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The Natural Frequencies of Vibration of Prismatic Blades with Particular Reference to a 12-stage Turbine

By

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SUMMARY

The natural frequencies of vibration of the blading of a 12 stage, 3000 r.p.m. turbine have been measured and compared with the values obtained by calculation.

In the calculations for the flexural modes, corrections have been introduced for shear and rotary inertia. An empirical correction is used for the influence of the increase in torsional stiffness, due to the platform, on the frequencies of torsional vibration.

The agreement of measured and computed frequencies is sufficient for the purpose of computing critical speeds up to a frequency of five kilocycles per second above which limit the discrepancy increases with the order of the mode.

CONTENTS

TABLES

ILLUSTRATIONS

1.0 Introduction

As a rotor blade passes through the wake of a stator blade or through the wake of any obstacle in the stream, there is a change in the gas bending load on the blade, the load on stator blading is subject to similar changes. Such periodic changes of load constitute a pulsating force on the blade and if one of the harmonic constituents of this force is in resonance with one of the natural modes of vibration of the blade, large amplitudes of vibration may be produced with corresponding high stresses. From the design of the compressor or turbine, the relation of the frequencies of the exciting forces to the rotational speed can be deduced, it remains to determine the natural frequencies of the blades in order to predict the critical speeds during operation.

It is obviously an advantage if the natural frequencies of the blades can be accurately calculated from the design data and in the case of prismatic blades a satisfactory solution can be obtained. The method used in the calculation of flexural vibration frequencies is that of Timoshenko[1] but in the higher modes of vibration it was found necessary to include the effect of shear and Jacobsens[2] solution has been modified to produce the shear effect as a correction factor to the purely bending solution. Corrections are also included for rotary inertia and stage temperature. The conventional method is followed in the calculations of torsional frequencies with the addition of an empirical correction for the effect of end constraint.

2.0 Symbols

The following symbols are used in the calculations in the body of this report, any symbols not included in this list have their usual significance or are defined in the text.

A	=	Area of Cross Section (in^2)
$2D$	=	Peak to peak tip amplitude (in)
E	=	Modulus of Elasticity (lb/in^2)
G	=	" " Rigidity (lb/in^2)
f_n	=	Frequency of n^{th} flexural mode (c.p.s.)
I	=	Moment of inertia of section about neutral axis (in^4)
d	=	Radius of gyration of section (in)
J_o	=	Polar moment of inertia of section (in^4)
p	=	Circular frequency (rads/sec)
T_n	=	Frequency of n^{th} torsional mode
ρ	=	Weight per unit volume (lb/in^3)

3.0 Measurement of Blade Frequencies

The blade frequencies were measured by clamping the blade to the driving cone of an electro-magnetic transducer driven by a power amplifier from a decade oscillator. The frequency was altered until resonance occurred, the mode of vibration being indicated by sand patterns, modes of vibration having frequencies higher than 11 Kc.s were not studied.

Measurements were taken at room temperature t_1 and the values corrected to the particular stage temperature t_2 on the basis given below,

$$f_{n_{t_2}} = f_{n_{t_1}} \sqrt{\frac{E_{t_2}}{E_{t_1}}}$$

$$T_{n_{t_2}} = T_{n_{t_1}} \sqrt{\frac{G_{t_2}}{G_{t_1}}}$$

The blade frequencies listed in Tables I and II were obtained in this way.

4.0 Calculation of Flexural Mode Frequencies. Ref. (1)

The blades of this turbine are prismatic and the differential equation of lateral or flexural vibration is the standard equation of a prismatic beam.

$$\frac{\partial^2 y}{\partial t^2} - \frac{a^2 \partial^4 y}{\partial x^4} = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

in which $a^2 = \frac{EIg}{A\rho}$

and x = co-ordinate of length.

Assuming the displacement at any point x is given by $y = X \cos(pt + 0)$

$$X = f(x)$$

Equation (1) becomes $\frac{d^4X}{dx^4} = \frac{p^2X}{a^2} = k^4X$

The general solution of this equation is

$$X = c_1 (\cosh kx + \cos kx) + c_2 (\cosh kx - \cos kx) + c_3 (\sinh kx + \sin kx) + c_4 (\sinh kx - \sin kx).$$

For a beam clamped at $x = \ell$ and free at $x = 0$, the end conditions are

$$x = \ell, \qquad X = \frac{dX}{dx} = 0,$$

$$x = 0, \qquad \frac{d^2X}{dx^2} = \frac{d^3X}{dx^3} = 0, \quad X = D.$$

Using these conditions

$$C_2 = C_4 = 0$$

and $\cosh k\ell \cos k\ell = -1 \qquad \ldots\ldots\ldots\ldots\ldots(2)$

The roots of equation (2) are

$$n = 1, 2, 3, 4, 5$$

$$k = 1.875, 4.694, 7.855, 10.996, 14.137$$

and the frequency of any mode is obtained by substitution in

$$f_n = \frac{p_n}{2\pi} = \frac{ak_n^2}{2\pi} = \frac{k_n^2}{2\pi}\sqrt{\frac{EIg}{A\rho}}$$

The general equation restricted to the present case becomes

$$X = \pm\frac{D}{2}\left[\operatorname{sech}\frac{k\ell}{2}\cosh k\left(x - \frac{\ell}{2}\right) - \operatorname{cosec}\frac{k\ell}{2}\sin k\left(x - \frac{\ell}{2}\right)\right] \quad \ldots\ldots\ldots(3)$$

or

$$X = \pm\frac{D}{2}\left[\operatorname{cosech}\frac{k\ell}{2}\sinh k\left(x - \frac{\ell}{2}\right) - \sec\frac{k\ell}{2}\cos k\left(x - \frac{\ell}{2}\right)\right] \quad \ldots\ldots\ldots(4)$$

dependent on n being odd or even. These equations are plotted in Fig (5) showing the deflection curves of the first four modes. The above reasoning is available in Ref. (1) but is included in order to derive equations (3) and (4) and leads to the following paragraph.

4.1 Correction for Shear Deflection (Ref. (2))

When the wave length of the vibration is not large in comparison with the depth of the beam a correction must be applied to the frequencies calculated as in the previous paragraph. This correction is to take into account the shear deflections of the beam which can no longer be neglected.

Defining a shear distribution factor S such that $\frac{\text{Shear Force}}{\text{Shear Slope}} = \frac{A.G.}{S}$ the corresponding differential equation to equation (1) is given in Ref. (1) as

$$\frac{\partial^4 y}{\partial x^4} - \frac{\rho S}{Gg}\frac{\partial^4 y}{\partial x^2 \partial t^2} + \frac{\rho A}{EIg}\frac{\partial^2 y}{\partial t^2} = 0$$

or

$$\frac{\partial^4 y}{\partial x^4} - \frac{1}{b^2}\frac{\partial^4 y}{\partial x^2 \partial t^2} + \frac{1}{a^2}\frac{\partial^2 y}{\partial t^2} = 0 \quad \ldots\ldots\ldots(5)$$

Introducing $y = X\cos(pt + \theta)$ the general solution of equation (5) is

$$X = c_1\cos qx + c_2\sin qx + c_3\cosh rx + c_4\sinh rx$$

where

$$q = \frac{p}{b}\sqrt{\frac{1}{2}\left(\sqrt{\left(1 + \frac{4b^4}{a^2p^2}\right)} + 1\right)}$$

and

$$r = \frac{p}{b}\sqrt{\frac{1}{2}\left(\sqrt{\left(1 + \frac{4b^4}{a^2p^2}\right)} - 1\right)}$$

Assuming the beam to be free at $x = 0$ and clamped at $x = \ell$, the following end conditions apply.

At, $\qquad x = 0, \qquad \frac{\partial^2 y}{\partial x^2} = \frac{1}{b^2}\frac{\partial^2 y}{\partial t^2}$

or

$$\frac{d^2 X}{dx^2} = -\frac{p^2}{b^2} X$$

since the bending moment and shear force must be zero at the free end.

$$x = 0, \qquad \frac{\partial^3 y}{\partial x^3} = \frac{1}{b^2}\frac{\partial^3 y}{\partial t^2 \partial x}$$

or

$$\frac{dX^3}{dx^3} = -\frac{p^2}{b^2}\frac{dX}{dx} .$$

At the clamped end of the beam, the displacement is zero and the slope of the beam is due to shear only.

Therefore at $x = \ell$, $\; y = 0, \quad \frac{\partial y}{\partial x} = \frac{1}{b^2}\int_0^\ell \frac{\partial^2 y}{\partial t^2}\, dx .$

or

$$\frac{dX}{dx} = -\frac{p^2}{b^2}\int_0^\ell X dx .$$

These end conditions may be used to eliminate the arbitrary constants c_1, c_2, c_3, c_4 and to derive the frequency equation with the following results.

$$X = \frac{D}{r^2 + q^2}\left[q^2 \cos qx + r^2 \cosh rx - B\,(q \sin qx + r \sinh rx)\right]$$

where

$$B = \frac{q^2 \cos q\ell + r^2 \cosh r\ell}{q \sin q\ell + r \sinh r\ell}$$

$$\cos q\ell \cosh r\ell - \frac{pab^2}{a^2p^2 + 2b^4} \sin q\ell \sinh r\ell + \frac{2b^4}{a^2p^2 + 2b^4} = 0 \quad \ldots\ldots\ldots(6)$$

Equation (6) is the corresponding frequency equation to equation (2) of paragraph 4.0.

If numerical values of S(1.5 for rectangular section) and $G/E = 0.375$ are introduced, and if p is replaced by $\phi_s p_b$ where p_b is the circular frequency previously calculated (shear neglected), equation (6) can be replaced by

$$\cos q\ell \cosh r\ell - \frac{2d^2\phi_s k^2}{1 + 8d^4\phi_s^2 k^4} \sin q\ell \sinh r\ell + \frac{1}{1 + 8d^4\phi_s^2 k^4} = 0 \quad \text{.......(7)}$$

in which $q = k\sqrt{\phi_s \left(\sqrt{1 + 4d^4\phi_s^2 k^4} + 2d^2\phi k^2\right)}$

$$r = k\sqrt{\phi_s \left(\sqrt{1 + 4d^4\phi_s^2 k^4} - 2d^2\phi k^2\right)}$$

Equation (7) may be solved for ϕ_s the dimensionless correction factor using the values of k given in paragraph 4.0. In the figure 6, ϕ_s is plotted against ℓ for the first four modes of vibration.

4.2 Correction for Rotary Inertia Ref. (3)

The elements of a beam during vibration undergo translatory motion and also rotate, the correction to the frequency of vibration due to the rotary inertia of the beam section increases with reduction of the ratio of wave length to depth of beam in a similar manner to the shear correction.

In ref. (3) the following expression for the correction is derived

$$p_{nri} = p_{nb}\left(1 - c_n\frac{d^2}{\ell^2}\right) \quad \text{.........................(8)}$$

where c_n is a number having values:-

$n =$	1,	2,	3,	4,	5
$c_n =$	2.32,	15.7,	38.7,	71.5,	114.1

The difference between p_{nri} and p_{nb} is negligible for the first mode of vibration of the blading used in this turbine, in the second mode the correction amounts to 3 % for the shortest blades, in general it is approximately one quarter of the shear correction.

Theoretical values of the flexural mode frequencies corrected for shear and rotary inertia, are listed in Tables II and III. In these tables ω is the theoretical value corrected in the above manner.

5.0 Calculation of Torsional Mode Frequencies

The differential equation of torsional vibration of a uniform bar is given in Ref. (1) as

$$C\frac{\partial^2\theta}{\partial x^2} - \frac{\rho J}{g}\frac{\partial^2\theta}{\partial t^2} = 0$$

where θ = angle of twist

assuming $\theta = X \sin (pt + e)$

where $X = f(x)$

The differential equation becomes

$$C \frac{d^2X}{dx^2} + \frac{p^2\rho}{g} JX = 0 \qquad \text{..............................(9)}$$

which has the complete solution $X = A \sin p\sqrt{\frac{\rho J}{Cg}}\, x + B \cos p\sqrt{\frac{\rho J}{Cg}}\, x$

The end conditions applicable to a cantilever clamped at x = o and free at $x = \ell$ are :-

$$x = o, \quad X = 0$$

$$x = \ell \quad \frac{dX}{dx} = o$$

$$\therefore \quad B = 0 \text{ and } T_n = \frac{p_n}{2\Pi} = \frac{(2n-1)}{4\ell}\sqrt{\frac{Cg}{\rho J}} \qquad \text{..............................(10)}$$

The torsional stiffness C of a complex section such as an aerofoil has been determined from the membrane analogy (Ref. 4) and is stated by Roark (Ref. 5) in the following form:-

$$C = GK = \frac{GF}{3 + \frac{4F}{Au^2}}$$

$$\text{where} \quad F = \int_o^u t^3\, du$$

δu = elementary length along median line

t = thickness normal to the median line.

For sections of low percentage thickness

$$F \ll Au^2$$

$$C = \frac{GF}{3}$$

The torsional stiffness obtained above assumes that the cross sections are free to warp as they rotate about the torsional centre, but at the blade root, the presence of the platform prevents such warping and the root cross section is constrained to remain plane. The result of this constraint is to increase the stiffness by an amount depending on the shape of the section. An approximate estimation of the increase in stiffness and its effect on the frequency of torsional vibration can be obtained by replacing the section by a uniform channel section having the same area, torsional stiffness and maximum moment of inertia. For a channel section of uniform thickness it has been shown by Timoshenko (Ref. 4) that the torque at any section at a distance x from the clamped, constrained end is given by

$$M = C\left(\frac{d\theta}{dx} - a^2 \frac{d^3\theta}{dx^3}\right)$$

in which $a^2 = \frac{Dh^2}{2C}\left(1 + \frac{bh^3}{4I_n}\right)$

D = flexural rigidity of a flange in its plane

h = length of web

b = thickness

I_n = moment of inertia about axis of symmetry

The first term $C \frac{d\theta}{dx}$ is that part of the torque which is balanced by shearing stresses due to twist and the second $- a^2 C \frac{d^3\theta}{dx^3}$ that part balanced by shearing stresses due to the bending of the flanges in their plane. To include the second term the differential equation becomes

$$\frac{d^4X}{dx^4} - \frac{1}{a^2}\frac{d^2X}{dx^2} - \frac{p^2 pJ}{Ca^2 g} = 0 \qquad \text{..............................(11)}$$

For the fundamental mode of vibration p may be replaced by

$$p = \phi_t \frac{2\Pi}{4\ell}\sqrt{\frac{Cg}{J}}$$

and equation 11 becomes

$$\frac{d^4X}{dx^4} - \frac{1}{a^2}\frac{d^2X}{dx^2} - \frac{\Pi^2}{4\ell^2}\phi_t^2 = 0$$

which has the complete solution

$$X = C_1 \cosh rx + C_2 \sinh rx + C_3 \cos qx + C_4 \sin qx$$

in which $q = \frac{1}{a\sqrt{2}}\sqrt{\left(\sqrt{\left(1 + \frac{\Pi^2 a^2 \phi_t^2}{\ell^2}\right)} - 1\right)}$

$$r = \frac{1}{a\sqrt{2}}\sqrt{\left(\sqrt{\left(1 + \frac{\Pi^2 a^2 \phi_t^2}{\ell^2}\right)} - 1\right)}$$

The end conditions are $x = o$, $X = o$, $\frac{dX}{dx} = o$;

$$x = \ell, \quad \frac{dX}{dx} = o, \quad \frac{d^3X}{dx^3} = o ;$$

since at the fixed end the whole of the torque is balanced by shearing stresses due to bending and at the free end the torque is zero. These end conditions are used to derive the frequency equation.

$$q \tan q\ell + r \tanh r\ell = o \quad \text{..................(12)}$$

For which values of ϕ_t corresponding to particular values of ℓ can be obtained for the fundamental mode. For this mode ϕ_t for the full section varied from $\phi_t = 1.08$ at $\ell = 5$, to $\phi_t = 1.25$ at $\ell = 2$, and for the part section $\phi_t = 1.2$ for $\ell = 1.85$.

As in the higher modes of vibration the nodal cross sections other than the root section are not constrained, values of ϕ_t cannot be obtained from equation 12 for modes other than the fundamental. An approximation to the effect of the increased root stiffness in the higher modes may be obtained by regarding the increased stiffness as equivalent to a decrease in the effective length of the beam, i.e. the correction is independent of the order of the mode and therefore for any given length, the value of ϕ_t used is that calculated for the fundamental mode.

The torsional centre of the section does not coincide with the centre of gravity and the value of J used in the above calculation is

$$J = J_o + AC^2$$

where C is the distance between the torsional centre and the centroid of the section. For the full section the increase in inertia amounted to 4%.

6.0 Flexural Vibration Stresses

The curvature and hence the stress at any point on a vibrating beam can be obtained by the differentiation of equations (3) and (4).

i.e. $$\frac{d^2X}{dx^2} = \frac{Dk^2}{2}\left(\operatorname{sech}\frac{k\ell}{2}\cosh k\left(x - \frac{\ell}{2}\right) + \operatorname{cosec}\frac{k\ell}{2}\sin k\left(x - \frac{\ell}{2}\right)\right)$$

when $n = 1, 3, 5$, etc.

and $$\frac{d^2X}{dx^2} = \frac{Dk^2}{2}\left(\operatorname{cosech}\frac{k\ell}{2}\sinh k\left(x - \frac{\ell}{2}\right) + \sec\frac{k\ell}{2}\cos k\left(x - \frac{\ell}{2}\right)\right)$$

$n = 2, 4, 6$, etc.

These expressions are plotted in Fig. 5. It can be seen that the maximum curvature is always at the root where the value is

$$\left.\frac{d^2X}{dx^2}\right|_{x = \ell} = \pm\, Dk_n^2$$

If shear deflection and rotary inertia are neglected the kinetic energy of the beam at maximum velocity

$$K.E. = \frac{\rho A}{2g} \; p_n^2 \int_0^{\ell} V^2 dx$$

$$= \frac{\rho A \ell}{8g} \; p_n^2 \; D^2$$

$$= \frac{EI\ell}{8} \; \mu_n^4 \; D^2$$

Therefore if successive modes of vibration are excited in such a way that the kinetic energy of the beam is the same for each, then the maximum strain will also remain constant.

7.0 Accuracy of Results

The factors causing differences between the computed and the measured values of blade frequencies may be divided into two classes, i.e. those producing errors of constant proportion and those in which neglect produces errors of increasing or decreasing proportion. In the first class fall the uncertainties in the values to be given the moduli of elasticity and to the section constants. Variations in the chemical composition of the blade material may cause a departure of 2-3% from the nominal values of the moduli with a much smaller variation in the material density. The errors in the section constants are due to tolerances in the manufacture of the blades and to the approximate nature of the expressions for torsional stiffness. Such tolerances result in a possible variation of 12-15% in I/A and C/J from the nominal value for this section. The maximum frequency spread to be expected from these factors, if the effect of the tolerance on blade length is taken as small, is ± 6%. Experiments on nominally identical blades have shown that the frequency variation is between ±7% of the mean. Factors of the second class are considered in paragraphs 7.1 and 7.2

7.1 Accuracy of Computed Frequencies of Flexural Modes

The computed and measured frequencies of Tables II and III are plotted in Fig. 7, and they show the agreement, within the limits of paragraph 7.0, up to a frequency of five kilocycles per second above which there is an increasing discrepancy. An explanation may be found in the departure of the actual end conditions from the assumed conditions as the transducer used in the measurement of the natural frequencies has a small amount of flexibility in the drive mechanism. The existence of freedom in tilt causes the end conditions to approach the hinged-free state, the elastic restraint in tilting of the clamp and drive is a function of frequency and will be a minimum at the resonant tilting frequencies of the system. The restraint at the hinge becomes of less importance as the order of the mode of vibration is increased, and at high orders the measured frequency will be that of the hinged-free state. In Figure 8 the ratio of the measured to the computed frequencies of the second flexural mode is plotted against the computed frequency, on the graph is also drawn a line at a ratio of 0.699, that is the ratio between the computed hinged-free first mode and the computed clamped-free second mode frequencies. In Figure 9 a similar graph is drawn for the third flexural mode, these Figures show the approach of the system to the hinged state. In confirmation of this explanation a free-free beam of similar constants placed in the same exciter gave agreement of measured and computed frequencies up to thirty kilocycles per second, the upper limit of measurement.

7.2 Accuracy of Computed Frequencies of Torsional Modes

The torsional frequencies listed in Tables II and III show a closer agreement of theoretical and measured values than the flexural modes. It is considered that the increased accuracy is due to the stiffness in torsion of the exciting mechanism.

8.0 Conclusion

From the frequencies listed in Tables II and III it can be concluded that the methods of calculation of the natural frequencies of prismatic blades used in this report are of sufficient accuracy up to a frequency of five kilocycles per second. It is probable that the actual frequencies of vibration would agree more closely with the theoretical values if the blade roots were encastred, the condition approached in practice by rotor blades at high rotational speeds.

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TABLE I

Blade Section Data

Section A.	Chord	b	1.25 in.
	Area	A	0.138 in^2
	Moment of Inertia (Bending)	I	1.01 x 10^{-3} in^4
	" " " Max.		12.23 x 10^{-3} in^4
	Polar Moment of Inertia	J	13.24 x 10^{-3} in^4
	Torsional stiffness factor	K	0.686 x 10^{-3} in^4
Section B.	Chord	b	1.15 in.
	Area	A	0.114 $in.^2$
	Moment of Inertia (Bending)	I	0.39 x 10^{-3} in^4
	" " " Max.		7.3 x 10^{-3} in^4
	Polar Moment of Inertia	J	7.69 x 10^{-3} in^4
	Torsional stiffness factor	K	0.576 x 10^{-3} in^4

Turbine Data

Density of Rex 326F at 20°C	0.289 lb. in^{-3}
" " F.C.B.T. " "	0.285 lb. $in.^{-3}$
" " H.R. Crown Max.	0.285 lb. $in.^{-3}$

Rotor Blades

Stage No.	Blade length[1]	Material	Temp. °C	E. lb/in.^{2}	G. lb/in.^{2}
1	2.03 in	Rex 326F	736	2.2x10^7	0.82x10^7
2	2.22 "	"	707	2.24 "	0.84 "
3	2.41 "	"	685	2.27 "	0.85 "
4	2.66 "	"	663	2.29 "	0.86 "
5	2.88 "	"	641	2.32 "	0.87 "
6	3.105 "	"	620	2.32 "	0.88 "
7	3.32 "	F.C.B.T.	598	2.17 "	0.81 "
8	3.59 "	"	576	2.20 "	0.82 "
9	3.95 "	"	555	2.23 "	0.84 "
10	4.31 "	"	533	2.26 "	0.85 "
11	4.67 "	"	511	2.29 "	0.86 "
12	5.03 "	"	589	2.32 "	0.87 "

Stator Blade

Stage No.	Blade length[1]	Material	Temp. °C	E. lb/in.^{2}	G. lb/in.^{2}
1	1.85 in	H.R.C.M.	750	2.04x10^7	0.76x10^7
2	2.02 "	"	722	2.07 "	0.78 "
3	2.22 "	"	696	2.10 "	0.79 "
4	2.40 "	"	674	2.12 "	0.80 "
5	2.62 "	"	652	2.14 "	0.80 "
6	2.84 "	"	631	2.16 "	0.81 "
7	3.06 "	F.C.B.T.	609	2.16 "	0.81 "
8	3.28 "	"	587	2.19 "	0.82 "
9	3.64 "	"	566	2.21 "	0.83 "
10	4.00 "	"	544	2.24 "	0.84 "
11	4.36 "	"	522	2.27 "	0.85 "
12	4.72 "	"	500	2.31 "	0.87 "

[1] Blade length in above Tables is the mean length to the blade platform.

TABLE II

Measured and Calculated Rotor Blade Frequencies

Flexural Modes

Stage	F1 (c.p.s.) Obs^d		F1 Calc^d		F2 (c.p.s.) Obs^d		F2 Calc^d		F3 (c.p.s.) Obs^d		F3 Calc^d	
	$F_{1_{t_1}}$	$F_{1_{t_2}}$	F_{1_b}	F_1	$F_{2_{t_1}}$	$F_{2_{t_2}}$	F_{2_b}	F_2	$F_{3_{t_1}}$	$F_{3_{t_2}}$	F_{3_b}	F_3
1	2163	1371	1990	1952	10060	8687	12450	10870				
2	1875	1634	1685	1660	9050	7890	10540	9420				
3	1645	1441	1440	1422	8240	7222	9020	8155				
4	1439	1267	1190	1176	7432	6554	7470	6850				
5	1218	1080	1020	1009	6481	5751	6130	5720				
6	1018	909	885	876	5945	5308	5340	5226	11820	10550	15510	13500
7	906	778	745	738	5208	4470	4650	4416	10920	9378	13020	11560
8	737	679	640	637	4664	4028	5005	3821	10130	8742	11200	10120
9	627	545	532	530	3741	3253	3335	3201	8515	7405	9338	8575
10	515	451	450	448	3109	2723	2820	2725	7316	6406	7895	7290
11	430	379	386	385	2629	2318	2415	2355	6419	5661	6762	6415
12	369	327	334	333	2265	2010	2100	2048	5713	5065	5880	5575

Torsional Modes

Stage	T1 (c.p.s.) Obs^d		T1 Calc^d		T2 (c.p.s.) Obs^d.		T2 Calc^d		T3 (c.p.s.) Obs^d		T3 Calc^d.	
	$T_{1_{t_1}}$	$T_{1_{t_2}}$	T_1	$\phi_t T_2$	$T_{2_{t_1}}$	$T_{2_{t_2}}$	T_2	$\phi_t T_2$	$T_{3_{t_1}}$	$T_{3_{t_2}}$	T_3	$\phi_t T_3$
1	4022	3473	2956	3627	12630	10907	6868	10910				
2	3753	3270	2737	3312	11360	9900	8211	9935				
3	3357	2945	2538	3020	10320	9050	7616	9063				
4	3062	2700	2303	2689	9374	8268	6927	8104				
5	2807	2490	2144	2455	8460	7502	6432	7365				
6	2610	2330	2004	2265	7846	7005	6011	6792				
7	2375	2040	1801	2017	7165	6156	5402	6050				
8	2144	1850	1677	1845	6645	5733	5032	5535	9820	8472	8586	9225
9	1986	1727	1534	1672	5918	5146	4602	5016	9184	7986	7670	8360
10	1843	1614	1715	1528	5347	4682	4245	4587	8660	7583	7079	7645
11	1675	1477	1315	1405	4822	4252	2947	4223	7827	6901	6580	7041
12	1534	1360	1226	1306	4351	3857	3678	3899	7162	6349	6130	6498

Note:- F_n is the final theoretical value of the flexural frequency of the n^{th} mode.

TABLE III

Measured and Calculated Stator Blade Frequencies

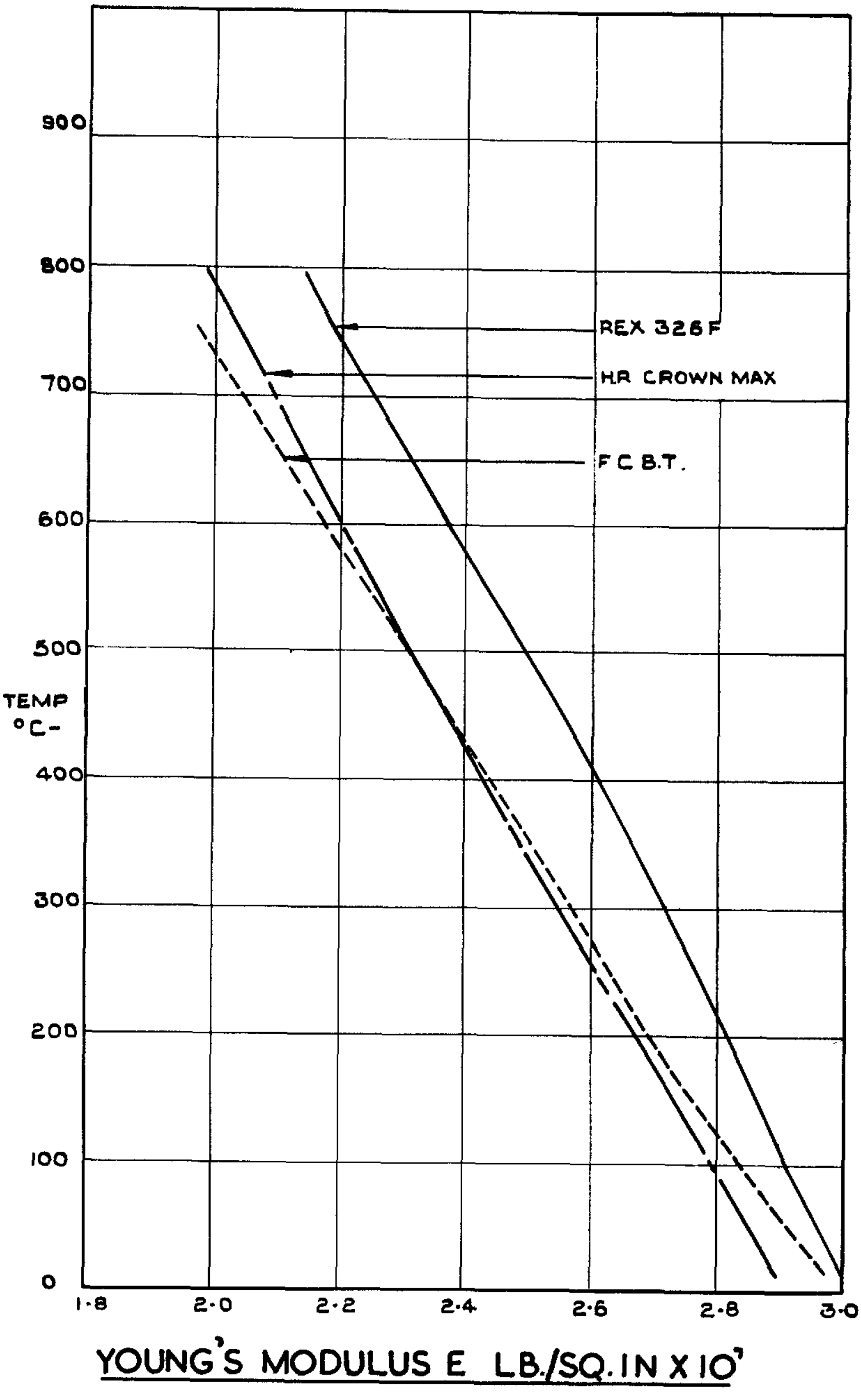
Flexural Modes

Stage	F_1 (c.p.s.) Obs^d. $F_{1_{t_1}}$	$F_{1_{t_2}}$	Calc^d. F_{1_b}	F_1	F_2 (c.p.s.) Obs^d. $F_{2_{t_1}}$	$F_{2_{t_2}}$	Calc^d. F_{2_b}	F_2	F_3 (c.p.s.) Obs^d. $F_{3_{t_1}}$	$F_{3_{t_2}}$	Calc^d. F_{3_b}	F_3
1	1734	1435	1587	1510	9110	7580	9945	9030				
2	2279	1909	1961	1918	9877	8272	12290	10700				
3	1965	1658	1636	1610	8793	7420	10250	9144				
4	1735	1472	1407	1385	7924	6721	8818	7985				
5	1472	1235	1187	1172	6980	5951	7439	6848				
6	1207	1035	1013	1001	6012	5143	6348	5908				
7	1003	858	872	863	5243	4486	5465	5141				
8	852	734	765	759	4638	3995	4795	4534	9773	8718	13430	11890
9	718	622	621	620	4032	3401	3911	3740	9042	7829	10950	9862
10	609	531	[illegible]	517	[illegible]	3140	3260	3137	8342	7267	9126	8379
11	511	448	441	439	3190	2799	2764	2676	7643	6705	7740	7181
12	435	385	379	378	2836	2510	2375	2316	7073	6260	6650	6219

Torsional Modes

Stage	T_1 (c.p.s.) Obs^d. $T_{1_{t_1}}$	$T_{1_{t_2}}$	Calc^d. T_1	$\phi_t^{T_1}$	T_2 (c.p.s.) Obs^d. $T_{2_{t_1}}$	$T_{2_{t_2}}$	Calc^d. T_2	$\phi_t^{T_2}$	T_3 (c.p.s.) Obs^d. $T_{3_{t_1}}$	$T_{3_{t_2}}$	Calc^d. T_3	$\phi_t^{T_3}$
1	4988	4150	3701	4367	13340	11100	11080	13300				
2	4064	3404	2856	3513	11030	9241	8553	10520				
3	3602	3040	2655	3212	10300	8692	7965	9637				
4	3275	2778	2452	2918	9682	8212	7357	8754				
5	2972	2534	2285	2661	8942	7623	6857	8022				
6	2729	2335	2104	2409	8281	7084	6312	7258				
7	2597	2162	1944	2197	7671	6562	5831	6589				
8	2350	2024	1837	2057	7132	6143	5510	6171				
9	2099	1818	1653	1818	6332	5482	4967	5464				
10	1907	1661	1516	1652	5724	4986	4542	4950	9751	8494	7580	8262
11	1741	1528	1408	1514	5198	4560	4226	4543	9020	7912	7008	7534
12	1605	1420	1309	1400	4791	4240	3928	4203	8401	7435	6548	7006

FIG.I.



YOUNG'S MODULUS FOR TURBINE BLADE MATERIALS

TURBINE BLADE SECTION

12½ T6/95 P.40

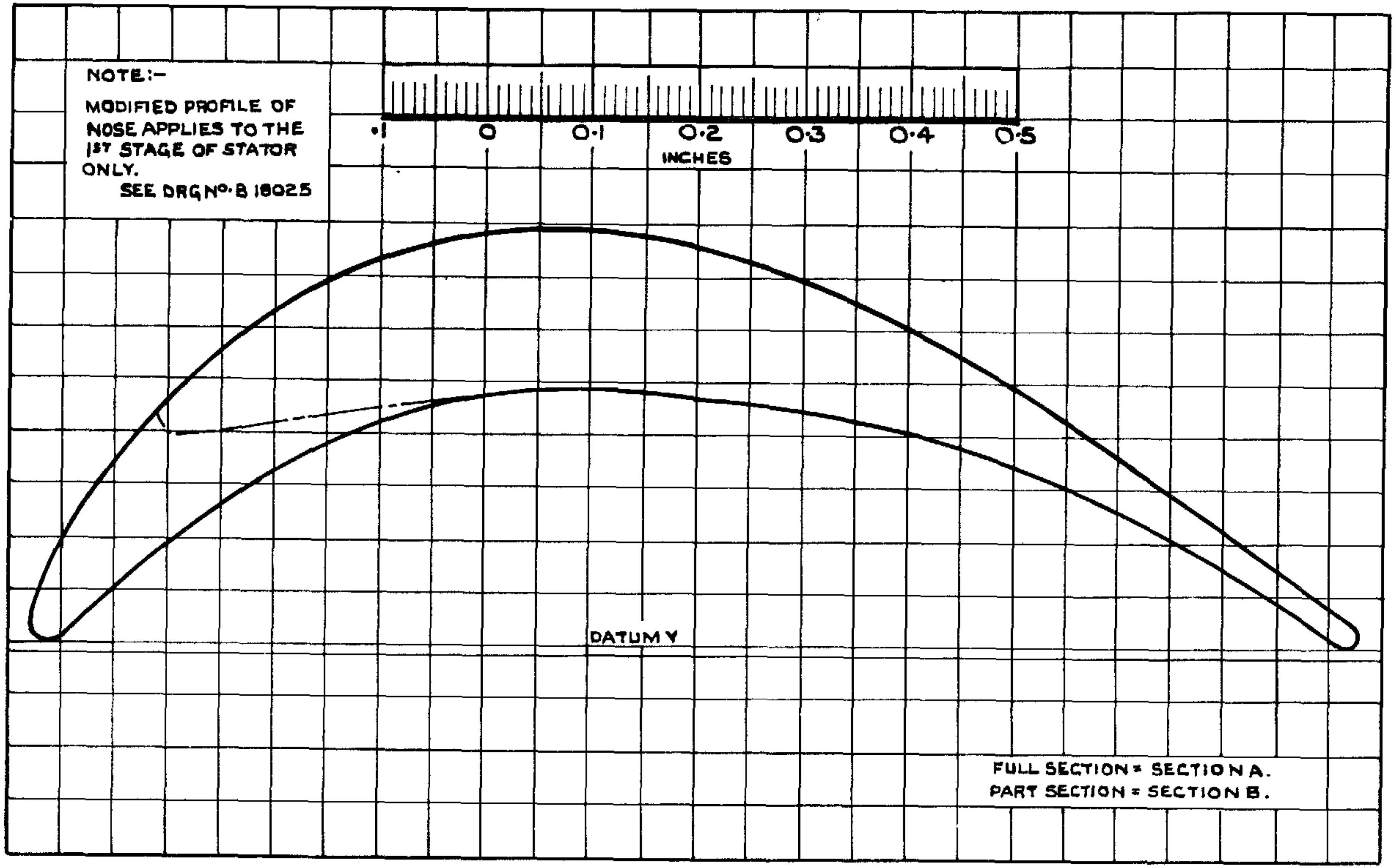
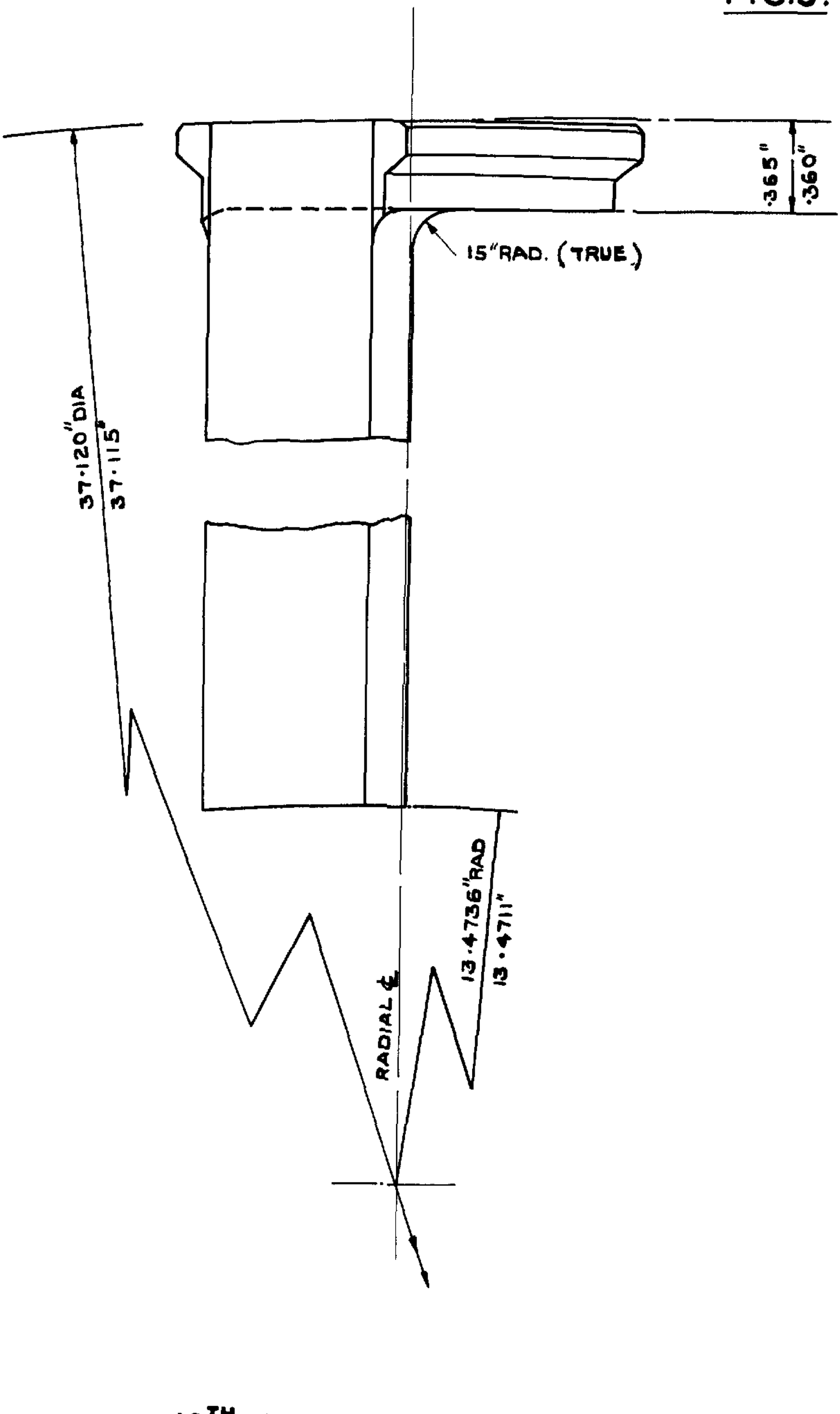
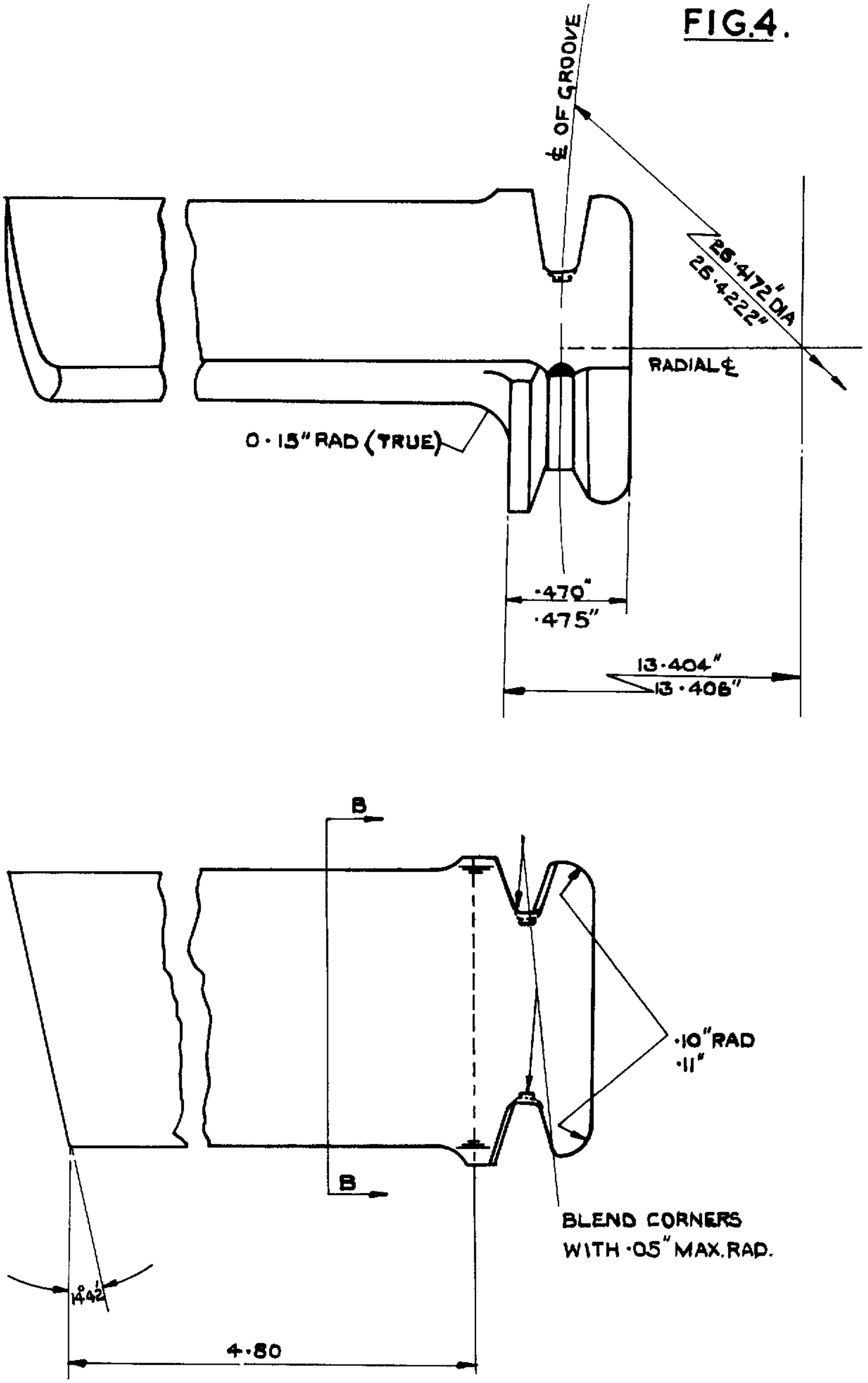


FIG. 2.

FIG.3.

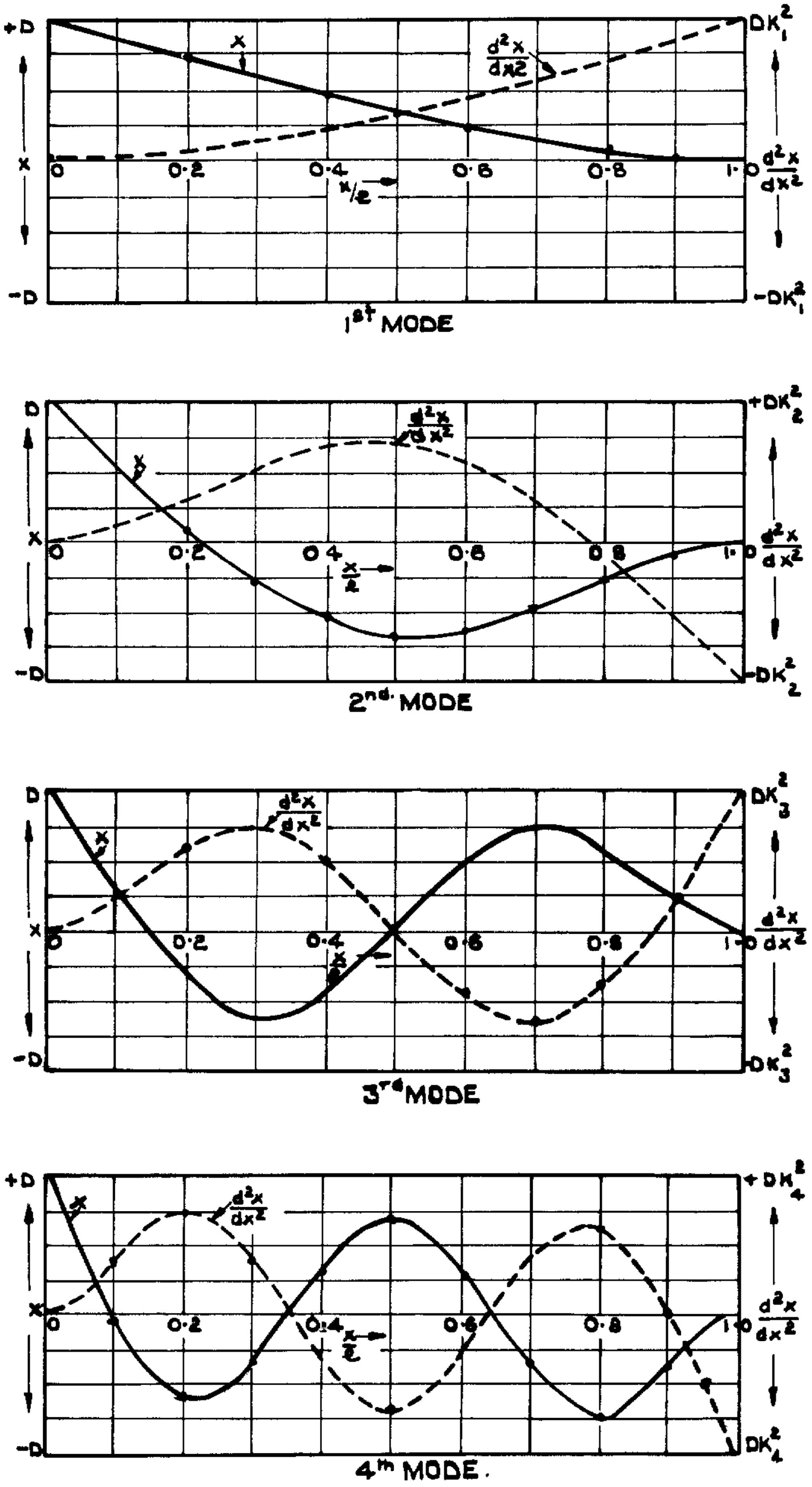


12TH. STAGE STATOR BLADE FOR TURBINE



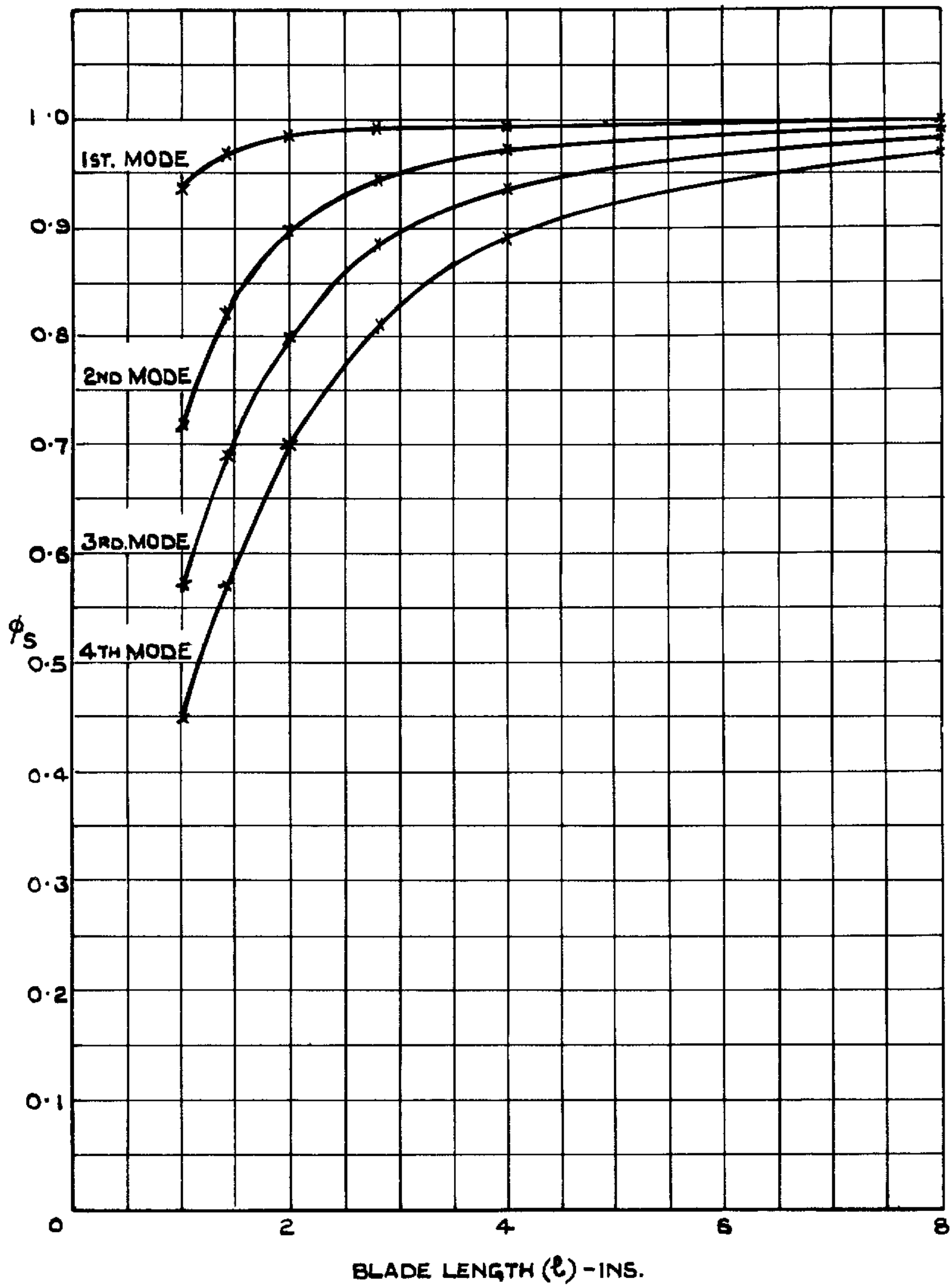
12th STAGE ROTOR BLADE
FOR TURBINE

FIG.5.



DEFLECTION CURVES OF A UNIFORM CANTILEVER

FIG.6.



SHEAR CORRECTION FACTOR

ROTOR BLADE FLEXURAL FREQUENCY.

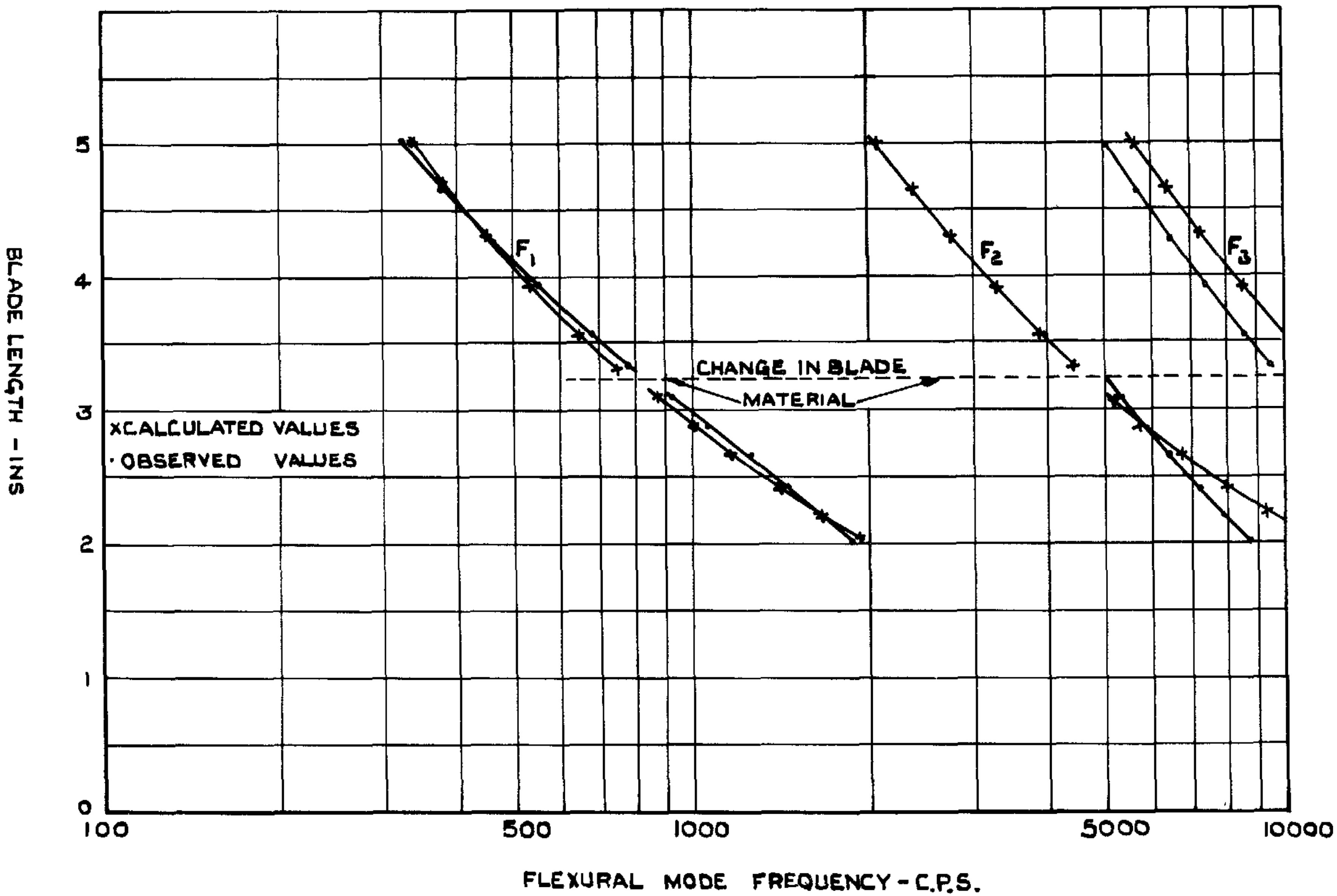


FIG.7.

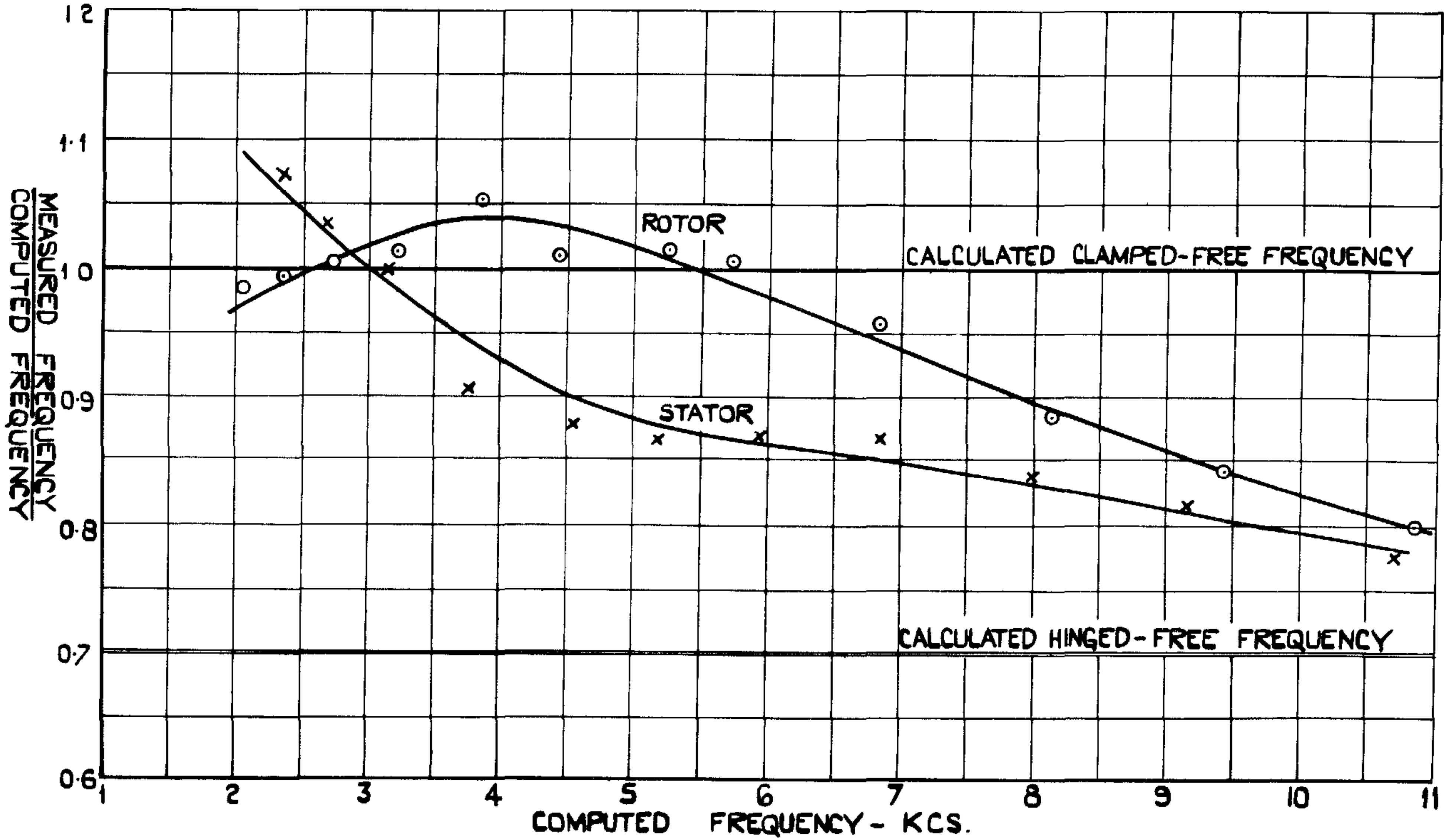
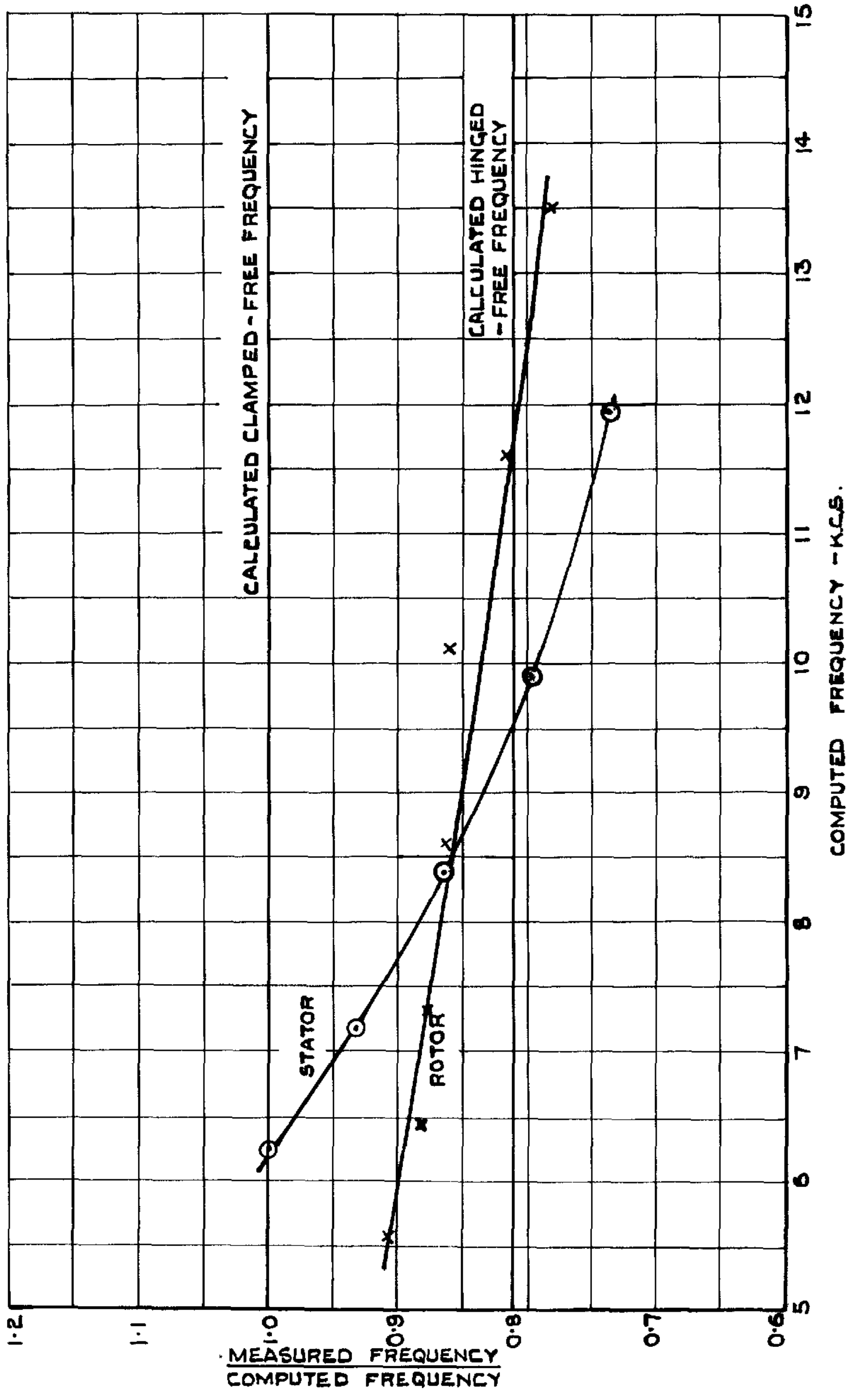


FIG 8.

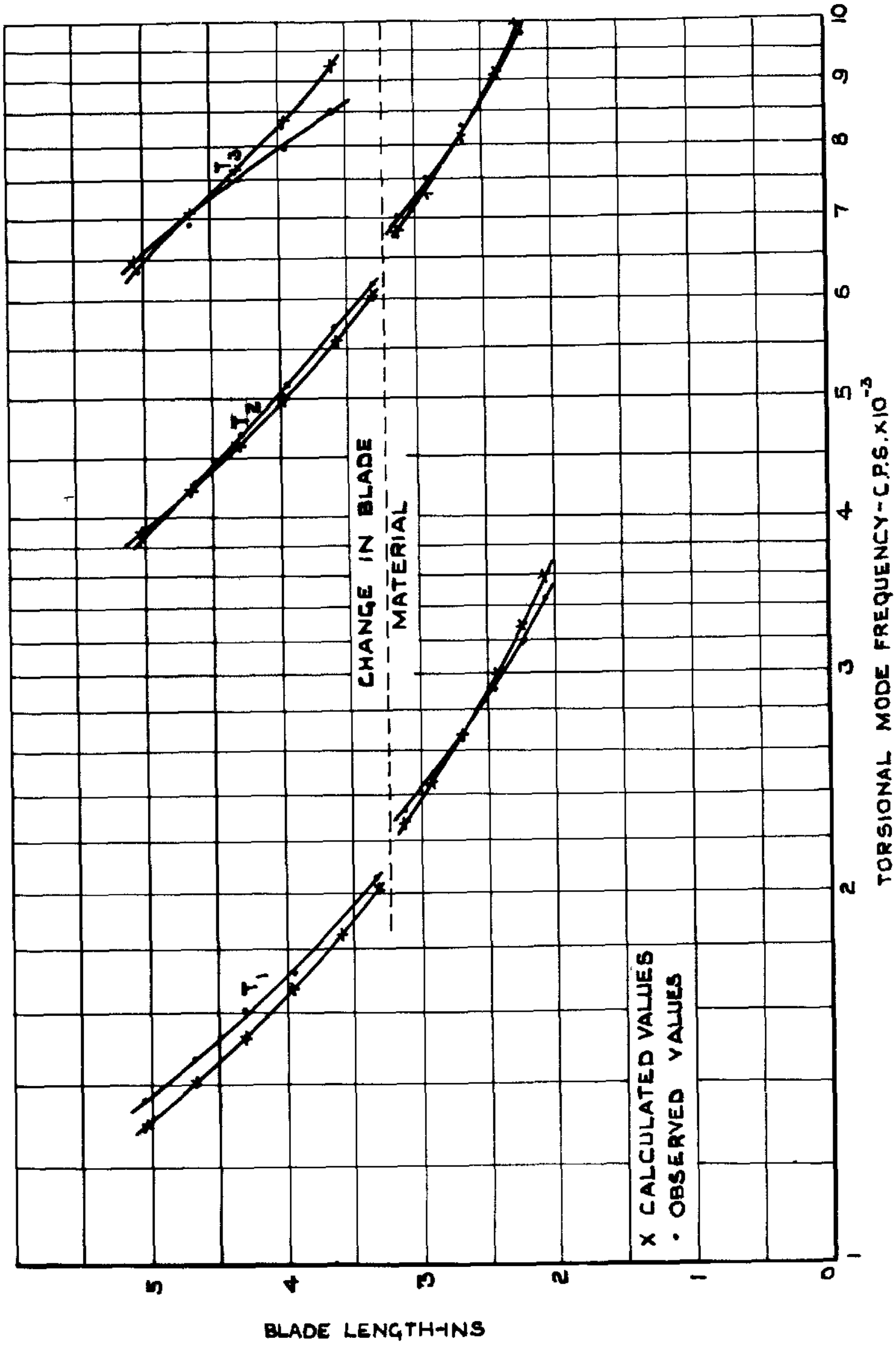
RATIO OF MEASURED COMPUTED FREQUENCY OF 2ND HARMONIC FLEXURAL MODE.

FIG.9.



RATIO OF MEASURED TO COMPUTED FREQUENCY OF OF 3RD. HARMONIC FLEXURAL MODE

FIG.10.



ROTOR BLADE TORSIONAL FREQUENCIES

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