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Method for the Determination of the Pressure Distribution over a Finite Thin Wing at a Steady Low Speed

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SUMMARY

For any given pressure distribution across a finite thin wing at low speed the wing surface can be obtained by direct double integration. Therefore the pressure distribution across a given wing surface may be obtained by the super position of a number of solutions in which the wing surface is known for a prescribed pressure distribution.

The method has been applied for the determination of the pressure distribution across a thin uncambered delta wing.

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1. Introduction

In the motion of a thin finite wing at a steady low speed two fundamental problems are presented, namely,

(i) to determine the load distribution of a given wing

(ii) to determine the shape of the central surface of the wing for a prescribed pressure distribution.

The design problem (ii) was considered in the author's previous paper[1]. Here the relationship which was established between the downwash velocity (defining the wing's central surface) and the pressure discontinuities over the wing surface and the trailing vortex sheet involved only a direct double integration. This avoids the difficulty of the formal potential approach which leads to a result containing an awkward limiting process.

By extending the ideas presented above an approach may be made to obtain the solution of problem (i). The method of design is to assume a load distribution p across the wing surface, obtaining the downwash velocity w by integration. This establishes the shape of the wing's central surface. It is suggested that the solution of problem (i) may be obtained by substituting a series of pressures $p_1, p_2 \dots p_n$ and calculating the corresponding values of $w_1, w_2, \dots w_n$. Therefore if the central surface of the wing defines a downwash velocity w such that

$$w = K_1 w_1 + K_2 w_2 + \dots + K_n w_n$$

where/

where $K_1, K_2, \dots K_n$ are constants, the appropriate pressure solution is

$$p = K_1 p_1 + K_2 p_2 + \dots + K_n p_n .$$

This method is similar to the solution presented by Garner[2] who used the formal velocity potential approach. Since the method suggested here avoids the difficulty in the region of the integral singularity in Garner's solution, the numerical work is reduced.

The paper ends with a qualitative investigation of the particular problem solved by Garner, namely, the determination of the load distribution over a thin uncambered 45° swept-back wing at a small angle of incidence. Comparisons are made between the results obtained from the two methods.

2. General Theory

Since we are to consider only the part of the pressure distribution corresponding to the camber, twist and incidence of a thin finite aerofoil, namely the part which is antisymmetrical about the plane of the wing, we are only interested in the shape of the central wing surface. Hence the problem reduces to an investigation of the steady flow past the central wing surface, determining the pressure discontinuities across this surface sheet.

Taking Cartesian co-ordinates of reference, with the origin fixed on the centre line of the wing surface so that the plane of the wing is identical to the plane $z = 0$, and assuming that the steady velocity V of the stream at infinity is in the direction of x increasing, the central wing surface is denoted by

$$z = f(x,y) .$$

The projection of the wing surface on the plane of the wing $(z = 0)$ is denoted by S_W, whilst the projection of the trailing vortex sheet on this plane is denoted by S_T.

All discontinuities in pressure and velocity are assumed to occur across the surface S_W and S_T. The boundary conditions of adjacent flow over the wing surface $z = f(x,y)$ is satisfied on S_W, also the condition of smooth flow over the trailing edge of the aerofoil is satisfied on the trailing edge of S_W.

If the disturbance velocities (u, v, w) due to the presence of the wing in the air stream, are small compared with V, then the linearized equations of motion reduce to

$V \dfrac{\partial u}{\partial x}$/

$$V \frac{\partial u}{\partial x} = -\frac{1}{\rho}\frac{\partial p}{\partial x}$$

$$V \frac{\partial v}{\partial x} = -\frac{1}{\rho}\frac{\partial p}{\partial y} \qquad \ldots(1)$$

$$V \frac{\partial w}{\partial x} = -\frac{1}{\rho}\frac{\partial p}{\partial z}.$$

It is noted that since pressure discontinuities occur across S_W only, then u is discontinuous only, whilst v is discontinuous across S_W and S_T. Outside of $S_W + S_T$ on the plane $z = 0$, $u = v = 0$ whilst w remains finite tending to zero at infinity.

The condition of adjacent flow over the wing surface is

$$\frac{(w_{S_W}}{V} = \frac{\partial f(x,y)}{\partial x} \qquad \ldots(2)$$

assuming that the rates of change of the surface $z = f(x,y)$ in both the x and y directions are small.

It has been shown[1] that the integral formula, relating the downwash velocity w to the velocity discontinuities, is

$$2\pi\, w(X,Y,0) = P \iint_{S_W+S_T} \frac{u(x,y,+0)\;(x-X) + v(x,y,+0)\;(y-Y)}{[(x-X)^2 + (y-Y)^2]^{3/2}} \qquad \ldots(3)$$

where P denotes the usual principal value to be taken.

3. Investigation of the Flow Past a Thin Uncambered 45° Delta Wing

This wing possesses a flat central wing surface at a small angle of incidence α to the main stream, therefore the central wing surface is

$$z = \alpha x.$$

The/

The plane form S_W of the flat surface on the plane $z = 0$ is defined by

(i) the equation of the leading edge $x = -6 + |y|$

(ii) the equation of the trailing edge $x = 1$

(iii) Semi-span, $s = 6$,

and the trailing surface S_T is the semi-infinite strip $(-1 \leqslant x, 6 \leqslant |y|, z = 0)$, as shown in Fig.1.

This wing has been chosen so that any results which are obtained may be compared with Garner's solution of the same problem.

It was explained in the introduction that a series of pressure discontinuities $p_1, p_2, \ldots p_n$ are to be assumed. The first question that arises concerns a suitable function for the first terms in a series expansion of the pressure distribution, corresponding to the leading term of the Birnbaum series in the two-dimensional theory. Obviously the first term, multiplied by an appropriate spanwise function cannot be taken as it stands since this would involve a discontinuity in the pressure derivatives giving rise to an infinite downwash along this line. It was shown by Ursell[3] that this implies a steep ridge on the centre line which violates the assumptions of the linearized theory. Since this is concerned with a flat surface the lines of constant pressure should be continuous, with continuous derivatives, across the centre line, depending on the geometry of the wing surface and not on a local rounding off effect.

The first expression for the pressure distribution is taken to be the first term of the Birnbaum expansion multiplied by a simple function which ensures that the pressure derivatives are continuous. Assuming

$$p_1 = -\rho V^2 \alpha \frac{x + 6 - \frac{1}{2}|y|}{x + 6} \left[\frac{(1 - x)\left(1 - \frac{y^2}{36}\right)}{(x + 6 - |y|)}\right]^{\frac{1}{2}}$$

then

$$\frac{u_1}{\alpha V} = \frac{x + 6 - \frac{1}{2}|y|}{x + 6} \left[\frac{(1 - x)\left(1 - \frac{y^2}{36}\right)}{(x + 6 - |y|)}\right]^{\frac{1}{2}} \qquad \text{on } S_W$$

$$= 0 \qquad \text{on } S_T$$

$\frac{v_1}{\alpha V}$/

$$\frac{v_1}{\alpha V} = \frac{\dfrac{-y}{36}}{\left(1 - \dfrac{y^2}{36}\right)^{\frac{1}{2}}} \left\{- 14 \left(\tan^{-1} \sqrt{\frac{1 - x}{x + 6 - |y|}} - \frac{\pi}{2}\right)\right.$$

$$+ 2\sqrt{7|y|} \left(\tan^{-1} \sqrt{\frac{|y|}{7} \cdot \frac{1 - x}{x + 6 - |y|}} - \frac{\pi}{2}\right)$$

$$\left. + 2 \left[(1 - x)(x + 6 - |y|)\right]\right\}$$

$$+ \frac{|y| \left(1 - \dfrac{y^2}{36}\right)^{\frac{1}{2}}}{4y} \left\{2 \sqrt{\frac{7}{|y|}} \tan^{-1} \sqrt{\frac{|y|}{7} \cdot \frac{1 - x}{x + 6 - |y|}}\right.$$

$$\left. - 2\sqrt{\frac{1 - x}{x + 6 - |y|}}\right\} \quad \text{on } S_W$$

$$= \frac{\dfrac{-y}{36}}{\left(1 - \dfrac{y^2}{36}\right)^{\frac{1}{2}}} \cdot \pi(7 - \sqrt{7|y|}) \cdot$$

The downwash velocity has been computed from (3) and is tabulated below for discrete points on the wing

Table of $\dfrac{w_1}{\alpha V}$

x	y = 0	y = 2	y = 4
-4	-1.26		
-2	-1.50	-1.04	
0	-1.68	-1.11	-0.53

It/

It is seen that this wing is in fact considerably cambered, and therefore, the next terms in the serious expansion for the pressure distribution, corresponding to $y^2 p_1$, xp_1, $xy^2 p_1$ are taken to counteract the twist in the spanwise direction and the camber in the chordwise direction.

Taking

$$p_2 = \frac{y^2}{36} p_1$$

$$p_3 = \frac{x + 6}{7} p_1$$

$$p_4 = \frac{y^2}{36} \cdot \frac{x + 6}{7} \cdot p_1 .$$

The corresponding downwash distributions are indicated as follows:-

Table of $\frac{w_2}{\alpha V}$

x	y = 0	y = 2	y = 4
-4	+0.15		
-2	+0.22	-0.09	
0	+0.31	-0.07	-0.55

Table of $\frac{w_3}{\alpha V}$

x	y = 0	y = 2	y = 4
-4	-0.50		
-2	-0.85	-0.73	
0	-1.05	-0.90	-0.41

Table of $\frac{w_4}{\alpha V}$

x	y = 0	y = 2	y = 4
-4	-0.16		
-2	-0.03	-0.15	
0	+0.05	-0.10	-0.40

Superimposing/

Superimposing these four solutions in such a way that the downwash condition is satisfied at the six discrete points with a minimum error, then the solution is

$$0.2p_1 - 0.5p_2 + 0.7p_3 + 2.3p_4$$

when the corresponding downwash distribution is

x	y = 0	y = 2	y = 4
-4	-0.95		
-2	-0.97	-0.96	
	-1.00	-0.96	-1.00

The value of the lift coefficient of this distribution is

$$\frac{\partial C_L}{\partial \alpha} = 2.70.$$

This should be compared to Garner's solution

$$\frac{\partial C_L}{\partial \alpha} = 3.04.$$

The graph of the circulation distribution (which is seen to be approximately elliptic) is shown in Fig.2, and the pressure distributions at various sections are shown in Fig.3.

In order to obtain a better result than the one deduced above more terms in the expansion for the pressure distribution than four expressions p_1, p_2, p_3, and p_4 must be taken so that the downwash conditions are satisfied more accurately. However, the investigation shows that this method is quite practicable, since the accuracy depends only on a double integration.

It is suggested that, in the application of this method to any other swept-back wing of small aspect ratio, the first term for the pressure distribution should incorporate the characteristics of the solution obtained above. That is, it should satisfy the conditions,

(i) approximately on elliptic circulation distribution

(ii) zero local lift on the trailing edge

(iii) infinite local lift on the leading edge

(iv) continuous pressure derivatives across the centre line.

Only/

Only (i) is not satisfied by p_1 above, whilst p_3 satisfies (i), (iii) and (iv). The choice of a first term satisfying all these four conditions should reduce the numerical work even further.

References

No.	Author(s)	Title, etc.
1	G. J. Hancock	The design of thin finite wings in incompressible flow. A.R.C. 15,185. September, 1952.
2	H. C. Garner	Theoretical calculations of the distribution of aerodynamic loading on a delta wing. R. & M. 2819. March, 1949.
3	F. Ursell	Notes on the linear theory of incompressible flow around symmetrical swept back wings at zero lift. Aero. Quart. Vol.1. (1949).

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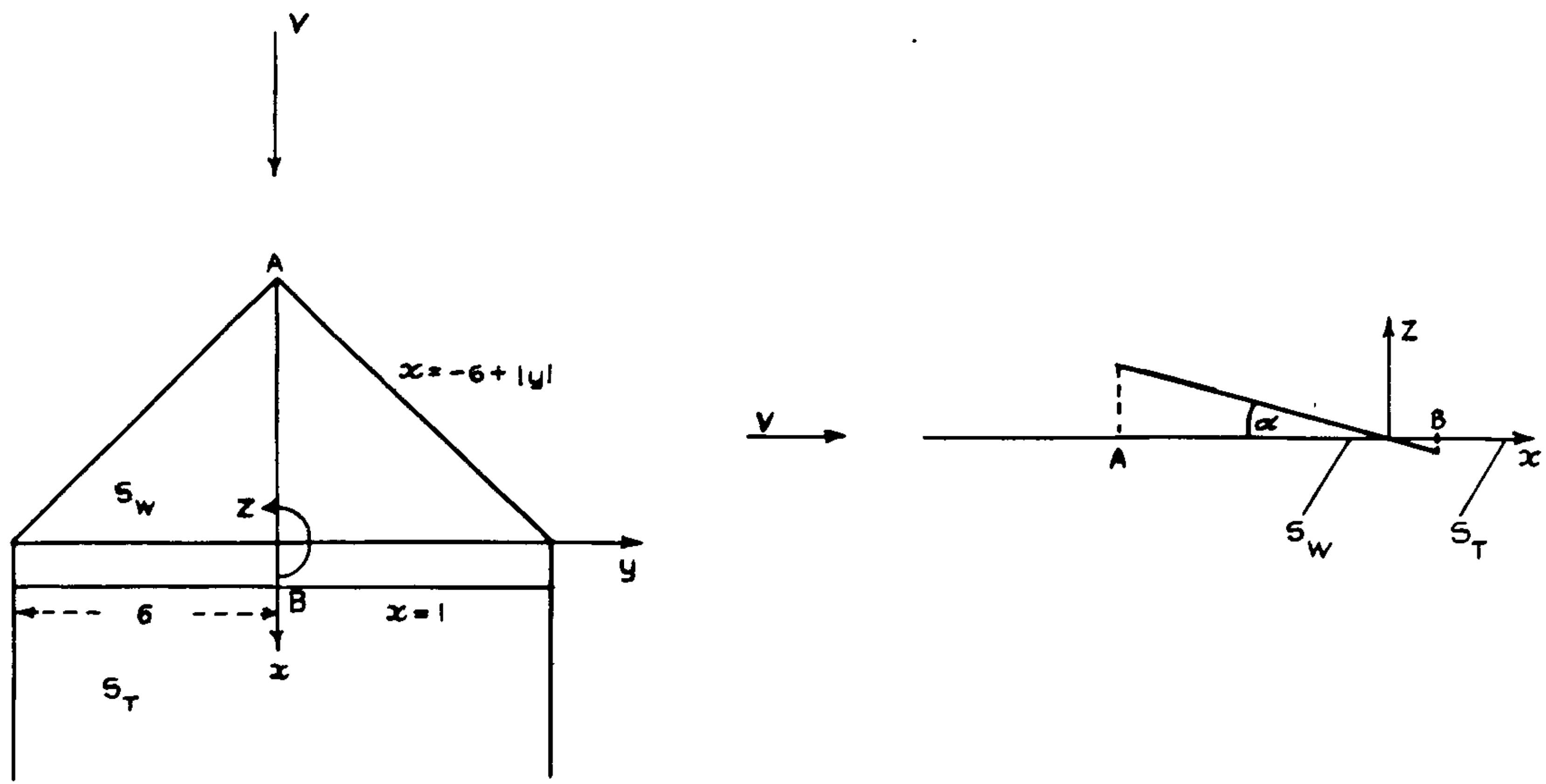
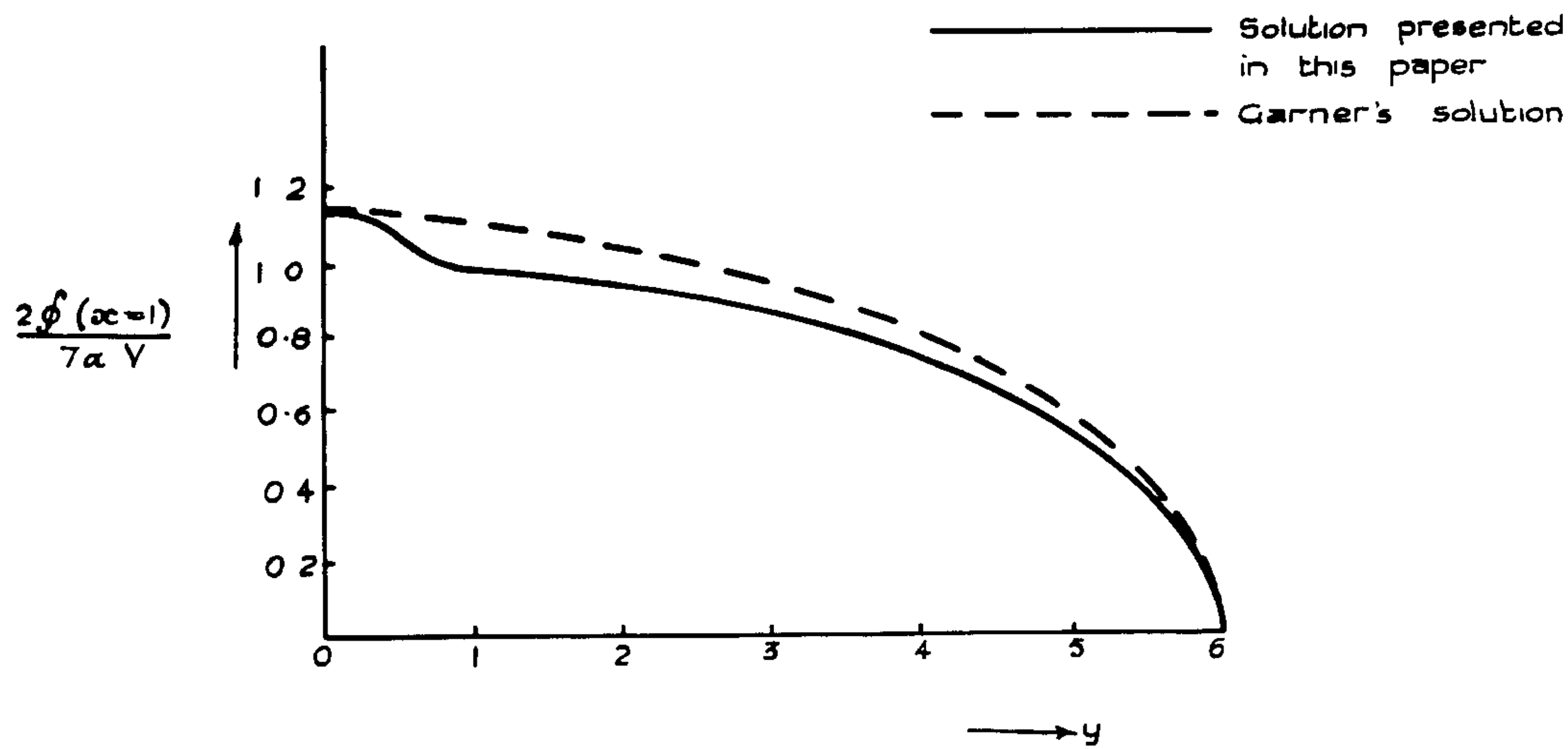


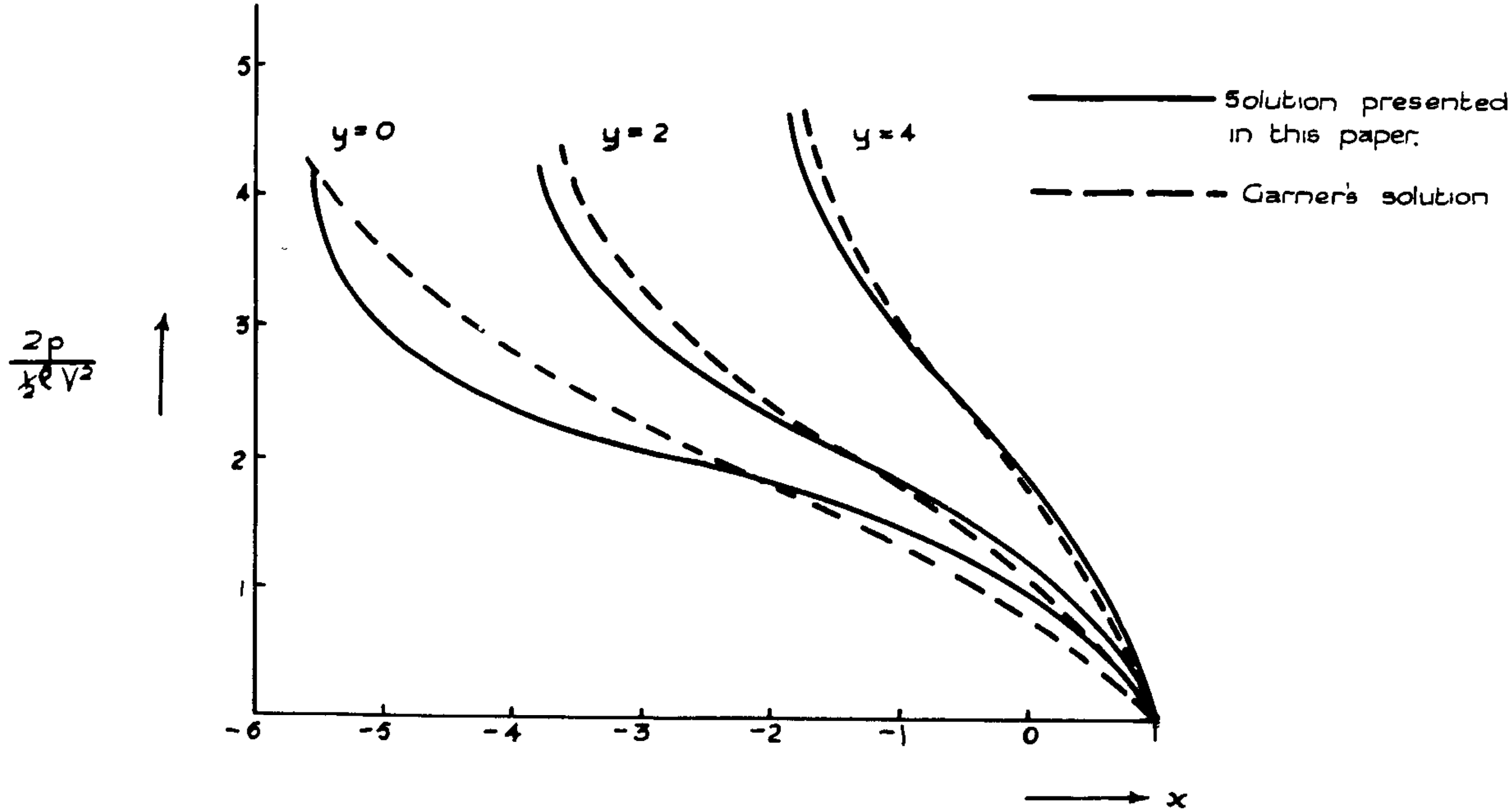
Plan and elevation of the Delta Wing

FIG 1

FIG 2



Circulation Distribution



Pressure distributions at various spanwise sections.

FIG 3

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