C.P. No. 58
13696
A.R.C. Technical Report

MINISTRY OF SUPPLY

AERONAUTICAL RESEARCH COUNCIL
CURRENT PAPERS

The Theoretical Lift and Pitching Moment of a Highly-Swept Delta Wing on a Body of Elliptic Cross-section.

By

T. Nonweiler, B.Sc.

LONDON · HIS MAJESTY'S STATIONERY OFFICE
1951
Price 3s. 6d. net.

C.P. No.58

Technical Note No. Aero 2055

June, 1950

ROYAL AIRCRAFT ESTABLISHMENT

The Theoretical Lift and Pitching Moment of a Highly-Swept Delta Wing on a Body of Elliptic Cross-Section

by

T. Norweiler, B.Sc.

SUMMARY

This note investigates the lift, pitching moment and induced drag coefficients of a highly-swept delta wing attached to an elliptic cylinder of constant cross-section. These coefficients are derived by treating the changes in perturbation velocity parallel to the free-stream direction as small compared with the velocity changes in transverse planes.

Curves are given which enable these coefficients to be determined for various values of the body width and body height.

LIST OF CONTENTS

LIST OF FIGURES

1 Introduction

The aerodynamic properties of slender delta wing + circular body combinations have been investigated by Spreiter[1] using the "transverse-strip" method of solution. This theory assumes that the term

$(1-M^2)\dfrac{\partial^2\phi}{\partial x^2}$ of the linearised equation for the potential is negligible,

and the solutions are constructed on the basis of a potential flow in planes normal to the stream direction. The method is strictly applicable only if the apex angle of the delta-wing and the slope of the surface of the body are vanishingly small.

The present note extends the work of reference 1 for a delta wing + body combination assuming the body to be an infinite elliptic cylinder and the wing to be of zero thickness. The results are obtained by a conformal transformation of the flow in transverse planes about the delta wing + circular body combination.

2 Mathematical Analysis

Consider flow in planes normal to the stream direction about a wing-body combination: the transformation

$$\xi = y + iz = \Xi + \frac{a^2\lambda}{\Xi} \tag{1}$$

where $\Xi = Y + iZ$, transforms a circular body + wing combination in the Y, Z plane into an elliptic body + wing combination in the y, z plane. If the centre of the body is at $Y = Z = 0$, the radius of the body is a, and the extremities of the wing are at $Y = \pm a$ and $Y = \pm s$, then in the y,z plane this boundary is transformed into:

$$\text{the ellipse, } \frac{y^2}{(1+\lambda)^2} + \frac{z^2}{(1-\lambda)^2} = a^2$$

and the straight line $z = 0$ for $a(1+\lambda) \leqslant |y| \leqslant s + \dfrac{\lambda a^2}{s}$

If the ellipse is a transverse section of a right elliptic cylinder with axis $y = z = 0$, and the wing-plan form is assumed triangular (see Fig. 1), then

$$\frac{\partial a}{\partial x} = \frac{\partial \lambda}{\partial x} = 0, \text{ and } \frac{\partial b}{\partial x} = \tan\Gamma$$

(where $b = s + \dfrac{\lambda a^2}{s}$) for $0<x<c$.

2.1 Lift

From Bernouilli's Equation for linearised flow the pressure difference between the upper and lower surfaces of the wing or body is given by

$$\frac{\Delta p}{q} = \frac{4}{U}\frac{\partial \varphi}{\partial x} \tag{2}$$

and integrating over the wing surface

$$C_{L_W} = \frac{4}{US}\iint \frac{\partial\varphi}{\partial x}\,dx\,dy = \frac{8}{US}\int_{a(1+\lambda)}^{b_m} \varphi_{TE}\,dy \tag{3}$$

where φ_{TE} is the value of the potential at the wing trailing edge, since the potential at the wing leading edge is zero, and b_m is the value of b at the wing trailing edge. From Ref. 1, by a transformation of the potential due to flow past a flat plate, it is shown that

$$\varphi = U\alpha\sqrt{\left(s + \frac{a^2}{s}\right)^2 - \left(Y + \frac{a^2}{Y}\right)^2} \tag{4}$$

On the wing surface where $z = 0$ and $Z = 0$, we have from (1)

$$dy = \left(1 - \frac{a^2\lambda}{Y^2}\right) dY \tag{5}$$

Therefore on substituting from (4) and (5) in (3).

$$C_{L_W} = \frac{8}{US}\int_a^{s_m} \varphi_{TE}\frac{dy}{dY}\,dY$$

$$= \frac{2\alpha}{S}\left\{(1+\lambda)\left[\left(s_m + \frac{a^2}{s_m}\right)^2 \sin^{-1}\left(\frac{s_m^2 - a^2}{s_m^2 + a^2}\right) - 2a\left(s_m - \frac{a^2}{s_m}\right)\right] + (1-\lambda)\left(s_m - \frac{a^2}{s_m}\right)^2\frac{\pi}{2}\right\} \tag{6}$$

where s_m is the value of s at the wing trailing edge.

Likewise, integrating over the body surface

$$C_{L_B} = \frac{8}{US}\int_0^{a(1+\lambda)} \left(\varphi\big|_{x=c} - \varphi\big|_{x=0}\right)dy \tag{7}$$

Where $\varphi|_{x=c}$ and $\varphi|_{x=0}$ are the values of the potential on the surface of the body at the planes $x = c$ and $x = 0$. (See Fig. 1).

From Ref. 1

$$\varphi \Big|_{Y^2+Z^2=a^2} = U\alpha \sqrt{\left(s + \frac{a^2}{s}\right)^2 - 4Y^2} \tag{8}$$

On the body surface where

$$Z^2 = a^2 - Y^2 \text{ or } \frac{z^2}{(1-\lambda)^2} = a^2 - \frac{y^2}{(1+\lambda)^2}$$

we have from (1)

$$dy = (1 + \lambda)\, dY \tag{9}$$

Therefore, substituting from (8) and (9) in (7).

$$C_{L_B} = \frac{8}{US}(1 + \lambda) \int_0^a \left(\varphi\Big|_{\substack{Y^2+Z^2=a^2 \\ s = s_m}} - \varphi\Big|_{\substack{Y^2+Z^2=a^2 \\ s = a}} \right) dY$$

$$= \frac{2\alpha}{S}(1+\lambda) \Bigg[2a\left(s_m - \frac{a^2}{s_m}\right) + \left(s_m - \frac{a^2}{s_m}\right)^2 \frac{\pi}{2} - \left(s_m + \frac{a^2}{s_m}\right)^2 \sin^{-1}\left(\frac{s_m^2 - a^2}{s_m^2 + a^2}\right) \Bigg] \tag{10}$$

Thus from (6) and (10), the total lift coefficient is

$$C_L = C_{L_W} + C_{L_B} = \frac{4\alpha}{S} s_m^2 \left(1 - \frac{a^2}{s_m^2}\right)^2 \frac{\pi}{2} \tag{11}$$

Since $b_m = s_m + \dfrac{\lambda a^2}{s_m}$, from definition

$$\text{i.e.} \quad s_m = \frac{b_m + \sqrt{b_m^2 - 4\lambda a^2}}{2}$$

we have

$$C_L = \frac{2\pi\alpha}{S}\left[\frac{b_m^2}{\lambda^2} + \left(1 - \frac{1}{\lambda^2}\right)\frac{b_m^2 - 2\lambda a^2 + b_m\sqrt{b_m^2 - 4\lambda a^2}}{2} - 2\left(1 + \frac{1}{\lambda}\right)a^2\right]$$

$$\therefore \frac{\partial C_L}{\partial \alpha} = \frac{\pi A}{2}\left[\left(\frac{1 + \lambda^2}{2\lambda^2}\right) - \frac{\sigma^2}{\lambda} - \left(\frac{1 - \lambda^2}{2\lambda^2}\right)\sqrt{1 - \frac{4\lambda}{(1 + \lambda)^2}\sigma^2}\right] \qquad (12)$$

where $\sigma = (1+\lambda)\frac{a}{b_m}$ is the ratio of body width to gross wing span,

and $A = \frac{4b_m^2}{S}$ is the gross wing aspect ratio.

For particular cases we have

(i) $\sigma = 0$ (i.e. there is no body) $\qquad \frac{\partial C_L}{\partial \alpha} = \frac{\pi A}{2}$

(ii) $\sigma = 1$ (i.e. there is no wing) $\qquad \frac{\partial C_L}{\partial \alpha} = 0$

(iii) $\lambda = 1$ (i.e. the body is a laminar strip) $\qquad \frac{\partial C_L}{\partial \alpha} = \frac{\pi A}{2}(1 - \sigma^2)$

(iv) $\lambda = 0$ (i.e the body is circular) $\qquad \frac{\partial C_L}{\partial \alpha} = \frac{\pi A}{2}(1 - \sigma^2)^2$

(v) $\lambda = -1$ (i.e. the body is replaced by two infinite walls perpendicular to the wing) $\qquad \frac{\partial C_L}{\partial \alpha} = \frac{\pi A}{2}(1 - \sigma)^2$

The value of $\frac{2}{\pi A}\frac{\partial C_L}{\partial \alpha}$ from equation (12) is plotted in Fig. 2 as a function

of

$$\frac{d}{2b_m} = \sigma, \text{ and } \frac{h}{d} = \frac{1 - \lambda}{1 + \lambda}$$

where d is the width of the body and h is its height - as shown in Fig. 1.

We see from Fig. 2 that $\frac{\partial C_L}{\partial \alpha}$ is reduced by increasing, separately, either the body width or the body height. It is pertinent, therefore, to investigate whether for a given body frontal area, there is an optimum ratio of body height to body width.

In Fig. 3, $\frac{2}{\pi A} \cdot \frac{\partial C_L}{\partial \alpha}$ is plotted as a function of $\frac{\text{body frontal area}}{(\text{gross wing span})^2} = \frac{\pi}{4} \; \sigma^2 \left(\frac{1 - \lambda}{1 + \lambda}\right)$, and $\frac{h}{d}$.

We see that there is no optimum in the strict sense, since for a given ratio $\frac{\text{body frontal area}}{(\text{gross wing span})^2}$, $\frac{\partial C_L}{\partial \alpha}$ increases progressively as the ratio $\frac{\text{body height}}{\text{body width}}$ increases, $\frac{\partial C_L}{\partial \alpha}$ being equal to the wing alone value of $\frac{1}{2}\pi A$ in the extreme case when the body is of infinite height and zero width. Even this conclusion only applies to the particular type of body considered in this note, namely an infinite cylinder. A more realistic body would have a pointed nose, and Ward[2] has shown that the lift on a pointed body of elliptic cross section due to the nose is $\frac{1}{4}\pi\rho U^2 d^2 \alpha$. The extra lift that should be added to our results to allow for the nose of a pointed body may therefore be written as

$$\Delta\left(\frac{\partial C_L}{\partial \alpha}\right) = \frac{\pi A}{2} \; \sigma^2$$

and Fig. 4 repeats the comparison of Fig. 3, but for a body having a pointed nose. Again there is no optimum in the strict sense, since for a given ratio $\frac{\text{body frontal area}}{(\text{gross wing span})^2}$ $\frac{\partial C_L}{\partial \alpha}$ increases progressively as the ratio $\frac{\text{body height}}{\text{body width}}$ decreases, $\frac{\partial C_L}{\partial \alpha}$ now being equal to the wing alone value of $\frac{1}{2}\pi A$ in the extreme case when the body diameter is equal to the gross wing span (i.e. there is no nett wing).

These conclusions display the danger of making any general deductions as to the benefit or otherwise of using wide or tall bodies. In any case, a true appraisement should include a discussion of the drag of the combination.

2.2 Pitching Moment

If $C_L(x) = L(x)/qS$, where $L(x)$ denotes the total lift on the wing + body ahead of the plane x,

$$\left(\frac{\bar{c}}{c}\right) \; C_M = \int \left(1 - \frac{x}{c}\right) dC_L(x) = \int \; C_L(x) d\left(\frac{x}{c}\right) = \frac{1}{b_m - \frac{d}{2}} \int_a^{s_m} C_L(x) \frac{db}{ds} \, ds \quad (13)$$

But

$$b = s + \frac{\lambda a^2}{s}$$

and

$$C_L(x) = \frac{2\pi\alpha}{S} s^2 \left(1 - \frac{a^2}{s^2}\right)^2,$$

from (11).

Hence, performing the integration in (13).

$$\frac{dC_M}{dC_L} = \frac{c}{3\bar{c}} \frac{\left(1 - \frac{a}{s_m}\right)^2}{\left(1 + \frac{a}{s_m}\right)^2} \left[1 + \frac{4a}{b_m}\left(\frac{1 - \frac{\lambda a}{s_m}}{1 - \frac{a}{s_m}}\right)\right] \Bigg/ \left(1 - \frac{d}{2b}\right)$$

i.e.

$$\frac{dC_M}{dC_L} = \frac{2}{3} \left\{1 - \frac{2\sigma^2}{(1+\sigma)^2}\left[1 + \frac{4\lambda\sigma}{(1+\lambda)^2} + \left(\frac{1-\lambda}{1+\lambda}\right)\sqrt{1 - \frac{4\lambda\sigma^2}{(1+\lambda)^2}}\right]\right\} \quad (14)$$

For particular cases we have

(i) $\sigma = 0$ (i.e. there is no body) $\qquad \frac{dC_M}{dC_L} = \frac{2}{3}$

(ii) $\sigma = 1$ (i.e. there is no wing) $\qquad \frac{dC_M}{dC_L} = 0$

(iii) $\lambda = 1$ (i.e. the body is a laminar strip) $\qquad \frac{dC_M}{dC_L} = \frac{2}{3}(1-\sigma)\left(\frac{1+2\sigma}{1+\sigma}\right)$

(iv) $\lambda = 0$ (i.e. the body is circular) $\qquad \frac{dC_M}{dC_L} = \frac{2}{3}(1-\sigma)\frac{1+3\sigma}{(1+\sigma)^2}$

(v) $\lambda = -1$ (i.e. the body is replaced by two infinite walls perpendicular to the wing). $\qquad \frac{dC_M}{dC_L} = \frac{2}{3}(1-\sigma)$

In Fig. 5 the position of the aerodynamic centre as a fraction of the distance forward of the wing trailing edge is shown as a function of $\frac{d}{2b_m} = \sigma$, and $\frac{h}{d} = \frac{1-\lambda}{1+\lambda}$.

2.3 Induced Drag

The induced drag is equal to the resolved component of the normal force in the free stream direction, less the force due to the suction along the wing leading edge.

According to the theorem of Blasius, the suction force at a sharp point is given by

$$\lim_{\gamma \to 0} \tfrac{1}{2} i\rho \int_{\gamma} \left(\frac{dw}{d\xi}\right)^2 d\xi$$

where w is the complex potential of the flow in the (y, z) plane and γ is a small circle about the sharp point.

By by Cauchy's Theorem,

$$\lim_{\gamma \to 0} \tfrac{1}{2} i\rho \int_{\gamma} \left(\frac{dw}{d\xi}\right)^2 d\xi = -\pi\rho \lim_{\xi \to \xi_0} (\xi - \xi_0)\left(\frac{dw}{d\xi}\right)^2$$

where $\xi = \xi_0$ is the position of the sharp point.

Hence, the resolved component of the force at the wing leading edge (y = b) in the y-direction is

$$-\pi\rho \operatorname{Lim} (\xi - \xi_0)\left(\frac{dw}{d\xi}\right)^2 = -\pi\rho \lim (\Xi - \Xi_0)\left(\frac{d\Xi}{d\xi}\right)\left(\frac{d\xi}{d\Xi}\right)^2$$

$$= -\pi\rho \lim_{Y \to s} (Y - s)\left(\frac{dY}{dy}\right)\left(\frac{\partial\phi}{\partial Y}\right)^2$$

Hence, from (4) and (5), the force in the y-direction per unit chordwise length is

$$\pi\rho U^2\alpha^2 s^2 \left(1 + \frac{a^2}{s^2}\right)^2 \frac{\left(1 - \frac{a^2}{s^2}\right)^2}{\left(1 - \frac{\lambda a^2}{s^2}\right)} \lim_{Y \to s} \left\{(s - Y) \Big/ \left[\left(s + \frac{a^2}{s}\right)^2 - \left(Y + \frac{a^2}{Y}\right)^2\right]\right\}$$

$$= \frac{\pi}{2}\rho U^2\alpha^2 \; s \left(1 - \frac{a^4}{s^4}\right) \Big/ \left(1 - \frac{\lambda a^2}{s^2}\right).$$

To obtain the force perpendicular to the leading-edge per unit chordwise length, this expression must be multiplied by sec Γ. Integrating for all x between the plane of the wing-body junction and the wing trailing-edge, it follows that the component of drag force contributed by both sides of the wing is

$$\pi\rho U^2\alpha^2 \int_a^{s_m} s \frac{\left(1 - \frac{a^4}{s^4}\right)}{\left(1 - \frac{\lambda a^2}{s^2}\right)} \frac{db}{ds}\, ds = \frac{\pi}{2}\rho U^2\alpha^2 s_m^2 \left(1 - \frac{a^2}{s_m^2}\right)^2$$

By comparison with equation (11), it follows that,

$$C_{D_i} = \tfrac{1}{2}\alpha C_L \tag{15}$$

This simple expression may be stated in the more usual form:

$$\pi A \frac{C_{D_i}}{C_L^2} = 1 \Bigg/ \left(\frac{2}{\pi A} \frac{\partial C_L}{\partial \alpha}\right)$$

where the term on the right-hand side can be obtained from Fig. 2.

LIST OF SYMBOLS

A = $4b_m^2/S$ (gross wing aspect ratio)

C_{D_i} = drag coefficient of wing + body combination $(= C_L\alpha + \Delta C_{D_i})$

C_{L_B} = lift coefficient of body } $= \frac{\text{lift}}{qS}$

C_{L_W} = " " " wing }

C_L = $C_{L_B} + C_{L_W} = \frac{L(c)}{qS}$

C_M = pitching moment of wing + body combination about wing trailing edge $\left(= \frac{\text{moment}}{qS\bar{c}}\right)$

$L(x_o)$ = lift on wing + body ahead of plane $x = x_o$.

S = gross wing area $\left(= \frac{b_m c}{1-\sigma}\right)$

U = speed of free-stream

Y, Z = axes about circular body + wing combination ($Y = Z = 0$ is body axis, $Z = 0$ is plane of wing)

a = radius of body in (Y, Z) plane $\left(= \frac{d + h}{4}\right)$

b = local wing gross semi-span in (y,z) plane

b_m = value of b at wing trailing edge (i.e. at $x = c$)

c = root chord of wing (i.e. value of x at wing trailing-edge)

$\bar{c}$ = gross wing standard mean chord = $\tfrac{1}{2}c/(1-\sigma)$

d = width of elliptic body

h = height of elliptic body

i = $\sqrt{-1}$

Δp = pressure difference between upper and lower surfaces of wing or body.

q = $\frac{1}{2}\rho U^2$

s = local wing gross semi-span in (Y, Z) plane $\left(= \frac{b + \sqrt{b^2 - 4\lambda a^2}}{2}\right)$

s_m = value of s at wing trailing-edge

w = complex potential $(= \phi + i\psi)$

x, y, z = axes about elliptic wing + body combination ($y = z = 0$ is body axis, $z = 0$ is plane of the wing)

Γ = semi-apex angle of delta wing $\left(= \tan^{-1} \frac{db}{dx}\right)$

Ξ = $Y + iZ$

α = incidence of wing to free-stream

λ = $\frac{d - h}{d + h}$ (a variable parameter of the conformal transformation)

ξ = $y + iz = \Xi + \frac{\lambda a^2}{\Xi}$

ρ = air density

σ = $\frac{d}{2b_m}$

φ = potential function

ψ = stream function.

REFERENCES

No.	Author	Title etc.
1	Spreiter, J.	Aerodynamic Properties of Slender Wing-Body Combinations at Subsonic, Transonic and Supersonic Speeds. NACA TN 1662, 1948
2	Ward, G.N.	Supersonic Flow past slender Pointed Bodies Quart. Journ. Mech. and Applied Math, Vol. II, Pt. 1 (1949).

Wt.2078.CP58.K3. Printed in Great Britain.

FIG. 1

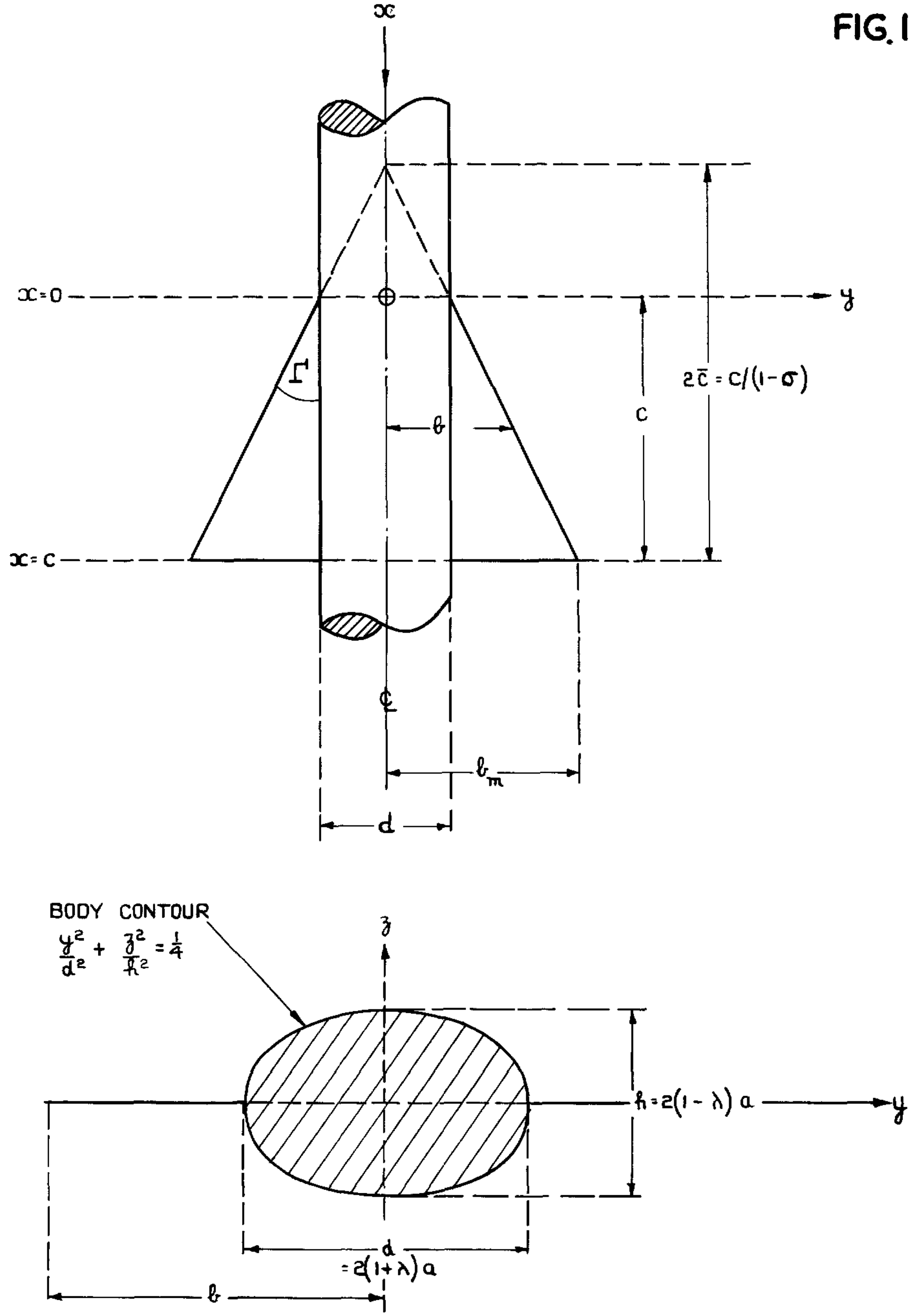

FIG. 1 NOMENCLATURE

FIG. 2

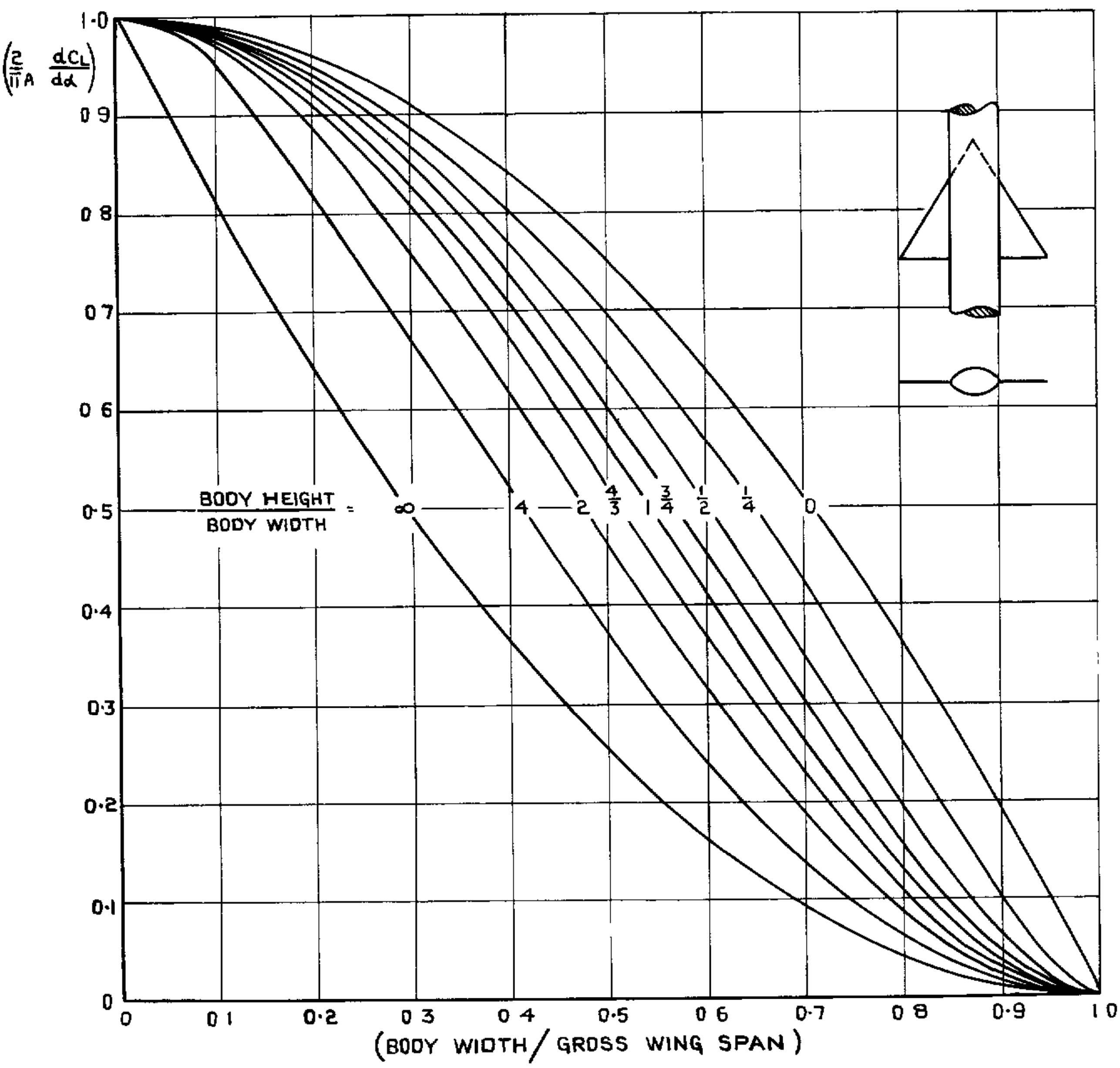

FIG. 2 VARIATION OF LIFT CURVE SLOPE WITH BODY WIDTH AND BODY HEIGHT.

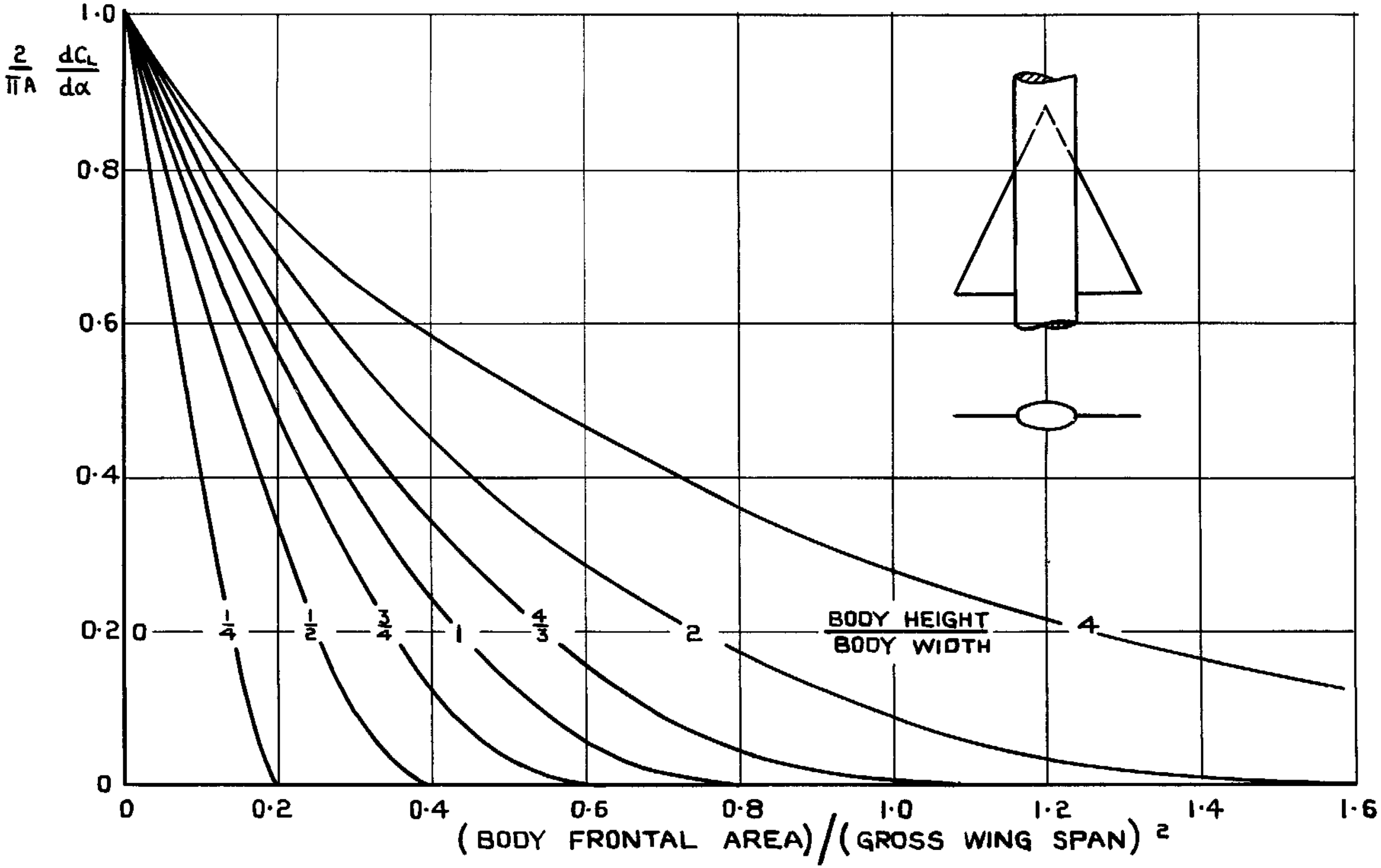

FIG 3 VARIATION OF LIFT CURVE SLOPE WITH BODY FRONTAL AREA & BODY HEIGHT/BODY WIDTH RATIO.
(ASSUMING THE BODY IS AN INFINITE ELLIPTIC CYLINDER.)

FIG. 3

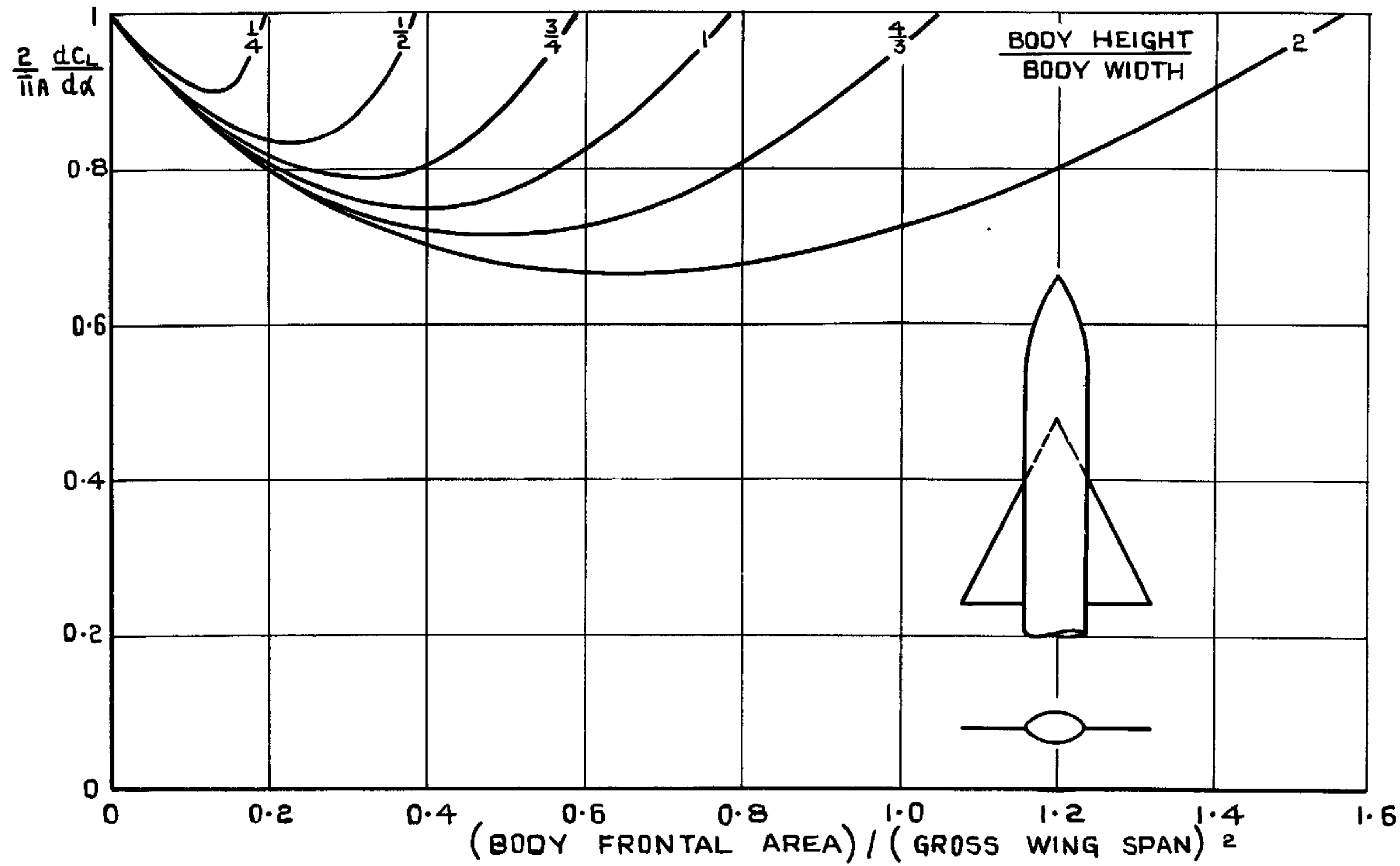

FIG. 4 VARIATION OF LIFT CURVE SLOPE WITH BODY FRONTAL AREA & BODY HEIGHT / BODY WIDTH RATIO.
(INCLUDING THE EXTRA LIFT DUE TO THE NOSE OF A POINTED BODY.)

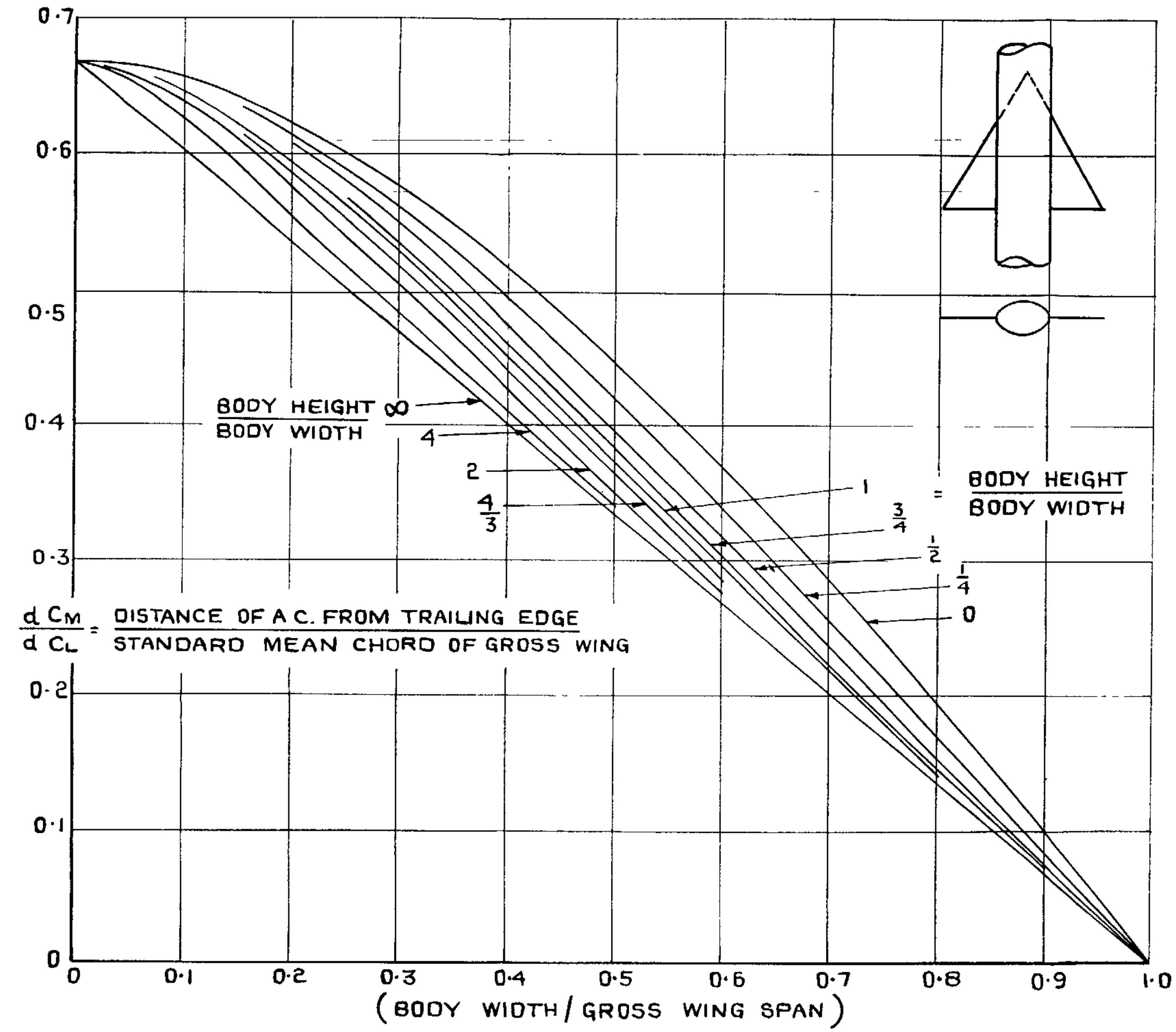

FIG.5. VARIATION OF AERODYNAMIC CENTRE POSITION WITH BODY WIDTH & HEIGHT.

C.P. No. 58
13696
A.R.C. Technical Report

PUBLISHED BY HIS MAJESTY'S STATIONERY OFFICE
To be purchased from
York House, Kingsway, LONDON, W C 2, 429 Oxford Street, LONDON, W 1,
P.O. Box 569, LONDON, S E 1,
13a Castle Street, EDINBURGH, 2 | 1 St Andrew's Crescent, CARDIFF
39 King Street, MANCHESTER, 2 | 1 Tower Lane, BRISTOL, 1
2 Edmund Street, BIRMINGHAM, 3 | 80 Chichester Street, BELFAST,
or from any Bookseller

1951
Price 3s. 6d. net
PRINTED IN GREAT BRITAIN

S.O. Code No. 23-9006-58