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Designing a Slot for a Given Wall Velocity

By

A. THOM, M.A. and Laura Klanfer, B.Sc

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C.P. No. 76

Designing a Slot for a given Wall Velocity+

- By -
A. Thom, M.A. and Laura Klanfer, B.Sc.

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Summary

This paper gives the results obtained arithmetically for the wall shape of an expanding passage with specified **constant wall** velocities. A slot is assumed to withdraw fluid from the passage at the velocity discontinuity and the shape of the slot entry is obtained. A cusp develops at the entrance to the slot and the effect on the remainder of the field of rounding this cusp is considered in detail.

1. The Slot with Cusped Edge - The Basic Field

The problem considered is shown in Figure 2 which depicts one half of a symmetrical expanding passage having an entry velocity of e (2.718) and an exit velocity of unity. The higher velocity (e) is assumed to remain constant along the wall up to and into the slot which is assumed to draw off one ninth of the total fluid entering the passage. On the other side of the slot and along the remainder of the passage wall the velocity is again constant and equal to unity[x].

1.1 Method

The method used was to "square" log 1/q on the ϕ, ψ grid. The resulting field when settled will be referred to as the Basic Field. Since symmetry exists across the centre line and the values on the walls were known no further conditions were necessary. The only difficulty encountered was the large amount of detail required around the cusp which developed on the downstream edge of the slot. The field was subdivided six times in this neighbourhood. One of the sheets showing this subdivision is given in Figure 1. The final result obtained is shown in Figure 2 which gives the wall shape and approximate streamlines, equipotentials and contours of velocity.

/2.

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+This published version of the original report has been lengthened by incorporating some material from A.R.C. 12,953 (Reference 1 of this report).

x An earlier approximate solution has been given (Reference 1).

2. The Slot with Rounded Nose

The cusp which developed in the solution of the above mentioned problem makes the resulting outline quite impracticable. If we round the nose of the cusp arbitrarily, the velocities on all the walls will be affected in an unknown manner. So it was decided to attempt to produce a solution which would contain a rounded nose and would at the same time give the specified velocities on the other parts of the walls. The point which was previously the cusp in the basic field becomes a stagnation point. A suitable velocity distribution was assumed near this point, actually in the range $0 < \phi < 1$. Outside these limits the wall velocities are the same as in the basic field. In order to ensure that the final shape is rounded and does not have a sharp corner at the stagnation point, the velocity distribution on the wall in the immediate neighbourhood of this point was assumed to approximate to that of the flow in a right angle bend, which is given by $w = k.z^2$. Apart from that, the velocities are arbitrary, care being taken that they run smoothly into the boundary value $(q = 1)$ of the basic field. The actual values used are shown in Figure 3 and given in Table I. Figure 3 also shows the velocities for a right angle bend and illustrates the manner in which the assumed velocities diverge from these. The amount of rounding of the nose is, of course, controlled by the assumed velocity distribution and is unknown until the field is finally plotted. Any other value for k might have been chosen and the fairing off into the constant value $(q = 1)$ might have been made in any other way desired.

With the above assumptions regarding boundary values it would be possible to proceed to a solution by squaring on $\log 1/q$ operating in the ϕ, ψ field but an alternative suggested itself.

/Table I

Table I

Velocity Distribution along $\psi = 0$

ϕ	$\log_e \frac{1}{q}$	ϕ	$\log_e \frac{1}{q}$
0		3/8	0.053
1/64	1.079	7/16	0.041
1/32	0.733	1/2	0.033
3/64	0.530	9/16	0.026
1/16	0.386	5/8	0.020
3/32	0.245	11/16	0.015
1/8	0.184	3/4	0.011
5/32	0.146	13/16	0.007
3/16	0.121	7/8	0.004
7/32	0.104	15/16	0.002
1/4	0.090	1	0.000
5/16	0.068		

The values for q between $\phi = 0$ and $\phi = 1/16$ inclusive are identical with those of the right angle bend

As the solution for the basic field was already known it was found to be much more convenient to operate on the ratio of the velocities to those in the basic field. Thus we 'square' on the values $\log q_B/q$, operating, of course, in the ϕ , ψ - field, with boundary values which are everywhere zero except in the immediate neighbourhood of the stagnation point. The final values are then obtained by superimposing the values found on those of the basic field. Thus

$$\log q = \log q_B - \log \frac{q_B}{q} \qquad \dots (1)$$

where q_B is the velocity in the basic field.

Figure 4 shows one sheet in the $\log (q_B/q)$ field and Figure 5 shows the final wall shape obtained, with flow pattern and velocity contours inserted approximately.

2.1 Method of obtaining the Conjugate Function

When we are dealing with solutions of Laplace's equation of the kind dealt with here, a great saving in time can be effected in the process of differentiating across and integrating along a boundary[3]. This will be illustrated for the case of zero boundary values but the formula is obviously applicable to a boundary with constant values. There is also a similar expression for the general case but in our experience it is not so successful. Let the values of the function F on which we are operating be a , b , c and d along the line $\psi = 1$. See Figure 8.

Figure 8/

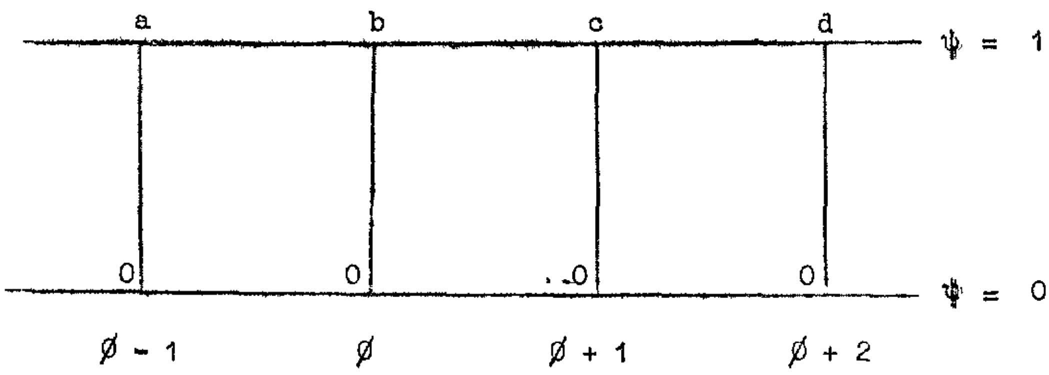


FIGURE 8

Then

$$\int_0^1 \frac{\partial F}{\partial \psi} \, d\phi = \frac{a + 11b + 11c + d}{24} . \qquad \dots (2)$$

This is easily proved by writing the values a, b, etc. as a Taylor expansion up to and including 4th order terms, using the fact that $\nabla^2 F = 0$ we can then obtain the above expression.

2.2 Treatment of a Singularity

During the process of squaring, the log (q_B/q) field was subdivided seven times, but, even so, special treatment was needed for the innermost sheet around the singularity, where the differential equation is not accurately represented by the difference equation.

Woods[2] has given a method of getting over the difficulty of 'squaring' near a stagnation point but in the present instance rather a different method of procedure was used. The field here was, in effect, at each step compared with the flow in a right angle bend given by $w = k.z^2$. Values of log q for a portion of the latter field are given in Fig. 6 (with $k = 1$). As it stands this comparison field does not 'square'; while it satisfies Laplace's equation it does not satisfy the difference equation. The mean of four surrounding points is not equal to the value at the centre. Thus, calling C the true value at the centre of a diamond and A_1, A_2, A_3 and A_4 the surrounding corner values, we must write

$$C = \frac{1}{4} \Sigma A + \Delta . \qquad \dots (3)$$

The arithmetical values of Δ are easily found from Figure 6 and are given in Figure 7. Since we are dealing with logs, changing k simply adds a constant amount to log q over the whole field so that the values of Δ are unaffected by the values of k or in fact by the

scale/

scale of the diagram. Any field with a stagnation point on a 'straight angle' will have the same values of Δ at corresponding points when we are close to the singularity and in fact there is no reason to suppose that this will not hold approximately when we get further out in the field, where the two fields (the actual and the comparison) have begun to diverge from one another.[x]

There still remains the difficulty, that the value of log q at the stagnation point is $-\infty$. This difficulty is easily overcome by the following consideration. We can proceed with the 'squaring' as soon as we have found some finite value for log q at this point which can be derived from the surrounding values and which will give them again when they are recalculated from equation (3). For the comparison field such a value is zero, because zero is the mean of the four surrounding values, and, with the values of Δ shown in Figure 7, equation (3) again gives zero at the four neighbouring points. Thus in the actual field we write at the stagnation point the mean of the surrounding values and use this value to recalculate them. The justification is that the process works for the comparison field with any value of k, and so will work for the actual field, since the two are identical near the singularity. When 'squaring' on $\log 1/q$ (or $\log q_B/q$) the same adjustments apply, except that the sign is changed, as the part containing the singularity stands in the denominator.

3. Conclusions

The velocity distribution assumed near the singularity is entirely arbitrary and the curvature of the nose produced is not known until the solution is completed. The nose is perhaps too much flattened and a better shape might be obtained by using a sharper and narrower dip in the velocity curve.

In Figure 3 the two profiles are compared. It appears that rounding the nose has not had a very serious effect on the general wall shape, the tendency being to widen the channel near the slot.

Presumably on an aerofoil with a similar slot the effect of rounding the cusp nose would be to reduce the thickness of the aerofoil locally.

List of Symbols

ϕ	Velocity Potential
ψ	Streamfunction
q	Velocity
q_B	Velocity in Basic Field.

References/

[x] After this paper was completed an investigation was made into the treatment of the stagnation point in problems of this nature. This will be published later.

References

No.	Author(s)	Title, etc.
1	A. Thom	Shape of a Slot for given wall Velocity. O.U.E.L.33. (10th February, 1950). A.R.C. 12,953.
2	L. C. Woods	The Numerical Solution of Two-dimensional Fluid Motion in the Neighbourhood of Stagnation Points and Sharp Corners. Communicated by Prof. A. Thom. O.U.E.L. No. 27. R. & M.2726, (October, 1949).
3	A. Thom	The Method of Influence Factors in Arithmetical Solutions of Certain Field Problems. R. & M.2440. (August, 1946).

SHEET No.4 $1000 \log q_B$

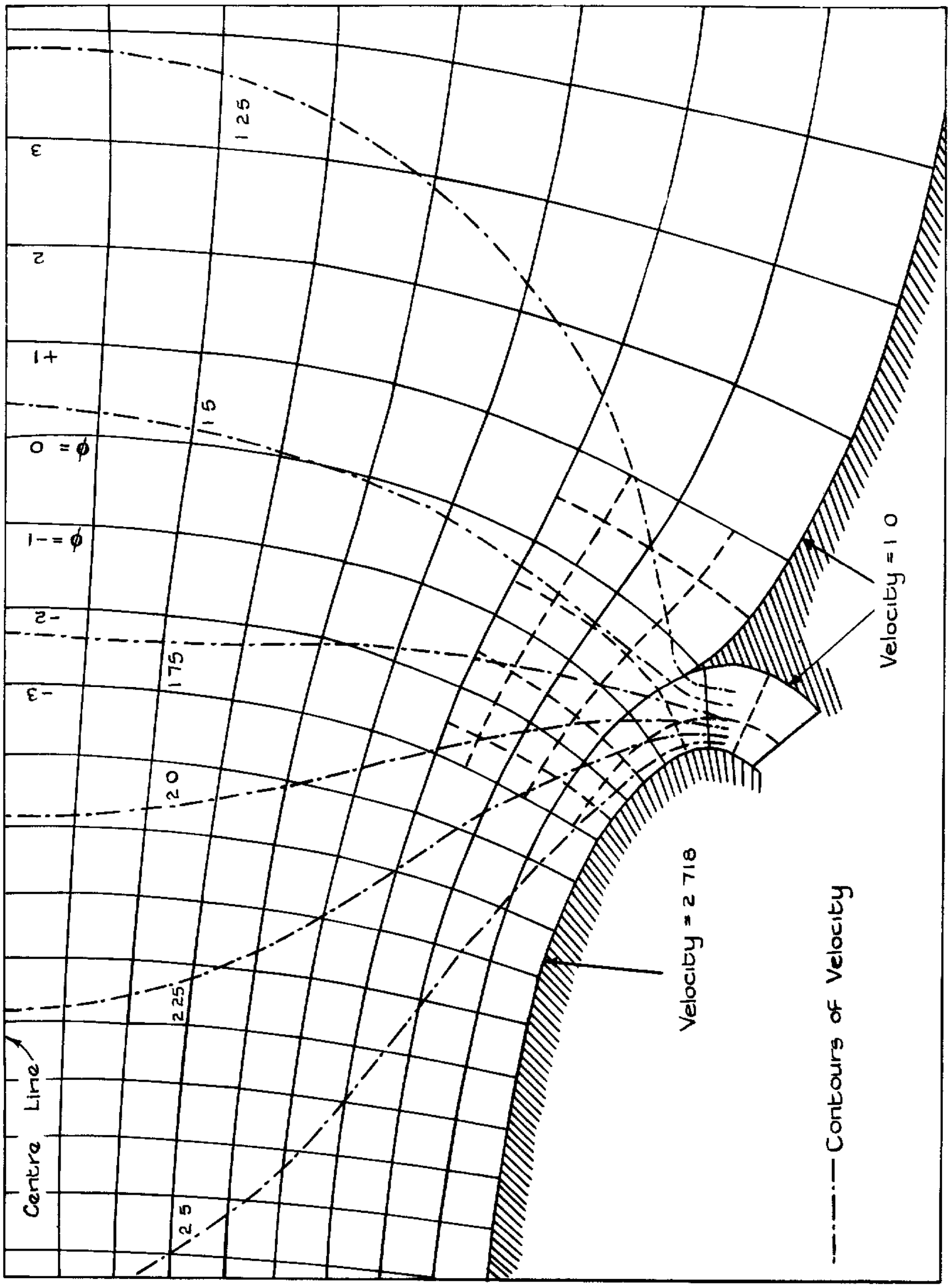
617		531		426		316		226		166		128
	598	550	493	429	360	293	236	190	156	131	112	98
672	629	577	514	437	349	260	189	141	111	91	77	67
	667	615	548	460	342	204	115	77	58	46	39	34
755	716	667	602	512	373	0	0	0	0	0	0	0
	776	734	679	603	497	372	294	267	256	253	251	250
868	845	815	776	724	661	597	549	522	510	504	501	500
	921	905	885	860	831	802	779	765	757	753	751	750
1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000

Figures outside the dotted rectangle come from next outer sheet and those inside the small rectangle from next inner sheet. Seven sheets were used

Fig 1.

FIG 2

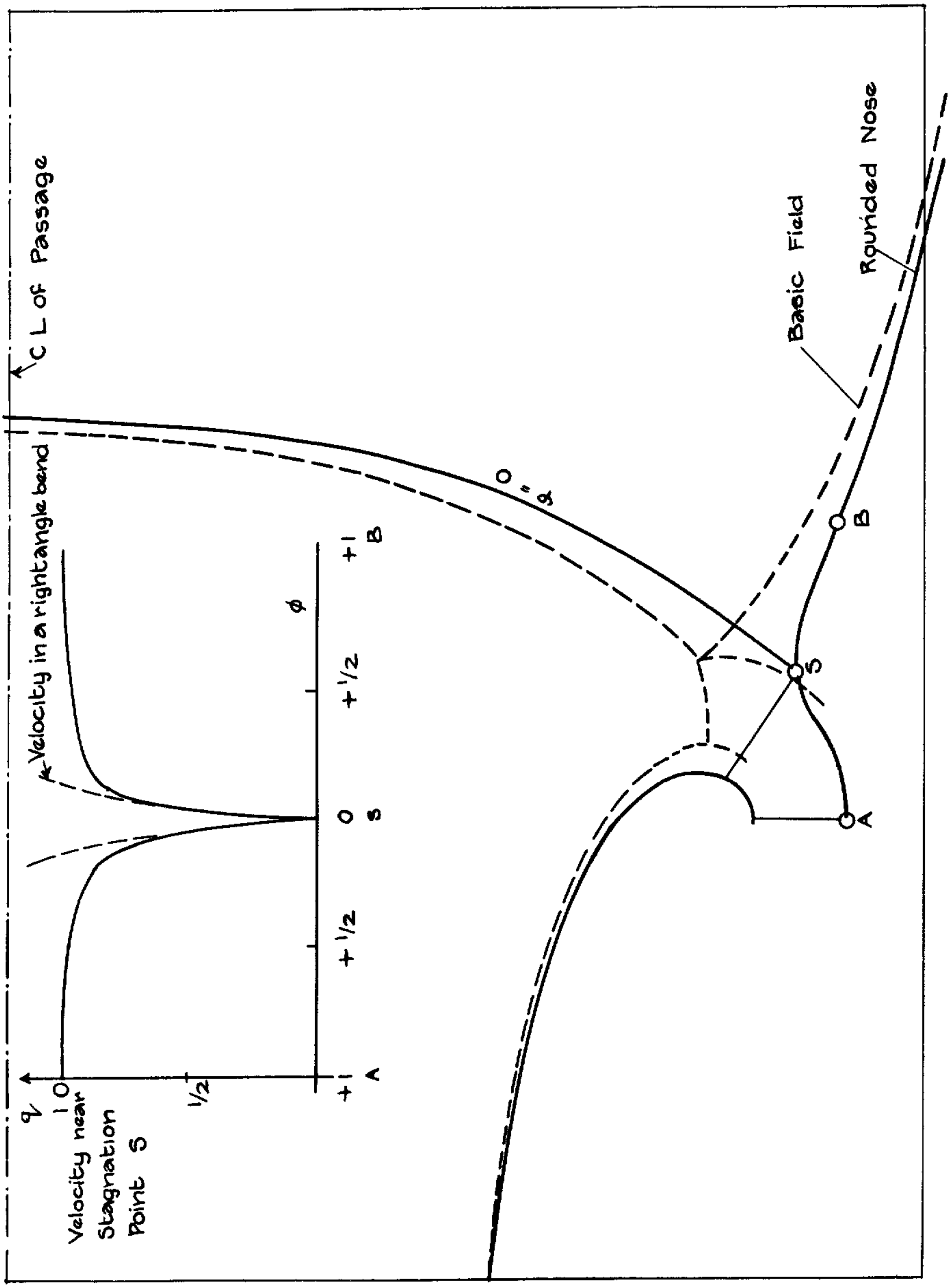
Width at Infinity = 2 718



Width at Infinity = 9/8

Ducted Expanding Passage with Constant Wall Velocities.

FIG 3.



Effect on the Boundary of Rounding the Nose.

FIG. 4

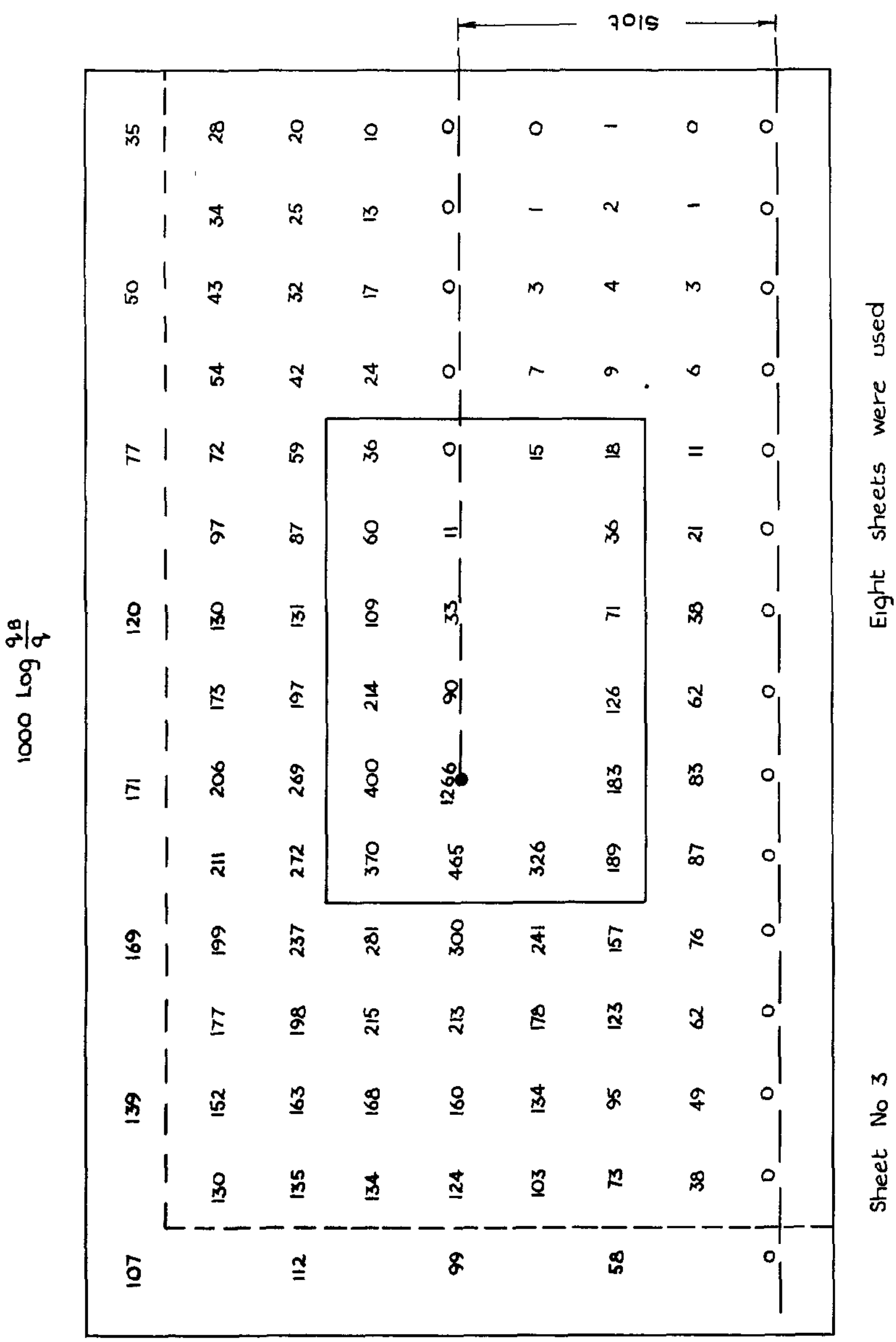
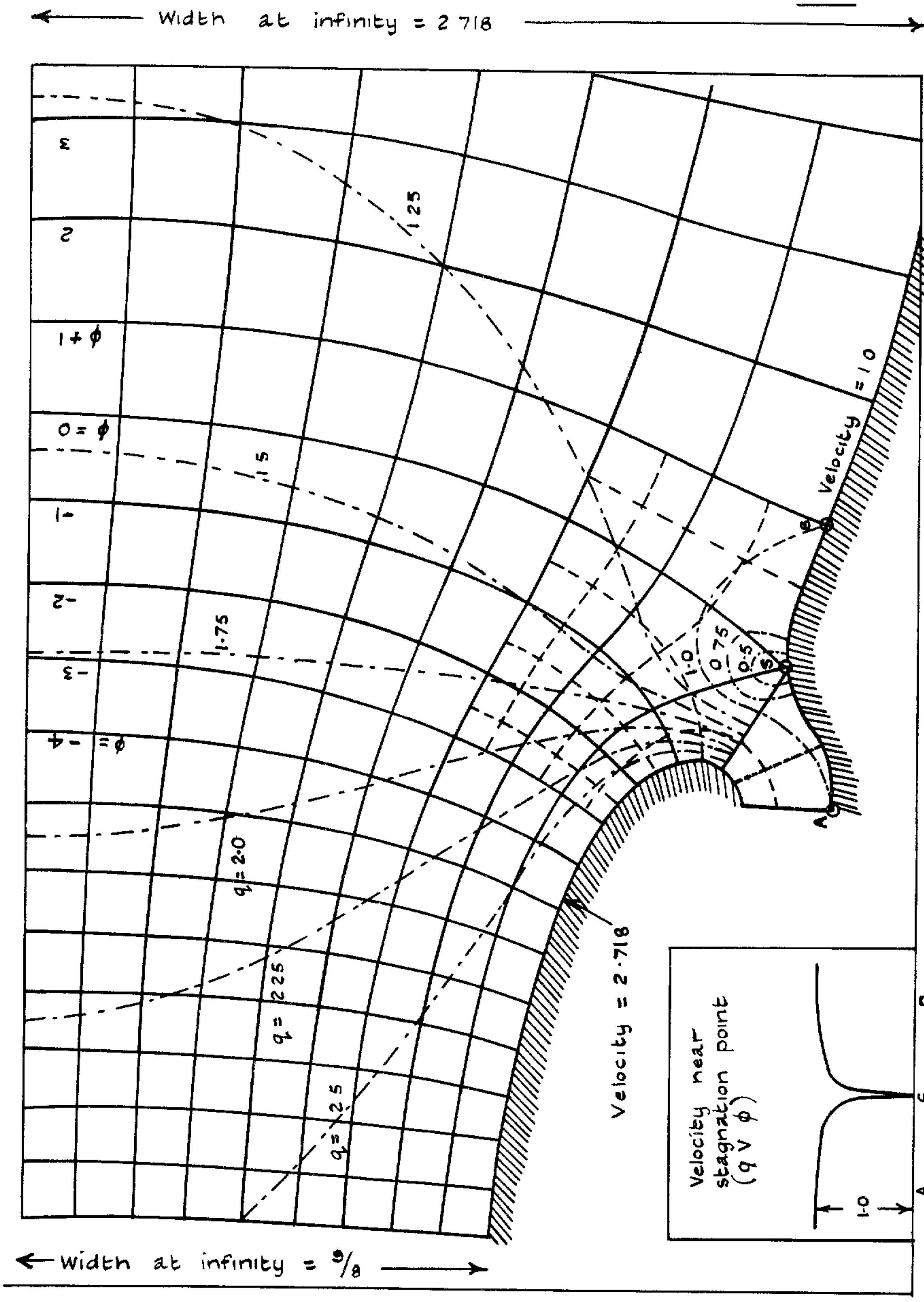


Fig. 5.



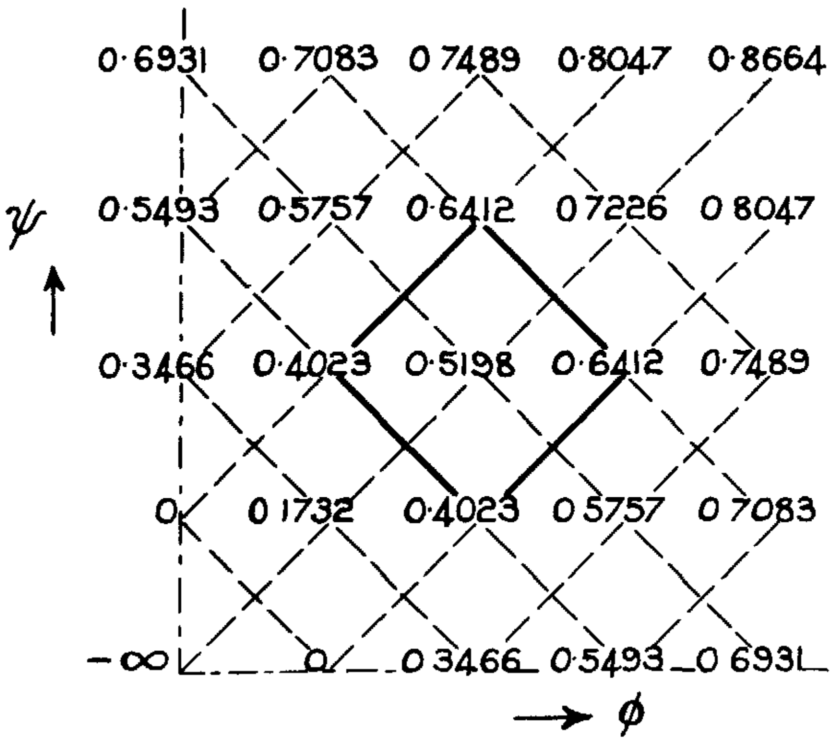
Ducted Expanding Passage with constant Wall Velocities except near the Stagnation Point

Comparison Field

Values of $\log_e q$

$w = Z^2, \; q^4 = \phi^2 + \psi^2$

Fig. 6.



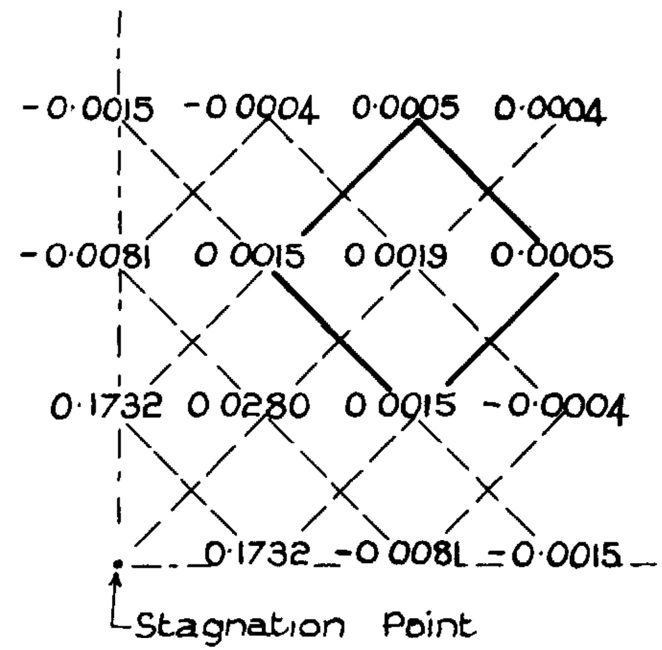
Adjustments used

(derived from Fig 6.)

Values of Δ

To be applied to $\log\frac{1}{q}$

Fig. 7.



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Figs 6 & 7.

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