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Growth of the Turbulent Wake Close Behind an Aerofoil at Incidence

By

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Cambridge University Aeronautics Laboratory.

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Growth of the Turbulent Wake Close Behind an Aerofoil at Incidence

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D. A. Spence,
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29th May, 1952.

Summary

Knowledge of the variation close behind a trailing edge of the wake displacement thickness

$$\delta^* = \int \left(1 - \frac{u}{U}\right) dy$$

is necessary in calculations of the circulation round an aerofoil. Examination of data now available reveals that in the wake: (i) velocity profiles on either side of the line of minimum velocity may be derived from the corresponding trailing-edge boundary-layer profiles by change of scale of each co-ordinate; (ii) the velocity defect at corresponding points follows a universal recovery law of the form $K(x - x_0)^{-\frac{1}{2}}$ even close to the trailing edge. An immediate consequence of these two empirical properties is a simple relation for the form parameter $H = \delta^*/\theta$ at points in the wake in terms of trailing edge values. In conjunction with the momentum equation this makes δ^* determinate. Agreement with experiment is very satisfactory.

/List of Symbols

List of Symbols

x, y	Streamwise and normal coordinates.
u, v	Streamwise and normal components of velocity in wake.
U	Velocity at edge of wake.
U_o	Velocity at infinity.
u_m, v_m	Velocity components on wake centre line.
δ^*	Displacement thickness $= \int (1 - \frac{u}{U})dy$.
θ	Momentum thickness $= \int \frac{u}{U} (1 - \frac{u}{U})dy$.
H	Form parameter $= \delta^*/\theta$.
$A(x)$	Universal function depending on velocity defect at centre of wake.
$b(x)$	Width of half wake.
I_1, I_2	Integrals defined from trailing edge profile.
T, u, l	Suffixes referring to trailing edge and to upper and lower halves of wake.
η	y - coordinate of centre line.
c	Chord of aerofoil.
σ	Rate of transfer of momentum across wake centre line.

/1. Introduction

1. Introduction

In theoretical investigations into the sectional characteristics of aerofoils it is necessary to know how the displacement flux

$$\psi^* = \int (U - u)\,dy = U\,\delta^*$$

varies in the trailing edge region. Methods are already available for calculating this variation for turbulent boundary layers, but the corresponding problem for turbulent wakes has hitherto received little attention. An investigation is now presented into the distribution of mean velocity in such wakes, and a method developed for predicting the variation of displacement thickness, starting from a trailing edge at which the boundary layer velocity profiles are known. Cases in which separation has occurred ahead of the trailing edge are excluded from the investigation, and the velocity $U(x)$ at the edge of the wake is assumed known.

As for turbulent boundary layers, the calculation proceeds by means of the momentum equation together with an expression for the form parameter H, and the problem is to find this latter. A convenient coordinate system is that composed of the streamlines and equipotentials of ideal flow past the aerofoil. Since there is no skin friction, the momentum equation is simply

$$\frac{d\theta}{dx} + (H + 2)\,\frac{\theta}{U}\,\frac{dU}{dx} = 0 \qquad \ldots \quad \ldots \quad \ldots \qquad (1)$$

(The integrals defining θ and $\delta^* = H\theta$ now extend across the whole width of the wake.)

At a great distance, U and therefore θ become constant; moreover

$$\theta = \int_{-\infty}^{\infty} \frac{u}{U}\left(1 - \frac{u}{U}\right) dy \;\rightarrow \int_{-\infty}^{\infty} \left(1 - \frac{u}{U}\right) dy = \delta^*$$

and $H \rightarrow 1$.

Squire and Young[(1)] gave the form parameter relation

$$\log \frac{U_0}{U} \;\alpha\; (H - 1) \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (2)$$

for use in calculating profile drag, which depends on θ and involves H only indirectly. The equation clearly expresses the correct behaviour of H as $U \rightarrow U_0$ downstream, but unfortunately it is not sufficiently accurate in the neighbourhood of the trailing edge. The relation was derived from the limited data then available, obtained by B. M. Jones[(2)] on a wing at small incidences for which the trailing edge value H_T did not exceed 1.4. Measurements with considerably higher values of H_T made at the N.P.L. by Preston [(3), (4)] and at Langley Field by Mendelsohn[(5)] do not agree at all well with the relation, which was in any case tentative. It has therefore seemed profitable to make a fresh investigation in the light of these measurements, and two empirical properties of the velocity profiles have emerged: (i) geometrical similarity of the half profiles on either side of the wake centre to the corresponding trailing edge profile with its cusp removed, and (ii) a universal law for recovery of velocity at the centre of the wake in terms of distance downstream. Taken together, these lead to a relation

/for

for the variation of H in terms of H_T and distance downstream.

2. Velocity Profiles in the Wake

Initially the wake is asymmetrical because of the different histories of the upper and lower boundary layers; as it spreads, its asymmetry is decreased by transverse mixing in the fully turbulent core. The typical state of affairs is shown diagrammatically in Figure 1.

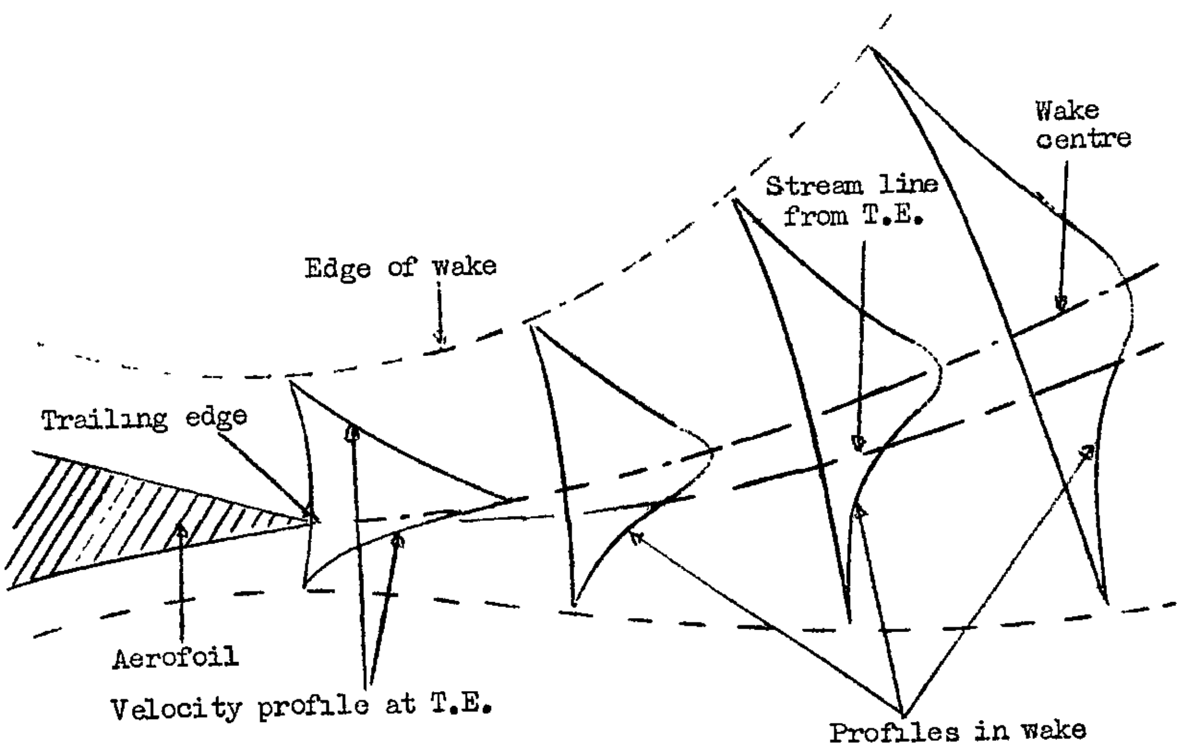

Fig.1

The dotted line passing through the points of minimum velocity u_m on succeeding profiles will be called the "wake centre line". In the region of asymmetry this is not expected to be a streamline, but since $\partial u/\partial y = 0$, shear stress vanishes along it, except at the actual trailing edge, where there is a discontinuity in shear, and a very complicated mixing process takes place. For convenience it will be assumed throughout that, as happens on an aerofoil at positive incidence, the upper boundary layer is thicker at the trailing edge than the lower one, and is nearer to separation in the sense of having a higher H.

At great distances from the aerofoil the velocity distribution and turbulence pattern may be expected to resemble those at comparable distances behind a circular cylinder, the asymmetry due to aerofoil incidence having been removed by turbulent mixing. The latter case has recently been fully investigated by Townsend[(6)], whose construction of the wake appears to justify the conventional assumption that the mean profiles sufficiently many diameters from the cylinder are geometrically similar, with the form

$$U_0 - u = U_0 A(x) f(y/b) \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

/where

where $b = g(x)$ and f is a universal function. This form has always been assumed in mathematical investigations such as those by Schlichting and others described in Ref.7. From purely dimensional considerations it may be shown that

$$A(x) \propto (x - x_0)^{-\frac{1}{2}}$$
$$b(x) \propto (x - x_0)^{\frac{1}{2}} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

where $x - x_0$ is the distance from the virtual origin of the wake. $A(x)$ measures the velocity defect at the centre of a profile, which is in fact found experimentally to fall off as $(x - x_0)^{-\frac{1}{2}}$. (Schlichting used a mixing length relation to find the form of f, thus assuming dynamical similarity of the turbulent motion. This is open to considerable doubt, except at great distances from the cylinder.)

The similarity relation (3) does not hold in the dead water region immediately behind a circular cylinder, but it is not obviously impossible for such a relation to exist for the flow immediately behind a streamline body. Clearly profiles of velocity from one side of the wake to the other cannot be expected to be similar, because of the initial difference in thickness of the two halves (which ultimately disappears). On the other hand half profiles appear to be so, each having broadly speaking a "half error curve" shape. Successive half profiles in the downstream direction differ only in being broader and in having smaller velocity defects $(U - u_m)$ at their centres. A detailed examination of profiles measured by Preston[3], [4], Mendelsohn[5] and Jones[2] shows that for these at least, such similarity exists very closely. The function f is no longer universal, being determined now by the boundary layer profiles at the trailing edge.

The set of profiles in Figure 2 illustrates this. The points are taken from experimental profiles, and the solid lines which fit their upper and lower halves belong to families derived by affine transformation (i.e. by change of scale of each coordinate axis) from the upper and lower halves respectively of the initial profile. This latter has been chosen as the trailing edge profile, faired across a very narrow part of its centre to remove the cusp; the fairing is not sufficient to alter the values of δ^* and θ. The profile at $x/c = .40$ in Figure 2 departs slightly from the interpolated curve, in such a way as to restore symmetry between the two halves, and it must be concluded that the shape imposed on the velocity profile by trailing edge conditions, although preserved for distances of the order of a quarter chord or more, is ultimately modified by turbulent mixing to a universal asymptotic form.

The calculations of the next section concern only the region close to the trailing edge, and the velocity distribution is assumed to be expressible there by

$$U - u = U\,A(x)\,f(y/b) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

where $b(x)$ is a representative width and y is measured from the wake centre line. f and b are different for the upper and lower halves, but the velocity $U(x)$ at the edge of the wake is the same for each half. Without loss of generality f may be normalized so as to make $f(0) = 1$. Then

$$A(x) = (U - u_m)/U.$$

3. Velocity at Centre of Wake

It seemed possible that the curves of $A(x)$ obtained from different aerofoils might satisfy a general law for recovery of velocity at the

/centre

centre of the wake, and accordingly the ratio u_m/U was plotted against x/c, the non-dimensional distance behind the trailing edge, for the data obtained by Preston and by Jones. c is the aerofoil chord, an appropriate length to represent the history of the boundary layers. The result is shown in Fig.3. A single curve can in fact be drawn through the points from the five sets of measurements plotted, with very little scatter. Mendelsohn's points, all of which are in the immediate vicinity of the trailing edge ($x/c < 0.10$) also agree well with the curve given, but they have been omitted to avoid over-crowding the figure. We conclude that at any rate as a good approximation Λ is a universal function of x/c. The interpolated curve in the figure is

$$(U - u_m)/U \;=\; 0.1265\,(x/c + 0.025)^{-\frac{1}{2}} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (6)$$

The form is chosen in order to give the variation, already referred to, proportional to $(x - x_o)^{-\frac{1}{2}}$ at large distances, and the constants 0.1265 and 0.025 give the best fit with experiment. The trailing edge value ($x/c = 0$) is then $u_m/U = 0.2$; this gives a consistent means of fairing the trailing edge velocity profiles so as to treat them as belonging to the wake. It must be remarked that all the data lies in the Reynolds number range $0.5 \times 10^6 - 5 \times 10^6$. The similarity of profiles is probably independent of Reynolds number, but the curve of velocity recovery on the centre line would perhaps show a scale effect outside this range.

4. Variation of H in a half wake

The typical width b may most conveniently be defined as the width (assumed) finite) of a half velocity profile, so that $f(1) = 0$. The integral characteristics of the half wake are then

$$\delta^* = \int_0^b \left(1 - \frac{u}{U}\right) dy \;=\; b\,\Lambda(x) \int_0^1 f(\xi)\, d\xi$$

$$\theta = \int_0^b \frac{u}{U}\left(1 - \frac{u}{U}\right) dy \;=\; b\,\Lambda(x) \int_0^1 f(\xi)\,\{\,1 - \Lambda(x)\, f(\xi)\,\}\, d\xi\,.$$

Now let

$$\int_0^1 f(\xi)\, d\xi = I_1$$

$$\int_0^1 [\,f(\xi)\,]^2\, d\xi = I_2.$$

Then I_1, I_2 are independent of x, being fixed by the initial profile. The integrals become

/Equation

$$\delta^* = b\,A(x)\,.\,I_1$$

$$\theta = b\,A(x)\,[\,I_1 - A(x)\,I_2\,]$$

$$\frac{\theta}{\delta^*} = \frac{1}{H} = 1 - A(x)\,\frac{I_2}{I_1}$$

$$\therefore \left(1 - \frac{1}{H}\right) \propto A(x)$$

$$\propto (x/c + 0.025)^{-\frac{1}{2}}.$$

Therefore if H_T is the trailing edge value for the corresponding boundary layer

$$\frac{1 - \frac{1}{H}}{1 - \frac{1}{H_T}} = \left(\frac{\frac{x}{c} + 0.025}{0.025}\right)^{-\frac{1}{2}} = \left(1 + 40\,\frac{x}{c}\right)^{-\frac{1}{2}} \quad \dots \quad (7)$$

Thus the variation of H a given distance downstream in the half wake depends only on H_T. Curves of this variation, for initial values H_T = 1.4, 1.8, 2.2, 2.6 and 3.0 are shown in Fig.4. These illustrate the fact, also found experimentally, that most of the decrease from H_T to the asymptotic value 1 takes place close behind the trailing edge. At distances at which the velocity profile has begun to depart from the geometrically similar form described by (5), H has decreased almost to unity, and the resultant error is insignificant. The analysis for the asymptotic case as given in Modern Developments assumes H = 1, so cannot be used to predict the way in which this value is approached.

5. Variation of H in a Whole Wake

(Here subscripts u, l refer to upper (i.e., thicker) and lower halves of the wake respectively. Symbols without subscripts refer to the whole wake). From the definitions it follows that:-

$$\theta = \theta_u + \theta_l$$

$$\delta^* = \delta^*_u + \delta^*_l$$

and therefore

$$H = (H_u \theta_u + H_l \theta_l)/\theta \quad \dots \quad \dots \quad \dots \quad (8)$$

H_u and H_l may be calculated from the form parameter equation (7) but it is not possible to calculate θ_u and θ_l separately, since the momentum equation (1) applies only to the whole wake. In the Appendix it is shown that for the upper half wake, using η for the y co-ordinate of the centre line and v_m for the transverse velocity there, the momentum

/equation

equation is

$$\frac{d\theta_u}{dx} + (H_u + 2)\ \frac{\theta_u}{U}\ \frac{dU}{dx} = \frac{U - u_m}{U}\left(\frac{v_m}{U} - \frac{u_m}{U}\ \frac{d\eta}{dx}\right)$$

$$= \sigma / \rho\ U^2 \quad \dots \quad \dots \quad \dots \quad (9)$$

where

$$\sigma = \rho\ (U - u_m)\left(v_m - u_m\ \frac{d\eta}{dx}\right) \quad . \quad \dots \quad \dots \quad (10)$$

Similarly for the lower half it is

$$\frac{d\theta_l}{dx} + (H_l + 2)\ \frac{\theta_l}{U}\ \frac{dU}{dx} = -\frac{\sigma}{\rho\, U^2} \quad . \quad \dots \quad \dots \quad (11)$$

Adding (9) and (11) gives equation (1).

$-\sigma(x)$ then represents the rate of transport of momentum downwards across the wake centre line by turbulent mixing, which takes place in such a way as to restore symmetry. It is evident from Fig.2 that this occurs, since b_l groves more rapidly than b_u until ultimately they become equal. Unfortunately there is no way of finding the quantities $d\eta/dx$ and v_m/U, so (9) and (11) cannot be solved separately. Thus H for the whole wake cannot be found exactly. However limits are known between which $\theta_u/\theta = 1 - \theta_l/\theta$ varies, and thus limits may be obtained for the variation of H. In fact θ_u/θ decreases from its trailing edge value $(\theta_u/\theta)_T$ to $\frac{1}{2}$, so that the true H lies between the curves of

$$H = \left(\frac{\theta_u}{\theta}\right)_T H_u + \left(\frac{\theta_l}{\theta}\right)_T H_l \quad \dots \quad \dots \quad (12)$$

and

$$H = \tfrac{1}{2}(H_u + H_l) \quad \dots \quad \dots \quad (13)$$

where H_u and H_l are calculated from their trailing edge values by means of (7). The accurate solution for H is a curve which agrees with that given by (12) at the trailing edge but ultimately approaches that given by (13). However the curves given by (7) for different values of H_T all lie close together beyond about a quarter of a chord downstream, and the departure of the true H from that given by (12) is very small. Equation (12) expresses H as a linear interpolation between the curves of H_u and H_l, and thus gives to the first order a curve of the family to which H_u and H_l belong; the curve of this family, in fact, which has the initial value

$$H_T = (H_u\ \theta_u + H_l\ \theta_l)_T / \theta_T.$$

This curve could therefore be treated as an approximation to the true solution.

A rather extreme case has been calculated to estimate the error in this approximate procedure, and the result is shown in Figure 5. The trailing edge conditions assumed are $\theta_u = 4\ \theta_l$, $H_u = 2.4$, $H_l = 1.4$. Then $H_T = 2.2$. The difference between the faired curve, which is intended to represent the true solution, and the linear interpolation, is nowhere more than $2\frac{1}{2}$ per cent. The latter curve

/differs

differs by less than 1 per cent from that given by equation (7). Thus the theoretical variation of H in a whole wake is given with reasonable accuracy in terms of its trailing edge value by (7).

For one of the cases measured by Preston[3] namely the wake behind a symmetrical Joukowski aerofoil at 6° incidence, the satisfactory agreement between theory and experiment is illustrated in Figure 6. Such discrepancy as occurs would be compensated by the fairing process just described. Similar agreement has been found in all cases.

6. Discussion

The theory is purely empirical, but seems acceptable in the light of present knowledge of turbulent shear flows. It is most accurate at points moderately close to the aerofoil, and only such points need be considered in estimating the influence of the wake on aerofoil characteristics. The investigation has been made mainly to find a consistent way of carrying out this estimation, without necessarily laying great stress on accuracy. In fact it was found in the investigation of lift described in Reference 8 that the streamline shift produced by the displacement thickness usually accounts for only a small part of the loss of lift below that of ideal fluid theory, and the approximations used in this paper are not sufficient to introduce errors into lift predictions. The streamline shift from the pattern of ideal flow arises from the sharp contraction in displacement thickness behind the trailing edge. This is a consequence of the rapid drop in H, which is predicted by the theory, and is found experimentally.

Use of the predicted variation of H in the momentum equation should increase the accuracy of drag predictions by the Squire-Young method, although this is probably unnecessary. For lift calculations a further conjecture must be made for the position of the wake streamlines. Flow leaves the trailing edge as smoothly as possible, and this may be represented empirically by assuming the trailing streamline to agree initially with that of ideal flow, which bisects the trailing edge angle. In upper and lower halves the momentum equation takes the form (1) if shear stress on the streamline is neglected. Solution in each half separately is then justifiable by an argument similar to that of section 5. In numerical examples it has been found sufficiently accurate to calculate δ^* for each half keeping θ constant, since H accounts for the greater part of the variation.

The rate of transfer of momentum across the wake centre line, represented by the quantity σ defined in equation (10), cannot be determined by analysis which involves only mean velocity quantities. Investigation of the factors influencing this transfer would be an interesting contribution to the theory of turbulent shear flow.

7. Acknowledgment

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/Appendix

APPENDIX

Momentum Equation for a Half Wake

The equation of motion is

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{dU}{dx} + \frac{\partial}{\partial y}\left(\frac{\tau}{\rho}\right) . \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

Let $y = \delta$ be the edge of the wake, $y = \eta$ the centre line, U_m and v_m the velocity components on the centre line. We define the integral characteristics of the half wake by

$$\delta^* = \int_\eta^\delta \left(1 - \frac{u}{U}\right)dy = H\theta \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

$$\theta = \int_\eta^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right)dy . \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

The momentum equation is derived by integrating the equation of motion (1) with respect of y from η to δ , i.e. from the relation

$$\int_\eta^\delta dy\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \int_\eta^\delta dy\left(U\frac{dU}{dx} + \frac{1}{\rho}\frac{\partial \tau}{\partial y}\right) . \ \ldots \quad (4)$$

Consider the various terms:-

(i) τ vanishes at $y = \eta$ since $\partial u/\partial y = 0$ there; and also at the edge of the wake, so that the last term on the right does not contribute to the integral.

/(ii)

(ii)

$$\int_\eta^\delta v \frac{\partial u}{\partial y} dy = [vu]_\eta^\delta - \int_\eta^\delta u \frac{\partial v}{\partial y} dy$$

$$= [vu]_\eta^\delta + \int_\eta^\delta u \frac{\partial u}{\partial x} dy$$

by continuity.

Similarly

$$v_{y=\delta} = v_m + \int_\eta^\delta \frac{\partial v}{\partial y} dy = v_m - \int_\eta^\delta \frac{\partial u}{\partial x} dy .$$

Thus

$$\int_\eta^\delta v \frac{\partial u}{\partial y} dy = Uv_m - U \int_\eta^\delta \frac{\partial u}{\partial x} dy - v_m u_m + \int_\eta^\delta u \frac{\partial u}{\partial x} dy$$

and altogether (4) becomes

$$U^2 \int_\eta^\delta dy \left\{ \frac{1}{U}\frac{dU}{dx} + \left(1 - 2\frac{u}{U}\right)\frac{1}{U}\frac{\partial u}{\partial x} \right\} = v_m(U - u_m) . \qquad \ldots (5)$$

Now consider the terms on the left hand side of the momentum equation:-

(i)
$$\frac{d\theta}{dx} = \frac{d}{dx}\int_\eta^\delta \left(\frac{u}{U} - \frac{u^2}{U^2}\right) dy$$

$$= \frac{1}{U}\frac{dU}{dx}\int_\eta^\delta \left(-\frac{u}{U} + \frac{2u^2}{U^2}\right) dy + \int_\eta^\delta \left(\frac{1}{U} - \frac{2u}{U^2}\right)\frac{\partial u}{\partial x} dy - \left(\frac{u_m}{U} - \frac{u_m^2}{U^2}\right) \cdot \frac{d\eta}{dx} .$$

(ii)
$$(H + 2)\frac{\theta}{U}\frac{dU}{dx} = \frac{1}{U}\frac{dU}{dx}\int_\eta^\delta \left(1 + \frac{u}{U} - \frac{2u^2}{U^2}\right) dy$$

$$\therefore \frac{d\theta}{dx} + (H + 2)\frac{\theta}{U}\frac{dU}{dx} = \int_\eta^\delta \left\{\frac{1}{U}\frac{dU}{dx} + \left(1 - \frac{2u}{U}\right)\frac{1}{U}\frac{\partial u}{\partial x}\right\} dy - \frac{u_m}{U}\left(1 - \frac{u_m}{U}\right)\frac{d\eta}{dx} . \qquad \ldots (6)$$

/The

The integral in (6) is the same as that in (5), and eliminating it from the two equations gives

$$\frac{d\theta}{dx} + (H + 2)\frac{\theta}{U}\frac{dU}{dx} = \left(\frac{v_n}{U} - \frac{u_n}{U}\frac{d\eta}{dx}\right)\left(1 - \frac{u_n}{U}\right) \qquad \ldots (7)$$

or

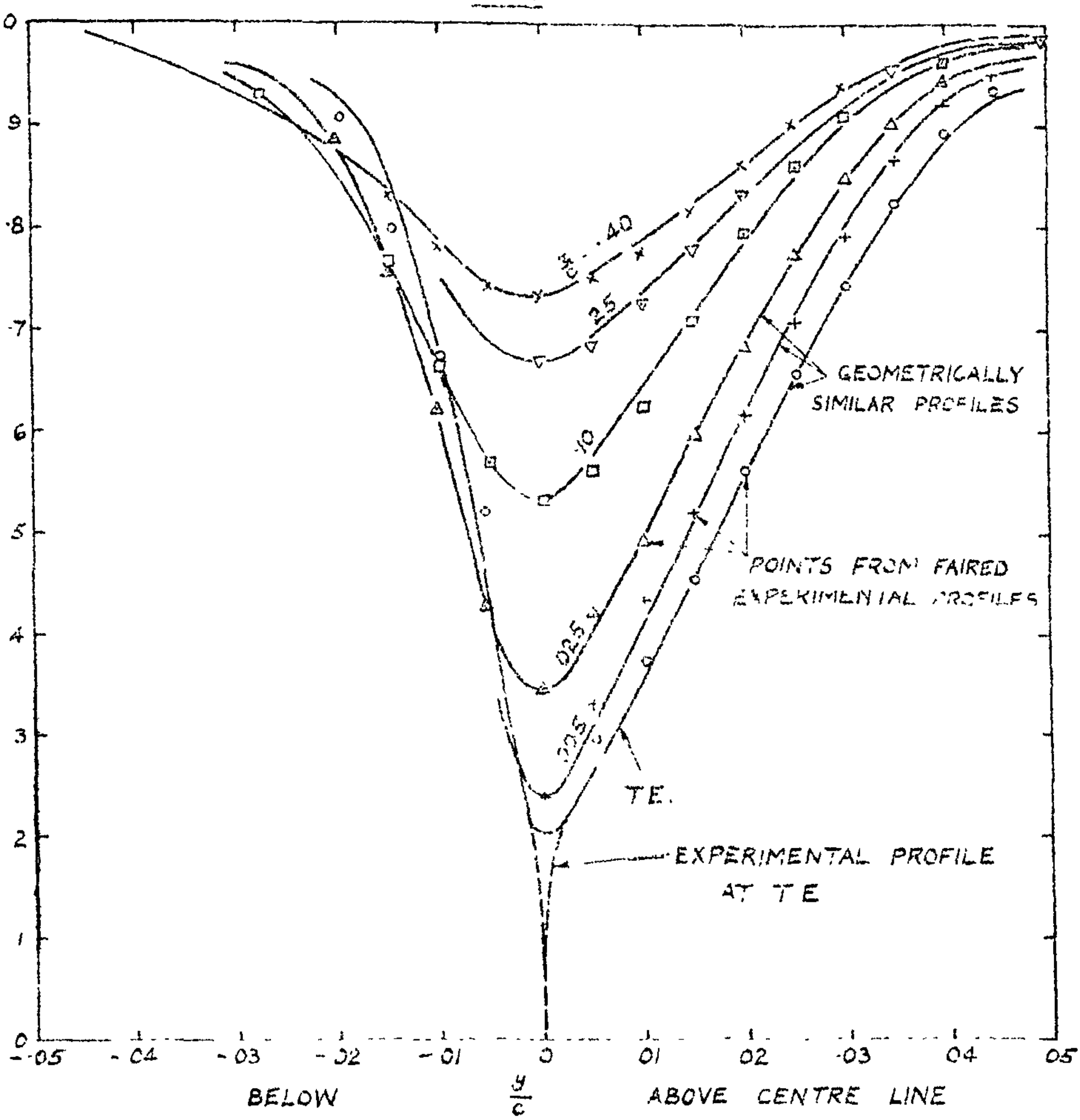
$$\frac{d\theta}{dx} + (H + 2)\frac{\theta}{U}\frac{dU}{dx} = \frac{\sigma}{\rho U^2} \qquad \ldots (8)$$

where

$$\sigma = \rho(U - u_n)\left(v_n - u_n\frac{d\eta}{dx}\right)$$

is the rate at which momentum crosses the wake centre line.

FIG 2



VELOCITY PROFILES IN A TURBULENT WAKE

VELOCITY ON WAKE CENTRE-LINE.

KEY

△ JOUKOWSKI 0°, ▽ JOUKOWSKI 6° (R M 1998)

⊙ C U (WIND TUNNEL) (R M 1688)

+ PIERCY 12/40 0°, × PIERCY 12/40 6° (R M 2013)

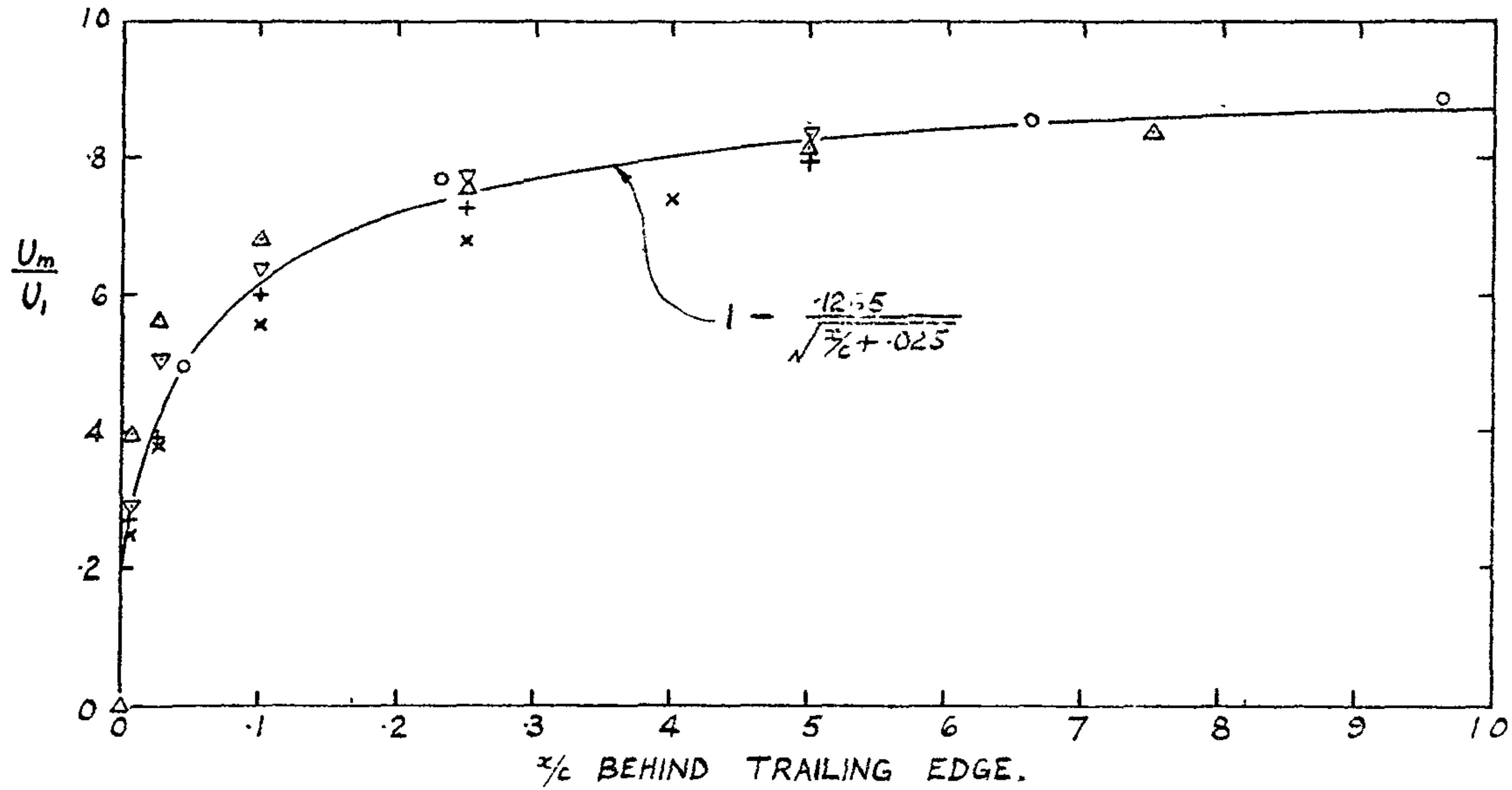
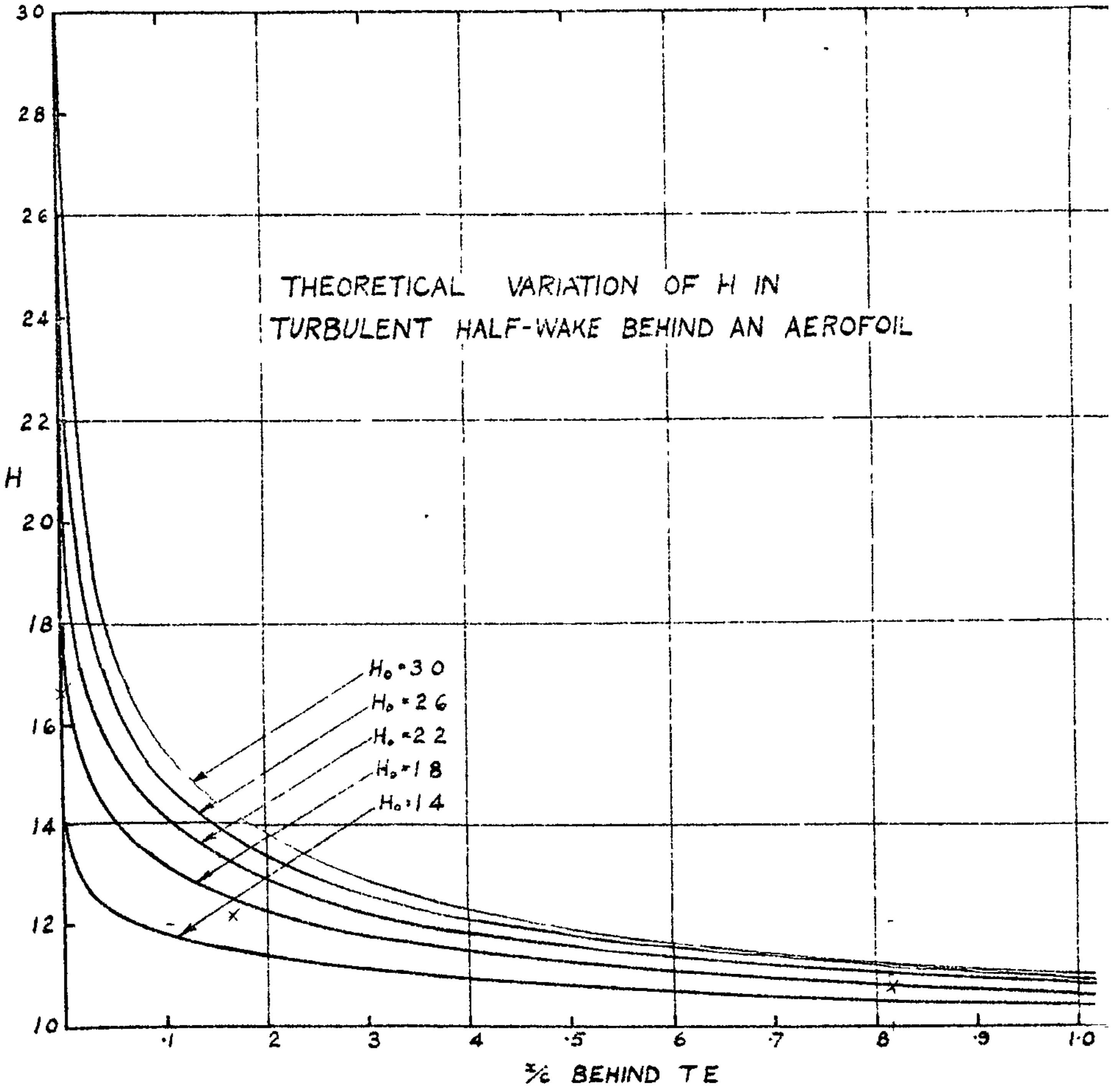


FIG 3

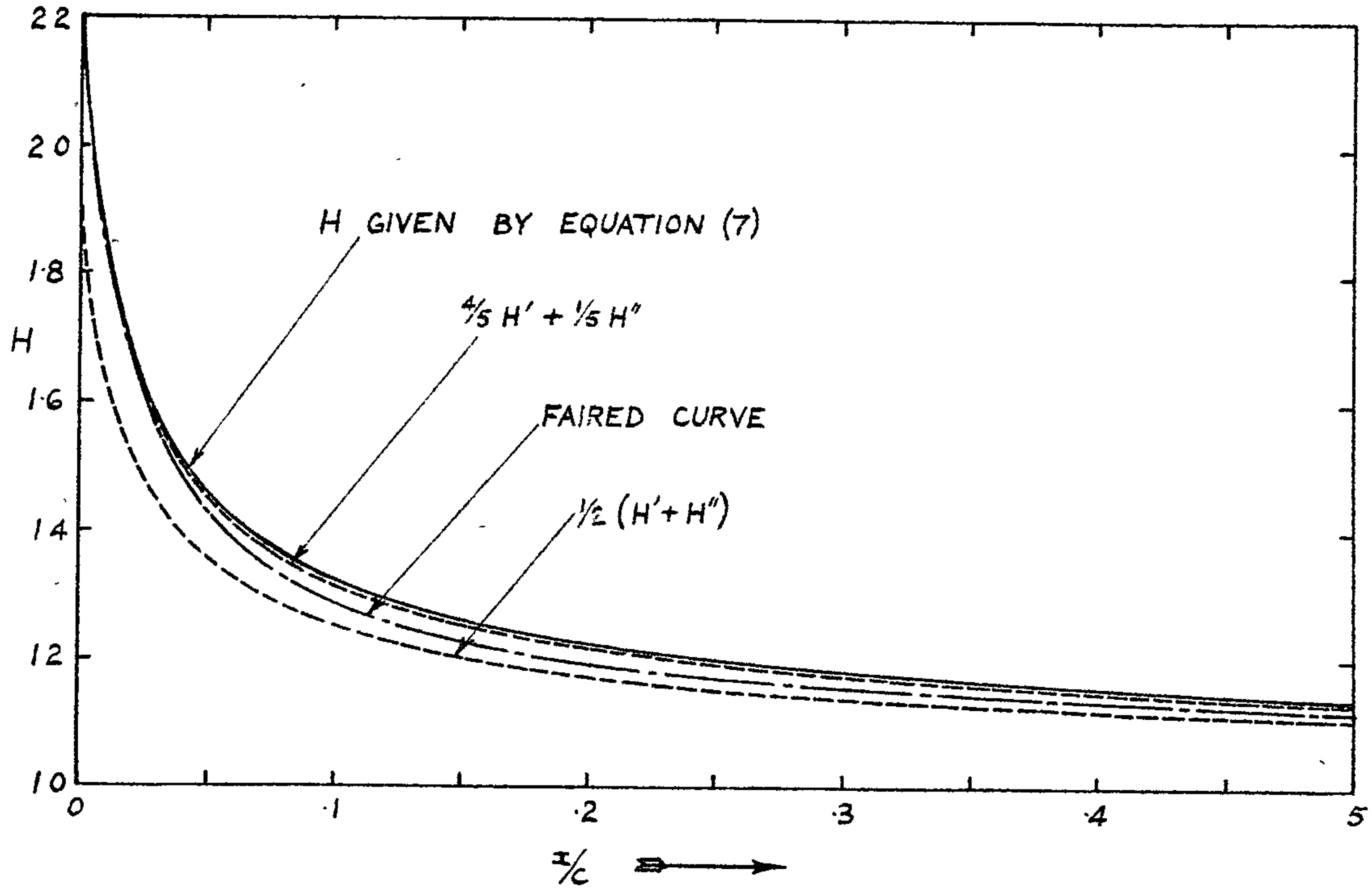
FIG 4

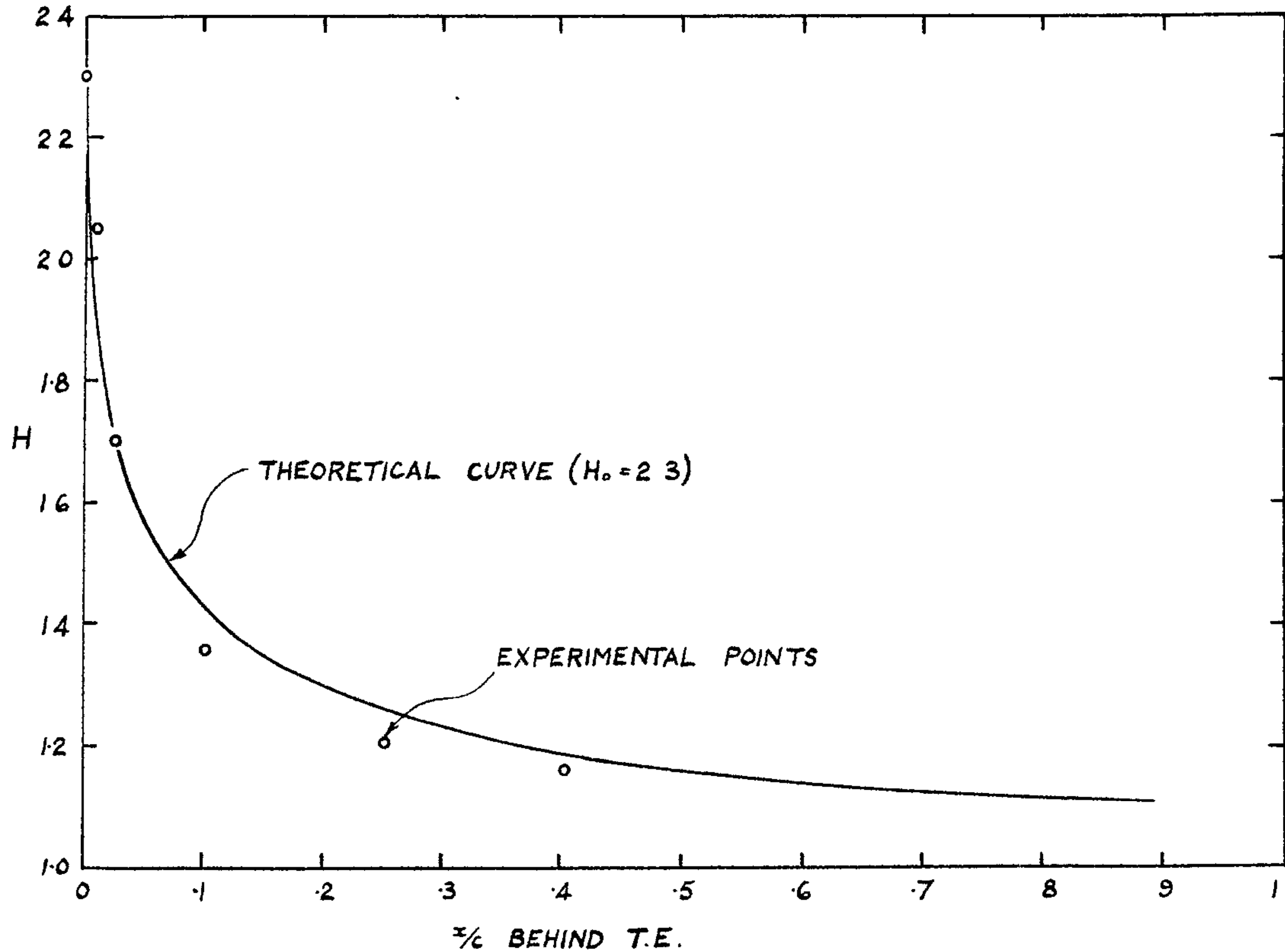


THEORETICAL VARIATION OF H IN TURBULENT HALF-WAKE BEHIND AN AEROFOIL
H
30
28
26
24
22
20
18
16
14
12
10
H₀ = 3·0
H₀ = 2·6
H₀ = 2·2
H₀ = 1·8
H₀ = 1·4
·1
·2
·3
·4
·5
·6
·7
·8
·9
1·0
x/c BEHIND TE

FIG. 5

VARIATION OF H IN WHOLE WAKE - EXAMPLE





VARIATION OF H IN WAKE BEHIND JOUKOWSKI AEROFOIL AT 6° INCIDENCE.

FIG. 6

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