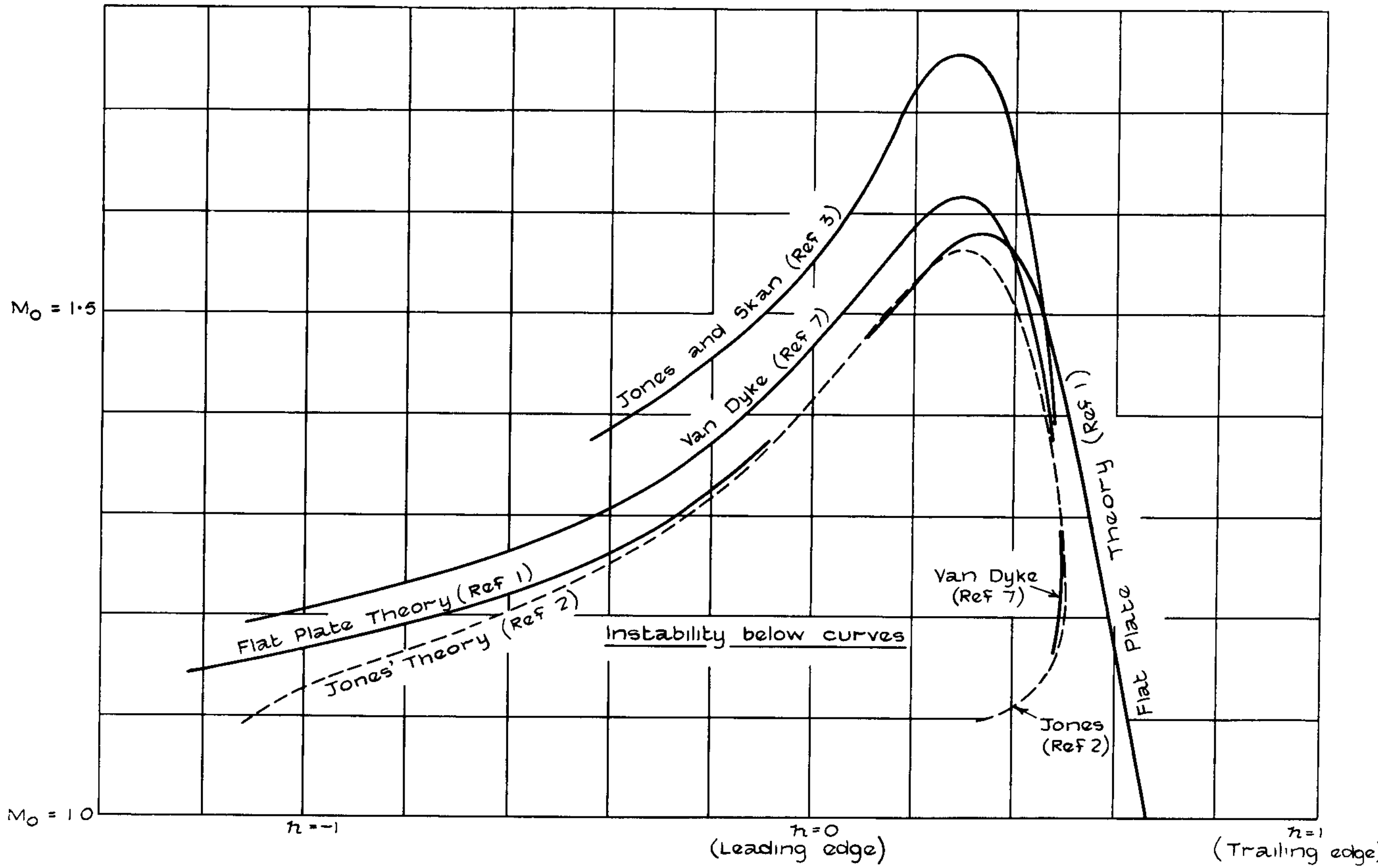
FIG 1



Comparison of stability diagrams for a 5% thick biconvex profile
(Low frequency)

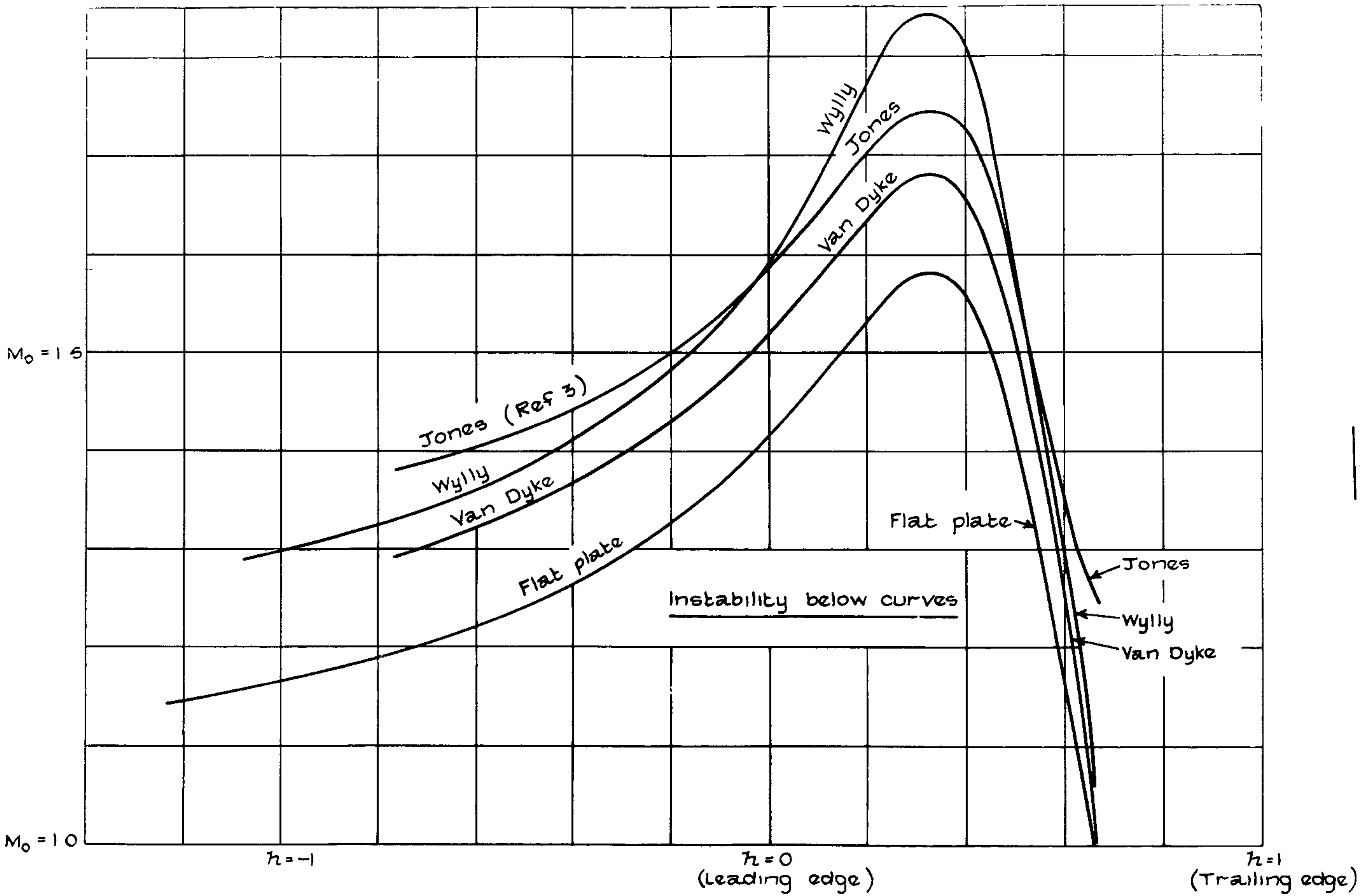


FIG 2

Comparison of stability diagrams for a 5° semi-apex angle single wedge profile. (Low frequency)

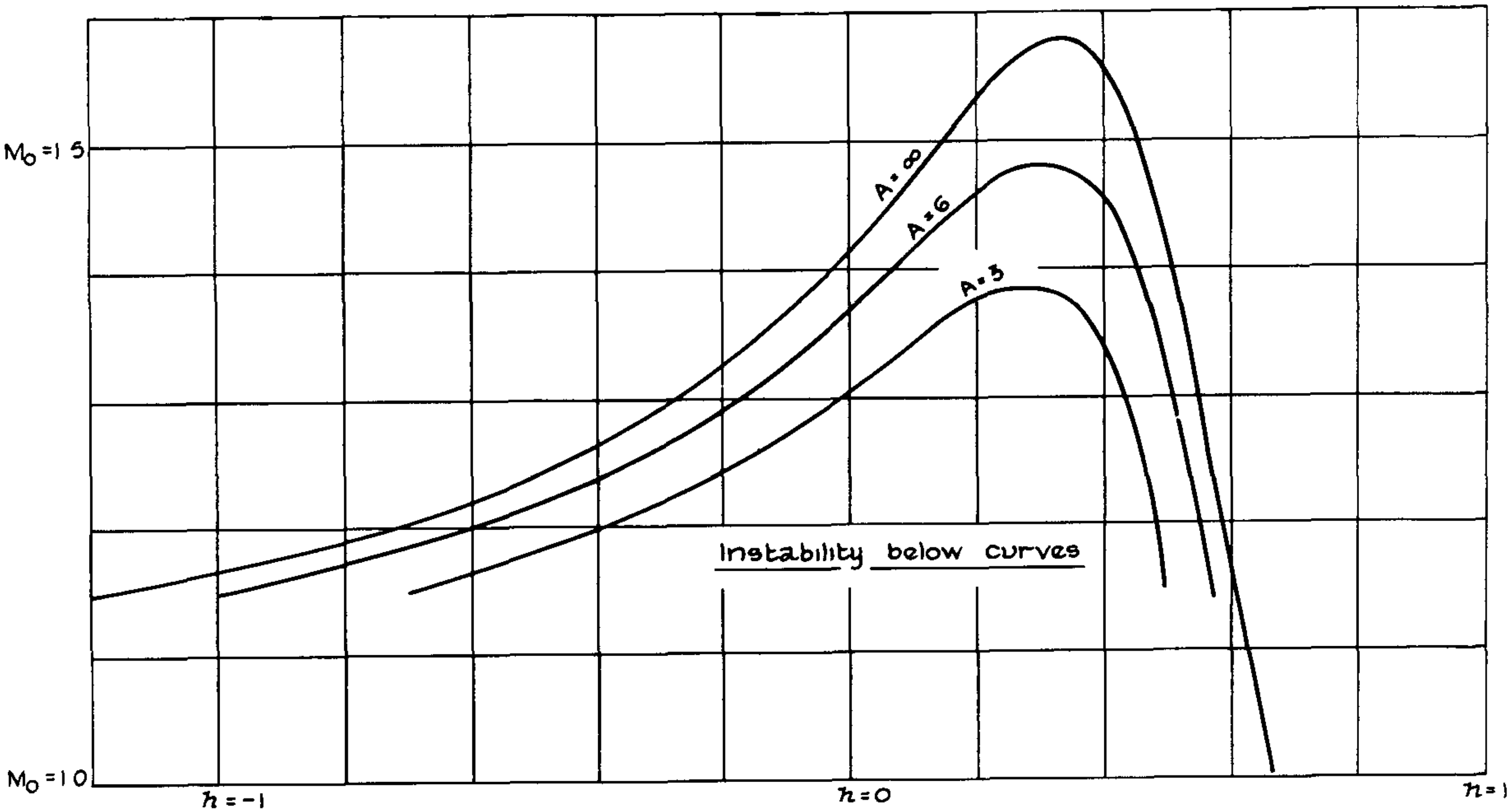
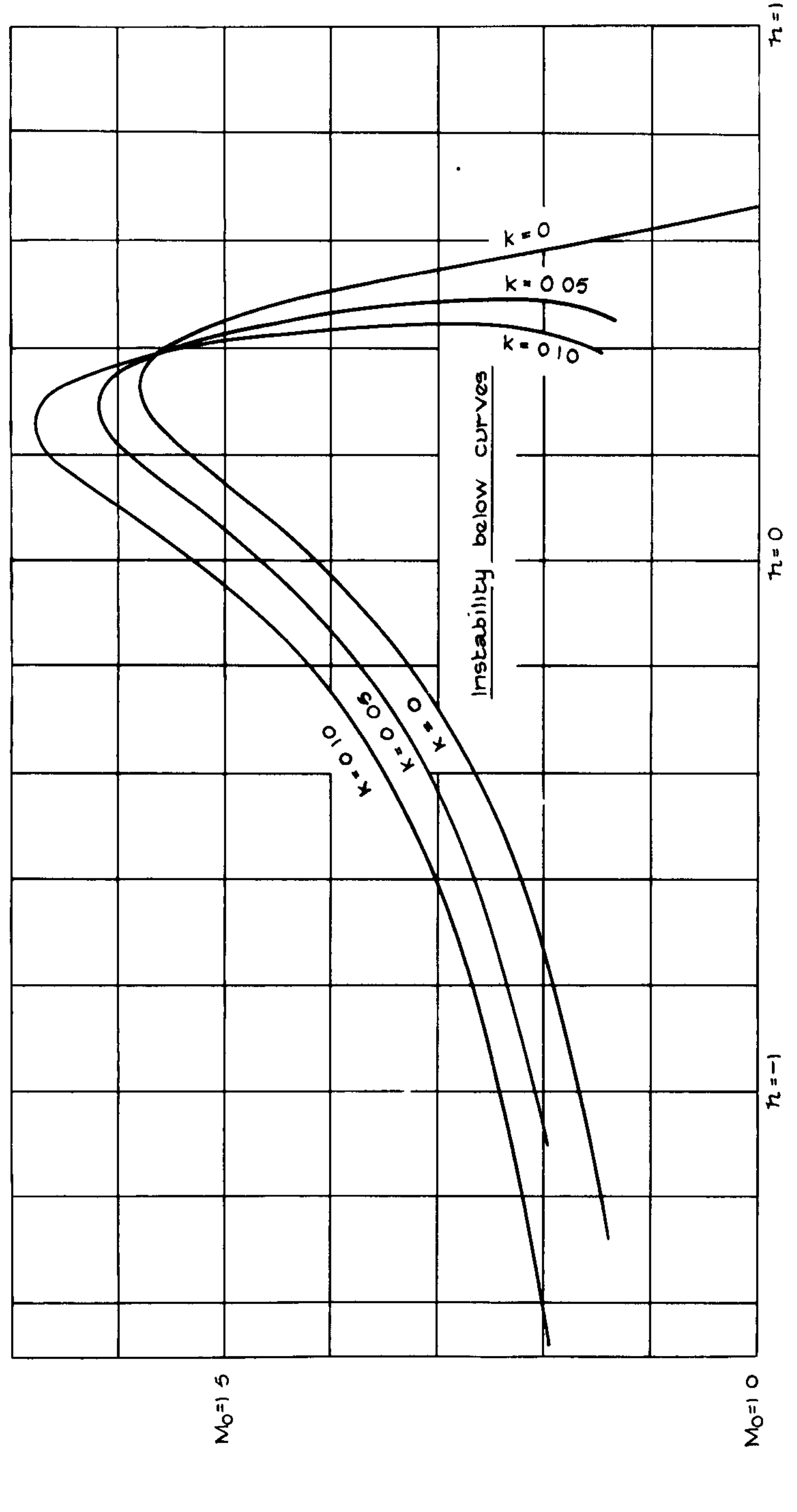


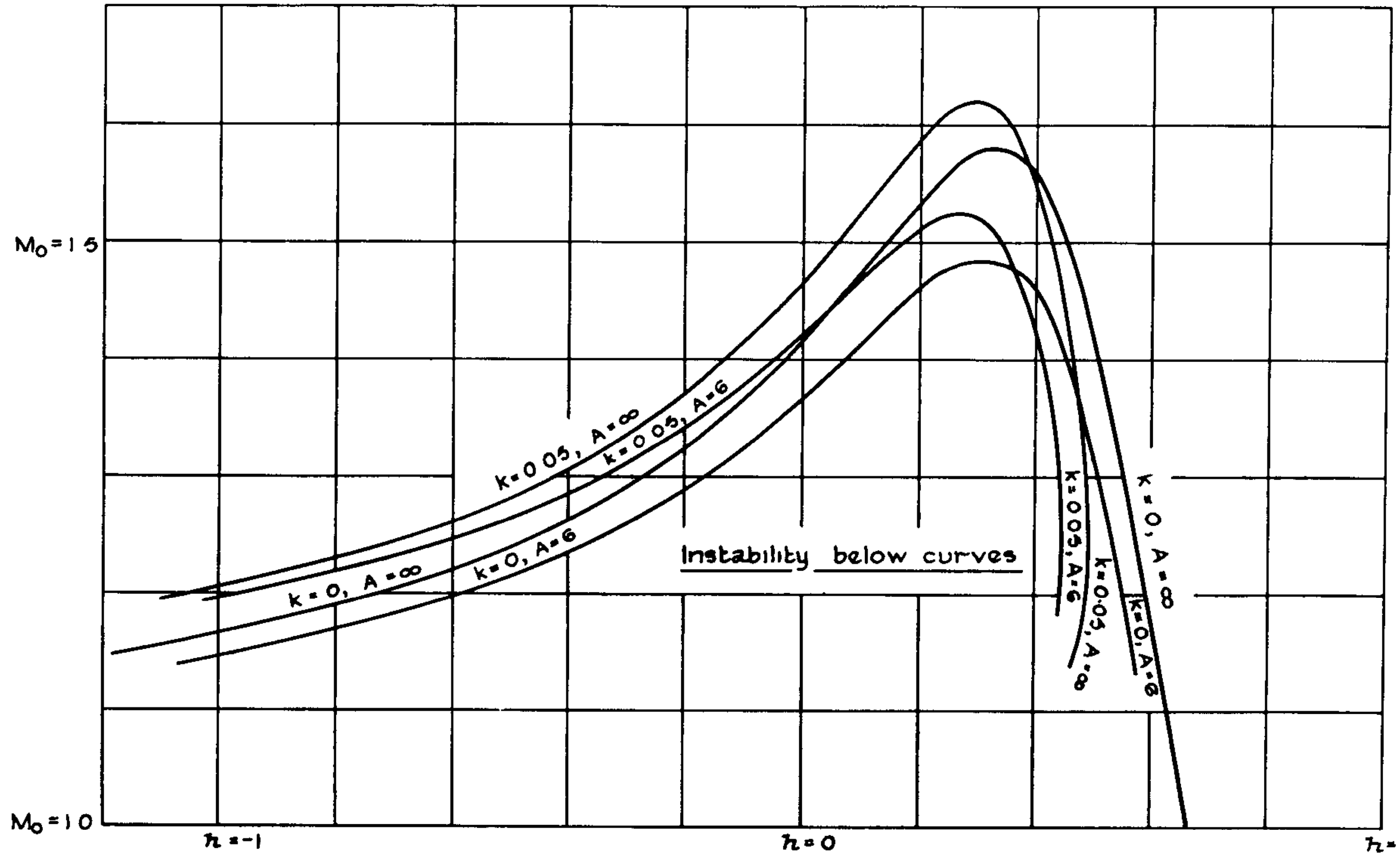
FIG 3

Stability diagrams for rectangular wings of zero thickness of various aspect ratios.

FIG 4



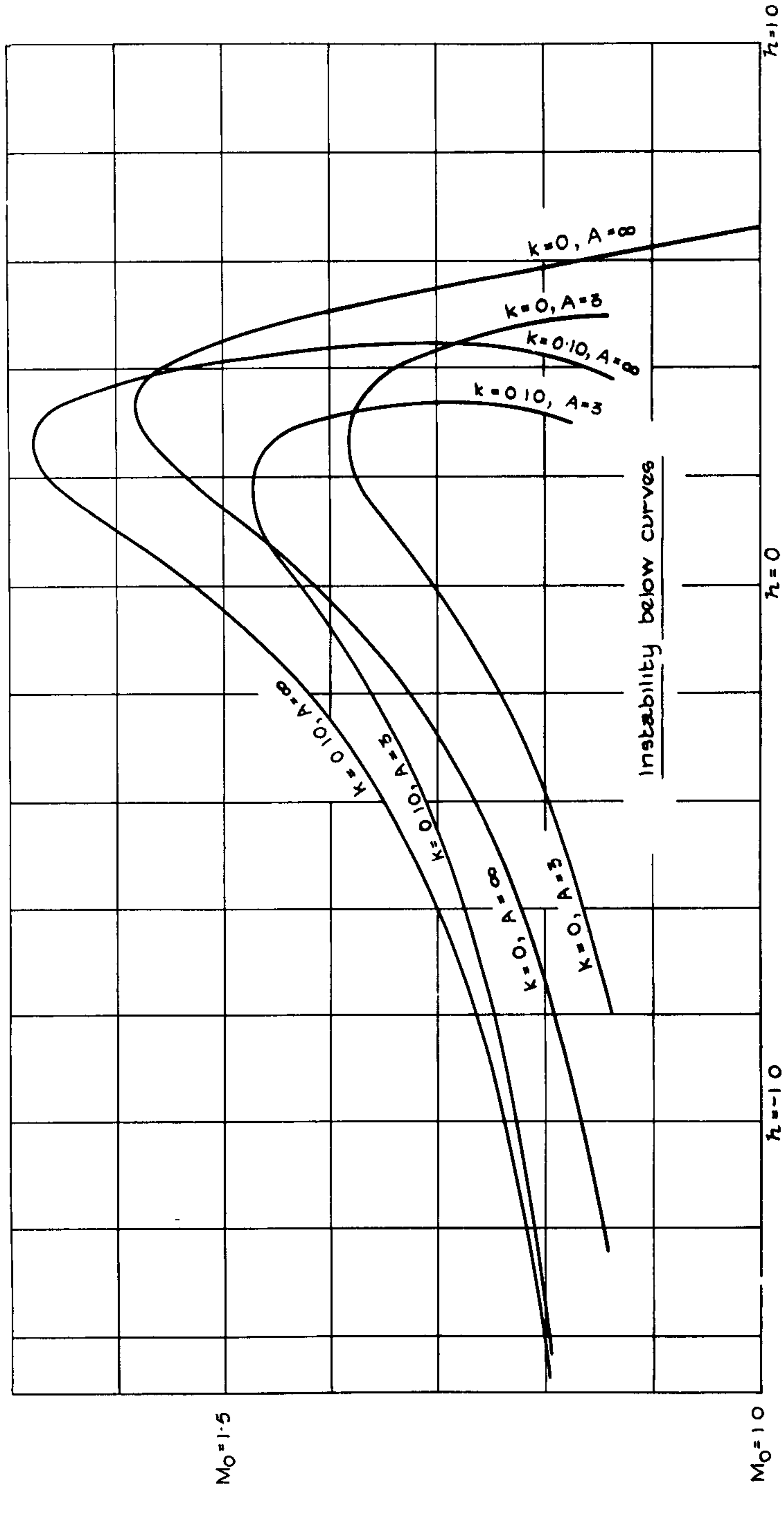
Stability diagrams, according to Van Dyke's theory, for two-dimensional biconvex wings of various thickness



Stability diagrams for rectangular wings of various aspect ratios and thickness parameters

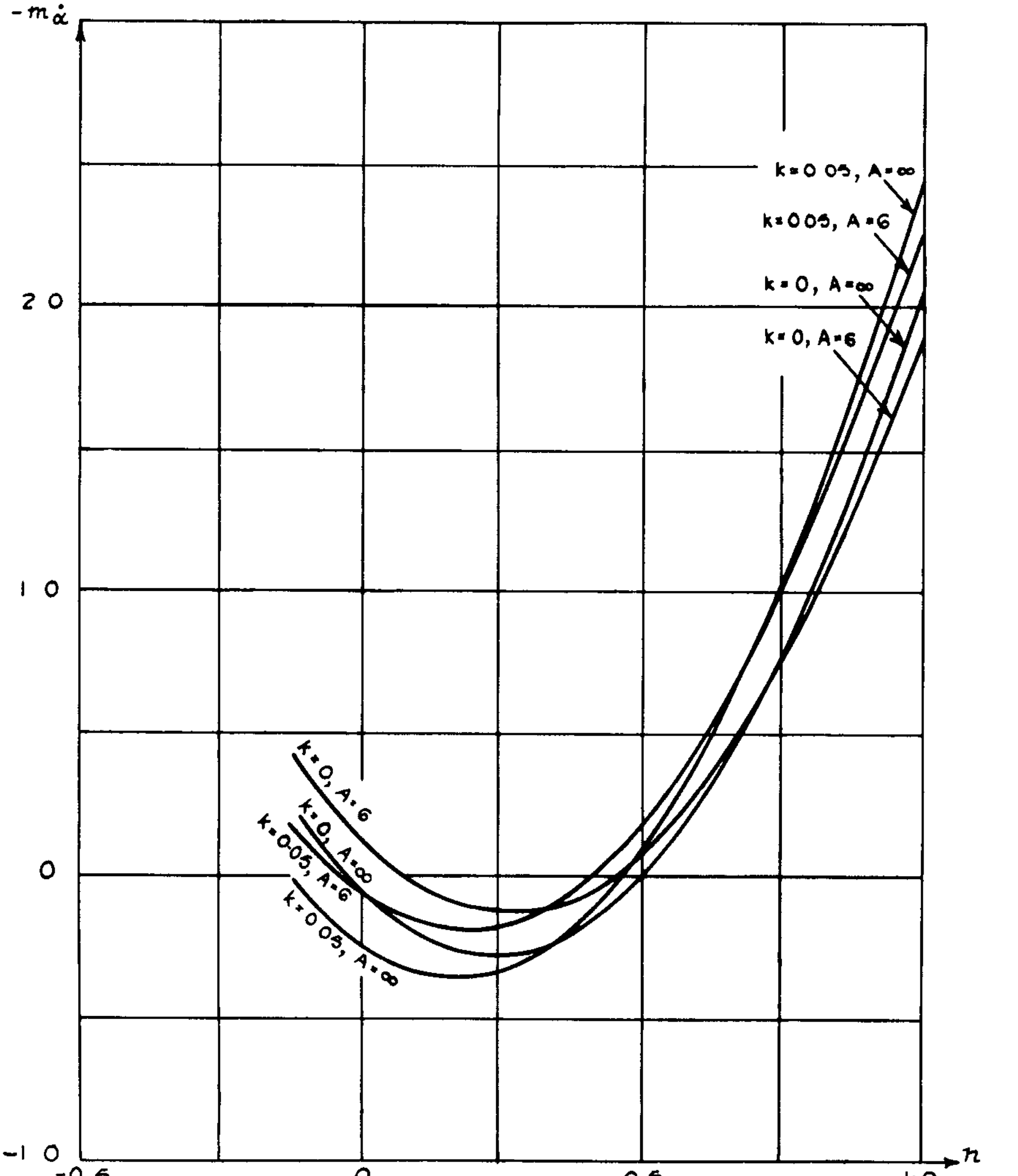
FIG. 5

FIG 6



Stability diagrams for rectangular wings of various aspect ratios and thickness parameters

FIG 7



Variation of damping in pitch with axis position for M = 1 4, for various combinations of aspect ratio, A, and thickness ratio, k

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