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By

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Comparison of Helicopter Rotor Model Tests of Aerodynamic Damping with Theoretical Estimates

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SUMMARY

The present report deals with the aerodynamic damping of a rotor oscillating in pitch (or roll) and is mainly concerned with the comparison between theory and experiment. Both the free and forced oscillations of a rotor system pivoted below the rotor centre are investigated.

The results can be summarized as follows:-

(a) The behaviour of a rotor oscillating in pitch or roll depends on a parameter, which is the ratio p of two non-dimensional quantities,

p = (frequency ratio of the oscillation)/(specific damping of the rotor blade).

(b) It is shown that the ordinary quasi-static theory holds only for a certain range of this parameter p. Generally, the oscillations of the full-scale aircraft lie inside and those of model tests outside this range. This means that the quasi-static theory is valid for the full-scale helicopter but in most cases not for model tests.

(c) The frequency response theory outlined in this report explains the results obtained from model tests.

(d) The interpretation of model tests for application to full scale work should be done by using the frequency response method given in this report.

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1 Introduction

If a rotor with hinged blades is subjected to angular oscillations in pitch or roll, the blades perform a periodic flapping motion which can be interpreted as an oscillation of the tip-path plane relative to the rotor shaft. In the first approximation it can be assumed that the thrust vector of the rotor is, at any time, normal to the tip-path plane, which means that the thrust vector oscillates in relation to the aircraft. It is obvious that this oscillation of the thrust vector is one of the important aerodynamic characteristics of the rotor.

The present report deals with the aerodynamic damping of an oscillating rotor in the hovering condition and is mainly concerned with a comparison between theory and experiment. Previous investigations, based on the ordinary quasi-static rotor theory, which seems to give the correct value for the full-scale aircraft, have shown that in some model tests the theory did not give good agreement. Until now a satisfactory explanation for this discrepancy could not be offered, though there were possibilities suggested such as the "down-wash lag" theory by Hohenemser, see Ref. 3. The present investigations indicate that the "quasi-static" theory holds only for a certain range of conditions and that outside this range the complete equation of motion for the flapping of the blades must be used.

2 The aerodynamic damping of a rotor oscillating in pitch

2.1 Determination of the frequency equation

The following dynamic problem is investigated: a rotor, with radius R and a flapping hinge offset eR, oscillates in pitch about a pivot located hR below the rotor centre. The system is constrained by spring forces and - in addition to the aerodynamic damping of the rotor - damped by a viscous damper. If α denotes the angular displacement about the pivot (positive nose-up), the equation of motion for the oscillation in pitch can be written as

$$I\ddot{\alpha} + D_0\,\dot{\alpha} + C\,\alpha - M_{a_1} \times a_1 = 0 \qquad (1)$$

The terms of equation (1) represent the moment of the inertia forces, the damping of the system with the rotor oscillating but not rotating, the restraining moment of the spring forces and finally the moment due to the longitudinal tilt a_1 of the tip-path plane. The latter gives the coupling term introducing the flapping motion of the blades and can be expressed as:-

$$M_{a_1} = ThR + \tfrac{1}{2}\, beRF_c \qquad (2)$$

where T = rotor thrust, assumed to be always normal to the tip-path plane

b = number of blades

F_c = centrifugal force of one blade

Equation (2) states that M_{a_1} consists of two components, one due to the tilt of the thrust vector, and another due to inertia forces. The latter is proportional to the flapping hinge offset and the centrifugal forces of all blades.

The second degree of freedom which has to be taken into account is the flapping motion of the blades. The equation of motion for the longitudinal tilt a_1 of the tip-path plane can be written as, see Ref. 1.

$$\dot{a}_1 + \dot{\alpha} = K\Omega(a_{1_\mu}\mu - a_1) \qquad (3)$$

In this equation

K = specific damping of the rotor blade, $K = \dfrac{\gamma B^4}{16}$

Ω = angular speed of rotor,

a_{1_μ} = rate of change of a_1 with tip speed ratio μ

Though equation (3) is exactly true for centrally arranged flapping hinges only, it can also be used for offset hinges (as a first approximation). It can easily be seen that for a state of steady rotation with the angular velocity $\dot{\alpha}$, equation (3) becomes the well known "quasi-static" equation:

$$a_1 = a_{1_\mu}\mu - \frac{16}{\gamma B^4}\frac{\dot{\alpha}}{\Omega} \qquad (4)$$

where in our case

$$\mu = - h\frac{\dot{\alpha}}{\Omega} \qquad (5)$$

Substituting from equation (5), equation (3) can be written as

$$\dot{a}_1 + K\Omega a_1 + \dot{\alpha}(1 + Kha_{1_\mu}) = 0 \qquad (6)$$

It will be seen later that in most cases $1 >> Kha_{1_\mu}$. This means that the effect of the linear velocity of the rotor centre can be neglected. Equations (1) (6) lead to the following frequency equation in λ

$$\lambda^3 + A_2\lambda^2 + A_1\lambda + A_o = 0 \qquad (7)$$

where

$$\left.\begin{aligned} A_2 &= K\Omega + D_o/I \\ A_1 &= \{ K\Omega D_o + C + M_{a_1}(1 + Kha_{1_\mu})\} /I \\ A_o &= CK\Omega/I \end{aligned}\right\} \qquad (8)$$

If the flapping motion of the blades is considered as a sequence of steady conditions, inserting equations (4) (5) in equation (1) gives

$$I K \Omega \lambda^2 + \{ D_o K \Omega + M_{a_1} (1 + K h a_{1_\mu}) \} \lambda + C K \Omega = 0 \tag{9}$$

This means that the damping coefficient k in the amplitude equation

$$\alpha = \alpha_o \, e^{-kt} \tag{10}$$

amounts to

$$k = \frac{D_o K \Omega + M_{a_1} (1 + K h a_{1_\mu})}{2 I K \Omega} \tag{11}$$

Since M_{a_1} is proportional Ω^2, it follows from equation (11) that for the quasi-static theory the damping coefficient k increases linearly with the angular speed of the rotor.

2.2 Comparison with model tests (free oscillations)

The only comprehensive measurements on oscillating rotors published to date are those recorded in Ref. 2. The tests were carried out on a model with the following principal data:-

Rotor radius	$R = 6$ ft
Number of blades	$b = 3$
Flapping hinge offset	$eR = 0.1876$ ft
Height of rotor centre above pitching axis	$hR = 1.475$ ft
Inertia number of rotor blade	$\gamma = 3.52$
Thrust at 200 r.p.m. (8^o pitch setting)	$T_{200} = 18.2$ lb
Centrifugal force at 200 r.p.m.	$F_{c200} = 289$ lb

From the data listed above and with $B = 0.97$ it follows

$$\left.\begin{aligned} K &= \frac{\gamma B^4}{16} = 0.195 \\ (M_{a_1})_{200} &= (T_{200} hR + \tfrac{1}{2} beR \, F_{c200}) = 108 \text{ lb ft} \\ M_{a_1} &= \left(\frac{n}{200}\right)^2 \times 108 \text{ lb ft} \end{aligned}\right\} \tag{12}$$

Further, with K = 0.195, h = 0.246, and the assumed figure $a_{1_\mu} = 0.4$ it follows that $Kha_{1_\mu} = 0.02$ which means that the linear velocity of the rotor centre can be neglected.

To cover a wide range of the frequency ratio $\bar{\nu}$, the tests were conducted with two rigs having different natural periods of oscillation:

Rig "A"

Period of oscillation	$T_o = 0.97$ s
Circular frequency	$\nu = 6.47\ s^{-1}$
Total moment of inertia	$I = 105$ lb ft s^2
Damping constant	$D_o = 3.57$ lb ft s
Spring constant:	
Component due to spring force	+ 4380 lb ft/rad
Weight moment without blades	+ 55 lb ft/rad
Weight moment of blades	- 47 lb ft/rad
	C = 4388 lb ft/rad

Rig "B"

Period of oscillation	$T_o = 4.0$ s
Circular frequency	$\nu = 1.57\ s^{-1}$
Total moment of inertia	$I = 266$ lb ft s^2
Spring constant:	
Component due to spring force	+ 648 lb ft/rad
Weight moment without blades	+ 55 lb ft/rad
Weight moment of blades	- 47 lb ft/rad
	C = 656 lb ft/rad

For comparison the damping coefficient k of the amplitude equation (10) has been calculated with

(1) The complete equation of motion for the flapping, and

(2) The ordinary "quasi-static" theory.

The theoretical results, together with the experimental ones for Rig. A and Rig B are given in Figs. 1 and 2 respectively. The curves show the damping coefficient k and the parameter $p = \bar{\nu}/K$ against the rotor revs. The importance of the parameter p is discussed in more detail in Para. 3.

Moreover, for "Rig A" the effect of the damping with the rotor oscillating but not rotating has been investigated. According to Table II of Ref. 2, the measured damping coefficient for n = 0 was k = 0.017 which means that the amplitude of an oscillation is halved in 40 secs. As no viscous damper had been installed, it must be assumed that this damping is mainly due to friction. For simplicity, however, in the theoretical investigation the friction has been replaced by a viscous damper with $D_o = 2Ik = 3.57$ lb ft s, which results in the same damping coefficient k.

The effect of D_o has been evaluated for each case (quasi-static and expanded theory) and the two curves have been plotted in Fig. 1.

In the curves "a" the term D_o has been taken into account, and in the curves "b" this effect has been neglected. Fig. 1 shows that the effect of D_o is independent of rotor speed, and results in an increase in k by approximately

$$\Delta k = D_o/2I \tag{13}$$

However, since the value of D_o was introduced to evaluate what was probably a friction influence, the effect of D_o may not exist to any extent at the higher rotational speeds and the curves of Fig. 1 should be interpreted accordingly.

In Fig. 2 where, due to the higher moment of inertia of "Rig B" the effect is much smaller, the damping with the rotor static (i.e. oscillating but not rotating) has been neglected.

Fig. 1 shows that the tests conducted with the Rig A lie in the range $p > 0.5$ and that for these tests the quasi-static theory breaks down. The expanded theory, however, is in a fairly good agreement with the experiment.

In the tests with Rig B, see Fig. 2, $0.13 < p < 0.39$. The curves of Fig. 2 show that in this range the results obtained with the quasi-static and the expanded theory lie very close together. Apart from the test with n = 600 r.p.m. the agreement between theory and experiment is very good. For n = 600 r.p.m. the measured damping coefficient is about 20% greater than the theoretical value. It may be that here another effect comes into the picture which has not yet been mentioned viz the changes in the induced velocity caused by changes in the distribution of the thrust around the rotor disc. This effect will be dealt with in a later report.

Another model test on oscillating rotors is reported in Ref. 3. In this case

$$I = 2.26 \text{ mkgs}^2$$

$$C = 67 \text{ mkg/rad}$$

$$D_o = 0.25 \text{ mkgs}$$

$$M_{a1} = 35 \text{ mkg/rad}$$

$$h = 0.34$$

$$\gamma = 8.8$$

$$\Omega = 40.8 \text{ s}^{-1}$$

With B = 0.97 and an assumed figure of $a_{1_\mu} = 0.48$ it follows

$$K = 0.51$$

$$Kha_{1_\mu} = 0.083$$

Inserting the figures listed above in equation (8) leads to the following frequency equation

$$\lambda^3 + 20.800\lambda^2 + 48.662\lambda + 612.42 = 0$$

which has the roots

$$\lambda_1 = - 19.90$$

$$\lambda_{2,3} = - 0.45 \pm 5.53\, i$$

The two complex roots correspond to a damped oscillation with $k = 0.45\ s^{-1}$ and $T_o = 1.13$ s. The experimental results were $k = 0.50\ s^{-1}$ and $T_o = 1.05$ s; the agreement between theory and experiment is again satisfactory.

3 Investigations on the motion of the tip path plane for a rotor subjected to forced oscillations

Another item dealt with in Ref. 2 is the oscillation of the tip path plane due to a forced oscillation of the shaft with constant amplitude α_o. In a forced oscillation the behaviour of a rotor is characterised by

(a) The amplitude ratio r = (amplitude of the oscillation of the tip-path plane)/(amplitude α_o of the whole system), and

(b) The phase angle ε between the two oscillations mentioned above.

The characteristic quantities r, ε can best be investigated by vector methods. If a_1 denotes the amplitude of the motion of the tip path plane relative to the shaft, its absolute amplitude can be expressed as the vector sum $\bar{\alpha}_o + \bar{a}_1$. The amplitude ratio r is obtained by dividing $(\bar{\alpha}_o + \bar{a}_1)$ by α_o, i.e.

$$r = \bar{1} + \overline{a_1/\alpha_o} \qquad (14)$$

In the vector diagram of Fig. 3, OC = 1 and OD = a_1/α_o which means that r is given by the length of the vector OE and that the phase angle ε = $\angle$COE. As shown in Ref. 1, the vector loci for a_1/α_o is a semicircle with 0.5 radius and centre at the point B (-0.5 + 0i). It follows that AO = OC, AD = OE, and $\angle$OAD = $\angle$COE. This means that the two characteristic quantities r, ε

can be obtained direct from the vector loci by connecting A with D where the point D is fixed by the parameter

$$p = \bar{\nu}/K = \nu/K\Omega \tag{15}$$

of the enforced oscillation. The length AD in Fig. 3 equals to the amplitude ratio r and $\angle OAD$ corresponds to the phase angle ε. As indicated in Fig. 3, all quantities of the vector diagram can be expressed as simple functions of the parameter p. With regard to r and ε the following equations hold

$$r = (1 + p^2)^{-\frac{1}{2}} \tag{16}$$

$$\tan \varepsilon = p \tag{17}$$

Another interesting feature is that - due to the geometric configuration of the vector loci - the time lag of the flapping motion $\angle DOG$ happens to be equal to the phase angle ε.

In the model test described in Ref. 2 the following results were obtained from Fig. 6.

$$\left.\begin{aligned} r &= 0.84 \\ \varepsilon_{average} &= 42^\circ \end{aligned}\right\} \tag{18}$$

With regard to the phase angle ε of this experiment, it must be noted, that the oscillations were manually excited and therefore not purely sinusoidal. The figure given above is the average lag of 5 oscillations, where the individual phase angles scatter between 26 and 58 deg. In this test

Period of oscillation $T_o = 0.9$ s

Angular speed of rotor $\Omega = 62.8$ rad/s

With K = 0.195 - see equation (12) of the present report - it follows from equation (15)

$$p = \frac{2\pi}{0.9 \times 62.8 \times 0.195} = 0.57$$

and from equations (16) and (17) the theoretical amplitude ratio and phase angle are:-

$$\left.\begin{aligned} r &= \frac{1}{\sqrt{1 + 0.57^2}} = 0.87 \\ \varepsilon &= \text{arc tan } 0.57 = 30 \text{ deg.} \end{aligned}\right\} \tag{19}$$

Comparison of the theoretical results, equation (19), with the experimental results of equation (18) shows that the agreement is quite satisfactory, especially if we bear in mind that the oscillations were probably not exactly sinusoidal. The theoretical figures can, of course, also be obtained by the graphical method described above and evaluated in the vector loci of Fig. 4.

Fig. 4 also gives some useful information about the validity range of the "quasi-static" theory. If the flapping motion of the blades is considered as a sequence of steady conditions, the time lag of the flapping motion becomes zero and its amplitude can be expressed as

$$a_1 = - \frac{1}{K} \frac{\dot{\alpha}}{\Omega} = - \alpha_o \cdot \frac{\bar{\nu}}{K} i \qquad (20)$$

which means that the vector loci coincide with the negative part of the imaginary axis. By comparison of the true vector loci with those of the quasi-static theory it follows that the latter holds good only for approximately $\bar{\nu}/K < 0.3$.

For an average present day full-scale helicopter

$$T_o = 15 \text{ s}$$

$$\Omega = 25 \text{ rad/s}$$

$$\gamma = 12$$

$$B = 0.97$$

i.e.

$$\bar{\nu} = \frac{2\pi}{T_o \Omega} = 0.017$$

$$K = \frac{\gamma B^4}{16} = 0.66$$

$$p = \bar{\nu}/K = 0.026$$

The figure $p = 0.026$ lies clearly in the validity range of the quasi-static theory. This means that for the full-scale helicopter - in opposition to most model tests - the simplified quasi-static theory can practically always be used.

4 Conclusions

It is shown that the dynamic characteristics of an oscillating rotor depend only on one parameter, namely the quantity $p = \bar{\nu}/K$ where $\bar{\nu}$ = *frequency ratio of the oscillation and* K = *specific damping of the* rotor blade.

The result can be summarized as follows. For $p < 0.3$ (appr.) the complete equation of motion for the flapping of the blades can be replaced by the ordinary "quasi-static" equation. This simplification however, no longer holds if values of $p>0.3$ occur. The apparent discrepancy between theory and experiment observed in previous investigations is mainly due to this fact. Existing model tests on oscillating rotors, especially those of RAE T.N. No Aero 2049, 1950, compare fairly well with theory if the complete equation of motion for the flapping of the blades is considered.

Unfortunately, the oscillations of most model tests lie in the range $p>0.3$ and those of the full-scale helicopter in the range $p<0.3$. This means that the results of model tests cannot be applied directly to the full-scale aircraft but must be converted by theory.

<u>List of Symbols</u>

R = rotor radius, ft.

hR = height of rotor centre above pivot, ft.

eR = flapping hinge offset, ft.

b = number of blades

γ = inertia number of blade

B = tip loss factor

K = specific damping of blade, $K = \frac{\gamma B^4}{16}$

n = rotor speed, revs. per minute

Ω = angular rotor speed, rad/sec

T = rotor thrust, lb.

F_c = centrifugal force of one blade, lb.

α = angular displacement about pivot, positive nose up, rad. $\alpha = \alpha_o \sin \nu t$

α_o = amplitude of pitching oscillation, rad.

ν = circular frequency of pitching oscillation, s^{-1}

$\bar{\nu}$ = frequency ratio, $\bar{\nu} = \nu/\Omega$

p = non-dimensional parameter, $p = \bar{\nu}/K$

r = amplitude ratio in a forced oscillation with constant amplitude α_o

r = (amplitude of the oscillation of the tip-path plane)/(amplitude α_o of the pitching oscillation).

ε = phase angle, rad

I = total moment of inertia of the oscillating system about the pivot, ft.lb.s^2

List of Symbols (Contd)

C = spring constant, lb.ft/rad

D_o = damping constant, lb.ft/rad/sec

a_1 = longitudinal tilt of tip-path plane, rad.

μ = tip speed ratio, $\mu = - h\dot{\alpha}/\Omega$

a_{1_μ} = rate of change of a_1 with μ, rad.

M = pitching moment about pivot, positive nose up, lb.ft.

M_{a_1} = rate of change of M with a_1, lb.ft/rad
$M_{a_1} = (ThR + \frac{1}{2} beRF_c)$

A_1, A_2, A_3 = coefficients of frequency equation, see equation (8)

t = time, s

T_o = period of oscillation, s. $T_o = 2\pi/\nu$

k = damping coefficient in the amplitude equation, s^{-1}
$\alpha = \alpha_o e^{-kt}$

The subscript 200 refers to a rotor speed of 200 r.p.m.

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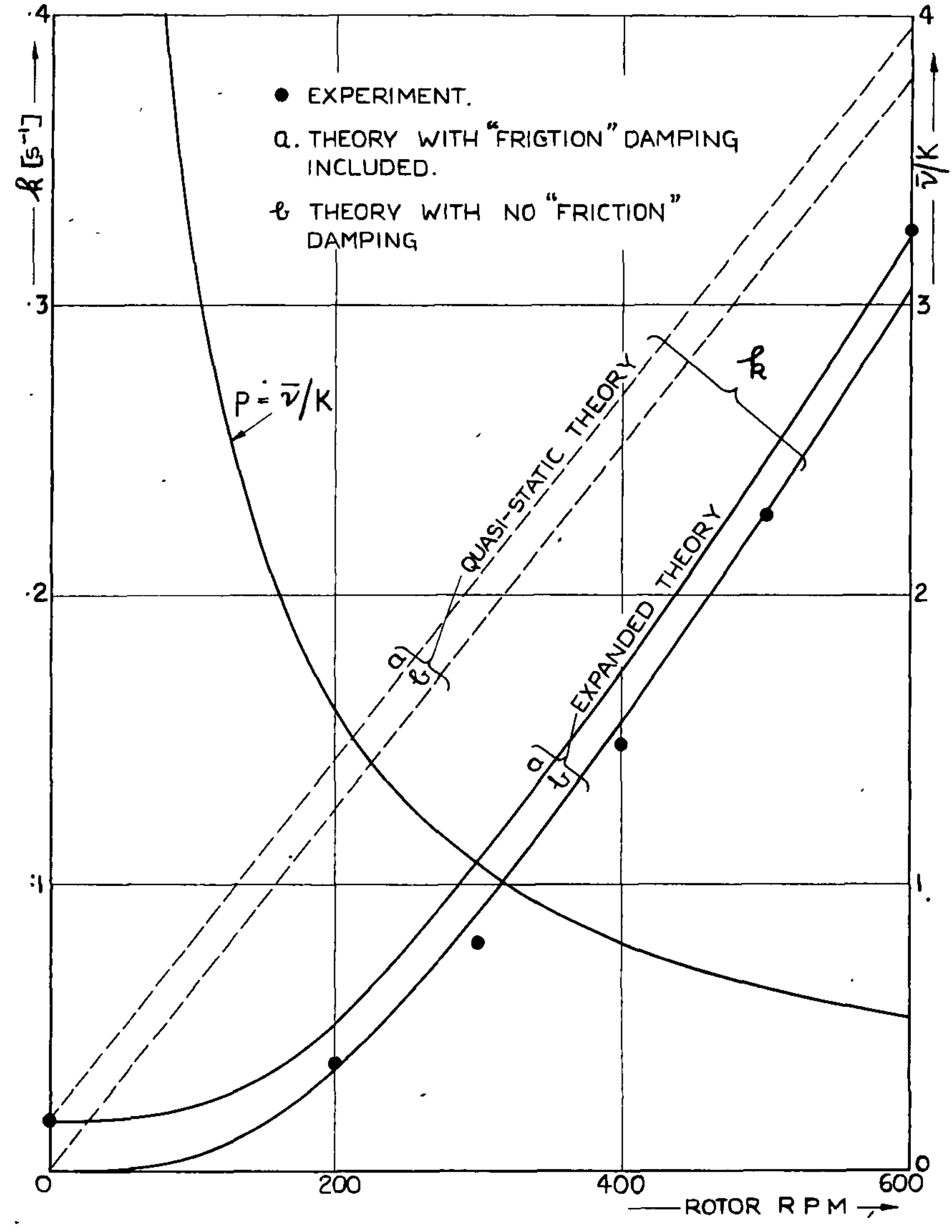


FIG. 1. VARIATION OF DAMPING WITH ROTOR SPEED (RIG A, REF. 2.)

FIG. 2.

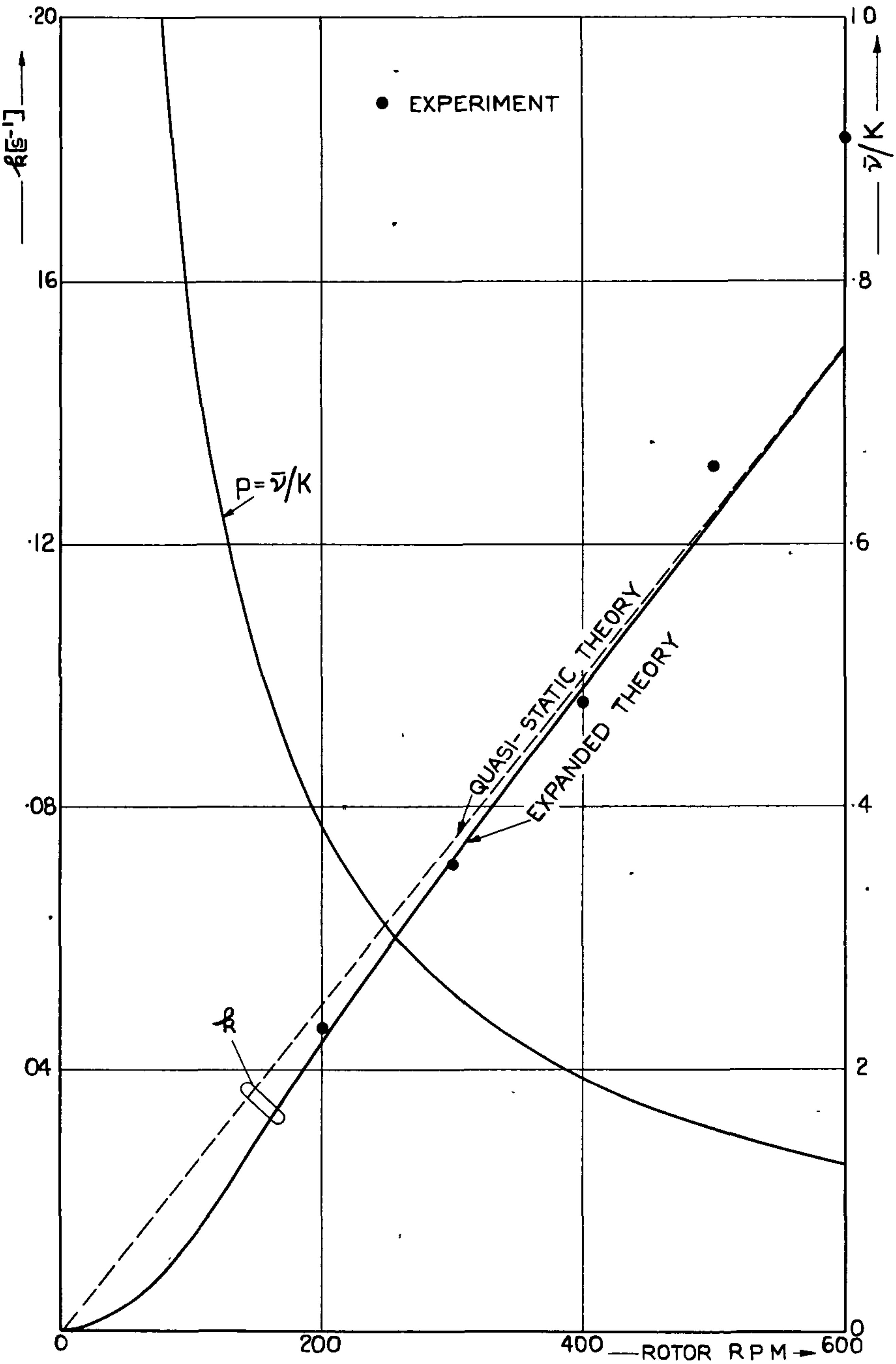
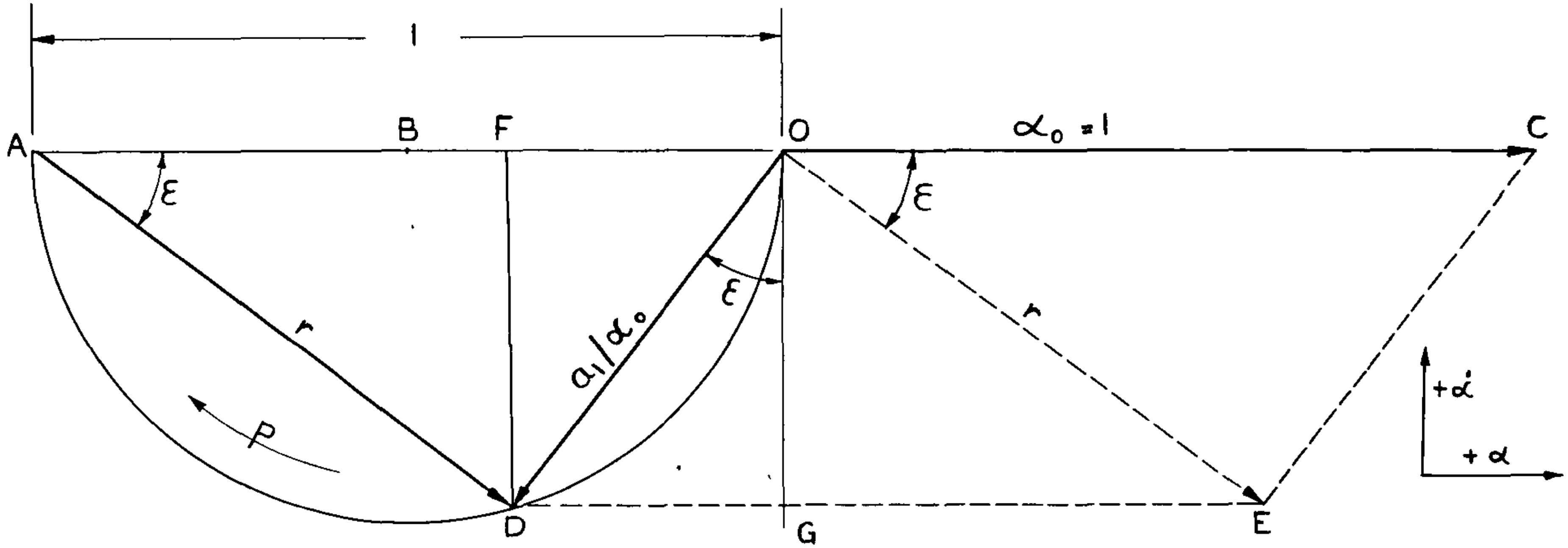


FIG. 2. VARIATION OF DAMPING WITH ROTOR SPEED (RIG B, REF. 2.)



$$AF = \frac{K^2}{K^2+\bar{\nu}^2} = \frac{1}{1+p^2}$$

$$DO = \frac{\bar{\nu}}{\sqrt{K^2+\bar{\nu}^2}} = \frac{p}{\sqrt{1+p^2}}$$

$$FO = \frac{\bar{\nu}^2}{K^2+\bar{\nu}^2} = \frac{p^2}{1+p^2}$$

$$AD = \frac{K}{\sqrt{K^2+\bar{\nu}^2}} = \frac{1}{\sqrt{1+p^2}}$$

$$FD = \frac{\bar{\nu}K}{K^2+\bar{\nu}^2} = \frac{p}{1+p^2}$$

WHERE $p = \bar{\nu}/K$

FIG. 3. VECTOR DIAGRAM FOR ROTOR OSCILLATION.

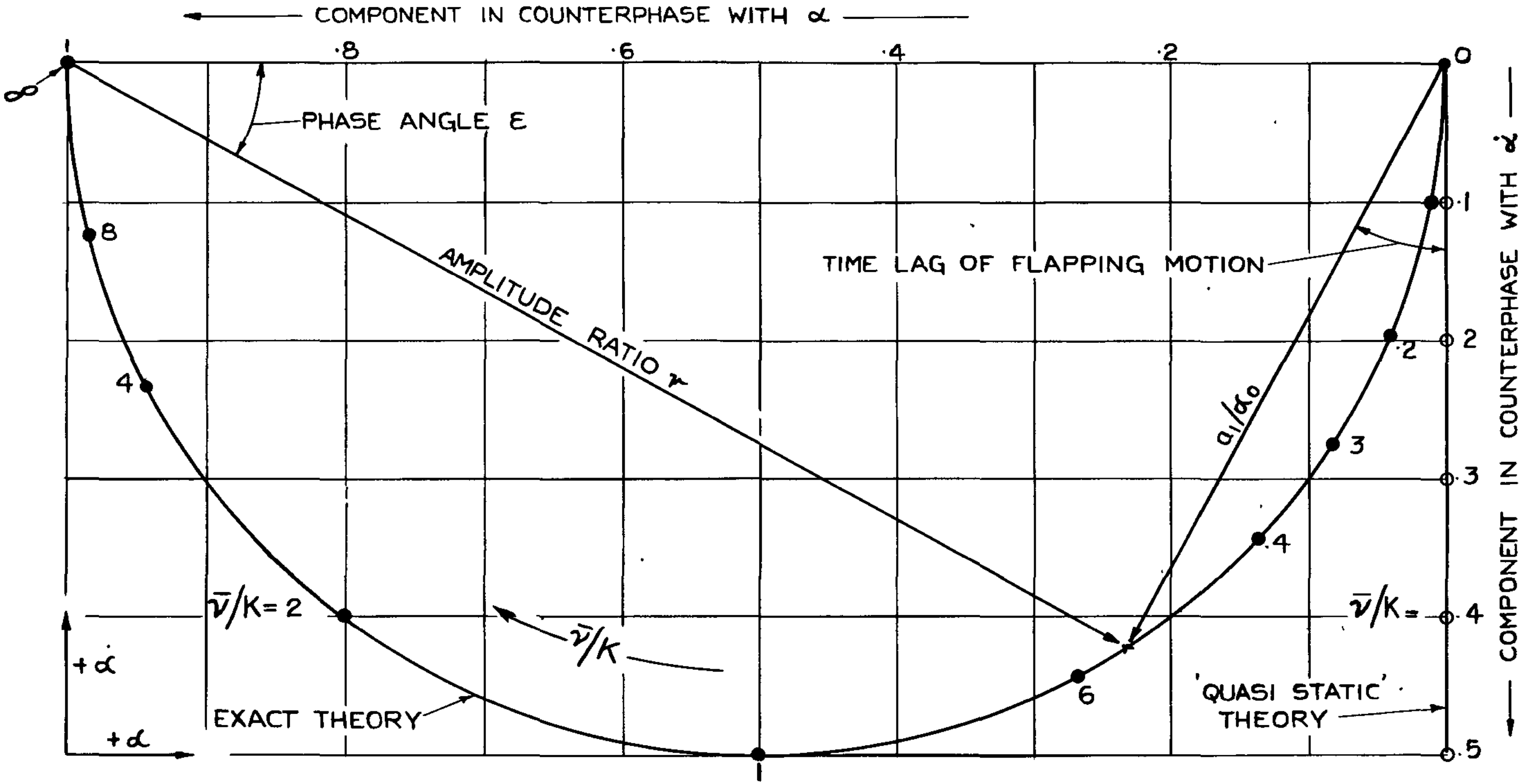


FIG. 4. VECTOR LOCI OF FLAPPING MOTION
(LONGITUDINAL TILT DUE TO PITCHING OSCILLATIONS)

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