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Some Notes on The Flapping Motion of Rotor Blades

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Some Notes on the Flapping Motion of Rotor Blades

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SUMMARY

By considering the stability of the flapping motion of a hinged rotor blade at any one fixed azimuth, this report derives simple expressions for the condition which just causes the flapping motion at a particular azimuth to tend to become unstable. It shows that a decrease in pitch as blades flap up has a considerable stabilising influence. Effects such as the offset of the blade C.G. behind the flexural axis have the reverse effect but the analysis of the main text is not extended beyond this because its primary purpose is to draw attention to the character of flapping motion. It is suggested that computational methods now available should be used for further studies of the flapping equations which are of the Mattieu-Hill type. These equations are deduced in a fairly general form in appendices which are largely self-contained.

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1 Introduction

Some recent rotor blade instability troubles, associated with blade twisting and flapping, suggested to the writer the desirability of reviewing the work which has been done on blade flapping stability. Sissingh[1] (1944), who refers to previous work by Adam[2] (1934) and Hohenemser[3] (1938), concludes that the motion is very heavily damped and in a numerical example he took found that forward speed had little effect on his results. Rotors, however, are frequently reported to become more and more "rough" as speed increases, and Lock[4] (1928) shows that harmonic terms generally tend to increase as the tip speed ratio increases. It seemed therefore desirable to look at Sissingh's analysis in this light. He by-passes the difficulties associated with the solution of the equation for the flapping motion of a rigid rotor blade, hinged at its root, in the way Glauert[5] (1926) originally dealt with the autogyro problem, viz: by expanding the blade flapping motion into a Fourier series. Like Glauert he considers only terms up to the first harmonic terms in the expansion and giving them an exponential form derives a sextic. In the two examples he chooses the roots of this sextic indicate that motion up to the first harmonic term is heavily damped and very little affected by forward speed. Extending Sissingh's work to deal with higher order harmonic terms would involve even more laborious calculations than those he has already done and it therefore seemed desirable to turn again to the differential equations of flapping motion and see if an alternative approach were possible. The derivation of these equations is therefore given briefly in Appendix I. The analysis given there may be extended to study what would usually be classed as flutter problems but the main object of the report is to draw attention to the character of the flapping motion equations. Detail consideration is given only to results which can be obtained by simple substitutions. Even a Fourier substitution is not made and consideration is restricted to the nature of the flapping motion at any one particular azimuth. Then only the roots of a quadratic equation have to be studied. This approach indicates that forward speed may have a considerable influence on the character of the flapping motion and shows that reducing the pitch of the blades as they flap up markedly increases flapping stability. An extension of the analysis shows that the offset of the blade centre of mass behind the flexural centre has the reverse effect. The investigation is not extended in detail further than this because the notion of investigating flapping motion at a fixed azimuth is not a rigorous method of dealing with this Mattieu-Hill equation for the flapping motion. It is felt that the computational facilities now available might be first brought to bear on the problem to indicate the significance of the simple treatment adopted here.

2 Flapping motion of rigid fixed pitch blades with root hinges

When the hinged root of a rigid fixed pitch blade is constrained to move along a straight line with steady velocity, it is shown in Appendix I that the dynamical equation of flapping motion reduces to

$$\ddot{\beta} + 2k\left(1 + \frac{4}{3}\mu\sin\psi\right)\Omega\,\dot{\beta} + \left\{1 + \frac{8}{3}k\,\mu\cos\psi\left(1 + \frac{3}{2}\mu\sin\psi\right)\right\}\Omega^2\beta$$

$$= \text{a function which is independent of } \beta \qquad \ldots\ldots\ldots (1)$$

when the blade is uniform along its length. The general nature of the equation will be much the same if the blades are not uniform except that the coefficients will have different values. In this equation β is the flapping angle, k = $3 \rho ca R/16m$, a positive non-dimensional quantity having a value of about 0.7 for the C.30 autogyro,

Ω = $d\psi/dt$, the constant rotor angular velocity,

R = the rotor radius, i.e. blade length,

μ = the tip speed ratio and

ψ = Ωt, defines the azimuth of the blade from down wind in the direction of rotation.

It will be observed that the coefficients of $\dot{\beta}$ and β in this equation vary with blade azimuth ψ; but, if attention is concentrated on one particular azimuth at a time then the coefficients remain sensibly constant.

The smallest value of the damping coefficient

$$2k(1 + \frac{4}{3} \mu \sin \psi)$$

will occur when $\sin \psi = -1$ and μ is greatest. Provided

$$\mu < \frac{3}{4} \qquad \ldots\ldots\ldots \quad (2)$$

this damping coefficient will always be positive. Such a high value for μ is well outside the present working range of helicopters and outside the scope of the present analysis which neglects stalling. It is evident then that for all practical purposes

$$2k\left(1 + \frac{4}{3} \mu \sin \psi\right) \qquad \ldots\ldots\ldots \quad (3)$$

will always be positive.

The smallest value of the "spring stiffness" coefficient $\left\{1 + \frac{8}{3} k \mu \cos \psi (1 + \frac{3}{2} \psi)\right\}$ is about $(1 - \frac{8}{3} k\mu)$, so that the least effective stiffness occurs at higher rather than lower forward speeds.

Considering now the nature of flapping motion at any one azimuth position, i.e. freezing ψ at any chosen value, the nature of the flapping motion will be determined by the nature of the operational roots of (1) which are

$$\left[-k(1 + \frac{4}{3} \mu \sin \psi) \pm \sqrt{k^2(1 + \frac{4}{3} \mu \sin \psi)^2 - \left\{1 + \frac{8}{3} k \mu \cos \psi (1 + \frac{3}{2} \mu \sin \psi)\right\}}\right] \Omega \quad \ldots \quad (4)$$

Since from (3), $k(1 + \frac{4}{3} \mu \sin \psi)$ will be positive, the first positive root of (4) will occur when

$$1 + \frac{8}{3} k \mu \cos \psi (1 + \frac{3}{2} \mu \sin \psi)$$

is just negative. The condition that at no azimuth position flapping motion should tend to be divergent is then that

$$1 + \frac{8}{3} k \mu \cos \psi (1 + \frac{3}{2} \mu \sin \psi) > 0 \qquad \ldots\ldots\ldots \quad (5)$$

which gives approximately

$$\mu < \frac{3}{8k} \text{ or } \mu < \frac{2m}{\rho acR} \qquad \ldots\ldots\ldots \quad (6)$$

It does not follow, however, that if μ is greater than this value flapping motion as a whole will be unstable; it will only first tend to be so in the region where the blade is approaching the straight ahead position and it may not be long enough in this region for a disturbance to be catastrophic but a loss of smoothness might be expected. The "critical" value of μ given by (6) for the C.30 autogyro is about 0.53, well beyond its top speed.

3 Motion when pitch varies as the flapping angle

Nowadays it is frequently the practice to decrease the blade pitch as the blade flaps up; thus the pitch of a rigid blade might be expressed by

$$\theta = \theta_o - s\beta \qquad \ldots\ldots\ldots \quad (7)$$

where s is a positive constant. It is shown in Appendix II that the only change introduced in equation (1) is in the coefficient of β and the critical condition (5) now becomes

$$1 + k\left[\frac{8}{3} \mu \cos \psi(1 + \frac{3}{2} \mu \sin \psi) + 4s \left\{(\mu \sin \psi + \frac{2}{3})^2 + \frac{1}{18}\right\}\right] > 0 \quad (8)$$

Since the last term is always positive the effect of this pitch change is to increase the "critical" speed. A rough indication of the increase is obtained by considering the motion in the region of $\psi = 180°$. Then the criterion (8) gives $1 + k\left[-\frac{8}{3}\mu + 2s\right] > 0$ or

$$\mu < \frac{3}{8k} + \frac{3}{4} s \qquad \ldots\ldots\ldots \quad (9)$$

If in the C.30 a pitch decrease equal to one third the increase in the flapping angle were introduced ($s = \frac{1}{3}$) the theoretical critical value of μ would increase from 0.53 to (0.53 + 0.25).

This most marked effect of pitch change suggests that if the blade pitch tended to increase only slightly with the flapping angle the effect would be very serious. With torsionally flexible blades such effects might occur.

4 Effect of a torsional flexibility

Twisting of a blade may be caused by the centre of lift, drag or mass not coinciding with what is often called the flexural axis, i.e., the load positions which produce no twist. Beavan and Lock[5] (1936), using harmonic analysis evaluated the effect of the inertia axis being aft of the flexural axis of the C.30 blades and also the effect of the presence of a large pitching moment coefficient. The former results in a twist which is a function of β and therefore affects the nature of the blade flapping motion. Twist due to the latter is not directly a function of β. Lift and flexural centres in the C.30 coincided and such a choice, together with small values of pitching moment coefficient seems possible for other blades unless regions of high Mach numbers are encountered. Experience indicates, however, that offsets between the position of the inertia and flexural axis will occur unless meticulous care is exercised in manufacture and while other offsets are not improbable it is proposed here to consider only the effect of an inertia offset which is amenable to simple treatment.

Assuming, in the same way as Beavan and Lock[5] (1936), that the twists produced are equal to those which would occur if the twisting moments were applied statically, it is shown in Appendix III that the effective pitch at a radius r is

$$\theta = \theta_0 + \frac{mbc}{NJ}\left\{(\ddot{\beta} + \Omega^2\beta)\frac{r^3}{6} + \beta.gi\cos\psi\frac{r^2}{2}\right\} \qquad \ldots\ldots. (10)$$

due to an offset of the centre of mass a distance bc behind the flexural axis, omitting all additional terms which are not functions of β. Usually the gravity contribution, which is the last term in (10) will be small and then as shown in Appendix III the left hand side of equation (1) takes the form

$$\left[1 - \kappa\left\{(\mu\sin\psi + \frac{5}{6})^2 + \frac{5}{252}\right\}\right]\ddot{\beta} + 2k(1 + \frac{4}{3}\mu\sin\psi)\,\Omega\dot{\beta}$$
$$+ \left[1 + \frac{8}{3}k\,\mu\cos\psi(1 + \frac{3}{2}\mu\sin\psi) - \kappa\left\{(\mu\sin\psi + \frac{5}{6})^2 + \frac{5}{252}\right\}\right]\Omega^2\beta \qquad \ldots\ldots. (11)$$

where $\kappa = 4\,\Omega^2 bcm\,R^3 k/15NJ = \rho abc^2R^4\Omega^2/20NJ$, and has a value of about 2.5b, i.e., about 0.16 for the C.30 autogyro.

Comparing (7) and (10) it will be observed that, while b is positive, an undesirable negative value of an "effective s" has made its appearance and we should expect the roots of (11) at certain fixed azimuths to be adversely affected. The coefficient of $\ddot{\beta}$ will usually be positive and while this is so the criterion that there should be no tendency to local flapping instability is that

$$1 + \frac{8}{3}k\,\mu\cos\psi(1 + \frac{3}{2}\mu\sin\psi) - \kappa\left\{(\mu\sin\psi + \frac{5}{6})^2 + \frac{5}{252}\right\} > 0 \qquad \ldots (12)$$

In the region of $\psi = 180^\circ$ this becomes

$$1 - \frac{8}{3}k\,\mu - \frac{5}{7}\kappa > 0$$

which gives

$$\mu < \frac{3}{8k} - \frac{15}{56}\frac{\kappa}{k} \qquad \ldots\ldots. (13)$$

For the C.30 the decrease in the critical value of μ is from $0.5\dot{3}$ to 0.53-0.06, in itself not a large effect. C.30 blades had, however, a tubular steel spar which was very stiff in torsion and with other types of construction and thinner blades it is possible that blade torsional stiffness values will not be so large and the effect of the twist produced by the inertia axis being aft of the flexural axis will be of greater importance. Provided torsional flexibility is such that root pitch changes are transmitted throughout the blade it would appear possible from (9) to nullify the adverse effect of elastic twist by decreasing the blade pitch as it flaps up. The criterion for flapping to be stable at $\psi = 180^\circ$ then becomes that

$$\mu < \frac{3}{8k} + \frac{3}{4} s - \frac{15}{56} \frac{\kappa}{k} \qquad \ldots\ldots\ (14)$$

Most early testing of rotors is done at very low values of the tip speed ratio μ. *In the particular case of a torsionally flexible but* otherwise rigid blade the left hand side of the flapping equation of motion at zero forward speed reduces to:-

$$\left(1 - \frac{5}{7}\kappa\right)\ddot{\beta} + 2k\Omega\dot{\beta} + \left\{1 + 2k\,s - \frac{5}{7}\kappa\right\}\Omega^2\beta \qquad \ldots\ldots\ (15)$$

and if instability just appears then

$$\left(1 - \frac{5}{7}\kappa\right)\left(1 + 2k\,s - \frac{5}{7}\kappa\right) = 0.$$

In the case of the C.30 blades if the blade torsional stiffness were reduced to a third its value and the distance between the flexural axis and inertia axis increased to 0.2c, conditions which might perhaps be obtained by a very clumsy redesign in say wood, then instability would appear before the working r.p.m. were reached. This curious result has been obtained because the coefficient of $\ddot{\beta}$ in (15), which had been obtained from (11), is now no longer positive as assumed in deriving (12) and (14).

5 Conclusions

The results obtained indicate that if it is permissible to consider the stability of flapping motions at a fixed azimuth, then, provided $\left(1 - \frac{5}{7}\kappa\right)$ is positive, flapping instability will first appear when the tip speed ratio μ has a value of about

$$\frac{3}{8k} + \frac{3}{4} s - \frac{15}{56} \frac{\kappa}{k}$$

for a uniform blade which is rigid in bending but flexible in torsion. An appreciation of the significance of this result in relation to flapping-cum-rotational stability might be obtained by making numerical calculations, possibly step by step, or using tables and computational aids now available, of blade motion at values of μ above and below this critical value. This seems desirable before extending further the approach of these notes.

(Since compiling these notes the writer has come across work by Horvay and Yuan (J.Ae.Sc., October, 1947) which gives an analytical step-by-step treatment of the problem. He is also indebted to Mr. Shapiro for passing him a copy of work by Parkus, of the Vienna Technical Institute, which is awaiting publication, and which deduces a criterion for flapping stability using Floquet's theory (1883) and a power series substitution. In both treatments the elegant analysis involved tends to obscure the physical picture.

In a discussion with Mr. I.T. Minhinnick, who has been considering rotor flutter problems, it transpired that ℓ_z terms, viz: aerodynamic forces associated with displacements such as $r\beta$ and z', considerably affected the results obtained. In the past such terms have been omitted in studies of rotor aerodynamics and are omitted in the present note. It thus appears that this omission is justifiable in considering low frequency and divergent motions only.)

LIST OF SYMBOLS

a	=	slope of the lift curve
b	=	fraction of chord C.G. is aft of the flexural axis
c	=	blade chord
f	=	any disturbing velocity through disc
g	=	acceleration due to gravity
i	=	disc incidence
k	=	$3\rho caR/16m$, a non dimensional constant
m	=	blade mass per unit length (slugs/ft. run)
r	=	distance from the blade root to an element of the blade measured along the flapping line (Fig.1)
s	=	a constant = the ratio of pitch change/flapping angle change
t	=	time
v	=	induced velocity measure positive downwards
z'	=	deflection of a blade element perpendicular to the "Flapping line" (Fig.1)
I	=	moment of inertia about blade root (slugs. ft.^{2})
M	=	mass moment about the blade root (slugs ft.)
NJ	=	torsional stiffness of the blade per unit run
R	=	tip radius
V	=	constant forward velocity of aircraft
X, Y, Z	=	reference axes, see Fig.1
β	=	flapping angle, the angle between "flapping line" and XY disc plane
$\dot{\beta}, \ddot{\beta}$	=	its successive derivatives with respect to time
θ	=	blade pitch from no lift at root
$\theta_o + \theta_r$	=	" " " " at distance r from root
κ	=	$\dfrac{8\,\Omega^2 kmbcR^3}{NJ} = \dfrac{\rho abc^2R^4\,\Omega^2}{20NJ}$ = a torsional flexibility constant
μ	=	$V\cos i/\Omega R$ = tip speed ratio
ρ	=	air density
ϕ	=	angle of incidence of a blade section from no lift
ψ	=	Ωt = azimuth measured from down wind in direction of rotation
Ω	=	angular velocity of rotor, assumed constant.

REFERENCES

Ref. No.	Author	Title, etc.
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2	Adam	On the stability of the motion of an autogyro blade. Revista de Aeronautica (Aerotechnia) (1934) No.30 page 478.
3	Hohenemser	On the dynamic stability of helicopters with hinged blades. Ing. Archiv. Vol.9 (1938) page 419.
4	C.N.H Lock	Further developments of autogyro theory R. & M.1127 (1928).
5	H. Glauert	A general theory of the autogyro. R. & M.111 (1926).
6	J.A Beavan, & C.N.H. Lock	The effect of blade twist on the characteristics of the C.30 autogyro. R. & M.1727 (1936).

Attached:-

Appendices I, II and III
Fig.I.

APPENDIX I

Derivation of Flapping Motion Equations

I.1 To avoid continual cross reference to earlier work the flapping equations used in the text are derived in this appendix from elementary considerations. Fig.1 shows the relative directions of the aircraft velocity, blade chord, etc. and from it the following direction cosines with respect to the orthogonal axes shown, can be derived:-

(1) Lengthwise tangent to blade drawn outwards

$$\cos(\beta + dz'/dr)\cos\psi, \quad \cos(\beta + dz'/dr)\sin\psi, \quad \sin(\beta + dz'/dr)$$

(2) Chordwise, leading edge to trailing edge

$$\sin\psi, \quad -\cos\psi, \quad 0$$

(3) Normal (upwards)

$$-\sin(\beta + dz'/dr)\cos\psi, \quad -\sin(\beta + dz'/dr)\sin\psi, \quad \cos(\beta + dz'/dr)$$

(4) Centrifugal force

$$\cos\psi, \quad \sin\psi, \quad 0$$

(5) Forward speed, V, reversed

$$\cos i, \quad 0, \quad \sin i$$

(6) Gravity

$$\sin i, \quad 0, \quad -\cos i$$

In the above β is the flapping angle at azimuth ψ and dz'/dr the small slope of the blade at radius r relative to the "flapping line", see Fig.1. The aircraft is taken to be flying straight and level at a constant velocity V and the rotor disc which contains the axes OXY is inclined at an incidence i.

I.2 The component of the constant forward velocity V along the blade chord, viz: $V\cos i\sin\psi$, and the angular rotation give a net chordwise velocity of

$$(\Omega r + V\cos i\sin\psi) \qquad \text{....... I (1)}$$

The component of the velocity V normal to the blade chord is

$$-V\cos i\cos\psi\sin(\beta + dz'/dr) + V\sin i\cos(\beta + dz'/dr)$$

upwards and this is modified by the presence of flapping angular velocity $\dot\beta$, normal velocity $\dot z'$ and induced velocity v, the latter taken positive downwards. The net wind velocity, relative to the blade at radius r, up through the disc is, therefore, when $(\beta + dz'/dr)$ is small

$$(V\sin i - v) - (\beta + dz'/dr)V\cos i\cos\psi - r\dot\beta - \dot z + f \qquad \text{....... I (2),}$$

where f stands for any arbitrary disturbance.

If the geometric pitch of the blade section from no lift is $\theta_o + \theta_r$, where θ_o is constant and θ_r varies with r, then the angle of incidence from I(1) and I(2) is given by

$$\phi = \theta_o + \theta_r + \frac{(V \sin i - v) - (\beta + dz'/dr)\, V \cos i \cos \psi - r\dot{\beta} - \dot{z} + f}{\Omega r + V \cos i \sin \psi} \qquad \ldots\ldots\ \mathrm{I}\ (3)$$

The element of lift on a length dr of the blade at r is then usually considered to be

$$\tfrac{1}{2}\, (\Omega r + V \cos i \sin \psi)^2\, ca\, \phi\, dr,$$

the presence of a velocity out along the blade being ignored. The moment of the lift load about the root is then

$$\int_o^R \tfrac{1}{2}\, \rho\, (\Omega r + V \cos i \sin \psi)^2\, ca\phi\, r\, dr.$$

When a, the slope of the lift curve, is constant, the blade chord c and the induced velocity v are also independent of r, this integral becomes

$$\begin{aligned}
\tfrac{1}{2}\, \rho\, (\Omega R)^2\, caR^2 \Bigg[& \tfrac{1}{4}\theta \left\{ 1 + \frac{8}{3}\, \mu \sin \psi + 2\mu^2 \sin^2 \psi \right\} \\
& + \frac{1}{3}\, \frac{V \sin i - v}{\Omega R} \left\{ 1 + \frac{3}{2}\, \mu \sin \psi \right\} \\
& - \frac{1}{3}\, \beta \mu \cos \psi \left\{ 1 + \frac{3}{2}\, \mu \sin \psi \right\} \\
& - \frac{1}{4}\, \frac{\dot{\beta}}{\Omega} \left\{ 1 + \frac{4}{3}\, \mu \sin \psi \right\} \\
& + \int_o^R \theta_r \left\{ \left(\frac{r}{R}\right)^3 + 2\mu \left(\frac{r}{R}\right)^2 \sin \psi + \mu^2 \sin^2 \psi \left(\frac{r}{R}\right) \right\} d\left(\frac{r}{R}\right) \\
& + \int_o^R \left\{ \frac{f}{\Omega R} - \frac{dz'}{dr}\, \mu \cos \psi - \frac{\dot{z}'}{\Omega R} \right\} \left(\frac{r}{R} + \mu \sin \psi\right) \frac{r}{R} \Bigg]\, d\left(\frac{r}{R}\right)
\end{aligned} \qquad \ldots\ldots\ \mathrm{I}\ (4)$$

In this equation the β term is present due to the forward velocity of the aircraft having a component normal to the blade. The $\dot{\beta}$ term is due to blade flapping. The integration neglects the effects of stalling and tip losses.

I.3 The inertia load on the blade due to

(i) flapping as a rigid body (i.e., inertia loads due to the motion of the "flapping line", Fig.1) is $mr\,\ddot{\beta}\,dr$

(ii) bending away from the rigid body flapping position (i.e., the "flapping line") is $m\ddot{z}'\,dr$

(iii) gravity is $mg\,dr$

(iv) centrifugal force is $m\Omega^2(r\cos\beta - z'\sin\beta)\;dr$.

(Inertia loads due to angular acceleration in pitch $\ddot{\theta}$ are omitted here; so also are the small $z'\beta$ and z'^2 terms).

The moment of these forces about the blade root is then

$$\int_0^R \Big[m(r\ddot{\beta} + \ddot{z}')r$$
$$+ mg\left\{\cos i\cos\beta + \sin i\cos\psi\sin\beta\right\}r$$
$$- mg\left\{\cos i\sin\beta - \sin i\cos\psi\cos\beta\right\}z'$$
$$+ m\Omega^2(r\cos\beta - z'\sin\beta)(r\sin\beta + z'\cos\beta)\Big]\,dr.$$

When β is small this reduces to

$$\int_0^R \Big[\Omega^2\left\{(r^2 - z'^2)\,\beta + rz'\right\} + r^2\ddot{\beta} + r\ddot{z}'$$
$$+ g\Big\{r(\cos i + \beta\sin i\cos\psi)$$
$$+ z'(\sin i\cos\psi - \beta\cos i)\Big\}\Big]\,m\,dr$$
$$= (\ddot{\beta} + \Omega^2\beta)\,I + (\beta\sin i\cos\psi + \cos i)\;gM$$
$$- \beta g\cos i\int_0^R mz'\,dr \qquad \ldots\ldots\ \mathrm{I}\,(5)$$

where I = the moment of inertia of the blade about the flapping hinge
= $mR^3/3$ for a uniform blade

M = the blade mass moment about the flapping hinge
= $mR^2/2$ for a uniform blade

By choosing the "flapping line" appropriately it is possible to make the inertia term $\int_0^R mz'dr$ vanish and so this term may be dropped.

I.4 Equating the lift root moment given by I(4) to the inertia moment given by I(5), collecting terms and dividing through by I gives the flapping equation of motion:-

$$\ddot{\beta} + 2k\left(1 + \frac{4}{3}\mu\sin\psi\right)\Omega\dot{\beta} + \left[1 + \left\{\frac{gM\sin i}{I\,\Omega^2} + \frac{8}{3}k\mu\left(1 + \frac{3}{2}\mu\sin\psi\right)\right\}\cos\psi\right]\Omega^2\beta$$

$$= -g\cos i\,M/I$$

$$+ 8\Omega^2 k\left[\frac{1}{4}\theta\left(1 + \frac{8}{3}\mu\sin\psi + 2\mu^2\sin^2\psi\right) + \frac{1}{3}\left(\frac{V\sin i - v}{\Omega R}\right)\left(1 + \frac{3}{2}\mu\sin\psi\right)\right.$$

$$+ \int_0^R \theta_r\left\{\left(\frac{r}{R}\right)^3 + 2\mu\left(\frac{r}{R}\right)^2\sin\psi + \mu^2\sin^2\psi\left(\frac{r}{R}\right)\right\}d\left(\frac{r}{R}\right)$$

$$\left. + \int_0^R\left\{\frac{f}{\Omega r} - \frac{dz'}{dr}\mu\cos\psi - \frac{\dot{z}'}{\Omega R}\left(\frac{r}{R} + \mu\sin\psi\right)\frac{r}{R}\right\}d\left(\frac{r}{R}\right)\right] \qquad \ldots\ldots\ \text{I (6)}$$

This differs from equation (1) of the main text by the retention of the small β term $gM\sin i/I\Omega^2$ which is neglected there. In it

$$k = \tfrac{1}{2}\rho(\Omega R)^2\,caR^2/8I\Omega^2 = \rho caR^4/16I = 3\rho acR/16m$$

= 0.71 for the C.30 autogyro when a = 5.72 per radian.

APPENDIX II

Effect of Pitch Change with Flapping

It is frequently a common practice nowadays to reduce the pitch of blades as they flap up so that the pitch may be written as

$$\theta = \theta_o - s\beta \qquad \text{....... II (1)}$$

where s is a positive. Putting in I(6) $\theta_r = -s\beta$ * an additional term in β may now be transferred to the r.h.s. This term is

$$8\,\Omega^2 k \int_0^R s\beta \left\{\left(\frac{r}{R}\right)^3 + 2\mu\left(\frac{r}{R}\right)^2 \sin\psi + \mu^2 \sin^2\psi\left(\frac{r}{R}\right)\right\} d\left(\frac{r}{R}\right)$$

$$= 8\,\Omega^2 ks\,\beta\left\{\frac{1}{4} + \frac{2}{3}\mu\sin\psi + \frac{1}{2}\mu^2\sin^2\psi\right\}$$

$$= 4ks\left\{\left(\mu\sin\psi + \frac{2}{3}\right)^2 + \frac{1}{18}\right\}\Omega^2\beta \qquad \text{....... II (2)}$$

* Or identifying θ_o with $-s\beta$

APPENDIX III

Effect of Offset Between Inertia and Flexural Axes

III.1 The inertia loading in the direction normal to the blade chord is composed of

(i) the component of the centrifugal force which gives

$$-m\,\Omega^2(r\cos\beta - z'\sin\beta).\ \sin(\beta + dz'/dr)dr.$$

(ii) flapping and bending inertia loads, $-m(r\ddot{\beta} + \ddot{z}')\,dr.$

(iii) the component of gravity, viz.

$$-mg\left\{\sin i\,\sin(\beta + dz'/dr)\cos\psi + \cos i\,\cos(\beta + dz'/dr)\right\}dr.$$

When i and $(\beta + dz'/dr)$ are small the upward inertia loading is

$$-m\left[\Omega^2(r - z'\beta)\,(\beta + dz'/dr) + r\ddot{\beta} + \ddot{z}' + g\left\{i(\beta + dz'/dr)\cos\psi + 1\right\}\right]dr$$

or collecting rigid body and "elastic" terms

$$-m\left[\left\{r\ddot{\beta} + (\Omega^2 r + g\,i\cos\psi)\,\beta + g\right\} + \left\{\ddot{z}' - \Omega^2\beta z'\,(\beta + dz'/dr) + \Omega^2\,r dz'/dr + g\,i\cos\psi\,dz'/dr\right\}\right]dr$$

III.2 Omitting "elastic" terms the nose up torque due to this load, if it is a distance hc aft of the position it should occupy to produce no torque on any one chosen section, is

$$d\tau = mhc\left[r\ddot{\beta} + (\Omega^2 r + g\,i\cos\psi)\,\beta + g\right]dr$$

The twist $d\theta$ in a short length dr of the blade due to a torque τ at the section is given by

$$\frac{d\theta}{dr} = \frac{\tau}{NJ}$$

where NJ is the torsional stiffness. When this is constant then

$$\frac{d^2\theta}{dr^2} = \frac{d}{dr}\left(\frac{\tau}{NJ}\right) = \frac{1}{NJ}\,\frac{d\tau}{dr} = \frac{mhc}{NJ}\left[r\ddot{\beta} + (\Omega^2 r + g\,i\cos\psi)\beta + g\right]$$

Integrating twice and omitting all terms not containing β

$$\theta = \frac{mhc}{NJ}\,(\ddot{\beta} + \Omega^2\beta)\,\frac{r^3}{6} + g\,i\cos\psi\,\frac{r^2}{2}\,\beta.$$

Substituting this expression for θ_r in the θ_r integral of I (4) gives

$$\frac{mbcR^3}{NJ}\left[\frac{1}{6}(\ddot{\beta}+\Omega^2\beta)\left(\frac{1}{7}+\frac{1}{3}\mu\sin\psi+\frac{1}{5}\mu^2\sin^2\psi\right)+\frac{g\,i\cos\psi.\beta}{2R}\left(\frac{1}{6}+\frac{2}{5}\mu\sin\psi+\frac{1}{4}\mu^2\sin^2\psi\right)\right]$$

$$=\frac{mbcR^3}{NJ}\left[\frac{1}{30}(\ddot{\beta}+\Omega^2\beta)\left\{\left(\mu\sin\psi+\frac{5}{6}\right)^2+\frac{5}{252}\right\}+\frac{1}{8}.\frac{g\,i\cos\psi}{R}\beta\left\{\left(\mu\sin\psi+\frac{4}{5}\right)^2+\frac{2}{75}\right\}\right].$$

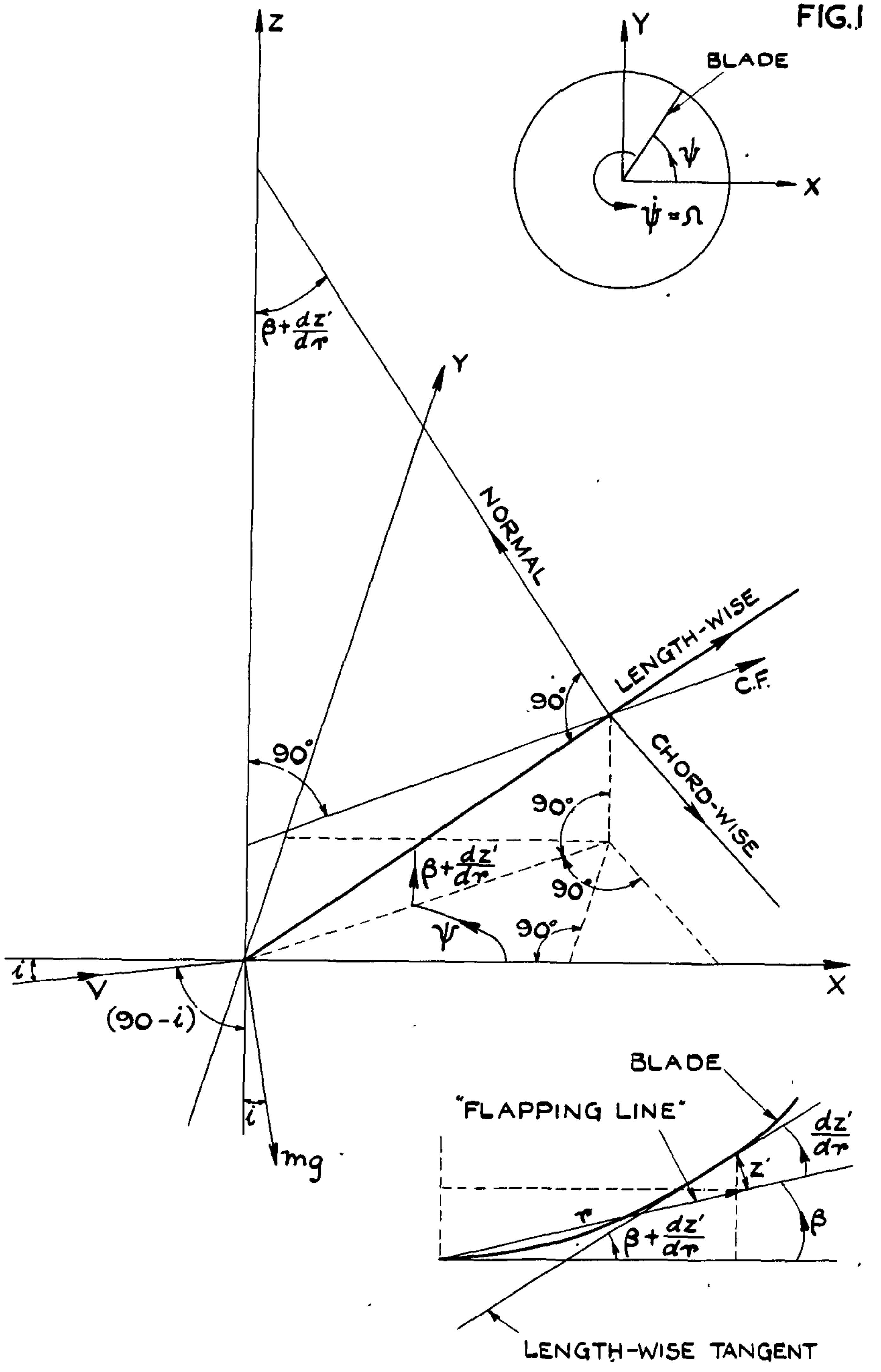
As a first approximation it will generally be permissible to neglect the gravity term in β which will be small compared with the Ω^2 term so that the θ_r term in I (6) now gives

$$\frac{8\,\Omega^2 kmbcR^3}{NJ}\left[\frac{1}{30}(\ddot{\beta}+\Omega^2\beta)\left\{\left(\mu\sin\psi+\frac{5}{6}\right)^2+\frac{5}{252}\right\}\right]$$

Putting $\kappa = \dfrac{8\,\Omega^2 kmbcR^3}{30NJ} = \dfrac{\rho abc^2R^4\Omega^2}{20NJ} = 2.5.b \approx 0.16$ for the C.30, and transferring the $\ddot{\beta}$ and β terms to the r.h.s. of I (6) we obtain the result quoted in (11).

(It will be observed that aerodynamical torsional terms in $\dot{\theta}$ are omitted in the above).

FIG. 1



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